GAME-THEORETIC ANALYSIS OF THE QUALITY ASSURANCE PROBLEM IN A TWO-ECHELON SUPPLY CHAIN WITH A RETAILER AS THE QUALITY GATEKEEPER

LI ZAICHEN

MPHIL

LINGNAN UNIVERSITY

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by

LI Zaichen

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ABSTRACT

Game-Theoretic Analysis of the Quality Assurance Problem in a Two-Echelon Supply Chain with a Retailer as the Quality Gatekeeper

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LI Zaichen

Master of Philosophy

We consider a two-level supply chain involving a manufacturer and a retailer who serves as the quality gatekeeper. The manufacturer determines a wholesale price and a defective rate and announces his decisions to the retailer, who then makes her decisions on the retail price and identification rate that means the percentage of the defects identified by the retailer and reflects the retailer's gatekeeping effort on her quality assurance. We accordingly develop a leader-follower game and solve it to find Stackelberg equilibrium for the manufacturer and the retailer. In order to examine whether or not the supply chain benefits from the retailer's quality gatekeeping effort, we also develop and solve another leader-follower game where the manufacturer still announces its wholesale pricing and defective rate decisions but the retailer only decides on the retail price. We show that the manufacturer's equilibrium defective rate for the game with the retailer's gatekeeping is higher than that for the game without the retailer's gatekeeping. The ratio of the manufacturer's profit to the retailer's profit is increased when the retailer serves as the gatekeeper. Moreover, the retailer reduces her price when she acts as the gatekeeper, if and only if the supply chain-wide cost decreases as a result of the retailer's gatekeeping effort. We also perform sensitivity analysis of each parameter in our game models to further examine the impacts of the retailer's gatekeeping on the manufacturer's and the retailer's decisions and profits. We find that the retailer's penalty cost per defect has more significant impacts than the manufacturer's unit penalty cost. The paper ends with a summary of managerial insights.

Key words: Quality; supply chain; gatekeeper; defective rate; identification rate; price; Stackelberg equilibrium.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

(LI Zaichen)
Date:

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1 Introduction

Quality has been playing an increasingly important role in highly competitive industries, and it has been widely treated as one of the most important determinants of a firm's success in domestic and international markets. In the past two decades, many firms in both the manufacturing and the service fields have paid intensive attention to the improvement in the quality of their products and services. Without a continuous focus on the quality improvement, a firm may have to experience a great loss in its profitability due to product or service quality problems. For example, as Tabuchi [26] reported, in 2010, Toyota received complaints from customers about its defects resulting in traffic accidents, and the firm was thus the only major automaker to experience a sales decline in the United States. In January 2011, Toyota decided to globally recall 1.7 million vehicles that have defective fuel lines and high-pressure fuel pumps, drawing renewed attention to the quality of the company's products.

There are a number of different but similar definitions on the product or service quality. In [25], Stevenson simply defined the quality as the *ability* of a product or service to consistently meet or exceed customer expectations. That is, for the sake of quality assurance, each firm should reduce the number (or, the percentage) of defects (for the manufacturing systems) or complaints (for the service systems), thus satisfying the customer needs. For a manufacturing system, we learn from `BusinessDictionary.com` a more specific definition on the quality of a tangible product, which indicates that the product quality is a measure of excellence or a state of being free from defects, deficiencies, and significant variations. In this paper, we focus on the control of a tangible product's quality in a two-echelon supply chain involving a manufacturer and a retailer.

In practice, many manufacturers spend great efforts on the quality assurance to prevent defects by, e.g., adopting the deployment of a quality management system and preventative activities such as failure mode and effects analysis (FMEA).

