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Yue Ma

Lingnan University
Hong Kong

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Dr. Yue Ma is Associate Professor at Department of Economics, Lingnan University, Hong Kong

Centre for Public Policy Studies
Lingnan University
Tuen Mun
Hong Kong
Tel: (852) 2616 7432
Fax: (852) 2591 0690
Email: cpps@LN.edu.hk
http://www.LN.edu.hk/cpps/

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Yue Ma

Economics Department
Lingnan University
Tuen Mun
Hong Kong

Tel: (+852) 2616 7202
Fax: (+852) 2891 7940
Email: yuema@Ln.edu.hk
http://www.Ln.edu.hk/econ/staff/yuema

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Abstract

In this paper, we prove that the noncooperative solution with mutually consistent conjectures is the same as the contract curve of the cooperative solution in a two-person game. A numerical example is also presented.

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1. Introduction

Economic conflict, or competitive behaviour between a finite and possibly small number of agents, is best modelled as a game. There are obvious applications in oligopoly theory (Friedman, 1977, 1983), international policy coordination (Oudiz and Sachs, 1985), wage bargaining (Calmfors and Driffill, 1988), environmental issues (Pethig, 1992), development economics (Li and Ma, 1996) as well as in many other areas.

In two-person games, two distinct behaviour patterns are possible: cooperative vs. non-cooperative. In a non-cooperative game (e.g. Nash or conjectural variations, see Basar and Olsder, 1982), each player will try to minimise his own cost function, conditional on the opponent's decision rule. But it is known that such non-cooperative decisions are socially inefficient (Da Cuhna and Polak, 1967). In contrast, a cooperative game is to minimise a joint cost function by the two players working together and will generate Pareto optimal outcomes.

In this paper, we show that, under certain conditions, non-cooperative behaviour can turn out to be Pareto efficient as well, i.e. the players in a non-cooperative game <u>can</u> behave as if they were playing a cooperative game, even though there is no pre-commitment for them to do so. That implies a built-in self-enforcement mechanism which would lead them to behave cooperatively out of self-interest. To show this, we work with one particular form of non-cooperative game: the conjectural variations equilibrium. Our result therefore extends earlier work by De Bruyne (1979), which showed that the Nash noncooperative equilibrium is Pareto optimal <u>only</u> when the targets and preferences are identical for all players.

2. Conjectural Variations Solutions

Non-cooperation implies that each player in the game maximises his self-interest, subject to his perception of the constraints on his decision variables, conditional on rational expectations about the decisions taken by others. For simplicity, we shall restrict attention to two-player static games here. Each decision maker has one

instrument, x_i to reach one target y_i (i=1,2). The private interests of each player can then be represented by the cost functions:

$$w_i = w_i (y_i , x_i) , i = 1, 2.$$

The players face a static reduced-form economic system:

$$y_i = f_i (x_1 , x_2) , i = 1, 2.$$

Given a suitable starting point, conjectural variations can now be used to deduce appropriate reaction terms within the optimisation process. Like in chess, each player recognises that his decisions affect the targets and hence the policy choice of his opponent, and that variations in the latter then influence his targets again and hence what he (the first player) should decide to do. Instead of optimising conditional on fixed reactions by the opponent ('given he does that, what should I do?'), each player must now anticipate his opponent's reactions ('if I do this, how will he react and how should I best accommodate that?, *etc.*'). The conjectural variations employed by each player here are captured by $\partial x_1/\partial x_2$ and $\partial x_2/\partial x_1$ (while the Nash equilibrium, of course, sets $\partial x_1/\partial x_2 = \partial x_2/\partial x_1 = 0$ *ex ante*). By optimising the decisions associated with each variation in the

conjectured responses to the currently envisaged choices, and then constraining those variations so that the new pair of decisions represent an improvement over the old pair, we reach the closed-loop conjectural variations equilibrium. That equilibrium holds only when both players perceive that no further gains can be made by varying their reactions to the decisions currently expected from their opponent, because to do so would trigger counter-reactions (in the opponent's interest) which would more than offset the gains of unilateral action.

There has been extensive discussion of the interpretation of conjectural variations model (see, for example, Bresnahan, 1981; Kamien and Schwartz, 1983). It has been argued by Friedman (1983), among others, that it arises in a static one-shot game, cannot be taken as a literal expectations of future reactions. Many game theorists prefer to utilise repeated game models, which, however, have their own weaknesses. A paper by Kalai and Stanford (1985) explicitly analyzes conjectural variations in a repeated game

framework, showing that various conjectural variations can be maintained as credible equilibria.   While Brander and Spencer (1985) and Dixit (1988) hold the view that the conjectural variations are simply a useful and intuitive summary measure of market conduct. Recent applications of conjectural variations model in microeconomics can be found in the papers by Brander and Zhang (1989, 1993) who assessed the post-deregulation competitiveness of the US airline industry.   Applications of conjectural variations model in the international macroeconomics can be found in Sterdyniak and Villa (1993) who investigated the international policy coordination issues.

