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A RISK-AVERSE NEWSVENDOR MODEL WITH PRICING CONSIDERATION

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A RISK-AVERSE NEWSVENDOR MODEL WITH PRICING CONSIDERATION

by
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ABSTRACT

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A decision maker who is facing a random demand for a perishable product, such as newspapers, decides how many units to order for a single selling period. This single-period inventory problem is often referred to as the "classic newsvendor problem", in which the selling price is fixed, the order must be made before the selling period, and the decision maker is risk-neutral. If the decision maker orders too many (overage), the inventory cost will be too high. If the decision maker orders too few (underage), the potential profit will be lost. The optimal order quantity is a balance between the expected costs of overage and underage.

This thesis investigates an extension of the classic newsvendor problem. In this extension the demand depends on the selling price, the decision maker may obtain an additional order at a higher price during the selling period, and the decision maker is risk-averse (not risk-neutral). The problem is to find optimal order quantity and selling price so that the expected utility of the risk-averse decision maker is maximized.

This thesis examines the relationship between the order quantity and the selling price for different risk-averse decision makers in this extended newsvendor problem defined above. The result shows that the relationships are consistent for some decision makers but not for others. For example, if the decision maker exhibits a constant absolute risk aversion (CARA), the optimal order quantity will decline when the selling price increases. If the decision maker has constant

relative risk aversion (CRRA), the relationship is complex. This thesis finds that if it is just known that the decision maker is risk-averse, the optimal order quantity placed is less than that made by a risk-neutral decision maker. Further more, the risk-averse decision maker's optimal order quantity falls when her/his risk aversion increases. However, the relationship between order quantity and selling price is still indeterminate in this case.

This extension of the classic newsvendor problem provides a more realistic dynamic setting than before, therefore providing an excellent framework for examining how the inventory problem interacting with the marketing issue (selling price) will influence decision makers at the firm level. It also provides an integrated framework for investigating different variations of newsvendor problems. Thus, this thesis will motivate and encourage more applications of the newsvendor problem which is a foundation of many supply chain management problems.

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

(YE Zuobin)
September 2004

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$\boldsymbol{w}$**:** Wealth of a decision maker (newsvendor) which is a random variable over $R = (-\infty, +\infty)$.

$\boldsymbol{w_0}$**:** The initial wealth of a decision maker (newsvendor).

$\boldsymbol{u(w)}$**:** The total utility of wealth w.

$\boldsymbol{u'(w)}$**:** Marginal utility of wealth w.

$\boldsymbol{u''(w)}$**:** The rate of change of marginal utility with respect to wealth w.

$\boldsymbol{r_u(w)}$**:** The Arrow-Pratt Measure of Absolute Risk-Aversion under utility $u(w)$.

$\boldsymbol{R_u(w)}$**:** The Arrow-Pratt Measure of Relative Risk-Aversion under utility $u(w)$.

$\boldsymbol{q}$**:** The initial order quantity at the beginning of a single period.

$\boldsymbol{c}$**:** The purchasing cost of unit product at the beginning of a single period.

$\boldsymbol{\hat{c}}$**:** The purchasing cost of unit product at late time if the demand exceeds the initial order quantity q. Usually $\hat{c} > c$.

$\boldsymbol{p}$**:** The selling price of unit product. Usually $p > \hat{c}$.

$\boldsymbol{s}$**:** The unit salvage price for unsold products during the selling season. Usually $0 < s < c$.

$\boldsymbol{\tilde{\epsilon}}$**:** is a random variable defined in the range $[-A, B]$ ($A \geq 0,\ B \geq 0$) and price-independent.

$\boldsymbol{F(\cdot)}$**:** is the cumulative distribution function of $\tilde{\epsilon}$.

$\boldsymbol{\tilde{D}}$**:** denotes Price-dependent Demand and is defined as $\tilde{D}(p, \tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0,\ b > 0$).

$\boldsymbol{\alpha}$**:** defined as $\alpha = q - (a - bp)$, which is a transformation of q.

$\boldsymbol{\alpha^\star_u(p)}$**:** The optimal choice of α for a risk-averse decision maker (newsvendor) under utility $u(w)$.

$\boldsymbol{\alpha^\star_{rn}(p)}$**:** The optimal choice of α for a risk-neutral decision maker (newsvendor) under utility $u(w)$.

$\boldsymbol{\alpha^\star_{ca}(p)}$**:** The optimal choice of α for a CARA decision maker (newsvendor).

$\boldsymbol{\alpha^\star_{cr}(p)}$**:** The optimal choice of α for a CRRA decision maker (newsvendor) under utility $u(w) = ln(w)$.

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1 Introduction

The classic newsvendor model considers a type of problem that many decision makers (newsvendors) encounter in the business world. Facing uncertain demands for limited-useful-life products (such as mobile phones, fashionable goods etc.), a decision maker (newsvendor) needs to decide how many units of these goods to order for a single selling period. Intuitively, if she/he orders too many (overage), this may cause unnecessary inventory cost. The unsold items have to be salvaged at low prices (generally less than order prices), or just to be thrown away (the salvaged prices are equal to 0). Thus, the cost will be too high. Whereas, if the decision maker (newsvendor) orders too few (underage), it will miss opportunities for additional profits because some customers have no chance to buy the goods. The optimal solution to this problem is characterized by a balance between the expected costs of overage and underage.

This classic inventory problem includes four assumptions: the uncertain demand is price-independent, the only decision for the decision maker (newsvendor) is the order quantity, the decision maker (newsvendor) is allowed to place the order only once in the whole selling season, and the decision made is based on the expected monetary value criterion. However, by considering the following issues, this thesis extends the classic newsvendor problem to include more satisfactory aspects.

First, a price-dependent demand might be considered. It is common sense that the selling price affects a consumer's willingness to buy. The higher the selling price, the lower the willingness to buy. On the other hand, when the selling price is set lower, the willingness increases. Therefore, the sellers generally face price-dependent demands for their goods. The higher the selling price of a product is, the lower the demand is, and vice-versa. That is the reason why discount strategies are often used when sellers promote their goods. The phenomenon that demand depends on selling price has been discussed in many economics books. Economists use the Demand Curve to describe this relationship (Such as Stiglitz and Walsh, 2002). This thesis investigates how selling price affects order quantity if a price-dependent demand is considered in the newsvendor problem.

Second, in addition to determining the order quantity of materials, the decision maker (newsvendor) might consider the selling price. In the situation where a delay in production exists, the producer faces uncertain selling prices. At the time of production, the producer knows with certainty the costs that must be incurred and the production that will result. However, what is unknown is the selling price at which the product can be sold. As mentioned above, selling price might affect demand, thus affecting profit. Therefore, selling price must be set carefully to result in the best income of the producer. This thesis considers jointly the order quantity and the selling price.

Third, a second order during the selling season might be considered. In some cases, when the producers launch a new product, such as a new type of mobile phone, they may have no clear idea about the demand at the beginning of production. Initially, the producers might produce a certain amount of the product according to their estimation. After a certain period, the producers might adjust their production according to the data collected from the market. In this situation, the producers have clearer information about the demand. If the demand exceeds the initial production, the producers might want to produce more. In contrast, the producers might want to produce less. For example, one of the telecommunication manufacturers in mainland China, Z.X. Telecommunication Equipment Co., Ltd., always takes such a strategy when it launches a new mobile phone. When the demand exceeds the initial order in the selling season, not only may the producers have to replenish their materials for production, but also the distributors may have to replenish the goods to sell. Generally, the initial order must be sent before the level of demand is revealed. If the replenishment is allowed, both the producers and the distributors may have more accurate demand information when the replenishment is needed. Thus, some costs may be eliminated. This thesis examines how selling price affects order quantity in a two-order scenario.

Finally, decision makers might use the expected utility criterion instead of the expected monetary value criterion. According to the expected-utility hypothesis, which is the standard paradigm for analyzing economic behavior under uncertainty, this thesis aims to examine the newsvendor problem for a different point of view and to see if different results exist. Some interesting results are listed in later sections of this thesis.

If this thesis can address these issues by considering a situation with a price-dependent demand, two decision variables (order quantity and selling price), and allowing replenishment when necessary, then the classic newsvendor model can be refined. The following extension model of the classic newsvendor problem is proposed. The newsvendor, who has initial wealth of w_0, orders product at unit cost c. The newsvendor sells the product at unit price p. All unsold product can be salvaged for unit price $s < c$. And the newsvendor is allowed to replenish the product if demand exceeds the initial order quantity, but at a higher unit price $\hat{c} > c$. A natural assumption is that $0 \leq s < c < \hat{c} \leq p$. Demand is random price-dependent, and is defined as $\tilde{D}(p, \tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0$, $b > 0$), where $a - bp$ is a decreasing function of p that captures the dependency between demand and price, $\tilde{\epsilon}$ is a random variable that shows randomness in demand and is price-independent. The newsvendor considers jointly order quantity and selling price to maximize the expected utility of profit.

In the proposed setting, the newsvendor is allowed to place a second order when the demand exceeds the initial order. Although the higher second order cost may raise the cost for the newsvendor, we can see if $p - \hat{c}$ is greater than $\hat{c} - c$, the newsvendor would gain positive profit, thus she/he might want to replenish when necessary.

The key contribution lies in the fact that this thesis extends the newsvendor model closer to real life than before. Thus the results may have more significance in the real business world. As demonstrated in the literature review (next section), this problem and the proposed extension have not been discussed in previous studies.

In the third section, the theoretical framework will be introduced. The extension of the classic newsvendor model is proposed in section 4. Section 5 will analyze the effect of change in selling price on the optimal order quantity. Finally, the conclusion is drawn in section 6.

2 Literature Review

The newsvendor problem has a rich history that goes back to the economist Edgeworth (1888), who applied a variant to a bank cash-flow problem. However, it was not until the 1950s that this problem, like many other OR/MS models seeded by the war effort, became a topic of serious and extensive academic study. This simple problem, with its intuitively appealing optimal solution, is a crucial building block of many stochastic inventory problems.

Porteus (1990) provides an excellent review of the newsvendor problem. Typically, the focus of this extensive literature is on operational efficiency to minimize expected cost. Demand or market parameters often are taken to be exogenous. Whitin (1955) was the first to formulate a newsvendor model with price effects. In his model, selling price and order quantity are set simultaneously. Whitin adapted the newsvendor model to include a probability distribution of demand that depends on the unit selling price, where price is a decision variable rather than an external parameter. He assumed that the expected demand is a function of price and using incremental analysis, derived the necessary optimality condition. Whitin then provided closed-form expressions for the optimal price, which is used to find the optimal order quantity for a demand with a uniform distribution. He established a sequential procedure for determining the optimal order quantity as a function of selling price first and then the corresponding optimal selling price. Mills (1959; 1962) refined the formulation by explicitly specifying mean demand as a function of the selling price. He assumed demand to be a random variable with an expected value that is decreasing in price and with constant variance. Mills derived the necessary optimality conditions and provided further analysis for the case of demand with uniform distribution. Unlike the version of the newsvendor problem in which selling price is exogenous, this more strategic variant has received limited attention since the 1950s. Khouja (1999) builds a taxonomy of the newsvendor problem literature and delineates the contribution of the different extensions of the newsvendor problem. He also suggests some future directions for research in the newsvendor problem.

Petruzzi and Dada (1999) apply the newsvendor framework to analyze a firm that sets a selling price and order quantity prior to facing random demand in a single period. In their paper they examine an extension of the newsvendor model (PD model) in which order quantity and selling price are set simultaneously. Petruzzi and Dada validate Zabel's (1970) method of first optimizing selling price for a given order quantity, and then searching over the resulting optimal trajectory to maximize the expected profit. Through this method, they find the optimal order quantity and the optimal selling price for a risk-neutral newsvendor. However, they just consider the situation where the demand exceeds the initial order, the newsvendor would be asked to pay the penalty, instead of allowing replenishment when shortage comes. Moreover, they just consider the case for a risk-neutral newsvendor, but not for a risk-averse newsvendor.