Accordingly, a number of scholars have investigated the manufacturer's diverse quality assurance problems from the academic perspective, as shown by our literature review in Section 2. However, very few existing papers consider the involvement of a retailer into the quality assurance. In fact, a retailer may serve as the quality gatekeeper to assure the quality by identifying the defects among the products bought from the manufacturer. For example, Wal-Mart allocates considerable human resources for the quality assurance, acting as the quality gatekeeper. Jeff Macho, the senior vice president and managing director of Wal-Mart Global Procurement, states that quality is a *top* priority for Wal-Mart and it is an area where Wal-Mart continuously challenge itself for improvement. On July 11, 2008, the Global Procurement Organization of Wal-Mart announced an enhancement to its quality assurance processes by selecting the Intertek Group—which is a leading European provider of quality and safety solutions serving a wide range of industries around the world; see `www.intertek.com`—to provide final inspection services for merchandise sourced in China. The Intertek Group is responsible for not only testing, inspecting, and certifying products but also helping customers improve performance, gain efficiencies in manufacturing and logistics, overcome market constraints, and reduce risk. As Jeff Macho further explained, the Intertek Group will provide Wal-Mart with an *independent* assessment of product quality, which helps assure that merchandise can meet or exceed Wal-Mart's specifications. For more information, see, e.g., a report released by Wal-Mart in [27].

In this paper, we are motived by Wal-Mart's quality gatekeeping effort to investigate the impacts of a retailer's gatekeeping in a two-echelon supply chain. In the supply chain, the manufacturer determines his wholesale price and defective rate that is the percentage of defects in his products and reflects the manufacturer's effort on the quality assurance. Note that the quality has been widely defined by academics as the defective rate; for a comprehensive discussion, see, e.g., Reeves and Bednar [21]. Moreover, in practice, it is not necessary for most manufacturers to promise zero defective rate; see, e.g., Schneiderman [23]. Responding to the manufacturer's decisions, the retailer decides on her retail price and identification rate, which is the percentage of the defects identified by the retailer in all defects bought from the manufacturer. The retailer's identification rate reflects her quality gatekeeping effort. The defects identified by the retailer will be returned by the retailer to the manufacturer without selling to customers. After receiving a returned defect, the manufacturer should pay a penalty to the retailer, replace the defect with a good-quality product for the retailer, and dispose the defect at a cost for repair, salvage, etc. If the retailer does not identify a defect and sells the defect to a customer, then the customer will eventually return the defect to the retailer who incurs a penalty cost. The retailer then returns the defect to the manufacturer and obtains a penalty from the manufacture, who provides the retailer with a good-quality product and disposes the defect at a cost. We plot Figure 1 to describe the process of quality assurance.

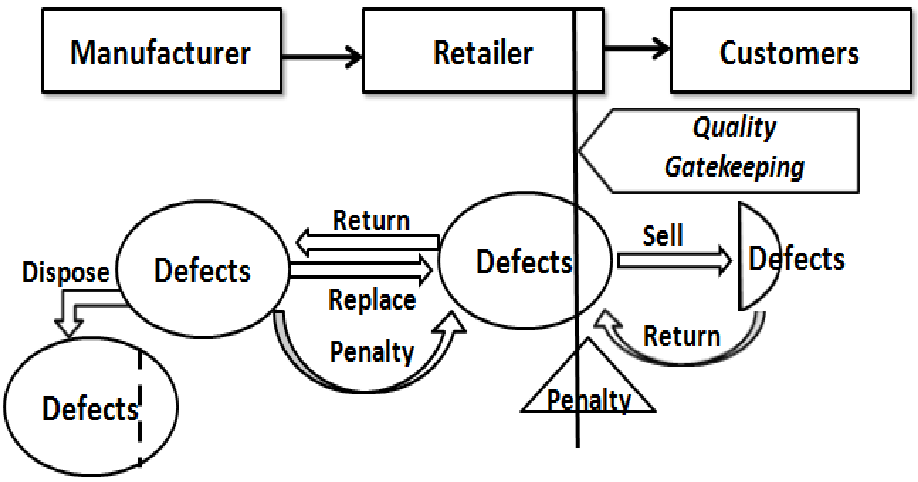

Figure 1: The process of quality assurance in the supply chain with the retailer as the quality gatekeeper.

In the supply chain, we consider each firm's joint pricing and quality assurance decisions, because of the following reason: if the firm spends more efforts on quality assurance, then he or she incurs a higher cost and thus has an incentive to increase the firm's price. But, the quality assurance results in an improvement of the product quality, thereby decreasing the number of defects sold to consumers and reducing the firm's penalty cost for sold defects. This may help the firm achieve a cost reduction, which induces the firm to decrease the price. From the above, we find that the wholesale and the retail prices may or may not increase as a result of the retailer's quality gatekeeping effort. Thus, an important research problem to be addressed in this paper is regarding the impacts of the retailer's gatekeeping on the manufacturer's and the retailer's pricing decisions.

