

6-2005

Optimal insurance brokerage commission

Arthur HAU

Follow this and additional works at: <http://commons.ln.edu.hk/hkibswp>



Part of the [Insurance Commons](#)

Recommended Citation

Hau, A. (2005). Optimal insurance brokerage commission (HKIBS Working Paper Series 058-045). Retrieved from Lingnan University website: <http://commons.ln.edu.hk/hkibswp/96>

This Paper Series is brought to you for free and open access by the Hong Kong Institute of Business Studies 香港商學研究所 at Digital Commons @ Lingnan University. It has been accepted for inclusion in Hong Kong Institute of Business Studies Working Paper Series by an authorized administrator of Digital Commons @ Lingnan University.

OPTIMAL INSURANCE BROKERAGE COMMISSION

ABSTRACT

This paper studies a principal-agent insurance brokerage problem with a risk-averse principal (an insured) and a risk-neutral agent (a broker). The concept of “mean-preserving spread-reducing effort” is introduced to delineate the broker's activities. Using the first-order approach, it is shown that under some common conditions, the insured may “concavify” the reward function to induce the risk-neutral agent to exert MPSR brokering effort. Surprisingly, these conditions together with an additional condition guarantee the validity of the first-order approach even when the monotone likelihood ratio condition (used exclusively to justify the first-order approach) is violated. The case with a risk-averse agent is also considered.

JEL Nos.:D82, G22

Keywords: insurance brokerage, principal-agent problem, maximum likelihood ratio condition, first-order approach, mean-preserving spread-reducing effort, risk management, brokerage commission.

Financial support from the University Research Grants and Academic Programme Research Grants (Business, grant reference DB04A2) of Lingnan University is highly appreciated.

1. INTRODUCTION

In a seminal paper, Spence and Zeckhauser (1971) formulate a moral hazard model under which a risk-neutral principal (an insurance company) alleviates the adverse incentive problem of a risk-averse agent (an insured) by structuring the payoff (indemnity) function to affect the agent's action (self-protection activity). They derive first-order conditions for this principal-agent problem using the calculus of variation technique. Later, Mirrlees (1975, 1999) points out that this first-order approach for solving principal-agent problems is often invalid. Grossman and Hart (1983) and Rogerson (1985) show that if the distribution of the risk faced by the principal satisfies both the monotone likelihood ratio condition (MLRC) and the convexity of distribution function condition (CDFC), the first-order approach is valid. Jewitt (1988) argues that the CDFC is unnatural.¹ He justifies the first-order approach by keeping the MLRC, substituting the CDFC with less restrictive conditions on the distribution function, and imposing a restriction on the agent's utility function.

It is well-known (see, e.g., Whitt, 1980; Rogerson, 1985) that the MLRC implies that a rise in the agent's effort raises the principal's outcome in the sense of first-order stochastic dominance and necessarily raises the principal's average outcome. The MLRC, which is exclusively relied upon in justifying the first-order approach, is generally believed to be natural in production management problems and insurance moral hazard problems as a rise in a manager's effort is expected to raise the average profit of his company and a rise in an insured's self-protection effort is expected to reduce the average insured loss. Unfortunately, not all interesting agency problems satisfy the MLRC. Particularly, in an insurance brokerage problem, in which an insured (a principal) transfers his risk by purchasing insurance through a broker (an agent), the MLRC is violated.² An insurance broker supposedly works on behalf of an insured to search for appropriate insurance products with sufficient coverage or to bargain with insurance companies to tailor-make new products with appropriate coverage. In the benchmark case where insurance is actuarially fair, a broker's search activities, though

¹Besides being unnatural, the CDFC is often incompatible with MLRC since many distribution functions that satisfy the latter do not satisfy the former. LiCalzi and Spaeter (2003) exhibit two classes of distribution function that satisfy both the MLRC and the CDFC although these distributions are still restrictive. For a good summary of the first-order approach and its related problems, see Laffont and Martimort (2002).

²The MLRC is also violated in some risk management problems. For example, a risk-averse exporter that has little or no knowledge in managing its foreign currency risk may rely on a risk management consultant, a bank, or an internal risk manager to perform professional hedging activities to reduce its foreign exchange rate risk. Such activities certainly do not increase the exporter's average outcome and hence do not satisfy the MLRC.

costly to the broker, preserve the mean outcome of an insured. At the same time, the spread of the risk distribution shrinks after the purchase of insurance. Therefore, the broker's activities can be called "mean-preserving, spread-reducing"(MPSR). It can be shown that such MPSR activities violate the MLRC such that the solution of an insurance brokerage problem derived using the first-order approach is no longer justified under Rogerson's (1985) or Jewitt's (1989) conditions.

Besides the nature of agents' activities, the risk attitudes of principals and agents also distinguish insurance brokerage problems from traditional principal-agent problems that focus on risk-neutral principals and risk-averse agents (see, e.g., Spence and Zeckhauser, 1971; Campbell and Kracaw, 1987; Oyer, 2000, etc.). In an insurance brokerage problem, an insured is often a small company (such as a small engineering consulting firm, a small audit firm, a sole proprietor company) or an individual who is risk-averse whereas a brokerage company often has a relatively large and diversified portfolio of clients and can thus be treated as risk-neutral.³ Clearly, a risk-neutral agent has no inherent incentive to exert costly effort to raise insurance coverage so as to reduce its principal's risk. It can be shown that a constant reward (commission) cannot induce a risk-neutral agent to exert an optimal level of MPSR effort to search for an optimal level of insurance coverage. More importantly, it is not clear whether the optimal reward function (commission schedule) under an insurance brokerage problem is or is not similar to those derived under traditional principal-agent problems.

The following questions will be answered in this paper. (i) What are the characteristics of the optimal reward function of an insurance brokerage problem in which the principal is risk-averse and the agent is risk-neutral and in which the agent's effort is MPSR? (ii) Under what conditions is the first-order approach justified under an insurance brokerage problem in which the MPSR condition is satisfied but the MLRC is violated? (iii) Can the above results still prevail when the agent becomes risk-averse (e.g., in an internal risk management problem in which the agent is a risk manager) and if not then what additional conditions are needed?

³According to Tommy (2000), the insurance brokerage market is dominated by several large brokers. Almost sixty percentage of global insurance has been brokered by the largest two insurance brokers, namely, MarshInc. and Aon Corp.

The rest of the paper is organized as follows. Section 2 states the principal-agent insurance brokerage problem with a risk-averse principal and a risk-neutral agent.

The solution of the model is derived using the first-order approach. A new concept called “mean-preserving spread-reducing effort” is then introduced. Section 3 discusses the incentive problem in insurance brokering. Section 4 characterizes the optimal reward function in the insurance brokerage problem. Section 5 derives the conditions that guarantee the validity of the first-order approach for solving the problem. Section 6 extends the results to the case in which both the principal and the agent are risk-averse. Section 7 concludes.