Lau and Lau (1988) introduced a model in which the newsvendor has the option of decreasing price in order to increase demand. The authors analyzed two cases for demand. Case A: A normally distributed demand with an expected value which decreases linearly with unit selling price. Case B: Demand distribution is constructed using a combination of statistical data analysis and experts' subjective estimates. For case A, the authors showed that the expected profit is unimodal and thus the golden section method can be used for maximization. For case B, there is no guarantee that the expected profit is unimodal. Thus, Lau and Lau developed a search procedure for identifying local maximums. They also solved the problem under the objective of maximizing the probability of achieving a target profit and considered both zero and positive shortage cost cases. For zero shortage cost and demand given by case A, the authors derived closed-form solutions for the optimal order quantity and optimal price. For zero shortage cost and demand given by case B, the authors developed a procedure for computing the probability of achieving a target profit and used a search procedure for finding a good solution. For positive shortage cost and demand given by case A or B, the probability of achieving a target profit may not be unimodal. Lau and Lau developed procedures for computing the probability of achieving a target profit and identifying a good solution. However, similar to the work of Petruzzi and Dada (1999), the authors do not consider a situation where replenishment is allowed when demand exceeds the initial order.

Also, they just consider the case for a risk-neutral newsvendor, but not for a risk-averse newsvendor.

Although much has been written about the newsvendor problem, relatively little has been done about the risk-averse newsvendor. It seems to be well known that risk aversion leads to a reduced initial order quantity. Unfortunately, only a scattering of other results can be found, and usually within very specific models. Horowitz (1970) for example, has examined a risk-averse newsvendor for specific utility functions.

An early paper looking at more general risk-averse preferences is by Baron (1973). Baron examines the comparative-static effects of changes in newsvendor risk aversion and changes in the salvage value of unsold newspaper on the optimal order quantity. Baron does not consider the newsvendor problem intrinsically, but his short section on piecewise-linear payoff functions can be interpreted to yield the above-mentioned analysis. Britney and Winkler (1974) and Lau (1980) also examine the optimal order for a risk-averse newsvendor, but for particular utility functions and in conjunction with particular demand distributions. Bouakiz and Sobel (1992) also examine the optimal stock policy for an exponential utility function.

Eeckhoudt, Gollier and Schlesinger (1995) consider the newsvendor problem under various risk-averse preferences. In their model (EGS model), the newsvendor (i.e. decision maker) is allowed to obtain additional newspapers if demand exceeds his original order quantity at a higher cost. Based on this model they examine the effects of risk and risk aversion in the newsvendor problem. Comparative-static effects of changes in the various price and cost parameters are determined and related to the newsvendor's risk aversion. The addition of a random background wealth and of an increase in the riskiness of newspaper demand are also examined. However, the analysis in this paper shows that many of the comparative effects generally are ambiguous, only some fairly simple restrictions on preferences and/or risk increases are shown to lead to qualitatively deterministic results. And, their model considers a price-independent demand only. If this thesis can address this gap by considering a situation with a price-dependent demand, the EGS model can be refined.

The purpose of this study is to extend the newsvendor model by combining the PD model and the EGS model, to find some deterministic results, which are different from existing results.

3 Theoretical Framework

This thesis investigates an extension of the classic newsvendor problem by using the theory of Risk Aversion. Before going ahead, the classic Newsvendor Problem and the Theory of Risk Aversion are briefly introduced in this section.

3.1 The Newsvendor Problem

Let us illustrate this classic inventory problem with a newsvendor who is selling newspapers every morning. Suppose the newsvendor sells newspapers at a unit price of \$1.00, after picking up the newspapers that morning at 70 cents for each. There is uncertainty in how many people will buy a newspaper. Assume that if the newsvendor has any leftover newspaper, she/he sells them at a discount, at 20 cents each. This leads to a cost of overage of 50 cents each. If the newsvendor fails to have enough newspapers one morning, a customer who expects to buy one that day, but couldn't because the newsvendor ran out, may decide to take a different route to office from then on, to insure that she/he is able to buy the newspaper she/he wants the second day. Thus, the newsvendor has lost not only that day's sale, but has also lost some potential future profit. In other words, when the newsvendor under-buy on the number of newspapers, she/he may lose more than the immediate profit. Thus, the newsvendor faces the problem that how many newspapers she/he should order every morning to maximize the profit. This is referred as the classic newsvendor problem.

Intuitively, this classic single-period inventory problem considers the following dilemma. The decision maker (newsvendor) facing random demand for a perishable product, such as newspaper, decides how many units to order for a single selling period. If the decision maker (newsvendor) orders too many (overage), the cost will be unnecessarily too high; whereas, if the decision maker (newsvendor) orders too few (underage), it will miss opportunities for additional profits. The optimal solution to this problem is characterized by a balance between the expected costs of overage and underage.

The newsvendor problem applies in a wide array of settings. For example, fashion apparel retailers often must submit orders well in advance of a selling season without any opportunity for submitting any other orders during the season. A manufacturer might need to choose its capacity (i.e., its order quantity) before the launch of a new product, knowing that the new product will become obsolete quickly (e.g., computers or mobile phones). Special promotions usually present a similar problem: order too little and the retailer faces irate customers, but order too much and the retailer incurs additional inventory holding costs as it slowly sells the excess inventory. The newsvendor model also applies to individual choice problems, such as health care financing and insurance purchasing (Rosenfield, 1986; Anvari, 1987; Chung, 1990; Eeckhoudt, Gollier and Schlesinger, 1991).

3.2 The Theory of Risk-Aversion

This section introduces the Theory of Risk-Aversion. Before the introduction, some terms and notations are defined as follows.

$\boldsymbol{w}$**:** Wealth of a decision maker (newsvendor) which is a random variable over $R = (-\infty, +\infty)$. For simplicity, this thesis takes wealth to be a single commodity and disregards the difficulties of aggregation over many commodities. For most purposes, w is taken to be the money value of commodity holdings (including holdings of money itself) at market prices. There is no loss of generality under perfect competition as long as prices remain constant.

$\boldsymbol{w_0}$**:** The initial wealth of a decision maker (newsvendor).

***u(w)*:** The total utility of wealth w. This thesis assumes that the utility of wealth is a differentiable function, indeed, twice differentiable. Economists refer to the benefits of consumption as the utility that decision makers (newsvendors) obtain from the combination of goods they consume. A simply way to measure utility will suffice: We ask how much a decision maker (newsvendor) would be willing to pay to be in one situation rather than another. Willingness to pay is a useful measure of utility, which is often helpful when considering how a decision maker (newsvendor) allocates her/his profit along her/his budget constraint.

***u′(w)*:** Marginal utility of wealth w. We can always assume that wealth is desirable, thus, $u'(w) > 0$ in general. So, $u(w)$ is a strictly increasing function of w.

***u″(w)*:** The rate of change of marginal utility with respect to wealth w.

Concavity is a geometric term which describes a curve. In mathematics, a function $f(x)$ is said to be concave on an interval $[a, b]$, if, for all x, y in $[a, b]$, $f(\frac{x+y}{2}) \geq \frac{f(x)+f(y)}{2}$.

From the time of Bernoulli (1738), risk aversion has been associated with concavity of utility functions. After the axiomatization of the expected utility hypothesis by John von Neumann and Oskar Morgenstern (1944), economists immediately began to see the potential applications of expected utility to economic issues like portfolio choice, insurance, etc. It was not until Pratt (1964) and Arrow (1965) that it was recognized that the Arrow-Pratt measures of absolute and relative risk aversion are excellent measures of the strength of risk aversion. Subsequently, the Arrow-Pratt measures of absolute and relative risk aversion have often demonstrated their usefulness in a wide range of both theoretical and empirical studies of behavior under uncertainty.

3.2.1 Risk Aversion, Neutrality and Loving

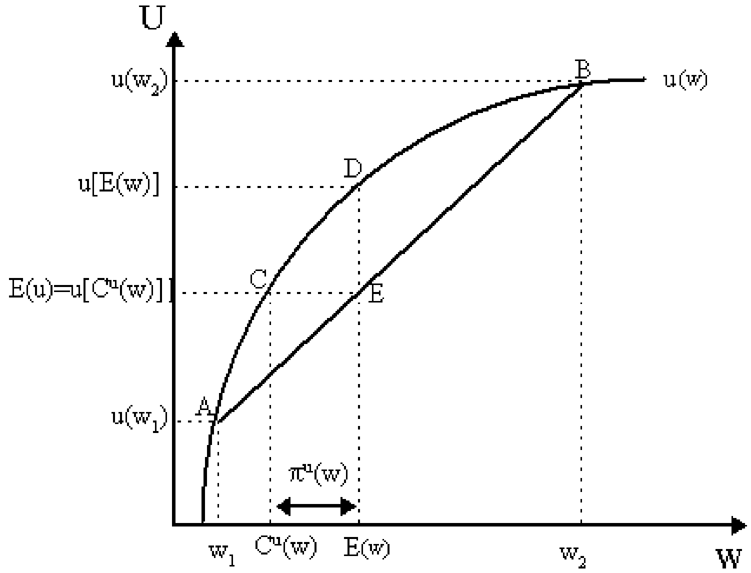

Figure 1: Risk-Aversion and Certainty Equivalence

Suppose w can take on two values, $\{w_1, w_2\}$, and let p_1 be the probability that w_1 happens and $(1 - p_1)$ the probability that w_2 happens. Consequently, expected outcome, or $E(w) = p_1w_1 + (1 - p_1)w_2$ which is shown in Figure 1 on the horizontal axis as the convex combination of w_1 and w_2. Suppose $u(w)$ is the utility function depicted in Figure 1 as concave. Thus, expected utility $E(u) = p_1u(w_1) + (1 - p_1)u(w_2)$, as shown in Figure 1 by point E on the chord connecting A = $\{w_1, u(w_1)\}$ and B = $\{w_2, u(w_2)\}$. The position of E on the chord depends, of course, on the probabilities p_1 and $(1 - p_1)$.

Notice by comparing points D and E in Figure 1 that the concavity of the utility function implies that the utility of expected income, $u[E(w)]$ is greater than expected utility $E(u)$, i.e. $u[p_1 w_1 + (1 - p_1) w_2] > p_1 u(w_1) + (1 - p_1) u(w_2)$. This represents the utility-decreasing aspects of pure risk-bearing.

As is obvious from Figure 1, we introduce $C^u(w)$ with certainty, which is equal to the expected utility of the random prospect, i.e. $u(C^u(z)) = E(u)$. In this term, the superscript "u" of C reminds us that the value of the certainty-equivalent lottery depends on the form of the utility function. However, notice that the income $C^u(w)$ is less than the expected income, i.e $C^u(w) < E(w)$. Yet we know that a decision maker (newsvendor) would be indifferent between receiving $C^u(w)$ with certainty and $E(w)$ with uncertainty. Thus, we can define the risk-premium below.

Risk-premium: is denoted by $\pi^u(w) = E(w) - C^u(w)$. This difference is equal to the maximum amount of income that an agent is willing to forego in order to obtain an allocation without risk (Pratt, 1964).

Turning to generalities, as described in Pratt's paper (1964), if a decision maker (newsvendor), who has initial wealth w_0, faces risk z, the risk-premium can be presented as follows under utility function $u(w)$.

$$\pi^u(w, z) = w_0 + E(z) - u^{-1}[Eu(w + z)].$$

We have the following definitions:

Risk-Aversion: A decision maker (newsvendor) is "risk-averse" if risk-premium $\pi^u(w, z) > 0$. Risk-Aversion, intuitively, implies that when facing choices with comparable returns, decision makers (newsvendors) tend to choose the less-risky alternative, a construction we owe largely to Milton Friedman and Leonard J. Savage (1948). Obviously, the greater $\pi^u(w, z)$ is, the more Risk-Aversion is.

Risk-Neutrality: A decision maker (newsvendor) is "risk-neutral" if risk-premium $\pi^u(w, z) = 0$ for all random variables.