We characterize the supply chain operations as a leader-follower (sequential) game, where the manufacturer first determines his wholesale price and defective rate as the "leader" and the retailer then responds by making her pricing and identification rate decisions as the "follower." Solving the game, we obtain the two supply chain members' decisions in Stackelberg equilibrium. We analytically show that the retailer's identification rate is determined such that the retailer's marginal quality-assurance cost is equal to her expected penalty cost for each defect sold to consumers. We also find that the manufacturer's defective rate is *independent* of his per defect penalty cost but *dependent* on the retailer's per defect penalty cost and his own cost of disposing each returned defect. This means that the value of the manufacturer's penalty cost for each defect does not affect his decision on the defective rate, when the retailer takes the gatekeeping role.

Next, we compare the manufacturer's and the retailer's decisions and profits with those when the retailer does not participate in the quality assurance, in order to investigate the impacts of the retailer's gatekeeping effort. That is, since the manufacturer always needs to assure the quality no matter whether or not the retailer serves as the gatekeeper, it is naturally necessary to investigate how the manufacturer's defective rate varies when the retailer is involved into the quality assurance. We also examine whether or not the manufacturer and the retailer benefit from the retailer's gatekeeping by achieving higher profits. Thus, we formulate and analyze another leader-follower game, where the manufacturer and the retailer still serve the leader and the follower, respectively, but the retailer does not act as the gatekeeper and does not determine the identification rate. For the process of the quality assurance in the game without the retailer's quality assurance, see Figure 2. We solve the game and find corresponding Stackelberg equilibrium, which is then compared with that when the retailer serves as a gatekeeper. We learn from our analysis that, if the manufacturer's marginal quality assurance cost is higher than the retailer's, then he should promise a smaller defective rate to the retailer. Moreover, in the game with the retailer as the quality gatekeeper, the manufacturer can achieve his profit twice as much as the retailer's profit.

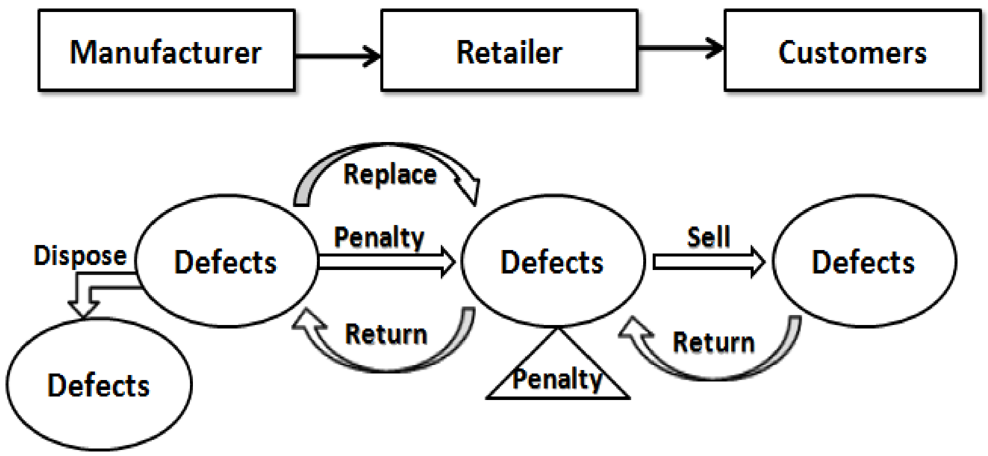

Figure 2: The process of quality assurance in the supply chain without the retailer's gatekeeping effort.

We analytically show that the manufacturer's equilibrium defective rate for the game with the retailer's quality gatekeeping is always higher than that for the game without the retailer's gatekeeping. That is, the retailer's gatekeeping effort induces the manufacturer to reduce his effort on the quality assurance. Furthermore, if the retailer's cost reduction made by her gatekeeping effort is larger than manufacturer's cost reduction, then the wholesale price for the game with the retailer's gatekeeping is higher than that for the game without the retailer's gatekeeping. We also find that, if the supply chain-wide cost is reduced as a result of the retailer's gatekeeping effort, then the retailer's retail price when the retailer serves as the gatekeeper is smaller than that when the retailer does not serve as the gatekeeper. Since, without the retailer's gatekeeping, the manufacturer's profit is less than twice as much as the retailer's profit, we find that the retailer's gatekeeping effort increase the ratio of the manufacturer' profit to the retailer's profit.