However, the conjectural variations equilibrium is indeterminate as the conjectures are formed outside the model.   Independent efforts have been made to narrow the possible market equilibria by imposing additional constraints that the conjectural variations be 'consistent' (Friedman, 1977; Laitner, 1980; Bresnahan, 1981; Kamien and Schwartz, 1983).   One way to solve this multiply solution problem

is by imposing *self* consistent conjectures for the players (see, for example, Bresnahan, 1981):   the conjecture in the reaction function is set to the slope of the reaction function.   Under certain conditions, the self consistent conjectures equilibrium is unique.   In this paper, we try to examine the conjectural variations solution by imposing an alternative assumption: the conjectures in the reaction function of each player are *mutually* consistent.   We find that this assumption will give us a general condition under which the conjectural variations solution becomes a cooperative solution.

3.   The Equivalence of Noncooperative Solution with Mutually Consistent Conjectures and Cooperative Solution

The necessary conditions for optimal decisions with conjectural variations are

Install Equation Editor and double-click here to view equation.   (1)

for i=1 and 2 (and i≠j), where $E_i(\partial x_j/\partial x_i)$ is the player i's expectation

on the player j's response.   From this we derive :

Theorem   Suppose player i (i=1,2) tries to minimise his objective

function w_i by choosing instrument x_i subject to a constraint f_i:

Then, the conjectural variations general solution is the same as the

contract curve, if the two players' expectations are mutually consistent,

i.e.

$$E_i(\partial x_j/\partial x_i) = \partial x_j/\partial x_i \qquad (2)$$

where $E_i(.)$ is player i's expectation (i=1,2 and i≠j).   This states that

the reaction player j chooses is consistent with what player i expect

him to choose.

Proof

For any given value of objective function k, the indifference curve for

player i is:

Therefore, the derivative of the indifference curve for player 1 is:

Similarly, the derivative of the indifference curve for player 2 is

The contract curve is the locus at which the derivatives of the two

indifference curves are the same, i.e. the locus of the tangent points of

the two players' indifference curves:

For the conjectural variations game in (1), player 1's decisions must

satisfy:

where $E_1(\partial x_2/\partial x_1) \neq 0$, and player 2's decisions

where $E_2(\partial x_1/\partial x_2) \neq 0$. But in equilibrium we are able to cancel the

terms of $E_2(\partial x_1/\partial x_2)$ and $E_1(\partial x_2/\partial x_1)$ under mutually consistent

expectations condition, since:

Hence,   in equilibrium we have:

which is the same as the contract curve (6). That implies that there is

an one-to-one correspondence between the contract curve and the

mutually consistent conjectures solution.

[Q.E.D.]

To illustrate the theorem, we apply it to solve a particular problem

which is given as follows:

The conjectural variations solution is:

$$\partial w_1/\partial x_1 = 0 \text{ , i.e. } y_1 [1 + E_1(\partial x_2/\partial x_1)] + x_1 = 0 \text{ ,}$$

and

$$\partial w_2/\partial x_2 = 0 \text{ , i.e. } y_2 [1 + E_2(\partial x_1/\partial x_2)] + x_2 = 0 \text{ ,}$$

implying that the noncooperative solution with mutually consistent

conjectures must satisfy:

$$x_1^2 + x_2^2 + 3x_1x_2 - x_1 - x_2 = 0 \qquad\qquad (13)$$

Meanwhile the indifference curves of the two players are,

respectively:

$$(x_1+x_2-1)^2 + x_2^2 = k_1 \text{ and } (x_1+x_2-1)^2 + x_2^2 = k_2 \qquad \text{(for any given } k_1,$$

$$k_2)$$

The derivatives of the indifference curves are:

and the contract curve of the cooperative solution is

$$x_1^2 + x_2^2 + 3x_1x_2 - x_1 - x_2 = 0 \qquad\qquad (15)$$

which is exactly the same as the noncooperative solution with mutually consistent conjectures (13) (see Figure 1).

4. Conclusion

In this paper, we proved that the noncooperative solution with the mutually consistent conjectures is the same as the contract curve of the cooperative solution in the two-person game. This implies that the players in a non-cooperative game can sometimes behave like in a cooperative game, for their own self-interest, although there is no binding commitment for them to do so. The result clearly is important for the policymakers when they design the coordinated policy packages, for the study of the market behaviour and in other areas.

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Fig 1.

x2: Player 2

Noncoop sol'n

Coop sol'n

x1: Player 1

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