Risk-Loving: A decision maker (newsvendor) is a "risk-lover" if risk-premium $\pi^u(w, z) < 0$ for all random variables. Similarly, the less $\pi^u(w, z)$ is, the more Risk-loving the decision maker is.

Now, we have appealed to the ideas of concave, linear and convex utility functions to represent risk-aversion, risk-neutrality and risk-loving. We might be impressed in this aspect by the following theorem.

Theorem 3.1 *Let $u : R \mapsto R$ be an utility function representing preference $\succeq_u$ for all random variables and u is monotonically increasing. Then:*

(i) u is concave if and only if $\succeq_u$ displays risk-aversion.

(ii) u is convex if and only if $\succeq_u$ display risk-proclivity.

(iii) u is linear if and only if $\succeq_u$ is risk-neutral.

Proof. See Appendix A. □

3.2.2 Arrow-Pratt Measures of Risk-Aversion

How does one measure the "degree" of risk aversion of a decision maker (newsvendor)? The first instinct may be to appeal immediately to the concavity of the utility functions. However, as utility functions are not unique, second derivatives of utility functions are not unique, and thus will not serve to compare the degrees of risk aversion in any pair of utility functions. The risk premium is expressed in terms of "wealth", and might be a better magnitude. If these can be connected to the "concavity" of utility curves - adjusted to control for non-uniqueness - so much the better. The most famous measures of risk-aversion, named the Arrow-Pratt Measures of Absolute Risk-Aversion, were introduced by John W. Pratt (1964) and Kenneth J. Arrow (1965). We can note the measure of risk-aversion other than Risk-Premium as follows:

$\boldsymbol{r_u(w) = -\frac{u''(w)}{u'(w)}}$**:** For a decision maker (newsvendor), if u is monotonically increasing and strictly concave for the risk-averse decision maker (newsvendor), then $r_u(w) > 0$. The greater $r_u(w)$ is, the more Risk-Aversion is. $r_u(w) = 0$ for the risk-neutral decision maker (newsvendor) with a linear utility function and $r_u(w) < 0$ for the risk-loving decision maker (newsvendor) with a strictly convex utility function. The less $r_u(w)$

is, the more Risk-loving the decision maker is.

Consider the following theorem due to J.W. Pratt (1964).

Theorem 3.2 (Pratt) *Let u, v be two utility functions over wealth which are continuous, monotonically increasing and twice-differentiable. Then the following are equivalent:*

(i) $r_u(x) \geq r_v(x)$ *for every* $x \in R$.

(ii) $u(x) = T(v(x))$ *where* T *is a concave function.*

(iii) $\pi^u(w) \geq \pi^v(w)$ *for all random variables.*

Proof. See Appendix B. □

As we can see, the Arrow-Pratt measure of absolute risk aversion cannot capture a situation. As in Figure 2, the decision maker (newsvendor) switches from risk-aversion to risk-loving and then back to risk-aversion. Thus, an alternative would be to weigh the measure of risk aversion by the level of wealth, w. In this case another measure, named the Arrow-Pratt Measures of Relative Risk-Aversion, is introduced as follows:

$$\boldsymbol{R_u(w) = wr_u(w) = -\frac{wu''(w)}{u'(w)}:}$$

At the same time, the following terms are also defined:

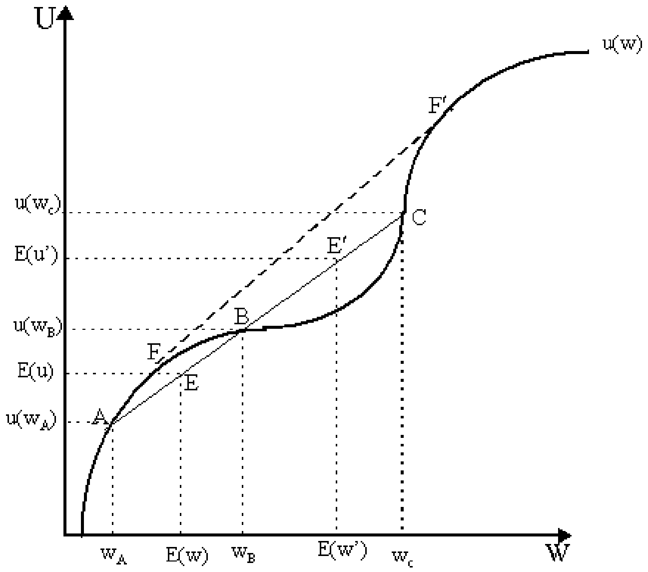

Figure 2: Friedman-Savage Double-Inflection Utility Function (Friedman and Savage, 1948)

Decreasing Absolute Risk Aversion (DARA) if $r'_u(w) < 0$. DARA means that the decision maker (newsvendor) has less risk aversion when it becomes wealthier. If absolute risk aversion decreased with wealth, it would follow that the willingness to engage in small bets of fixed size increases with wealth. Such a decision maker (newsvendor) might invest a larger dollar amount in risky assets as her/his wealth grows larger.

Constant Absolute Risk Aversion (CARA) if $r'_u(w) = 0$. CARA means that the decision maker (newsvendor) has no change in risk-aversion when her/his wealth changes.

Increasing Absolute Risk Aversion (IARA) if $r'_u(w) > 0$. IARA means that the decision maker (newsvendor) has more risk-aversion when she/he becomes wealthier. In contrast, the decision maker (newsvendor) has less risk-aversion when she/he becomes poorer. If absolute risk aversion increased with wealth, it would follow that when a decision maker (newsvendor) became wealthier, she/he would actually decrease the amount of risky assets held. IARA is not supported by everyday observation, so, this thesis will not discuss the case where the newsvendor has IARA.

Decreasing Relative Risk Aversion (DRRA) if $R'_u(w) < 0$. If a decision maker (newsvendor) exhibits DRRA, it will invest not only a larger dollar amount, but also a larger fraction of its wealth in risky assets as its wealth grows larger. As the reason Arrow (1965) states, this preference is seldom displayed in the real world. DRRA is not supported by everyday observation, so, this thesis will not discuss the case where the newsvendor has DRRA.

Constant Relative Risk Aversion (CRRA) if $R'_u(w) = 0$. CRRA means that a decision maker (newsvendor) has no change in relative risk-aversion when its wealth changes.

Increasing Relative Risk Aversion (IRRA) if $R'_u(w) > 0$. IRRA means that a decision maker (newsvendor) has more relative risk-aversion when it becomes wealthier. In contrast, it has less relative risk-aversion when it becomes poorer. IRRA also means that if both wealth and the size of the risk are increased in the same proportion, the willingness to accept the risk should decrease.

Let us now consider the following:

Theorem 3.3 *The following three conditions are equivalent:*

(i) $u(w)$ *displays decreasing absolute risk aversion (DARA)*

(ii) $u(w) = T_a[u(w+a)]$ *is a concave transformation in the level of wealth, where* $T'_a > 0$ *and* $T''_a < 0$ *for all* w *and all given* $a > 0$.

(iii) $r_u(w + a) \leq r_u(w)$ *for all* w *and* $a > 0$.

Proof. See Appendix C. □

The similar theorems can be applied to CARA and IARA. The proof for them is analogous and this thesis omit it.

3.2.3 Summary and Examples

According to the discussion above, we summary the Theory of Risk-Aversion in Table 1.

Table 1: Summary of the Theory of Risk-Aversion

$r_u > 0$						$r_u = 0$	$r_u < 0$
Risk-Aversion						Risk-Neutrality	Risk-Loving
r_u			R_u				
$r'_u > 0$	$r'_u = 0$	$r'_u < 0$	$R'_u > 0$	$R'_u = 0$	$R'_u < 0$		
IARA	CARA	DARA	IRRA	CRRA	DRRA		

Let us now consider the following examples to verify the Theory of Risk-Aversion so as to enhance the understanding of it.

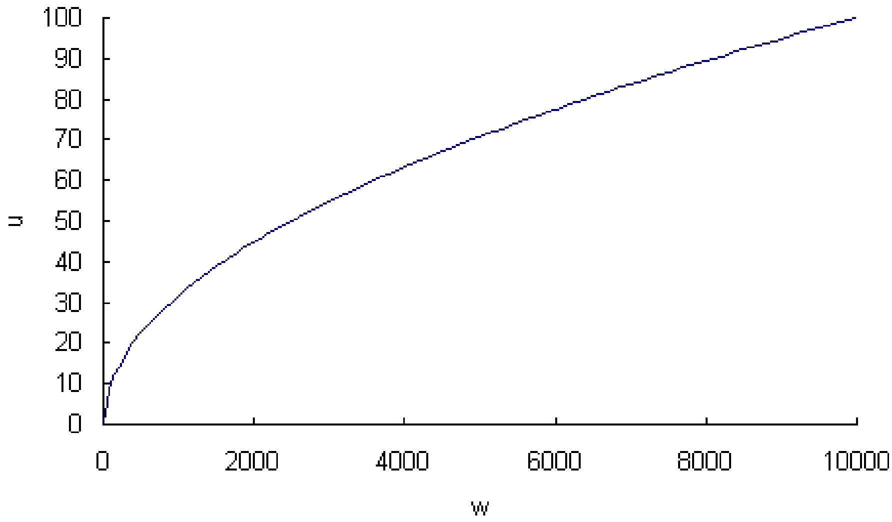

Figure 3: Utility Function $u(w) = w^{1-r}$ $(r = 0.5)$

Example 1: Notice that the famous utility function used in the macroeconomic consumption theory, $u(w) = \dfrac{w^{1-R}}{1-R}$, where $R \in (0,1)$, as described in Figure 3. Note that

$$
\begin{aligned}
u'(w) &= w^{-R}, \\
u''(w) &= -Rw^{-R-1},
\end{aligned}
$$

thus $r_u(w) = \dfrac{R}{w} > 0$, so this decision maker (newsvendor) with such a utility has absolute risk-aversion. Note that for a fixed R, with the increment of w, $r_u(w)$ declines. This means if the decision maker (newsvendor) becomes wealthier, it will have less risk-aversion. This also can be seen from

$r_u'(w) = -\dfrac{R}{w^2} < 0$ as $R \in (0,1)$, this utility function displays decreasing absolute risk aversion (DARA). Meanwhile,

$R_u(w) = R > 0$, thus $R_u'(w) = 0$, so this utility function also displays constant relative risk-aversion (CRRA). R is the coefficient of relative risk-aversion.

Lemma 3.4 *If the utility function $u(w)$ has CRRA, it has the form*

$$u(w) = \begin{cases} \frac{c_1}{1-R} w^{1-R}, & 0 < R < 1,\ 1 < R, \\ c_2 ln(w), & R = 1, \end{cases}$$

where $c_1, c_2 > 0$. R is the coefficient of CRRA.

Proof. See Appendix D. □

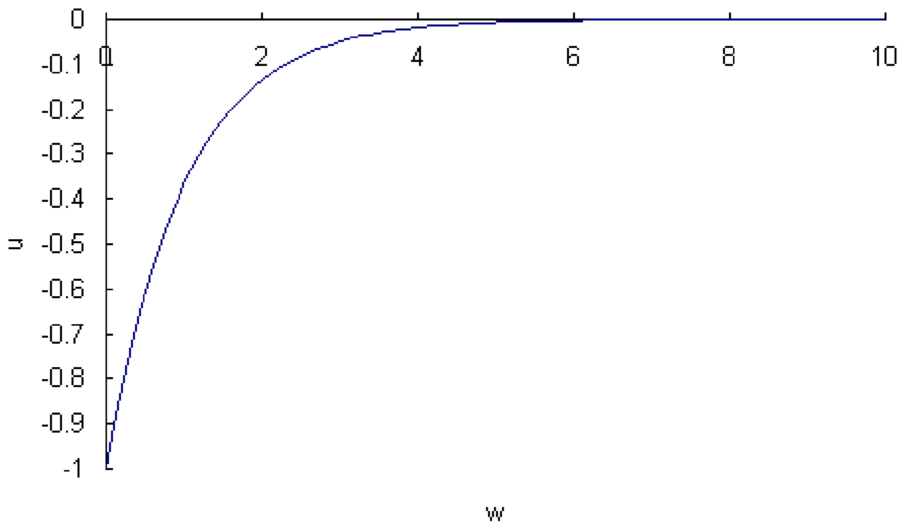

Figure 4: Utility Function $u(w) = -e^{-rw}$ $(r = 1)$

Example 2: Suppose a utility function, $u(w) = -e^{-rw}$, where $r > 0$, as described in Figure 4. Note that

$$\begin{aligned} u'(w) &= re^{-rw}, \\ u''(w) &= -r^2 e^{-rw}. \end{aligned}$$

Thus, $r_u = r > 0$, so this decision maker (newsvendor) has absolute risk-aversion. Note that r is the coefficient of absolute risk-aversion, the greater r is, the more risk-aversion is.