We then analytically and numerically perform sensitivity analysis of each parameter in our game model, to investigate the impacts of the parameter on the manufacturer's and the retailer's decisions and profits. From our analysis, we draw a large number of managerial insights. For example, we analytically show that the retailer's per defect penalty cost more significantly affects the two supply chain members' equilibrium decisions and profits, compared with the manufacturer's per defect penalty cost. We also find that the manufacturer's wholesale price is increasing in his per defect penalty cost, no matter whether or not the retailer serves as the quality gatekeeper. As the retailer's per defect penalty cost increases, the manufacturer reduces his wholesale price when the retailer serves as the quality gatekeeper; but, he increases his wholesale price when the retailer does not serve as the gatekeeper. When the retailer is involved into the quality assurance, the retailer's identification rate is increasing in her per defect penalty cost whereas the manufacturer's defective rate is decreasing in the cost. When the retailer does not serve as the quality gatekeeper, the manufacturer's defective rate is also decreasing in the retailer's penalty cost. Our results demonstrate that both the manufacturer's and the retailer's profits are decreasing in the retailer's penalty cost per defect.

When the manufacturer's unit acquisition cost is increased, we analytically show that, for the game with and the game without the retailer's quality gatekeeping effort, the manufacturer and the retailer increase their wholesale and retail prices, respectively, and the two supply chain members' profits decrease.

In addition, the reduction in the manufacturer's profit is greater than the reduction in the retailer's profit. Our analytic results also indicate that, as the manufacturer disposes each unit of defect at a higher cost, he should decrease his defective rate and the retailer should also reduce her identification rate. In addition, without the retailer's quality gatekeeping effort, both the wholesale price and the retail price are increasing in the manufacturer's disposing cost but their profits are both decreasing in such a cost. But, for the game with the retailer's gatekeeping, the wholesale price is decreasing in the disposing cost whereas the retail price is increasing in s, and both the manufacturer's and the retailer's profits are reduced as a result of increasing the manufacturer's disposing cost. In addition to the sensitivity analysis of the above important parameters, we also examine the impacts of the parameters in the demand function, the parameters in the manufacturer's and the retailer's quality-assurance cost functions.

The remainder of the paper is organized as follows: In Section 2, we review quality assurance-related publications to show the originality of our paper. In Section 3, we formulate and analyze the leader-follower game with the retailer's gatekeeping effort. Section 4 presents our analysis regarding the impacts of the retailer's gatekeeping effort on the supply chain. In the section, we first analyze the leader-follower game when the retailer does not serve as the quality gatekeeper, and then compare the corresponding Stackelberg equilibrium with that for the game with the retailer's gatekeeping. We also perform sensitivity analysis of major parameters in our game model to discuss the impacts of the parameters on the manufacturer's and the retailer's equilibrium decisions and profits. The paper ends with a summary of major managerial insights in Section 5. Moreover, we list all notations in Table 2, which is given in Appendix A.

2 Literature Review

In this section, we review major publications regarding the quality assurance in manufacturing systems. In the past three decades, a number of scholars have published some review papers that are concerned with the quality management. However, we find that most of the existing review papers were proposed to discuss the quality-related issues in some specific industries such as food, healthcare, etc. Though, there are still three major review paper concerning the quality assurance that does not apply to a specific industry. As an early publication associated with quality management, Rao and Monroe [20] provided an integrative review on previous research that had investigated experimentally the influence of price, brand name, and/or store name on buyers' evaluations of product quality. In addition, the authors used meta-analytic procedures to find that, for consumer products, the relationships between price and perceived quality and between brand name and perceived quality are positive and statistically significant. In [19], Powell reviewed existing empirical evidence to examine that total quality management (TQM) is a potential source of sustainable competitive advantage, and summarized major findings from a new empirical study of TQM's performance consequences. Chan and Wu [5] proposed a literature review of quality function deployment (QFD) using a reference bank of about 650 QFD publications established through searching various sources. The authors first mentioned the origination and historical development of QFD with a partial list of QFD organizations, software, and online resources. Then, they conducted a categorical analysis about QFD's functional fields, applied industries, and methodological development, and also suggested ten informative QFD publications.