2. A PRINCIPAL-AGENT INSURANCE BROKERAGE MODEL

A risk-averse principal (e.g., an individual or a small business) faces random outcome \tilde{x} , where \tilde{x} may be an insurable risk or a hedgeable risk. The principal hires a risk-neutral agent whose sole responsibility is to search for insurance or hedging instruments with appropriate coverage to reduce the risk. In the case of an insurable risk, the agent is a large insurance brokerage company. For convenience, therefore, this class of problems will be called “insurance brokerage problems” from now on. Assume \tilde{x} has realizations x , support $[\underline{x}, \bar{x}]$, distribution function F , and density function $f : [\underline{x}, \bar{x}] \times \mathfrak{R}_+ \rightarrow [0, 1]$ with $f(x, a) > 0, \forall x \in [\underline{x}, \bar{x}]$ and $\forall a$. Assume for simplicity that f is twice continuously differentiable in a and that both f_a and f are twice continuously differentiable in x . Here, $a \geq 0$ is agent’s effort with cost function c satisfying $c(\cdot) > 0, \lim_{a \rightarrow 0} c'(a) = 0$, and $c''(\cdot) > 0$.⁴ A larger a means that a larger portion of risk \tilde{x} is transferred via a change in f to be clarified later. Assume for simplicity that the principal is totally ignorant in risk management, and is thus incapable of observing a . This is particularly true in the property and liability insurance because the ultimate coverage received by the insured is often extremely complicated depending on a lot of factors, such as coinsurance rate, deductibles or excesses on different exposures, policy limits (e.g., per claim limits and aggregate limits), exclusions, endorsements, etc.

⁴These assumptions on the cost function of the agent's effort are standard in the literature. In the case of insurance brokerage, to increase coverage, an insurance broker has to dedicate more resources and effort to search amongst different insurance policies offered in the market and to bargain with insurers to raise indemnity rates, to lower deductibles, to delete exclusions, or to add endorsements to broaden the coverage under a standard policy. An alternative interpretation is that the insured has many small independent insurable risk units such that the agent's cost rises as a higher percentage of these risk units receives coverage.

The principal with utility function v satisfying $v' > 0$ and $v'' < 0$ chooses a and reward function s to maximize expected utility, that is,

$$\max_{\{s(\cdot), a\}} \int_{\underline{x}}^{\bar{x}} v[x - s(x)]f(x, a)dx \quad (1)$$

subject to the risk-neutral agent's incentive compatibility (**IC**) constraint given by

$$a \in \arg \max \int_{\underline{x}}^{\bar{x}} s(x)f(x, a)dx - c(a) . \quad (2)$$

In other words, the principle, though incapable of observing a , selects a via the agent's incentive compatibility constraint using reward function s . Besides the **IC** constraints, the principle's problem is also subject to the agent's individual rationality (**IR**) constraint given by

$$\int_{\underline{x}}^{\bar{x}} s(x)f(x, a)dx - c(a) \geq R, \quad (3)$$

where R is agent's reservation utility.

According to the first-order approach introduced by Spence and Zeckhauser (1971), the IC constraint may be replaced by the first-order condition of the agent's maximization problem (2), namely,

$$\int_{\underline{x}}^{\bar{x}} s(x)f_a(x, a)dx - c'(a) = 0. \quad (4)$$

The solution of the optimal control problem is given by:⁵

$$v'[x - s(x)] = \lambda + \mu \frac{f_a(x, a)}{f(x, a)}; \quad (5)$$

$$\int_{\underline{x}}^{\bar{x}} v[x - s(x)]f_a(x, a)dx + \mu \left[\int_{\underline{x}}^{\bar{x}} s(x)f_{aa}(x, a)dx - c''(a) \right] = 0 . \quad (6)$$

⁵ The Lagrangian function of the optimal control problem is given by

$$L = v(x - s)f(x, a) + \mu s f_a(x, a) + \lambda s f(x, a).$$

The optimal control solution is given by

$$\frac{dL}{ds} = -v'(x - s)f(x, a) + \mu f_a(x, a) + \lambda f(x, a) = 0$$

which upon simplifying gives rise to (5). Static maximization by the choice of a gives

$$\begin{aligned} & \int_{\underline{x}}^{\bar{x}} v[x - s(x)]f_a(x, a)dx + \mu \left[\int_{\underline{x}}^{\bar{x}} s(x)f_{aa}(x, a)dx - c''(a) \right] + \lambda \left[\int_{\underline{x}}^{\bar{x}} s(x)f_a(x, a)dx - c'(a) \right] \\ &= \int_{\underline{x}}^{\bar{x}} v[x - s(x)]f_a(x, a)dx + \mu \left[\int_{\underline{x}}^{\bar{x}} s(x)f_{aa}(x, a)dx - c''(a) \right] \\ &= 0. \end{aligned}$$

The first equality is due to (4).

Since Lagrange multipliers λ and μ are independent of x , (5) can be used to solve for s as a function of x .⁶

To characterize the optimal solution of the insurance brokerage problem, it is necessary to specify the nature of the agent's effort and hence the distribution function of \tilde{x} . Let us focus on risk transfer activities of an agent that is actuarially fair or unbiased. For the purchase of fair insurance, for example, a rise in the broker's effort raises coverage and hence reduces the spread of the risk without changing the mean loss of the insured. This suggests that distribution F should satisfy the following condition:

Definition An agent's effort is said to be Mean-Preserving, Spreading-Reducing (**MPSR**) if

$$\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0, \forall a \text{ and } \forall x \in (\underline{x}, \bar{x}) \text{ and } \int_{\underline{x}}^{\bar{x}} F_a(\theta, a) d\theta = 0, \forall a.^7$$

Rothschild and Stiglitz (1970) show that a risk-averse individual strictly prefers distribution G to distribution F if the latter represents a strict mean-preserving spread of the former (i.e., for any a , $\int_{\underline{x}}^x [F(\theta, a) - G(\theta, a)] d\theta > 0, \forall x \in (\underline{x}, \bar{x})$ with $\int_{\underline{x}}^{\bar{x}} [F(x, a) - G(x, a)] dx = 0$). Let $G(\theta, a) = F(\theta, a + \Delta a)$, for any fixed a . It can be checked that $\int_{\underline{x}}^x [F(\theta, a + \Delta a) - F(\theta, a)] d\theta < 0, \forall x \in (\underline{x}, \bar{x})$. Divide the left side by Δa and take limit as Δa tends to zero to get $\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0$. Similarly, $\int_{\underline{x}}^{\bar{x}} [F(\theta, a + \Delta a) - F(\theta, a)] d\theta = 0$ gives rise to $\int_{\underline{x}}^{\bar{x}} F_a(\theta, a) d\theta = 0$. A rise in MPSR effort a clearly results in a mean-preserving, spread-reducing change in the distribution of \tilde{x} .

⁶More correctly, s is a function of x and a . Argument a does not play any role in the analysis that follows and is, therefore, suppressed for convenience following the common practice in the literature (see, e.g., Jewitt (1988)).