$r'_u = 0$, thus this utility function displays constant absolute risk-aversion (CARA).

$R_u = rw$, $R'_u = r > 0$, so this utility function also displays increasing relative risk aversion (IRRA).

Lemma 3.5 *If the utility function $u(w)$ has CARA, it has the form*

$$u(w) = -c_1 e^{-rw},$$

where $r > 0$, $c_1 > 0$. r is the coefficient of CARA.

Proof. See Appendix E. □

Example 3: Quadratic utility function $u(w) = w - rw^2$, where $r > 0$, which is described in Figure 5. Notice that

$$\begin{aligned} u'(w) &= 1 - 2rw, \\ u''(w) &= -2r. \end{aligned}$$

Thus, $r_u(w) = \dfrac{2r}{1 - 2rw}$.

For being absolute risk-averse, i.e. $r_u(w) > 0$, we need $1 > 2rw$, thus it only

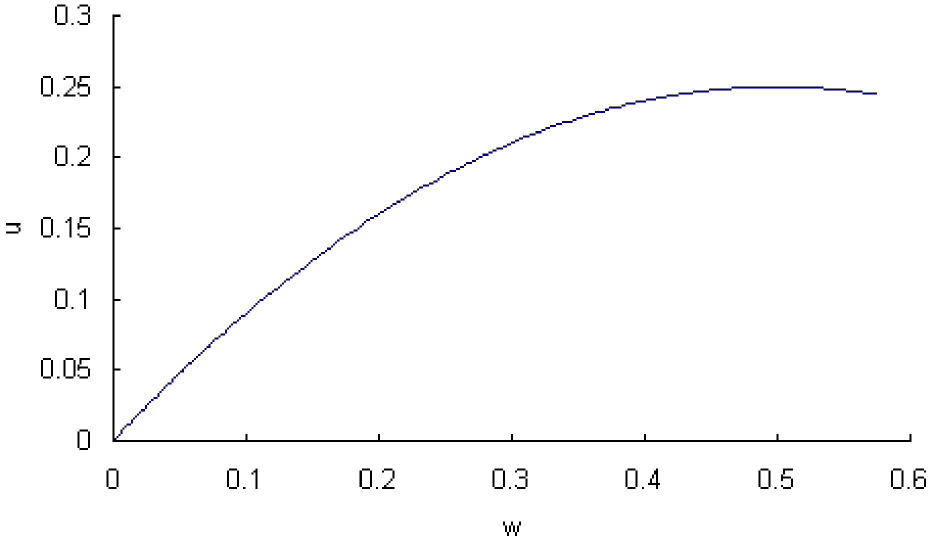

Figure 5: Utility Function $u(w) = w - rw^2$ $(r = 1)$

applies for a limited range of $w < \dfrac{1}{2r}$.

$r_u'(w) = \dfrac{4r^2}{(1-2rw)^2} > 0$, where $w < \dfrac{1}{2r}$. So, this utility function displays increasing absolute risk-aversion (IARA) when $w < \dfrac{1}{2r}$.

$R_u(w) = \dfrac{2rw}{1-2rw}$, $R_u'(w) = \dfrac{2r}{(1-2rw)^2} > 0$, for $w < \dfrac{1}{2r}$. So this utility function also displays increasing relative risk aversion (IRRA).

Note that, where $w > \dfrac{1}{2r}$, marginal utility $u'(w) = 1 - 2rw$ is negative - i.e. beyond this level of wealth, utility declines, and it would be better to throw some away. Namely, that the willingness to take risks decreases as wealth increases, i.e. that richer people are more unwilling to take risks. John Hicks (1962) and Kenneth Arrow (1965) have assaulted the quadratic utility function on this basis.

A summary of the examples above is shown in Table 2.

Table 2: Summary of the Examples

$\mathbf{u}$	$\mathbf{r_u}$	$\mathbf{R_u}$	$\mathbf{r'_u}$	$\mathbf{R'_u}$	**Property**
$\frac{w^{1-R}}{1-R}$, $(0 < R < 1)$	$\frac{R}{w}$	R	$-\frac{R}{w^2}$	0	DARA & CRRA
$-e^{-rw}$, $(r > 0)$	r	rw	0	r	CARA & IRRA
$w - rw^2$, $\left(r > 0 \text{ \& } w < \frac{1}{2r}\right)$	$\frac{2r}{1-2rw}$	$\frac{2rw}{1-2rw}$	$\frac{4r^2}{(1-2rw)^2}$	$\frac{2r}{(1-2rw)^2}$	IARA & IRRA

4 The RAPD model

This section investigates an extension of the classic newsvendor problem, in which a Risk-Averse newsvendor who faces a Price-Dependent demand (RAPD) is considered. Thus, this thesis names the extension as the RAPD model hereafter.

4.1 Notations

This thesis defines the following notations.

***q*:** The initial order quantity at the beginning of a single period.

***c*:** The purchasing cost of unit product at the beginning of a single period.

$\hat{c}$: The purchasing cost of unit product at late time if the demand exceeds the initial order quantity q. Usually $\hat{c} > c$.

***p*:** The selling price of unit product. Usually $p > \hat{c}$. We would not consider the case where $p = \hat{c}$ in this thesis, because this case where $p = \hat{c}$ means no re-order is possible equivalently.

***s*:** The unit salvage price for unsold products during the selling season. Usually $0 < s < c$.

Price-dependent Demand is defined as $\tilde{D}(p, \tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0$, $b > 0$) (Mills, 1959), where $a - bp$ is a decreasing function of p that captures the dependency between demand and selling price. Generally, an increasing selling price p leads to a decreasing demand. $\tilde{\epsilon}$ is a random variable defined in the range $[-A, B]$ ($A \geq 0$, $B \geq 0$) and price-independent. Let $F(\cdot)$ be the cumulative distribution function of $\tilde{\epsilon}$. Thus, $F(-A) = 0$ and $F(B) = 1$. In order to assure that positive demand is possible for some range of selling price p, this thesis requires $A \leq a - bp$.

4.2 The Description of the RAPD Model

In the PD model (Petruzzi and Dada, 1999), the authors consider a price-setting firm that faces a random price-dependent demand and determines jointly an initial order quantity and selling price to maximize the expected profit. After placing the initial order, the firm cannot re-order if demand exceeds the initial order and a penalty cost will be charge. They also assume that the firm (i.e. newsvendor) is risk-neutral and do not analyze this model with other risk preferences. For details please refer to Table 3 for a summary of the PD model.

In the EGS model (Eeckhoudt, Gollier and Schlesinger 1995), the authors illustrate that a newsvendor, with initial wealth w_0, buys newspapers at a unit cost c and sells them at unit price p. All unsold newspapers are returned to the publisher at unit salvage price s. The newsvendor is allowed to obtain additional newspapers when needed at a higher unit cost $\hat{c}$. The newsvendor faces a price-independent demand. However, in this model, only a few comparative effects can be determined with restrictions. For example, for an increase in the selling price of a newspaper, the comparative-static analysis is complex. If the newsvendor is CARA, increasing selling price will lead to a decrease optimal order quantity. Whereas, if the newsvendor prefers to risk aversion or DARA, the results are still indeterminate. For details please refer to Table 3 for a summary of the EGS Model.

Based on the literature and reasoning, the new model is raised. Let us consider such an extension newsvendor model, the RAPD model, which combines the elements in the PD model and the EGS Model. That is to say, a decision maker (newsvendor), with initial wealth w_0, orders products at unit cost c. The decision maker (newsvendor) sells the products at unit price p. All unsold products can be salvaged for $s < c$. And it is allowed to replenish the products if demand exceeds the initial order quantity, but at a higher price, $\hat{c}$. A natural assumption is that $0 \leq s < c < \hat{c} \leq p$. Demand is random price-dependent. Randomness in demand is price-independent. The decision maker (newsvendor) determines jointly an order quantity and selling price to maximize expected utility of profit. One interpretation of the model is that a single product, such as a type of sunglass, is quite popular this summer. The sunglasses have sold out so fast that the retailer has to re-order the sunglasses. Other retailers also want to re-order this type of sunglass during the season. This leads to a higher second order cost for this type of sunglass before the end of this selling season. The retailers have to decide their initial orders of the sunglass carefully. For details please refer to Table 3 for a summary of the RAPD model.

In the RAPD model, this thesis considers the situation where the newsvendor faces price-dependent demand. Thus, the newsvendor will decide more than the initial order quantity, she/he may also consider selling price at the same time. That is to say, the model has two decision variables, order quantity and selling price. This model also considers the situation allowing re-order when the initial order is less than the demand. When the newsvendor places the second order, she/he might have more accurate information on the demand than she/he has at the beginning of the selling season. This helps her/him to save cost. Based on such a model, this thesis investigates how the selling price will affect the optimal order quantity under variant risk preferences by using the Theory of Risk-Aversion.

A comparative summary among the RAPD model, the EGS Model and the PD model is showed in Table 3.

Table 3: Comparative Summary among the Models

Factors	PD Model	EGS Model	RAPD model
initial wealth	–	w_0	w_0
unit cost	c	c	c
unit re-order cost	–	$\hat{c}$	$\hat{c}$
unit salvage price	s	s	s
unit penalty cost	c_p	–	–
demand	$a - bp + \epsilon$, $(b > 0)$	$a + \epsilon$	$a - bp + \epsilon$, $(b > 0)$
distribution function of ϵ	$F(\cdot)$	$F(\cdot)$	$F(\cdot)$
density function of ϵ	$f(\cdot)$	$f(\cdot)$	$f(\cdot)$
utility function	–	u	u
unit selling price	p	p	p
initial order quantity	q	q	q

4.3 Basic Equations

Based on the RAPD model above, at the beginning of the selling season, q units are ordered at a cost of cq. If the demand during this period does not exceed q, then the revenue is $p\tilde{D}$ and each of the $(q - \tilde{D})$ leftovers are salvaged at the unit price s. If the demand exceeds q, then the revenue is $p\tilde{D}$, and each of the $(\tilde{D} - q)$ shortages are replenished at unit cost $\hat{c} > c$. The newsvendor, therefore, is endowed with the following profit function:

$$\Pi(p, q, \tilde{D}) \triangleq p\tilde{D} - cq + s\max(0, q - \tilde{D}) - \hat{c}\max(0, \tilde{D} - q),$$

or equivalently,

$$\Pi(p, q, \tilde{D}) = \begin{cases} \Pi_-(p, q, \tilde{D}) \triangleq (p - s)\tilde{D} - (c - s)q, & \tilde{D} \leq q, \\ \Pi_+(p, q, \tilde{D}) \triangleq (p - \hat{c})\tilde{D} + (\hat{c} - c)q, & \tilde{D} > q. \end{cases}$$

A convenient expression for this profit function is obtained by substituting

$\tilde{D}(p,\tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ and, consistent with Ernst (1970) and Thowsen (1975), defining $\alpha = q - (a - bp)$:

$$\Pi(p,\alpha,\tilde{\epsilon}) = \begin{cases} \Pi_-(p,\alpha,\tilde{\epsilon}) & \triangleq & (p-s)(a-bp+\tilde{\epsilon}) \\ & & -(c-s)(a-bp+\alpha), \tilde{\epsilon} \in [-A,\alpha], \\ \Pi_+(p,\alpha,\tilde{\epsilon}) & \triangleq & (p-\hat{c})(a-bp+\tilde{\epsilon}) \\ & & +(\hat{c}-c)(a-bp+\alpha), \tilde{\epsilon} \in (\alpha, B]. \end{cases}$$

This transformation of variables provides an alternative interpretation of the order decision: If the choice of α is greater than the realized value of $\tilde{\epsilon}$, then overage occurs. If the choice of α is less than the realized value of $\tilde{\epsilon}$, then underage occurs.

The newsvendor's objective is to maximize the expected utility of her/his final wealth $w_0 + \Pi$, where w_0 is the initial wealth, while Π is the profit. That is to say, the objective is to maximize $E[u(w_0 + \Pi)]$ for $u(w_0, \Pi) = u(w_0 + \Pi)$. The newsvendor is assumed to be risk-averse and this thesis thus defines $u(w_0, \Pi)$ to be increasing and concave. For analytical ease, $u(w_0, \Pi)$ also is assumed to be twice differentiable.

The expected utility of the final wealth $w_0 + \Pi$ for the newsvendor can be presented as follows:

$$\begin{aligned} & H_u(p,\alpha) \\ & \quad \triangleq \quad E[u(w_0, \Pi(p,\alpha,\epsilon))] \\ & \quad = \quad \int_{-A}^{\alpha} u(w_0, \Pi_-(p,\alpha,\epsilon))\, dF + \int_{\alpha}^{B} u(w_0, \Pi_+(p,\alpha,\epsilon))\, dF. \end{aligned} \tag{1}$$

To find the maximum of $H_u(p, \alpha)$, this thesis examines that the Hessian for $H_u(p, \alpha)$, $\nabla H_u(p, \alpha)$, is negative definite. Note that other than determining the optimal order quantity and the optimal selling price to maximize the expected utility, this thesis examines only the effect of change in selling price p on the optimal order quantity $q = a - bp + \alpha_u^\star(p)$, where $\alpha_u^\star(p)$ is to maximize $H_u(p, \alpha)$ for a given p. That is to say,

$$H_u(p, \alpha_u^\star(p)) = \max_\alpha H_u(p, \alpha).$$

Therefore, this thesis omits the calculation of $\nabla H_u(p, \alpha)$.

To find $\alpha_u^\star(p)$, this thesis can use the basic calculus method. $\alpha_u^\star(p)$ satisfies the first-order condition of $H_u(p, \alpha)$ is zero.

Specifically consider the first partial derivative of $H_u(p, \alpha)$ taken with respect to α (by Equation (1)),

$$\begin{aligned}
H_u'(p, \alpha) &\triangleq \frac{\partial H_u(p, \alpha)}{\partial \alpha} \\
&= -(c - s) \int_{-A}^{\alpha} u_\alpha'(w_0, \Pi_-(p, \alpha, \epsilon))\, dF + u(w_0, \Pi_-(p, \alpha, \alpha)) f(\alpha) \\
&\quad + (\hat{c} - c) \int_{\alpha}^{B} u_\alpha'(w_0, \Pi_+(p, \alpha, \epsilon))\, dF - u(w_0, \Pi_+(p, \alpha, \alpha)) f(\alpha) \\
&= -(c - s) \int_{-A}^{\alpha} u_\alpha'(w_0, \Pi_-(p, \alpha, \epsilon))\, dF \\
&\quad + (\hat{c} - c) \int_{\alpha}^{B} u_\alpha'(w_0, \Pi_+(p, \alpha, \epsilon))\, dF. \qquad (2)
\end{aligned}$$

For a fixed p, $\alpha_u^\star(p)$ is determined uniquely by the following equation.

$$H_u'(p, \alpha_u^\star(p)) = 0. \qquad (3)$$

The corresponding optimal order quantity is to order $q = a - bp + \alpha_u^\star(p)$ units to sell at the unit price p. This thesis investigates how $\alpha^\star(p)$ would change with selling price p, therefore, to see how the change in selling price p affects the optimal order quantity q.

The second partial derivative of $H_u(p, \alpha)$ is as follows.

$$
\begin{aligned}
H_u''(p,\alpha) &\triangleq \frac{\partial^2 H_u(p,\alpha)}{\partial \alpha^2} \\
&= -(c-s)\left[-(c-s)\int_{-A}^{\alpha} u_\alpha''(w_0, \Pi_-(p,\alpha,\epsilon))\,dF + u_\alpha'(w_0, \Pi(p,\alpha,\alpha))f(\alpha)\right] \\
&\quad +(\hat{c}-c)\left[(\hat{c}-c)\int_{\alpha}^{B} u_\alpha''(w_0, \Pi_+(p,\alpha,\epsilon))\,dF - u_\alpha'(w_0, \Pi(p,\alpha,\alpha))f(\alpha)\right] \\
&= (c-s)^2\int_{-A}^{\alpha} u_\alpha''(w_0, \Pi_-(p,\alpha,\epsilon))\,dF + (\hat{c}-c)^2\int_{\alpha}^{B} u_\alpha''(w_0, \Pi_+(p,\alpha,\epsilon))\,dF \\
&\quad +(s-\hat{c})u_\alpha'(w_0, \Pi(p,\alpha,\alpha))f(\alpha)
\end{aligned}
$$

Due to $u_\alpha'(w_0, \Pi) > 0$, $u_\alpha''(w_0, \Pi) < 0$ for risk aversion and $\hat{c} > s$,

$$H_u''(p,\alpha) < 0, \tag{4}$$

for a given p.

Notice from Inequality (4) that $H_u(p, \alpha)$ is concave in α. Thus, $\alpha_u^\star(p)$ maximizes $H_u(p, \alpha)$ for a given p.

5 The Effect of Change in Selling Price on the Optimal Order Quantity in the RAPD model

In this section, this thesis examines how selling price p affects the optimal order quantity q under different risk preferences in the RAPD model.

5.1 Risk-Neutral Newsvendor

Before going to the case where the newsvendor is risk-averse, we consider the case where the newsvendor is risk neutral. The result might be regarded as a reference point for further discussion.

For risk-neutral newsvendor, let $\alpha^\star_{rn}(p)$ be the optimal solution to maximize $H_u(p, \alpha)$ while p is fixed.

$$H_u(p, \alpha^\star_{rn}(p)) = \max_{\alpha} H_u(p, \alpha).$$

Thus, $\alpha^\star_{rn}(p)$ satisfies the following Equation by Equation (2).

$$\begin{aligned} & H'_u(p, \alpha^\star_{rn}(p)) \\ = \;& -(c-s)\int_{-A}^{\alpha^\star_{rn}(p)} u'_\alpha(w_0, \Pi_-(p, \alpha^\star_{rn}(p), \epsilon))\, dF \\ & +(\hat{c}-c)\int_{\alpha^\star_{rn}(p)}^{B} u'_\alpha(w_0, \Pi_+(p, \alpha^\star_{rn}(p), \epsilon))\, dF \\ = \;& 0. \end{aligned} \tag{5}$$

For risk neutrality, $u'_\alpha(w_0, \Pi) = c_1$ is a constant independent of α. We have the following by Equation (5).

$$
\begin{aligned}
&-(c-s)\int_{-A}^{\alpha^\star_{rn}(p)} c_1\, dF + (\hat{c}-c)\int_{\alpha^\star_{rn}(p)}^{B} c_1\, dF = 0 \\
&\quad\Rightarrow\quad c_1[-(c-s)[F(\alpha^\star_{rn}(p)) - F(-A)] + (\hat{c}-c)[F(B) - F(\alpha^\star_{rn}(p))]] = 0 \\
&\quad\Rightarrow\quad -(c-s)[F(\alpha^\star_{rn}(p)) - 0] + (\hat{c}-c)[1 - F(\alpha^\star_{rn}(p))] = 0 \\
&\quad\Rightarrow\quad F(\alpha^\star_{rn}(p)) = \frac{\hat{c}-c}{\hat{c}-s}.
\end{aligned}
$$

This condition states that the probability of excess capacity is a constant for all risk-neutral newsvendors. This probability is a simple function of cost parameters and salvage price. In Petruzzi and Dada's paper (1999), the authors yield the similar fraction rule for determining α,

$$
F(\alpha^\star_{rn}(p)) = \frac{p + c_p - c}{p + c_p - s}.
$$

However, this probability is price-dependent.

We note that $\alpha^\star_{rn}(p)$ for a risk-neutral newsvendor is independent of the selling price. From Equation (2) we know the net money cost of initially ordering one more newspaper when the demand happens to be less than the initial order quantity is $c-s$. If the newsvendor initially orders one more newspaper when the demand is greater than the initial order quantity, she/he will earn an additional benefit of $\hat{c}-c$. Both these two terms are independent of selling price p. Thus, $\alpha^\star_{rn}(p)$, is a balance between the cost and the benefit, and is independent of selling price p. However, we note that the optimal order quantity $q = a - bp + \alpha^\star_{rn}(p)$ is decreasing with p while $\alpha^\star_{rn}(p)$ is fixed because it is independent of p.

5.2 Risk-Averse Newsvendor

Let us now turn back to the case where a risk-averse newsvendor is considered in the RAPD model.

Lemma 5.1

$$u'_\alpha(w_0, \Pi(p, \alpha, \alpha)) \begin{cases} \leq u'_\alpha(w_0, \Pi_-(p, \alpha, \tilde{\epsilon})), & \tilde{\epsilon} \in [-A, \alpha], \\ > u'_\alpha(w_0, \Pi_+(p, \alpha, \tilde{\epsilon})), & \tilde{\epsilon} \in (\alpha, B]. \end{cases}$$

Proof. We have the following from the profit equation.

$$\Pi(p, \alpha, \alpha) \begin{cases} \geq \Pi_-(p, \alpha, \tilde{\epsilon}), & \tilde{\epsilon} \in [-A, \alpha], \\ < \Pi_+(p, \alpha, \tilde{\epsilon}), & \tilde{\epsilon} \in (\alpha, B]. \end{cases}$$

When the newsvendor is risk-averse, her/his utility function $u(w_0, \Pi)$ is strictly concave. $u'_\alpha(w_0, \Pi) > 0$ and $u''_\alpha(w_0, \Pi) < 0$. Thus, $u'_\alpha(w_0, \Pi)$ is a decreasing function of wealth. Therefore,

$$u'_\alpha(w_0, \Pi(p, \alpha, \alpha)) \leq u'_\alpha(w_0, \Pi_-(p, \alpha, \tilde{\epsilon})), \text{if } \tilde{\epsilon} \in [-A, \alpha],$$

$$u'_\alpha(w_0, \Pi(p, \alpha, \alpha)) > u'_\alpha(w_0, \Pi_+(p, \alpha, \tilde{\epsilon})), \text{if } \tilde{\epsilon} \in (\alpha, B].$$

This completes the proof. □

Theorem 5.2 *A risk-averse newsvendor orders less than a risk-neutral one does for a given selling price p.*

Proof. If we replace $u'_\alpha(w_0, \Pi_-(p, \alpha_u^\star(p), \tilde{\epsilon}))$ and $u'_\alpha(w_0, \Pi_+(p, \alpha_u^\star(p), \tilde{\epsilon}))$ with $u'_\alpha(w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p)))$ in Equation (3), according to Lemma 5.1, we have

$$\begin{aligned}
0 &= -(c-s)\int_{-A}^{\alpha_u^\star(p)} u'_\alpha(w_0, \Pi_-(p, \alpha_u^\star(p), \epsilon))\, dF \\
&\quad +(\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} u'_\alpha(w_0, \Pi_+(p, \alpha_u^\star(p), \epsilon))\, dF \\
&< -(c-s)\int_{-A}^{\alpha_u^\star(p)} u'_\alpha\left[w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p))\right]\, dF \\
&\quad +(\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} u'_\alpha\left[w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p))\right]\, dF \\
&\Rightarrow u'_\alpha\left[w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p))\right]\left[-(c-s)\int_{-A}^{\alpha_u^\star(p)} dF + (\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} dF\right] > 0 \\
&\Rightarrow -(c-s)\int_{-A}^{\alpha_u^\star(p)} dF + (\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} dF > 0 \\
&\Rightarrow F(\alpha_u^\star(p)) < \frac{\hat{c}-c}{\hat{c}-s} = F(\alpha_{rn}^\star(p)) \\
&\Rightarrow \alpha_u^\star(p) < \alpha_{rn}^\star(p).
\end{aligned}$$

Thus, for a given p, $a - bp + \alpha_u^\star(p)$, the order made by a risk-averse newsvendor, is less than $a - bp + \alpha_{rn}^\star(p)$, the order placed by a risk-neutral one. □

More generally, we have the following theorem.