⁷The MPSR concept can be modified slightly to become more general without affecting the conclusions in this paper. Particularly, one only needs to specify that $\int_{\underline{x}}^x F_a(\theta, a) d\theta \leq 0, \forall a$ and $\forall x \in (\underline{x}, \bar{x})$ with

$$\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0, \forall a \text{ for some subsets of } (\underline{x}, \bar{x}) \text{ having positive measures, and } \int_{\underline{x}}^{\bar{x}} F_a(\theta, a) d\theta = 0.$$

It can be checked that if an insurance broker's effort is to raise the indemnity rate, to reduce the deductible, or to raise the policy limit of a fair insurance policy, distribution F satisfies the **MPSR** condition. To see this, let w be the initial wealth of the principal and \tilde{y} be an insurable risk (with density function h distribution function H , and support $[\underline{y}, \bar{y}]$) faced by the principal. The insurance broker exerts effort $a \in [0,1]$ to arrange for coinsurance-type insurance with indemnity rate a .⁸ A similar argument applies to actuarially fair deductible-type insurance or insurance with a policy limit and is thus omitted. Now, the actuarially fair premium of the coinsurance-type insurance equals $aE\tilde{y}$, where E is the expectation operator. The principal's random net worth, in the presence of broker-arranged insurance, equals

$$w - \tilde{y} + a(\tilde{y} - E\tilde{y}) - s(\tilde{x}).$$

Let $\tilde{x} = w - \tilde{y} + a(\tilde{y} - E\tilde{y})$. The support of \tilde{x} can be written as $[\underline{x}, \bar{x}] = [w - \bar{y} + a(\bar{y} - E\tilde{y}), w - \underline{y} + a(\underline{y} - E\tilde{y})]$. The principal's net payoff is again equal to $\tilde{x} - s(\tilde{x})$.

To see that the distribution of \tilde{x} satisfies the MPSR condition, it can be checked that

$$F(x, a) = \int_{\underline{x}}^x f(x, a) = \int_y^{\tilde{y}} h(y) dy = \int_{\frac{w-x-aE\tilde{y}}{1-a}}^{\tilde{y}} h(y) dy$$

such that when $a < 1$,

$$F_a(x, a) = -(1-a)^{-2} (w-x-E\tilde{y}) \cdot h\left(\frac{w-x-aE\tilde{y}}{1-a}\right). \quad (7)$$

Now, $F_a(\bar{x}, a) = 0$. The right side of (7) changes from negative to positive (i.e., $-(w-x-E\tilde{y})$ changes from negative to positive) when x increases. This certainly implies that the MPSR condition is satisfied as $\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0$ for all $x \in (\underline{x}, \bar{x})$ because $\int_{\underline{x}}^{\bar{x}} F_a(x, a) dx = 0$. The last equality is due to

$$\int_{\underline{x}}^{\bar{x}} x f(x, a) dx = E\tilde{x} = E[w - \tilde{y} + a(\tilde{y} - E\tilde{y})] = w - E\tilde{y}$$

regardless of the value of a such that

$$0 = \frac{dE\tilde{x}}{da} = \int_{\underline{x}}^{\bar{x}} x f_a(x, a) dx = \bar{x} F_a(\bar{x}, a) - \int_{\underline{x}}^{\bar{x}} F_a(x, a) dx = - \int_{\underline{x}}^{\bar{x}} F_a(x, a) dx.$$

⁸In other words, it is assumed that it takes more effort to arrange for insurance with a higher indemnity rate.

3. THE INCENTIVE PROBLEM OF A RISK-NEUTRAL BROKER

Can a state-independent reward that is commonly found in actual insurance brokering be optimal? From the previous section, it can be checked that any reward of the form $s(x) = s_0$ for all $x \in [\underline{x}, \bar{x}]$ where s_0 is independent of x cannot be optimal. To see this, consider

$$\int_{\underline{x}}^{\bar{x}} s(x) f_a(x, a) dx - c'(a) = \int_{\underline{x}}^{\bar{x}} s_0 f_a(x, a) dx - c'(a) = s_0 F_a(\bar{x}, a) - c'(a) = -c'(a) < 0$$

as $F_a(\bar{x}, a) = 0$ and $c' > 0$ violating the agent's first-order condition (4).

The next question is whether the solution derived in Section 2 can ever coincide with the first-best solution under perfect information. According to Wilson (1968) and Holmstrom (1979), when there is perfect information, the IC constraint is absent. Therefore, in the insurance brokerage problem, the Pareto optimal contract specifies reward function s such that

$$v'[x - s(x)] = \lambda \quad (8)$$

when the principal can contract with the agent on effort a perfectly. Under this first-best contract, the principal's payoff $x - s(x)$ is constant as $v'' < 0$. In other words, the principal will shift as much risk to the risk-neutral agent as possible provided that reservation utility R is just met according to (3).

It can be checked from (5) that under asymmetric information, the first-best solution given by (8) can be attained only if f_a / f is constant (i.e., $(f_a / f)_x = 0$) for all a and for all $x \in [\underline{x}, \bar{x}]$ because $\mu \neq 0$ as will be shown in Lemma 1 to be presented in the next section. However, it can be checked that $a > 0$ cannot be optimal to the agent when $(f_a / f)_x = 0$ for all $x \in [\underline{x}, \bar{x}]$. To see this, suppose $(f_a / f)_x = 0, \forall x \in [\underline{x}, \bar{x}]$. This implies that $f_a / f = M, \forall x \in [\underline{x}, \bar{x}]$, where M is a constant. It can be checked that $M = 0$. Suppose by contradiction that $M \neq 0$. By taking expectation on both sides of $f_a / f = M$, it can be checked that $0 \neq M = \int_{\underline{x}}^{\bar{x}} f_a dx = F_a(\bar{x}, a) = 0$. A contradiction. Now, $0 = M = f_a / f, \forall x \in [\underline{x}, \bar{x}]$, which in turn implies that $\int_{\underline{x}}^{\bar{x}} s(x) f_a(x, a) dx = 0$, violating the agent's first-

order condition (4) regardless of the shape of distribution F . This gives rise to the following result:

Theorem 1 The first-best solution, $d(x - s(x))/dx = 0$, will never be achieved under the insurance brokerage problem in which the principal is risk-averse and the risk-neutral agent's effort a cannot be observed by the principal.

Clearly, to induce the risk-neutral agent to exert any MPSR effort, the principal has to design reward function S based solely upon observable realization x of \tilde{x} . Intuition suggests that the optimal reward function should create some spread in the agent's reward so that the agent will take the principal's risk attitude into account. The shape of the reward function is the subject of investigation of the next section.

4. CHARACTERIZING THE OPTIMAL REWARD FUNCTION

Before investigating the optimal reward function, it is necessary to first check that multipliers λ and μ are strictly positive. Jewitt (1988) shows that when the principal is risk-neutral, the agent is risk-averse, and the distribution satisfies MLRC, the multipliers are strictly positive. The following lemma states that his result can be extended to the insurance brokerage problem with a risk-averse principal and a risk-neutral agent:

Lemma 1: Suppose the agent's effort is MPSR. When the principal is risk-averse and the agent is risk-neutral, $\lambda > 0$ and $\mu > 0$

Proof: See Appendix.

To compare the solution of the insurance brokerage problem with that of the traditional principal-agent problem, denote the reward function under the traditional problem with a risk-neutral principal and a risk-averse agent by t . It is well-known (see, e.g., Jewitt (1988))

that $\text{Sgn}[t'(x)] = \text{Sgn}[(f_a/f)_x]$ such that t rises (c.f. falls) as f_a/f rises (c.f. falls.)⁹ When f_a/f is monotonically increasing satisfying the monotone likelihood ratio condition (to be defined formally in the next section), so is t . It is not clear whether this result holds under the principal-agent insurance brokerage problem.