Theorem 5.3 *For a given selling price p, the uniformly more risk-averse the newsvendor is, the less the newsvendor orders.*

Proof. Suppose a utility function $v(w_0, w)$ is uniformly more risk-averse than $u(w_0, w)$, we define the optimal order quantity as q_v, the optimal choice of α as $\alpha_v^\star(p)$ for a given p under $v(w_0, w)$. $q_v = a - bp + \alpha_v^\star(p)$.

Pratt (1964) indicates that an increase in risk aversion is equivalent to a concave transformation of the utility function. Thus, $\exists k[u(w_0, w)]$ such that $v(w_0, w) = k[u(w_0, w)]$, where $k'_u(w_0, w) > 0$, $k''_u(w_0, w) < 0$. We denote the expected utility of profit under $v(w_0, w)$ as $H_v(p, \alpha)$, and by Equation (2) we obtain

$$\begin{aligned} H'_v(p,\alpha) &\triangleq \frac{\partial H_v(p,\alpha)}{\partial \alpha} \\ &= -(c-s)\int_{-A}^{\alpha} v'_\alpha(w_0, \Pi_-(p,\alpha,\epsilon))\, dF \\ &\quad +(\hat{c}-c)\int_{\alpha}^{B} v'_\alpha(w_0, \Pi_+(p,\alpha,\epsilon))\, dF \\ &= -(c-s)\int_{-A}^{\alpha} k'_u(w_0, \Pi_-(p,\alpha,\epsilon)) u'_\alpha(w_0, \Pi_-(p,\alpha,\epsilon))\, dF \\ &\quad +(\hat{c}-c)\int_{\alpha}^{B} k'_u(w_0, \Pi_+(p,\alpha,\epsilon)) u'_\alpha(w_0, \Pi_+(p,\alpha,\epsilon))\, dF. \end{aligned} \quad (6)$$

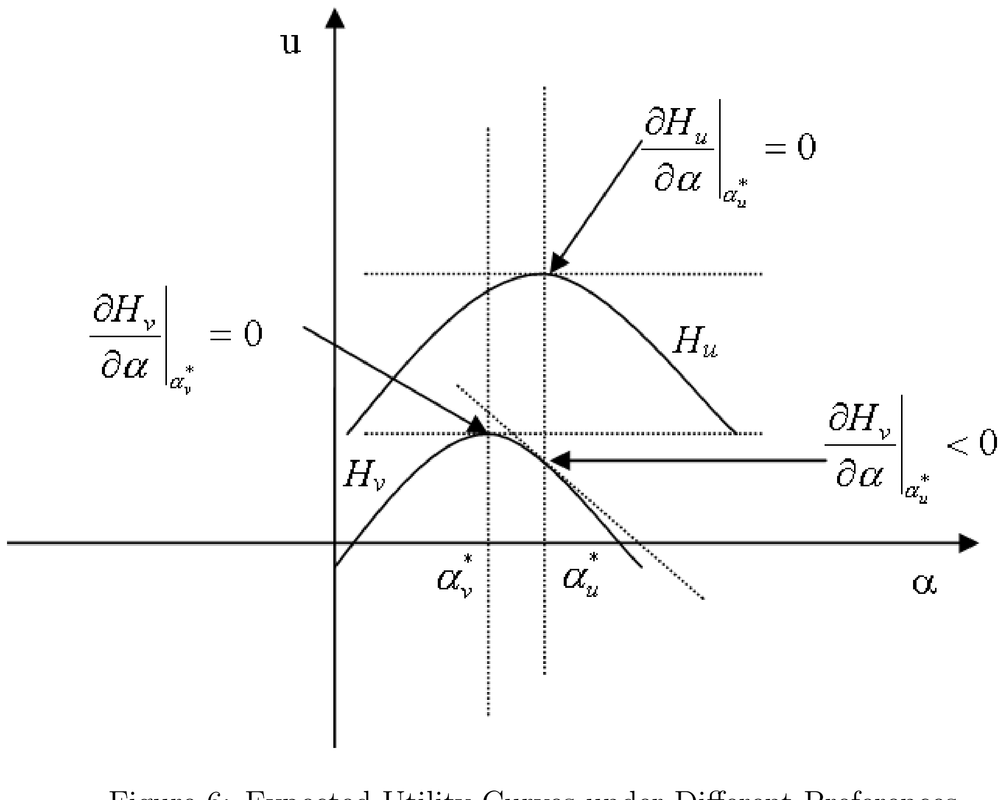

Figure 6: Expected Utility Curves under Different Preferences

For $k''_u(w_0, w) < 0$, we know $k'_u(w_0, w)$ is a decreasing function of wealth. Replacing $u(w_0, w)$ with $k[u(w_0, w)]$ in Equation (6), we obtain

$$
\begin{aligned}
& H'_v(p, \alpha_u^\star(p)) \\
= \quad & -(c-s)\int_{-A}^{\alpha_u^\star(p)} [k'_u(u(w_0, \Pi_-(p, \alpha_u^\star(p), \epsilon))) \\
& u'_\alpha(w_0, \Pi_-(p, \alpha_u^\star(p), \epsilon))] \ dF \\
& +(\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} [k'_u(u(w_0, \Pi_+(p, \alpha_u^\star(p), \epsilon))) \\
& u'_\alpha(w_0, \Pi_+(p, \alpha_u^\star(p), \epsilon))] \ dF.
\end{aligned}
$$

Due to Lemma 5.1,

$$
\begin{aligned}
& H'_v(p, \alpha_u^\star(p)) \\
< \quad & -(c-s)\int_{-A}^{\alpha_u^\star(p)} k'_u \left[u(w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p)))\right] \\
& u'_\alpha(w_0, \Pi_-(p, \alpha_u^\star(p), \epsilon)) \ dF \\
& +(\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} k'_u \left[u(w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p)))\right] \\
& u'_\alpha(w_0, \Pi_+(p, \alpha_u^\star(p), \epsilon)) \ dF \\
= \quad & k'_u[u(w_0, \Pi(p, \alpha_u^\star(p), \alpha_u^\star(p)))] \\
& \Bigg[-(c-s)\int_{-A}^{\alpha_u^\star(p)} u'_\alpha(w_0, \Pi_-(p, \alpha_u^\star(p), \epsilon)) \ dF \\
& +(\hat{c}-c)\int_{\alpha_u^\star(p)}^{B} u'_\alpha(w_0, \Pi_+(p, \alpha_u^\star(p), \epsilon)) \ dF\Bigg] \\
= \quad & 0, \text{ (by Equation (3)).}
\end{aligned}
$$

From Inequality (4), we know both $H_u(p, \alpha)$ and $H_v(p, \alpha)$ are concave in α for a given p. $H'_v(p, \alpha_v^\star(p)) = 0$, while $H'_v(p, \alpha_u^\star(p)) = 0$, for a given p. Thus, for a given p, $\alpha_v^\star(p) < \alpha_u^\star(p)$ as described in Figure 6, therefore, $\alpha_v^\star(p) + a - bp = q_v < q = \alpha_u^\star(p) + a - bp$ for a given p.

If the newsvendor has a utility function which is more risk-averse than $v(w_0, w)$, we can apply another concave transformation to $v(w_0, w)$, i.e. $g[v(w_0, w)]$ for g is increasing and concave. We can make the conclusion that the newsvendor orders less if she/he has more risk aversion for a given p. $\square$

Table 4: Optimal Order Quantity under Different Levels of Risk Aversion

Risk aversion	**Optimal order**
$r = 0.00016$	20
$r = 0.00019$	18
$r = 0.00020$	11
$r = 0.00210$	4
$r = 0.00240$	0

To see the effect that risk aversion can have on the optimal order quantity, consider the following simple example of a risk-averse newsvendor, whose preference satisfies CARA. Such preference can be represented by the utility function $u(w) = -e^{-rw}$ $(r > 0)$, where r represents the newsvendor's degree of risk aversion, an increase in r means more risk aversion. Let $w_0 = 1860$, $s = 5$, $c = 20$, $\hat{c} = 30$, $p = 190$, and let $\tilde{\epsilon} \in \{-10, 10\}$, where the probability of -10 is 0.25 and the probability of 10 is 0.75. Let $a = 105$, $b = 0.5$. Straightforward calculations yield the optimal newspaper orders for different levels of risk aversion (assuming newspaper order quantities are around to the nearest integer) in Table 4.

Note that, for the case where $r = 0.0024$, the newsvendor is so risk-averse that she/he does not order even a single newspaper, for fear of losing the cost of 20.

The property of Decreasing Absolute Risk Aversion means, for a fixed risk, decision makers (newsvendors) are willing to pay less to avoid the risk when they are wealthier. If the newsvendor's preference exhibits DARA, the above analysis further implies that wealthier newsvendor (i.e., higher w_0) will order more newspapers, since the higher initial wealth implies lower risk aversion over the support of the newsvendor's distribution of final wealth. The optimal order quantity will fall due to either increased risk aversion or decreased wealth under DARA, were shown by Baron (1973) for the case where $p = \hat{c}$ (equivalently, the case where no second newspaper purchase is possible).

From above, we know that, given risk-averse newsvendor only, we cannot estimate the effect of change in selling price p on $\alpha_u^\star(p)$. Therefore, we cannot estimate the effect of change in selling price p on optimal order quantity q.

5.3 CARA Newsvendor

In this section, this thesis considers a CARA newsvendor.

CARA is presented as utility function $u(w) = -e^{-rw}$ (lemma 3.5), where r represents the newsvendor's degree of absolute risk aversion, w is the wealth of the newsvendor. Note that this utility function also displays Increasing Relative Risk Aversion (IRRA).

For CARA newsvendor, let $\alpha^\star_{ca}(p)$ be the optimal solution of α to maximize $H_u(p,\alpha)$ while p is fixed.

$$H_u(p,\alpha^\star_{ca}(p)) = \max_{\alpha} H_u(p,\alpha).$$

The corresponding optimal order quantity is $q^\star_{ca}(p) = a - bp + \alpha^\star_{ca}(p)$.

For simplicity, this thesis hereafter considers that the demand, $\tilde{D}(p,\tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0$, $b > 0$), is a Bernoulli Distribution only, where $\tilde{\epsilon}$ follows Bernoulli Distribution, corresponding to high demand and low demand. $P(\tilde{\epsilon} = -A) = p_b$, $P(\tilde{\epsilon} = B) = 1 - p_b$.

We have the following theorem.

Theorem 5.4 *A newsvendor has CARA, i.e. her/his utility function is $u(w) = -e^{-rw}$, where $r > 0$, w is the wealth. The demand is defined as $\tilde{D}(p,\tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0$, $b > 0$), where $\tilde{\epsilon}$ is a random variable and it follows Bernoulli Distribution. Then, the optimal order quantity $q^\star_{ca}(p)$ is decreasing with the selling price p.*

Proof. $\tilde{\epsilon}$ follows Bernoulli Distribution. Then, by Equation (1) we obtain

$$\begin{aligned} & H_u(p,\alpha) \\ = \; & p_b \left[-e^{\,-\,r(w_0,\Pi_-(p,\alpha,-A))} \right] + (1-p_b) \left[-e^{\,-\,r(w_0,\Pi_+(p,\alpha,B))} \right]. \end{aligned}$$

The optimal value $\alpha^\star_{ca}(p)$ satisfies the first-order condition of $H_u(p,\alpha)$ is zero.