Denote the Arrow-Pratt coefficient of absolute risk aversion for utility function v by $A_v = -v''/v'$. v is said to exhibit decreasing absolute risk aversion (**DARA**) if $A'_v < 0$, non-increasing absolute risk aversion (**NIARA**) if $A'_v \leq 0$, and constant absolute risk aversion (**CARA**) if $A'_v = 0$. NIARA is a generally accepted assumption because it is believed to be consistent with many reasonable economic behaviors (see, e.g., Arrow, 1963; Mossin, 1968). The following theorem states the relation between the principal's net payoff (i.e., $x - s(x)$) and the f_a/f ratio, and the shape of reward function s :

Theorem 2 Suppose the principal is risk-averse and the agent is risk-neutral.

- (a) $1 - s'(x)$ and $[f_a(x, a)/f(x, a)]_x$ have opposite signs.
- (b) If v exhibits NIARA and $[f_a(x, a)/f(x, a)]_{xx} < 0$, then $s''(x) < 0$.

Proof: See Appendix.

Theorem 2(a) suggests that the principal's optimal net return, $x - s(x)$, and the f_a/f ratio move in opposite directions as x increases whereas the agent's optimal reward $s(x)$ may rise or fall when f_a/f rises. This result is obviously different from that of the traditional problem in which the risk-averse agent's optimal reward $t(x)$ and f_a/f move in the same direction. Theorem 2(b) suggests that reward function s is concave in x whenever f_a/f is concave in

⁹In the traditional problem with a risk-neutral principal and a risk-averse agent, the principal maximizes expected payoff $\int_x^{\bar{x}} [x - t(x)]f(x, a)dx$, instead of expected utility. The agent's expected payoff

becomes $\int_x^{\bar{x}} u[t(x)]f(x, a)dx - (c)a$, where u satisfies $u' > 0$ and $u'' < 0$. First-order condition

(ref{eq:laura04}) now becomes $\frac{1}{u'[t(x)]} = \lambda + \mu \frac{f_a(x, a)}{f(x, a)}$ which upon differentiation on both sides of the equality gives $\text{Sgn}[t'(x)] = \text{Sgn}[(f_a/f)_x]$.

x and the principal's utility function exhibits NIARA. Theorem 2(b) is intuitive. To induce the risk-neutral insurance broker to exert costly effort to search for appropriate insurance coverage to reduce the risk, the insured should concavify the broker's payoff function so that the broker will act as if it is risk-averse. It turns out that the conditions stated in Theorem 2(b) are also important for justifying the first-order approach for the insurance brokerage problem to be presented in the next section where it will be shown that $s'' < 0$ guarantees that the agent's expected payoff is concave in a .

Figures 1 and 2 allow us to visualize the relations between reward function s , the f_a / f ratio, and other related functions specified in Theorem 2. Here, it is assumed that $x^@ \in (\underline{x}, \bar{x})$ such that $(f_a / f)_x > 0, \forall x \in (\underline{x}, x^@)$ and $(f_a / f)_x < 0, \forall x \in (x^@, \bar{x})$.

[Please Insert Figure 1 and 2 Here!]

Notice that it is well-known that if a distribution second-order stochastically dominates another distribution in Hadar and Russell's (1969) terminology, then the two distribution functions satisfy the single-crossing property such that the density function of the first crosses that of the second twice, once from below and once from above. Since a mean-preserving reduction in spread between two distributions is a special case of second-order stochastic dominance, the double-crossing property of the density functions holds. This implies that the MPSR condition requires that f_a (and hence f_a / f) crosses the horizontal-axis twice, once from below and once from above as shown in Figure 1. Figure 1 also depicts a possible case in which the $(f_a / f)_x$ function crosses the horizontal-axis once from above such that the slope of the principal's payoff, $1 - s'$, crosses the horizontal-axis once from below. In this particular case, the corresponding shapes of the principal's net payoff function and the agent's payoff function are depicted in Figure 2.

5. JUSTIFYING THE FIRST-ORDER APPROACH TO THE INSURANCE BROKERAGE PROBLEM

The optimal solution of the insurance brokerage problem studied in this paper is derived using the “first-order approach” just like those of traditional principal-agent problems.¹⁰ However, as pointed out by Mirrlees (1975), the first-order approach may not be valid in general. Rogerson (1985) shows that the first-order approach is valid for solving traditional principal-agent problems with risk-neutral principals and risk-averse agents if both of the following conditions are satisfied:

Definition An agent's effort is said to satisfy the Monotone Likelihood Ratio Condition (**MLRC**) if $(f_a/f)_x \geq 0, \forall x \in [\underline{x}, \bar{x}]$ and $(f_a/f)_x > 0$ for some subsets of $[\underline{x}, \bar{x}]$ with positive measures.¹¹

Definition An agent's effort is said to satisfy the Convexity of Distribution Function Condition (**CDFC**) if $F_{aa}(x, a) \geq 0$ for all a and all $x \in [\underline{x}, \bar{x}]$.

Jewitt (1988, p.1177) argues that “most of the distributions commonly occurring in statistics (and economics) do not have the CDF property.” He therefore maintains the MLRC but replaces the CDFC with some less restrictive conditions on the distribution function of the risk. At the same time, he restricts the class of utility functions that a risk-averse agent can take.¹² On the other hand, the MLRC is generally believed to be natural and essential to

¹⁰ Araujo and Moreira (2001) show how the traditional principal-agent problem can be solved without using the first-order approach.

¹¹It can be shown using an argument similar to the one giving rise to Theorem 1 that $(f_a/f)_x = 0, \forall x \in [\underline{x}, \bar{x}]$ cannot be optimal when the agent is risk-averse instead of risk-neutral. The literature seems to have omitted this by defining the MLRC as a condition under which the distribution only needs to satisfy $(f_a/f)_x = 0, \forall x \in [\underline{x}, \bar{x}]$ (see, e.g., Rogerson (1986, p.1361)) and Jewitt (1989, condition 2.11).

¹² Jewitt's (1988) Theorem 2 states that when the principal is risk-neutral and the agent is risk-averse, the first-order approach is valid if

- (a) $\int_{\underline{x}}^x F(\theta, a) d\theta$ is non-increasing and convex in a for all x (Jewitt's (2.10a));
- (b) $\int_{\underline{x}}^x F(\theta, a) d\theta$ is non-decreasing and concave in a for all x (Jewitt's (2.10b));
- (c) $f_a(x, a)/f(x, a)$ is non-decreasing and concave in x for all a (Jewitt's (2.11));
- (d) $\omega''(z) \leq 0, \forall z > 0$, where $\omega(z) = u(u'^{-1}(1/z))$ (Jewitt's (2.12)).

the validity of the first-order approach.¹³ Unfortunately, whereas the MLRC is reasonable under share-cropping and some general moral hazard insurance models with a being work effort and self-protection effort, respectively, it is not satisfied under the insurance brokerage problem. Particularly, one can show the following:

Claim 2 The MPSR condition is incompatible with the MLRC.

Proof: See Appendix.

The question is whether the first-order approach can be justified under the principal-agent insurance brokerage problem in which the principal is risk-averse, the agent is risk-neutral, and the agent's effort is MPSR (violating the MLRC). The following theorem states the conditions under which the first-order approach is valid for the insurance brokerage problem:

Theorem 3: Suppose the principal is risk-averse, the agent is risk-neutral, and the agent's effort is MPSR. The first-order approach is valid such that there exists $a > 0$ being optimal and being sustained by the agent's first-order condition if

- (a) $\int_x^x F(\theta, a) d\theta$ is convex in a for all $x \in [\underline{x}, \bar{x}]$,
- (b) $[f_a(x, a)/f(x, a)]_{xx} < 0$ for all a and for all $x \in [\underline{x}, \bar{x}]$, and
- (c) v exhibits NIARA.

Proof: See Appendix.

Theorem 3 presents a new set of conditions that justifies the first-order approach to principal-agent problems without utilizing either the MLRC or the "unnatural" CDFC. This new set of conditions is much simpler than that proposed by Jewitt (1988). Two main reasons why such simplification is possible are: (i) the replacement of risk-neutral principals/risk-

¹³Jewitt (1988, p.1177) suggests that the MLRC has the "fairly natural interpretation of more effort, more output, and also serves to imply (when the first-order approach is valid) that the agent's payment is increasing in observed output. Sinclair-Desgagné (1994) justifies the first-order approach for the multi-signal principal-agent problem by maintaining the MLRC assumption. Brown et al.'s (1986) paper appears not to utilize the MLRC in part (i) of their theorem whereas the other three parts of their theorem all assume that the MLRC holds. Part (i) of Brown et al.'s (1986) theorem states that the first-order approach is valid when the risk-averse agent's utility is sufficiently concave. Unfortunately, it does not state concretely how concave the agent's utility function really needs to be.

averse agents by risk-averse principals/risk-neutral agents and (ii) the nature of MPSR activities giving rise to $F_a(\bar{x}, a) = 0$. Notice that condition (a) of Theorem 3 has been used by Jewitt (1988, Theorem 2) and is believed to be less restrictive than the CDFC. Condition (c) is similar but less complicated than condition (2.12) of Jewitt's Theorem 2 (1988). It can be checked from footnote (12) that Jewitt's condition (2.12) essentially requires that the agent's utility function has a coefficient of absolute risk aversion that must not decrease too quickly.¹⁴

The intuition of Theorem 3 becomes apparent when one compares it with Theorem 2. It should be recalled from part (b) of Theorem 2 that conditions (b) and (c) of Theorem 3 imply that reward function s is concave. The concavity of s in x renders the agent's expected payoff concave in a as shown in the proof of Theorem 3. With the now concave expected payoff, the risk-neutral agent is induced to choose an optimal level of positive MPSR effort as if he is risk-averse (or the agent is simply forced to take account of the risk-averse attitude of the principal as a result of the concave reward function). Notice from the proof of Theorem 3 (particularly, equation (23)) that a linear reward function cannot be optimal because $a > 0$ cannot be sustained by the agent's first-order condition when $s'' = 0$.

6. RISK-AVERSE RISK MANAGERS AND INTERNAL RISK MANAGEMENT PROBLEMS

Instead of hiring a large and hence risk-neutral insurance broker to search for appropriate insurance coverage, a risk-averse principal can rely on a small insurance brokerage company or an internal risk manager to handle the matter. In either case, the agent may be risk-averse.

¹⁴According to Jewitt (1988), $\omega''(z) \leq 0 \Leftrightarrow d[-u''(z)/(u'(z))^2]/dz \geq 0$. Further computation reveals that the latter condition is equivalent to $[A'_u(z)/A_u(z)] + A_u(z) \geq 0 \Leftrightarrow 2A_u(z) - P_u(z) \geq 0$, where $P_u(z) = -u'''(z)/u''(z)$ is the coefficient of absolute prudence in Kimball's (1990) terminology. The last inequality holds only if $A_u(z)$ does not decrease too quickly. Jewitt (1988, p.1181) also suggests that this is equivalent to a coefficient of relative risk aversion that is nondecreasing and bounded above by one half.

Denote a risk-averse agent's utility function by u with $u' > 0$ and $u'' < 0$. Following the common practice, assume that the agent's expected utility is additively separable given by:¹⁵

$$\int_{\underline{x}}^{\bar{x}} u[s(x)]f(x, a)dx - c(a)$$

such that the agent's first-order condition (4) becomes

$$\int_{\underline{x}}^{\bar{x}} u[s(x)]f_a(x, a)dx - c'(a) = 0 \quad (9)$$

and the principal's first-order condition (5) becomes

$$\frac{v'[x - s(x)]}{u'[s(x)]} = \lambda + \mu \frac{f_a(x, a)}{f(x, a)}. \quad (10)$$

Since $\mu > 0$ is no longer guaranteed when both the principal and the agent are risk-averse, assume for simplicity that $\mu > 0$. The following states a theorem analogous to Theorems 2 and 3:

Theorem 4 Suppose both the principal and the agent are risk-averse with CARA utility functions (i.e., $A_v = \overline{A}_v$ and $A_u = \overline{A}_u$ where \overline{A}_v and \overline{A}_u are constants).

(a) $\text{Sgn} \left[\left(\frac{\overline{A}_v + \overline{A}_u}{\overline{A}_v} \right) s' - 1 \right] = \text{Sgn} [(f_a/f)_x]$.

(b) If $[f_a(x, a)/f(x, a)]_{xx} < 0$, then $s''(x) < 0$.

(c) If, in addition to the above conditions, $\int_{\underline{x}}^x F(\theta, a)d\theta$ is convex in a for all $x \in [\underline{x}, \bar{x}]$, then

the first-order approach is valid with $a > 0$ being sustained by the agent's first-order conditions.

Proof: See Appendix.

It should be apparent from Theorem 4 that allowing the agent to be risk-averse makes the insurance brokerage problem more complicated. In particular, to yield results similar to those stated in Theorems 2 and 3, one may need to impose restrictions (such as CARA) on the utility functions of both the principal and the agent. Notice that similar to the case with a risk-neutral agent, a state-independent reward cannot be optimal (as stated in Theorem 1). However, a linear reward function can now be optimal, because $a > 0$ can be sustained by

¹⁵Alvi (1997) solves the traditional principal-agent problem without assuming additively separable utility on s and c .

the agent's first-order conditions given $u'' < 0$ even when $s'' = 0$. Both results can be inferred easily from the Proof of Theorem 4 (particularly, equation (29)).

Theorem 4 can be applied to a general principal-agent risk management problem. For example, the principal may be a risk-averse exporting company that hires a risk-averse risk manager who is responsible solely for transferring its exchange rate risk by hedging via the forwards market. Assume for simplicity that the company faces random exchange rate \tilde{e} with support $[\underline{e}, \bar{e}]$, density function $g : [\underline{e}, \bar{e}] \rightarrow \mathfrak{R}_+$, and distribution function G . The principal's net worth is given by $\tilde{x} = \omega + \tilde{e}q - a(\tilde{e} - E\tilde{e})q$, where w again represents initial wealth, q is the total value of exports denominated in a foreign currency, and a is the percentage of the exchange rate risk hedged by the agent. The support of \tilde{x} is given by $[\omega + \underline{e}q - a(\underline{e} - E\tilde{e})q, \omega + \bar{e}q - a(\bar{e} - E\tilde{e})q]$.