Consider the first partial derivative of $H_u(p, \alpha)$ taken with respect to α, we obtain

$$
\begin{aligned}
& p_b r\,(-c+s)\, e^{-r\,(w_0 + (p-c)\,(a-bp) - (p-s)\,A - (c-s)\,\alpha^\star_{ca}(p))} \\
& + (1-p_b)\, r\,(\hat{c}-c)\, e^{-r\,(w_0 + (p-c)\,(a-bp) + (p-\hat{c})\,B + (\hat{c}-c)\,\alpha^\star_{ca}(p))} \\
& \quad = \quad 0 \\
& \quad \Rightarrow \quad \alpha^\star_{ca}(p) = -\frac{(A+B)p}{\hat{c}-s} + \Phi_1 \\
& \quad \Rightarrow \quad \frac{d\alpha^\star_{ca}(p)}{dp} = -\frac{A+B}{\hat{c}-s} < 0,
\end{aligned}
$$

where $\Phi_1 = \dfrac{As + B\hat{c}}{\hat{c}-s} - \dfrac{1}{r\,(\hat{c}-s)} \ln\left(\dfrac{p_b\,(c-s)}{(\hat{c}-c)\,(1-p_b)}\right)$.

It follows that the optimal value $\alpha^\star_{ca}(p)$ is a decreasing function of p. Therefore, $q^\star_{ca}(p_1) < q^\star_{ca}(p_2)$ if $p_1 > p_2$. □

5.4 CRRA Newsvendor

In this section, this thesis considers CRRA newsvendor.

CRRA is presented as $u(w) = ln(w)$ or $u(w) = \dfrac{1}{1-r} w^{1-r}$ $(0 < r < 1$ or $1 < r)$ (lemma 3.4), where r represents the newsvendor's degree of relative risk aversion, w is the wealth of the newsvendor. Note that the above three utility functions also display Decreasing Absolute Risk Aversion (DARA).

Arrow (1965) declares two conclusions:

1. It is broadly permissible to assume that relative risk aversion increases with wealth, though theory does not exclude some fluctuations.

2. If, for simplicity, we wish to assume a constant relative risk aversion, then the appropriate value is 1.

According to the point 2 in Arrow's declaration, this thesis only considers the CRRA newsvendor's utility function $u(w) = ln(w)$. Let $\alpha^{\star}_{cr}(p)$ be the optimal solution of α to maximize $H_u(p, \alpha)$ while p is fixed.

$$H_u(p, \alpha^{\star}_{cr}(p)) = \max_{\alpha} H_u(p, \alpha).$$

The corresponding optimal order quantity is $q^{\star}_{cr}(p) = a - bp + \alpha^{\star}_{cr}(p)$.

We have the following theorem.

Theorem 5.5 *A newsvendor has CRRA, i.e. her/his utility function is $u(w) = ln(w)$, where w is the wealth. The demand is defined as $\tilde{D}(p, \tilde{\epsilon}) = a - bp + \tilde{\epsilon}$ ($a > 0$, $b > 0$), where $\tilde{\epsilon}$ is a random variable and it follows Bernoulli Distribution.*

(i) If $p_b \in [0, p_b^0)$ and $p > 0$, when p increases, the optimal order quantity $q^{\star}_{cr}(p)$ increases first and then decreases;

(ii) if $p_b \in [p_b^0, \Omega)$ and $p > 0$, when p increases, the optimal order quantity $q^{\star}_{cr}(p)$ decreases;

(iii) if $p_b = \Omega$ and $p > 0$, when p increases, the optimal order quantity $q^{\star}_{cr}(p)$ decreases;

(iv) if $p_b \in (\Omega, 1]$ *and* $p > 0$*, when* p *increases, the optimal order quantity* $q^\star_{cr}(p)$ *decreases first and then increases;*

where $p_b^0 = \dfrac{[A - (a + bc)]\,(\hat{c} - c)}{A(\hat{c} - c) - B(c - s) - (a + bc)(\hat{c} - s)}$, $\Omega = \dfrac{\hat{c} - c}{\hat{c} - s}$ *(It will be proved later that* $p_b^0 < \Omega$ *in Lemma 5.6).*

Proof. $\tilde{\epsilon}$ follows the Bernoulli Distribution. Then, by Equation (1) we obtain

$$
\begin{aligned}
& H_u(p, \alpha) \\
&= p_b \ln\left(w_0 + (p - c)\,(a - bp) - (p - s)\,A - (c - s)\,\alpha\right) \\
&\quad + (1 - p_b) \ln\left(w_0 + (p - c)\,(a - bp) + (p - \hat{c})\,B + (\hat{c} - c)\,\alpha\right).
\end{aligned}
$$

The optimal value $\alpha^\star_{cr}(p)$ satisfies the first-order condition of $H_u(p, \alpha)$ is zero. Consider the first partial derivative of $H_u(p, \alpha)$ taken with respect to α, we obtain

$$
\begin{aligned}
& \frac{p_b\,(-c + s)}{w_0 + (p - c)\,(a - bp) - (p - s)\,A - (c - s)\,\alpha^\star_{cr}(p)} \\
& + \frac{(1 - p_b)\,(\hat{c} - c)}{w_0 + (p - c)\,(a - bp) + (p - \hat{c})\,B + (\hat{c} - c)\,\alpha^\star_{cr}(p)} \\
&= 0 \\
& \Rightarrow \alpha^\star_{cr}(p) = x_1 p^2 + y_1 p + z_1,
\end{aligned}
$$

where $x_1 = \dfrac{b\,[p_b(\hat{c} - s) - (\hat{c} - c)]}{(\hat{c} - c)(c - s)}$,

$$y_1 = \frac{-(a + bc)[p_b(\hat{c} - s) - (\hat{c} - c)] - A(\hat{c} - c)(1 - p_b) - Bp_b(c - s)}{(\hat{c} - c)(c - s)},$$

$$z_1 = \frac{(ac - w_0)[p_b(\hat{c} - s) - (\hat{c} - c)] + As(\hat{c} - c)(1 - p_b) + Bp_b\hat{c}(c - s)}{(\hat{c} - c)(c - s)}.$$

(i) If $p_b \in [0, p_b^0)$, it follows that $x_1 < 0$ and $y_1 > 0$. Further it turns out that $\alpha_{cr}^{\star}(p)$ increases with p when $0 \leq p \leq -\frac{y_1}{2x_1}$ and then decreases when $p > -\frac{y_1}{2x_1}$. Therefore, the optimal order quantity $q_{cr}^{\star}(p)$ increases with p when $0 \leq p \leq \max\{0, -\frac{y_1 - b}{2x_1}\}$ and then decreases when $p > \max\{0, -\frac{y_1 - b}{2x_1}\}$.

(ii) If $p_b \in [p_b^0, \Omega)$, it follows that $x_1 < 0$ and $y_1 < 0$. Further it turns out that $\alpha_{cr}^{\star}(p)$ decreases with p when $p > 0$. Therefore, $q_{cr}^{\star}(p)$ decreases with p when $p > 0$.

(iii) If $p_b = \Omega$, it follows that $x_1 = 0$, $y_1 = -\frac{A + B}{\hat{c} - s} < 0$. Further it turns out that $\alpha_{cr}^{\star}(p)$ is a decreasing function of p when $p > 0$. Therefore, $q_{cr}^{\star}(p)$ is a decreasing function of p when $p > 0$.

(iv) If $p_b \in (\Omega, 1]$, it follows that $x_1 > 0$ and $y_1 < 0$. Further it turns out that $\alpha_{cr}^{\star}(p)$ decreases with p when $0 \leq p \leq -\frac{y_1}{2x_1}$ and then increases when $p > -\frac{y_1}{2x_1}$. Therefore, the optimal order quantity $q_{cr}^{\star}(p)$ decreases with p when $0 \leq p \leq \max\{0, -\frac{y_1 - b}{2x_1}\}$ and, $q_{cr}^{\star}(p)$ increases with p when $p > \max\{0, -\frac{y_1 - b}{2x_1}\}$.

□

We have the following lemma.

Lemma 5.6 *$p_b^0 < \Omega$ if $A < a + bc$.*

Proof. From above we know that $y_1 = \frac{[A(\hat{c} - c) - B(c - s) - (a + bc)(\hat{c} - s)]\, p_b + [(a + bc) - A]\,(\hat{c} - c)}{(\hat{c} - c)(c - s)}$ is a linear function of p_b. If $p_b > \Omega$, $y_1 < 0$. If $p_b = \Omega$, $y_1 < 0$. If $0 \leq p_b < \Omega$, $y_1 > 0$ because $A \leq a - bp < a + bc$. Thus, y_1 is a decreasing function of p_b if $p_b > 0$.

$y_1(p_b^0) = 0$, therefore, $0 < p_b^0 < \Omega$ if $A < a + bc$. □

We give a numeric example for illustration. Let $w_0 = 1860$, $s = 5$, $c = 20$, $\hat{c} = 30$, $a = 105$, $b = 0.5$. We further assume $\tilde{\epsilon} \in \{-10, 10\}$, which means $A = B = 10$. It follows that $p_b^0 = \dfrac{14}{39}$ and $\Omega = 0.4$. (i) when $p_b = 0.3 < p_b^0$. It turns out that $\alpha_{cr}^{\star}(p) = -0.0083p^2 + 1.1500p + 7.3333$, $q_{cr}^{\star}(p) = -0.0083p^2 + 0.6500p + 112.3333$. Therefore, when $0 < p < 39.0$, $q_{cr}^{\star}$ increases with p. When $p > 39.0$, $q_{cr}^{\star}$ decreases with p. (ii) when $p_b = 0.38$. It turns out that $\alpha_{cr}^{\star}(p) = -0.0017p^2 - 0.4100p + 12.6667$, $q_{cr}^{\star}(p) = -0.0017p^2 - 0.9100p + 117.6667$. Therefore, when $0 < p$, $q_{cr}^{\star}$ decreases with p. (iii) when $p_b = 0.4$. It turns out that $\alpha_{cr}^{\star}(p) = -0.8p + 14$, $q_{cr}^{\star}(p) = -1.3p + 119$. Therefore, when $0 < p$, $q_{cr}^{\star}$ decreases with p. (iv) when $p_b = 0.5$. It turns out that $\alpha_{cr}^{\star}(p) = 0.02083p^2 - 5.67500p + 30.66667$, $q_{cr}^{\star}(p) = 0.02083p^2 - 6.17500p + 135.66667$. Therefore, when $0 < p < 148.20$, $q_{cr}^{\star}$ decreases with p. When $p > 148.20$, $q_{cr}^{\star}$ increases with p.

From the above theorem we know that the optimal order quantity decreases with the selling price in case (ii) and (iii). In case (i) and (iv) the optimal order quantity decreases with the selling price partially. Economists have already explained the reason. i.e. higher prices lead to lower demand. Notice that both in case (i) and (iv) that the optimal order quantity is also increasing with the selling price partially. This thesis explains it in the following two aspects, referring to Qing Li and Hongtao Zhang's working paper. For one explanation, we can easily say that this phenomenon occurs due to the randomness of demand. Therefore, when q goes up, it is possible, and sometimes even optimal as in the theorem 5.5, to sell more at a higher margin. For the other explanation, we define $\alpha = q - (a - bp)$ before and think of it as the portion of inventory set aside to guard against uncertainty. Petruzzi and Dada (1999) point out that this α plays the role of safety stock factor. When demand is uncertain, for a given price, the decision maker (newsvendor) will be better off with a higher α as there is a possibility of selling more by reducing shortages. Considering the identity $q = (a - bp) + \alpha$, there are two possible responses to an increase in the initial order quantity. On the one hand, a higher order can be used to meet a greater average demand. That is to say, when q increases, $(a - bp)$ can be increased by decreasing p. On the other hand, a higher order can be used to reduce the expected shortage by increasing the safety stock factor α. That is to say, when q increases, α can be increased. We know from the newsvendor problem that a higher safety stock factor corresponds to a higher marginal shortage cost. Since here the marginal shortage cost equals $p - c$, the decision maker (newsvendor) should set a higher p to have a correspondly higher marginal shortage cost when α is greater.