Consider

$$F(x, a) = \int_{\underline{x}}^x f(x, a) = \int_{\underline{e}}^e g(\xi, a) d\xi = \int_{\underline{e}}^{\frac{x-w-aqE\tilde{e}}{(1-a)q}} g(\xi, a) d\xi$$

such that

$$F_a(x, a) = q^{-1}(1-a)^{-2} \cdot (x - w - qE\tilde{e}) \cdot g\left[\frac{x - w - aqE\tilde{e}}{(1-a)q}\right]. \quad (11)$$

It can be checked that $E\tilde{x} = w + qE\tilde{e}$ regardless of a so that $\int_{\underline{x}}^{\bar{x}} F_a(x, a) dx = 0$. Now, the right side of (11) changes from negative to positive as x increases. Therefore, $\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0, \forall x \in (\underline{x}, \bar{x})$. The MPSR condition is again satisfied such that Theorem 4 applies.

7. CONCLUSION

This paper has analyzed the principal-agent insurance brokerage problem by introducing the concept of "mean-preserving spread-reducing" (MPSR) effort. It has been shown that when the principal's utility exhibits NIARA, the concavity of the f_a/f ratio implies that the optimal reward function is concave. The concavity of the reward function induces the risk-neutral agent to exert MPSR effort. Surprisingly, these conditions together with an additional condition on the distribution function of the risk (similar but simpler than those imposed by

Jewitt (1988)) guarantee the validity of the first-order approach, without using either the MLRC or the CDFC. Similar results can be obtained for the case in which both the principal and the agent are risk-averse (e.g., in an internal risk management problem) if the CARA utility restriction is imposed.

In reality, however, a reward function that is concave in the realized value of \tilde{x} as predicted in Theorem 2 is seldom observed in insurance brokering. There are several possible explanations. First, the insurance brokering market is often subject to some degree of market imperfection as a result of market concentration and the expertise possessed by certain brokers (see, e.g., Tommy (2000)). Moreover, insureds may not have sufficient bargaining power so that brokerage commissions may be set by relatively large insurance brokers. Second, the insurance brokerage problem is often complicated by the moral hazard problem of a principal who can affect the loss distribution and hence the actual reward to be made to a broker under any pre-determined reward function. Third, besides purchasing insurance for its principal, a broker may provide other tied-in risk-management services, such as risk analysis, claim settlement, and provision of legal advice. In general, it is difficult to separate the reward made solely to insurance purchase and that made to other services.

After all, the commissions received by brokers are often not transparent. The Risk and Insurance Management Society Inc. in New York has reportedly urged that the fees (including the contingent commissions offered by insurers) received by brokers be disclosed if requested by policyholders (see Tommy (2000)). Recently, there have been allegations of insurance brokers' misconduct for possibly receiving non-disclosed incentive fees from insurers. It is obvious that insureds should be reminded to beware of any potential conflict of interest with insurance brokers. Moreover, they should also be better informed of the possibility of concavifying their state-contingent commissions to be made to insurance brokers (e.g., by requiring insurance brokers to share their losses) in order to give them proper incentives to search for optimal insurance coverage.

APPENDIX

Proof of Lemma 1: Integrating both sides of (5) and using the fact that $F_a(\bar{x}, a) = 0$ (as $F(\bar{x}, a) = 1, \forall a$) gives

$$\begin{aligned} \int_{\underline{x}}^{\bar{x}} [v'(x-s(x)) - \lambda] f(x, a) dx &= \int_{\underline{x}}^{\bar{x}} \mu f_a(\bar{x}, a) = \mu F_a(\bar{x}, a) = 0 \\ \Leftrightarrow \lambda &= \int_{\underline{x}}^{\bar{x}} v'(x-s(x)) f(x, a) dx > 0. \end{aligned} \quad (12)$$

Next, it will first be shown that $\mu \neq 0$. Suppose by contradiction that $\mu = 0$. Then (5) implies that $v'(x-s(x)) = \lambda$ such that $s(x) = x - K$ as $v''(\cdot) < 0$, where K is a constant. The last equality implies that

$$\int_{\underline{x}}^{\bar{x}} s(x) f_a(x, a) dx - c'(a) = \int_{\underline{x}}^{\bar{x}} (x - K) f_a(x, a) dx - c'(a) = -c'(a) < 0 \quad (13)$$

The second equality in (13) is because of $\int_{\underline{x}}^{\bar{x}} x f_a(x, a) dx = \int_{\underline{x}}^{\bar{x}} F_a(x, a) dx = 0$ and $F_a(\bar{x}, a) = 0$.

The inequality in (13) contradicts (4). Therefore, $\mu \neq 0$. With $\mu \neq 0$, (5) can be written as

$$f_a(x, a) = \left[\frac{v'(x-s(x)) - \lambda}{\mu} \right] f(x, a). \quad (14)$$

To check that $\mu > 0$, suppose by contradiction that $\mu < 0$. Now, using the fact that $\int_{\underline{x}}^{\bar{x}} x f_a(x, a) dx = 0$, one can rewrite the agent's first-order condition (4) as

$$\begin{aligned} 0 &= \int_{\underline{x}}^{\bar{x}} [x - s(x)] f_a(x, a) dx + c'(a) \\ &= \mu^{-1} \int_{\underline{x}}^{\bar{x}} [x - s(x)] [v'(x-s(x)) - \lambda] f(x, a) dx + c'(a) \\ &= \mu^{-1} \int_{\underline{x}}^{\bar{x}} [x - s(x)] \left[v'(x-s(x)) - \int_{\underline{x}}^{\bar{x}} v' f(y, a) dy \right] f(x, a) dx + c'(a) \\ &= \mu^{-1} \text{cov}[x - s(\tilde{x}), v'(x-s(x))] + c'(a) \\ &> 0 \end{aligned} \quad (15)$$

The second equality in (15) is due to (14). The third equality is due to (12). The last inequality is because of $c' > 0, \mu < 0$, and the fact that a rise in $x - s(x)$ coincides with a fall in $v'(x-s(x))$ as $v'' < 0$ such that $\text{cov}[x - s(x), v'(x-s(x))] < 0$. The last inequality in (15) gives rise to a contradiction! Therefore, $\mu > 0$. \square

Proof of Theorem 2: To prove part (a), differentiate both sides of (5) with respect to x to get

$$\nu''(x-s(x))(1-s'(x)) = \mu \left[\frac{f_a(x,a)}{f(x,a)} \right]_x. \quad (16)$$

According to Lemma 1, $\mu > 0$. Substitute this and $\nu'' < 0$ into (16) to get

$$\text{Sgn}[1-s'(x)] = -\text{Sgn}[(f_a/f)_x]$$

To prove part (b), rewrite (16) as

$$A_\nu \cdot (s'-1)\nu' = \mu \left[\frac{f_a(x,a)}{f(x,a)} \right]_x$$

and differentiate both sides with respect to x to get

$$-A'_\nu(s'-1)^2\nu' + A_\nu s''\nu' - A_\nu(s'-1)^2\nu'' = \mu \left[\frac{f_a(x,a)}{f(x,a)} \right]_{xx} \quad (17)$$