6 Conclusion

We have looked at the effect of change in selling price on optimal order quantity under different risk preferences for this extension of the classic

Table 5: Summary Comparative Statics Results as p Increases in the RAPD model

Utility Assumptions	**Change in Order**	**Distribution**
risk neutrality	decrease	–
risk aversion	indeterminate	–
CARA	decrease	Bernoulli Distribution
CRRA	i. increase first and then decrease ii. decrease iii. decrease first and then increase	Bernoulli Distribution

Table 6: Results as p Increases under Special DARA and IRRA Utility Functions in the RAPD model

Utility Assumptions	**Change in Order**	**Distribution**
IRRA	decrease	Bernoulli Distribution
DARA	i. increase first and then decrease ii. decrease iii. decrease first and then increase	Bernoulli Distribution

newsvendor problem. Except that the effect of selling price on optimal order quantity remains indeterminate for risk-averse decision maker (newsvendor), we can see selling price greatly affect the optimal order quantity in other cases. (i) In the case where the decision maker (newsvendor) is risk-neutral, increasing selling price leads to decreasing optimal order quantity; (ii) In the case where the decision maker (newsvendor) has CARA, increasing selling price leads to decreasing optimal order quantity; and (iii) In the case where the decision maker (newsvendor) has CRRA, the results are more complex. If the probability of low demand is in the range of $\left[0, \dfrac{[A-(a+bc)](\hat{c}-c)}{A(\hat{c}-c)-B(c-s)-(a+bc)(\hat{c}-s)}\right)$, when selling price increases, the optimal order quantity increases first and then decreases. If the probability of low

Table 7: Summary Comparative Statics Results as p Increases in the EGS Model

Utility Assumptions	**Change in Order**
risk neutrality	no change
risk aversion	indeterminate
CARA	decrease
DARA	indeterminate

demand is in the range of $\left[\frac{[A-(a+bc)](\hat{c}-c)}{A(\hat{c}-c)-B(c-s)-(a+bc)(\hat{c}-s)}, \frac{\hat{c}-c}{\hat{c}-s}\right]$, optimal order quantity decreases with selling price. If the probability of low demand is greater than $\frac{\hat{c}-c}{\hat{c}-s}$, when selling price increases, optimal order quantity decreases first and then increases. A summary of the results is shown in Table 5. This thesis also examines some special utility functions of DARA and IRRA at the same time. The results are similar and described in Table 6. All these results are different from those obtained in the EGS Model, which are shown in Table 7.

This extension of the classic newsvendor problem provides an excellent framework for examining how the inventory problem interacting with the marketing issue (selling price) will influence the decision maker at the firm level. The owners of some firms, especially small and medium-sized enterprises, are usually the managers as well, whose decisions greatly affect the fate of the firms. This thesis investigates the behavior of the firms by examining the behavior of the decision makers.

This extension of the classic newsvendor problem is closer to reality than before. It also provides an integrated framework for investigating different newsvendor problems. Thus, it might motivate and encourage more applications of the newsvendor problem which is a foundation of many inventory problems.

This thesis can be improved in the following aspects. First, this study has not determined p and α to maximize $H_u(p, \alpha)$ for simplicity. It might be worthy of further study to determine them in the RAPD model. Second, this thesis has discussed the randomness of demand follows the Bernoulli Distribution. To examine the RAPD model more accurately, suppose the randomness of demand follows the other distributions, such as normal distribution, is worthy of further study.

In addition to any benefits and costs, the risk-averse newsvendor also reacts to wealth effects for changes in prices, costs and risk. It thus becomes more difficult to determine the qualitative effects of changes in these parameters for the risk-averse newsvendor than for the risk-neutral newsvendor. Static models can only capture a part of reality. How risk aversion works in a more realistic dynamic setting is worthy of future examination. The analysis could be helpful in examining a dynamic model. Given the prominence of the static newsvendor problem in the literature, the RAPD model is useful in its own right as well.

Appendix

A Proof of Theorem 3.1

(i) Let u be concave. Then by definition of concavity $u(\alpha x + (1-\alpha)y) \geq \alpha u(x) + (1-\alpha)u(y)$ for all $x, y \in R$ and $\alpha \in [0,1]$. But $E(w) = \alpha x + (1-\alpha)y$ and $E(u(w)) = \alpha u(x) + (1-\alpha)u(y)$. Thus, this inequality implies $u(E(w)) \geq E(u(w))$. As by definition $E(u(w)) = u(C^u(w))$, then $u(E(w)) \geq u(C^u(w))$. As u is monotonically increasing, then $E(w) \geq C^u(w)$, which is the definition of risk-aversion. (ii) and (iii) follow analogously. □

B Proof of Theorem 3.2

We shall go $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$.

$(i) \Rightarrow (ii)$: First we must establish that T exists. Let $v(x) = t$. As v is monotonic and continuous, then the inverse v^{-1} exists and so $v^{-1}(t) = x$. Thus, $u(x) = u(v^{-1}(t))$. Let us define $T = uv^{-1}$, then $u(x) = T(t)$. But recall that $t = v(x)$, thus $u(x) = T(v(x))$. Thus T exists. Now, differentiating $u'(x) = T'(v(x))v'(x)$. Thus:

$$\frac{u'(x)}{v'(x)} = T'(v(x)) > 0,$$

as $u'(x), v'(x) > 0$ by assumption of monotonicity. Differentiating again:

$$u''(x) = T''(v(x))v'(x)^2 + T'(v(x))v''(x),$$

or substituting in for $T'(v(x))$:

$$u''(x) = T''(v(x))v'(x)^2 + \frac{u'(x)v''(x)}{v'(x)}.$$

Thus dividing through by $u'(x)$ and rearranging:

$$\frac{v''(x)}{v'(x)} - \frac{u''(x)}{u'(x)} = -\frac{T''(v(x))v'(x)^2}{u'(x)}.$$

Now, by assumption $v' > 0$ and $u' > 0$, thus let $\frac{v'(x)^2}{u'(x)} = \alpha > 0$ so:

$$\frac{v''(x)}{v'(x)} - \frac{u''(x)}{u'(x)} = -T''(v(x))\alpha. \tag{7}$$

But by (i), $\frac{v''(x)}{v'(x)} - \frac{u''(x)}{u'(x)} \geq 0$. Thus, as $\alpha > 0$, then it must be that $-T''(v(x)) \geq 0$, or simply, $T''(v(x)) \leq 0$. Thus, for all $x \in R$, $T' \geq 0$ and $T'' \leq 0$, thus T is concave. Q.E.D.

(ii) $\Rightarrow$ *(iii)*: Recall that $u(C^u(w)) = E(u(w))$. Thus, by (ii), as $u(x) = T(v(x))$, then $u(C^u(w)) = E(T(v(w)))$. As T is concave, then by Jensen's inequality:

$$E(T(v(w))) \leq T(E(v(w))),$$

but as $E(T(v(w))) = u(C^u(w))$ and $E(v(w)) = v(C^v(w))$ by definition, then this implies:

$$u(C^u(w)) \leq T(v(C^v(w))),$$

or, by (ii), as $u(\cdot) = T(v(\cdot))$, this becomes $u(C^u(w)) \leq u(C^v(w))$. Thus by monotonicity of u, $C^u(w)) \leq C^v(w)$ which implies, by definition, that $\pi^u(w) \geq \pi^v(w)$ which is (iii). Q.E.D.

$(iii) \Rightarrow (i)$: Let us show, equivalently, that "not (i)" $\Rightarrow$ "not (iii)". Thus, if "not (i)", then:

$$-\frac{u''(\bar{x})}{u'(\bar{x})} < -\frac{v''(\bar{x})}{v'(\bar{x})} \text{ for some } \bar{x} \in R.$$

By continuity, there is a neighborhood $N_\epsilon(\bar{x})$ for which this is true. Let z be a random variable which takes values only in $N_\epsilon(\bar{x})$, and elsewhere zero. Recall that in our earlier proof of (i) $\Rightarrow$ (ii), we obtained Equation (7). For $z \in N_\epsilon(\bar{x})$, $-\frac{u''(z)}{u'(z)} < -\frac{v''(z)}{v'(z)}$, thus, $-T''[v(z)]\alpha < 0$ for $z \in N_\epsilon(\bar{x})$, thus $T'' > 0$, and thus $T(\cdot)$ is convex. But, by earlier theorem (ii) $\Rightarrow$ (iii), we can see that $T'' > 0$ implies that $\pi^v(w) > \pi^u(w)$, i.e. "not (iii)". Thus, "not (i)" $\Rightarrow$ "not (iii)" or, equivalently, (iii) $\Rightarrow$ (i). Q.E.D.

Thus, $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$. □

C Proof of Theorem 3.3

Let $v_a(w) = u(w + a)$ and the rest follows by the theorem 3.2. Note that $\frac{d(-\frac{u''(w)}{u'(w)})}{dw} < 0$. □

D Proof of Lemma 3.4

As we all know, if the utility function $u(w)$ has CRRA, then

$$r_u(w) = -\frac{u''(w)}{u'(w)} = \frac{R}{w}, \tag{8}$$

$$R_u(w) = -\frac{wu''(w)}{u'(w)} = R, \tag{9}$$

where R is the coefficient of CRRA. For risk-aversion, Equation (8) > 0, so $R > 0$.

Let $y(w) = u'(w)$, and substitute $y(w)$ into Equation (9), we obtain

$$-\frac{wy'(w)}{y(w)} = R,$$

$$\Rightarrow \quad \frac{dy}{y} = -R\frac{dw}{w}.$$

Integral on both sides, we obtain

$$ln|y| = -Rlnw + c_3,$$

$$\Rightarrow \quad ln|y| = ln(w^{-R}e^{c_3}),$$

$$\Rightarrow \quad y = \pm e^{c_3}w^{-R},$$

where c_3 is a constant which is in $(-\infty, +\infty)$.

As we know, $u(w)$ has risk-aversion, so $y = u'(w) > 0$, thus

$$y(w) = u'(w) = e^{c_3}w^{-R} = c_2w^{-R}, \tag{10}$$

where $c_2 > 0$.

By Equation (10) we have

$$\frac{du(w)}{dw} = c_2 w^{-R},$$
$$\Rightarrow \quad u(w) = \begin{cases} \frac{c_1}{1-R} w^{1-R}, & 0 < R < 1, 1 < R, \\ c_2 ln(w), & R = 1, \end{cases}$$

where $c_1, c_2 > 0$. □

E Proof of Lemma 3.5

As we all know, if the utility function $u(w)$ has CARA, then

$$r_u(w) = -\frac{u''(w)}{u'(w)} = r, \tag{11}$$

where $r > 0$ for risk-aversion.

Let $y(w) = u'(w)$, and substitute $y(w)$ into Equation (11), we obtain

$$-\frac{y'(w)}{y(w)} = r,$$
$$\Rightarrow \quad \frac{dy}{y} = -rdw.$$

Integral on both sides, we obtain

$$ln|y| = -rw + c_2,$$

$$\Rightarrow \quad y = \pm e^{-rw+c_2},$$

where $r > 0$, $c_2 \in (-\infty, +\infty)$.

As we know, $u(w)$ has risk-aversion, $u(w)$ must be concave, so $y(w) = u'(w) > 0$, thus

$$y(w) = u'(w) = e^{c_2} e^{-rw}. \tag{12}$$

By Equation (12), we have

$$\frac{du}{dw} = e^{c_2} e^{-rw},$$
$$\Rightarrow \quad u(w) = -c_1 e^{-rw},$$

where $r > 0$, $c_1 > 0$. □

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