According to (17), $s'' < 0$ if and only if

$$-A'_\nu(s'-1)^2\nu' - A_\nu(s'-1)^2\nu'' > \mu \left[\frac{f_a(x,a)}{f(x,a)} \right]_{xx}. \quad (18)$$

The inequality in (18) is clearly satisfied given $\nu'' < 0$, $A_\nu > 0$, $A'_\nu < 0$ and $[f_a(x,a)/f(x,a)]_{xx} < 0$. \square

Proof of Claim 2: Suppose the MLRC is satisfied such that $(f_a/f)_x \geq 0, \forall x \in [\underline{x}, \bar{x}]$ with $(f_a/f)_x > 0$ for some subsets of $[\underline{x}, \bar{x}]$ with positive measures. Consider

$$0 = F_a(\bar{x}, a) = \int_{\underline{x}}^{\bar{x}} f_a(x, a) dx \quad (19)$$

implying that f_a/f must change sign on (\underline{x}, \bar{x}) at least once. This together with MLRC implies that f_a/f must change sign exactly once. Therefore, there exists $x_0 \in (\underline{x}, \bar{x})$ at which $f_a/f = 0$ such that $f_a/f < 0, \forall x \in (\underline{x}, x_0)$ and $f_a/f > 0, \forall x \in (x_0, \bar{x})$. The first inequality implies that $F_a(x, a) < 0, \forall x \in (\underline{x}, x_0)$.

Next, consider any $x_3 \in (x_0, \bar{x})$. Rewrite (19) as

$$0 = F_a(\bar{x}, a) = \int_{\underline{x}}^{x_0} (f_a/f) f dx + \int_{x_0}^{x_3} (f_a/f) f dx + \int_{x_3}^{\bar{x}} (f_a/f) f dx. \quad (20)$$

Now, $f_a/f > 0, \forall_x \in (x_0, \bar{x})$ implies that $\int_{x_3}^{\bar{x}} (f_a/f) f dx > 0$ and hence $F_a(x_3, a) = \int_{x_3}^{\bar{x}} (f_a/f) f dx < 0$ according to (20). Since x_3 can be chosen arbitrarily close to \bar{x} , one has $F_a(x, a) < 0, \forall_x \in (x_0, \bar{x})$. This together with $F_a(x, a) < 0, \forall_x \in (\underline{x}, x_0)$ implies that $\int_{\underline{x}}^x F_a(\theta, a) d\theta < 0, \forall_x \in (x_0, \bar{x})$ violating the MPSR condition at $x = \bar{x}$. Therefore, MLRC implies not MPSR and vice versa. \square

Proof of Theorem 3: As suggested by Jewitt (1988), to show that the first-order approach of principal-agent problems is valid, it suffices to show that the agent's second-order condition holds. It can be checked that the agent's second-order condition is given by

$$\int_{\underline{x}}^{\bar{x}} s(x) f_{aa}(x, a) dx - c''(a) < 0. \quad (21)$$

Integrating the first term on the left side of (21) by parts twice gives

$$\begin{aligned} & \int_{\underline{x}}^{\bar{x}} s(x) f_{aa}(x, a) dx \\ &= s(\bar{x}) \int_{\underline{x}}^{\bar{x}} f_{aa}(x, a) dx - \int_{\underline{x}}^{\bar{x}} s'(x) \left[\int_{\underline{x}}^x f_{aa}(\theta, a) x \theta \right] dx \\ &= - \int_{\underline{x}}^{\bar{x}} s'(x) \left[\int_{\underline{x}}^x f_{aa}(\theta, a) d\theta \right] dx \\ &= -s(\bar{x}) \int_{\underline{x}}^{\bar{x}} F_{aa}(x, a) dx + \int_{\underline{x}}^{\bar{x}} s''(x) \left[\int_{\underline{x}}^x F_{aa}(\theta, a) d\theta \right] dx. \\ &= \int_{\underline{x}}^{\bar{x}} s''(x) \left[\int_{\underline{x}}^x F_{aa}(\theta, a) d\theta \right] dx \end{aligned} \quad (22)$$

The second equality in (22) is because of $\int_{\underline{x}}^{\bar{x}} f_{aa}(x, a) dx = F_{aa}(\bar{x}, a) = 0$ ($as F_a(\bar{x}, a) = 0$). The fourth equality is because of $\int_{\underline{x}}^{\bar{x}} F_{aa}(x, a) dx = 0$ ($as \int_{\underline{x}}^{\bar{x}} F_a(\bar{x}, a) dx = 0, \forall_a$). Now, conditions (b) and (c) imply that $s'' < 0$ according to Theorem 2. This together with $\int_{\underline{x}}^x F_{aa}(\theta, a) d\theta > 0, \forall_x \in (\underline{x}, \bar{x})$ clearly implies that the right side of the last equality of (22) is negative. Therefore, (21) holds as $c'' > 0$.

Next, it can be checked that $a > 0$ can be sustained by the agent's first-order condition when effort a satisfies the MPSR condition. Replacing f_{aa} and F_{aa} by f_a and F_a in (22) and using the fact that $F_a(\bar{x}, a) = 0$ and $\int_{\bar{x}}^x F_a(x, a) dx = 0$ gives

$$\int_{\bar{x}}^x s(x) f_a(x, a) dx = \int_{\bar{x}}^x s''(x) \left[\int_{\bar{x}}^x F_a(\theta, a) d\theta \right] dx. \quad (23)$$

$\int_{\bar{x}}^x F_a(\theta, a) d\theta < 0$ for all a and all $x \in (x, \bar{x})$ together with $s'' < 0$ implies that the right side of (23) is strictly positive. This together with $c'' > 0$ and $\lim_{a \rightarrow 0} c'(a) = 0$ implies that there exists $a > 0$ such that the agent's first-order condition (4) holds.

Proof of Theorem 4: Differentiating both sides of (10) with respect to x gives

$$\begin{aligned} \left(\frac{v'}{u'} \right) &= \frac{v''}{u'} (1 - s') - \frac{u'' v'}{u'^2} s' = \mu \left(\frac{f_a}{f} \right) \\ \Leftrightarrow -A_v \left(\frac{v'}{u'} \right) (1 - s') + A_u \left(\frac{v'}{u'} \right) s' &= \mu \left(\frac{f_a}{f} \right)_x \\ \Leftrightarrow \left(\frac{A_v + A_u}{A_v} \right) s' - 1 &= \frac{\mu (f_a / f)_x (u' / v')}{A_v}. \end{aligned} \quad (24)$$

Suppose $A_v = \bar{A}_v$ and $A_u = \bar{A}_u$. Then given $\mu > 0, A_v > 0, A_u > 0$, and $(u' / v') > 0$, it can be checked that (24) implies that

$$\text{Sgn} \left[\left(\frac{\bar{A}_v + \bar{A}_u}{\bar{A}_v} \right) s' - 1 \right] = \text{Sgn} \left[(f_a / f)_x \right].$$

Differentiating the first line of (24) with respect to x gives

$$\begin{aligned} -A'_v (s' - 1)^2 \left(\frac{v'}{u'} \right) + A_v \left(\frac{v'}{u'} \right)' (s' - 1) + A_v \left(\frac{v'}{u'} \right) s'' \\ + A'_v s'^2 \left(\frac{v'}{u'} \right) + A_u \left(\frac{v'}{u'} \right)' s' + A_u \left(\frac{v'}{u'} \right) s'' &= \mu \left(\frac{f_a}{f} \right)_{xx}. \end{aligned} \quad (25)$$

Substituting the first line of (24) into (25) and rearranging yields

$$(A_u + A_v) \left(\frac{v'}{u'} \right) s'' = \mu \left(\frac{f_a}{f} \right)_{xx} + A'_v (s' - 1)^2 \left(\frac{v'}{u'} \right) - A'_u s'^2 \left(\frac{v'}{u'} \right) - \left[\mu \left(\frac{f_a}{f} \right)_x \right]^2 \left(\frac{u'}{v'} \right). \quad (26)$$

Substituting $A_u = \overline{A}_u$, $A_v = \overline{A}_v$, and $A'_v = A'_u = 0$ into (26) gives

$$\left(\overline{A}_v + \overline{A}_u\right)\left(\frac{v'}{u'}\right)s'' = \mu\left(\frac{f_a}{f}\right)_{xx} - \left[\mu\left(\frac{f_a}{f}\right)_x\right]^2\left(\frac{u'}{v'}\right) \quad (27)$$

such that $(f_a/f)_{xx} < 0$ as $\mu > 0$ and $u'/v' > 0$ imply that $s'' < 0$.

To prove part (c), substitute $s(x)$ in (21) by $u[s(x)]$ and integrate the new second-order condition by parts twice to obtain

$$\begin{aligned} & \int_{\underline{x}}^{\bar{x}} u[s(x)]f_{aa}(x, a)dx - c''(a) \\ &= u[s(\bar{x})]s(\bar{x})\int_{\underline{x}}^{\bar{x}} f_{aa}(x, a)dx - \int_{\underline{x}}^{\bar{x}} u' \cdot s' \cdot \left[\int_{\underline{x}}^x f_{aa}(\theta, a)x\theta\right]dx - c''(a) \\ &= -\int_{\underline{x}}^{\bar{x}} u' \cdot s' \cdot \left[\int_{\underline{x}}^x f_{aa}(\theta, a)d\theta\right]dx - c''(a) \\ &= -u'[s(\bar{x})]s'(\bar{x})\int_{\underline{x}}^{\bar{x}} F_{aa}(x, a)dx + \int_{\underline{x}}^{\bar{x}} (u''s'^2 + u's'')\left[\int_{\underline{x}}^x F_{aa}(\theta, a)x\theta\right]dx - c''(a) \\ &< 0 \end{aligned} \quad (28)$$

The second equality is because of $F_{aa}(\bar{x}, a) = 0$ (as $F_a(\bar{x}, a) = 0$). The last inequality is because of $\int_{\underline{x}}^{\bar{x}} F_{aa}(x, a)dx = 0$ (as $\int_{\underline{x}}^{\bar{x}} F_a(x, a)dx = 0, \forall a$), $s'' < 0$, $u'' < 0$, $\int_{\underline{x}}^x F_{aa}(\theta, a)d\theta > 0, \forall x \in (\underline{x}, \bar{x})$, and $c'' < 0$.

Next, it can be checked that (23) in the proof of Theorem 3 now becomes

$$\int_{\underline{x}}^{\bar{x}} u[s(x)]f_a(x, a)dx = \int_{\underline{x}}^{\bar{x}} (u''s'^2 + u's'')\left[\int_{\underline{x}}^x F_a(\theta, a)d\theta\right]dx. \quad (29)$$

$\int_{\underline{x}}^x F_a(\theta, a)d\theta < 0$ for all a and all $x \in (\underline{x}, \bar{x})$ together with $u'' < 0$ and $s'' < 0$ implies that the right side of (29) is strictly positive. This together with $c'' > 0$ and $\lim_{a \rightarrow 0} c'(a) = 0$ implies that there exists $a > 0$ such that the agent's first-order condition (9) holds. \square

REFERENCES

- Alvi, E.: First-Order Approach to Principal-Agent Problems: A Generalization. *Geneva Papers on Risk Insurance Theory* 22, 59-65 (1997).
- Arrow, K.J.: Uncertainty and the Welfare Economics of Medical Care. *American Economic Review* 53, 941-973 (1963).
- Brown, M., Chiang, S.H., Ghost, S., Wolfstetter, E.: A New Class of Sufficient Conditions for the First-Order Approach to Principal-Agent Problems. *Economics Letters* 21, 1-6 (1986).
- Campbell, T.S., Kracaw, W.A.: Optimal Managerial Incentive Contracts and the Value of Corporate Insurance. *Journal of Financial and Quantitative Analysis* 22, 315-28 (1987).
- Grossman, S., Hart, O.: An Analysis of the Principal-Agent Problem. *Econometrica* 51, 7-45 (1983).
- Hadar, J., Russell, W.: Rules for ordering uncertain prospects. *American Economic Review* 59, 25-34 (1969).
- Holmstrom, B.: Moral Hazard and Observability. *Bell Journal of Economics* 10, 74-91 (1979).
- Jewitt, I.: Justifying the First-Order Approach to Principal-Agent Problems. *Econometrica* 56, 1177-90 (1988).
- Kimball, M.S.: Precautionary Saving in the Small and in the Large. *Econometrica* 58, 53-73 (1990).
- Laffont, J., Martimort, D.: *The Theory of Incentives - The Principal-Agent Model*. New Jersey: Princeton University Press 2002.
- LiCalzi M., Spaeter, S.: Distributions for the First-order Approach to Principal-agent Problems. *Economic Theory* 21, 167-173 (2003).
- Mirrlees, J.: *The Theory of Moral Hazard and Unobservable Behavior - Part I*. mimeo, Nuffield College, Oxford (1975).
- Mirrlees, J.: *The Theory of Moral Hazard and Unobservable Behavior - Part I*. *Review of Economic Studies* 66, 3-21 (1999).
- Mossin, J.: Aspects of Rational Insurance Purchasing. *Journal of Political Economy* 76, 553-568 (1968).
- Oyer, P.: A Theory of Sales Quotas with Limited Liability and Rent Sharing. *Journal of Labor Economics* 18, 405-426 (2000).
- Rogerson, W.: The First-Order Approach to Principal-Agent Problems. *Econometrica* 53, 1357-68 (1985).

Sinclair-Desgagné, B.: The First-Order Approach to Multi-Signal Principal-Agent Problems. *Econometrica* 62, 259-66 (1994).

Spence, M., Zeckhauser, R.: Insurance, Information, and Individual Action. *American Economic Review*, Papers and Proceedings of the Eighty-Third Annual Meeting of the American Economic Association 61, 380-87 (1971).

Tommy, V.A.: The Mystery of Broker Remuneration. *Business Insurance* 34, 25 (2000).

Wilson, R.: The Theory of Syndicates. *Econometrica* 36, 119-32 (1968).

Whitt, W.: Uniform Conditional Stochastic Order. *Journal of Applied Probability* 17, 112-123 (1980).