From the above, we find that Rao and Monroe [20] considered the empirical study from the marketing perspective; Powell [19] also used the empirical evidence to discuss the important role of TQM in industry; and, Chan and Wu [5] focused on the review of the QFD that is a specific technique in the quality management.

This means that our review in this section differs from the existing quality-related review papers. Next, we briefly summarize major publications that are relevant to the assurance of product quality, with an emphasis on the analytic investigations.

2.1 Quality Assurance in a Single Operating System

We learn from our review that, prior to two decades, some analytic models were developed to analyze the quality problem for a single firm. For example, from the marketing perspective, Shapiro [24] treated reputation as an expectation of quality in the market where consumers are incompletely informed about the product quality before their purchases. The author found that the uncertainty about quality is a widespread and important feature of markets for most firms's good and services. Schneiderman [23] developed a simple model to investigate the optimal quality level in terms of the defective rate, assuming that, as the defective rate rises, the failure costs decline while appraisal plus prevention costs increase. The author concluded that each firm does not need to assure zero defects from the economic perspective, because a further improvement of quality above the optimum quality level increases total cost and decrease financial performance. Fine [11] reviewed some papers that had challenged the traditional literatures indicating that improving the quality of a product increases the unit production cost. The author showed that high quality and low costs may be consistent under the help of learning curve, and thus suggested that firms should be investing more in improving their products' quality if they have been neglecting the effectiveness of quality-based learning.

Next, we review the publications concerning the quality control in manufacturing processes. In [15], Lee and Rose dealt with the problem of simultaneous determination of the economic manufacturing quantity and the schedule of inspection for process maintenance, assuming that product quality is a function of the state of the production process. The authors considered the length of production run as a decision variable, and addressed the problem of joint optimization of production lot size and maintenance by inspections. Ng and Hui [18] considered the monitoring of an ongoing production process subject to complete inspection, which includes an in-control state and an out-of-control state. The production process was assumed to make products with a quality characteristic following a known probability distribution. If a product's quality characteristic exceeds an action limit, then the decision maker takes remedial action to restore the process to the in-control state; and, the decision maker has a learning opportunity to improve the process by investing in resources to identify and eliminate the causes of deviation from the target quality. The authors developed a cost model—describing the trade-off between quality cost and process improvement cost—to determine the optimal number of learning actions to be taken and the optimal action limit.

The following three publications focus on the methodologies for the quality control in a manufacturing system. Choo, Linderman, and Schroeder [7] investigates the method- and the psychologically-driven mechanisms of knowledge creation in Six Sigma projects. The authors showed that the method-driven mechanism directly influences learning behaviors, while the psychological-driven mechanism directly affects knowledge creation. Moreover, the authors found that, when a firm adopts a quality program such as Six Sigma, the method and the degree of its adherence influence the firm's innovations and knowledge creations. Chun [9] developed a maximum likelihood method and also a Bayesian method to estimate both the number of undiscovered errors in the product and the detection probability, and then compared their performances with that of the existing nonlinear regression method. The author found that it would be worthwhile to use various estimation methods and compare their estimates in a specific inspection environment. Chan and Spedding [4] used an on-line, non-disruptive neural network metamodel to obtain quality information from a manufacturing system which can thus be configured to provide optimal performance in terms of quality and productivity at the lowest cost. The authors also used a case to illustrate this approach.

2.2 Joint Pricing and Quality Assurance Decisions

We now provide a review of the publications regarding the joint pricing and quality decisions, which are involved into our model in this paper. As a seminal paper about the price and quality, Leavitt [14] experimentally examined buyers' tendencies to use price as an indicator of quality. Following Leavitt [14], a large number of empirical and experimental papers investigated the price-quality relationship from the marketing perspective; for a detailed review, see, e.g., Rao and Monroe [20].

White [29] published an early paper to analytically address the price-quality problem. The author supposed that an industry composed of n firms produces a basic output "transportation," and a second output "meals," which is a *quality* aspect of the primary output. That is, in [29], White considered two quantity decision variables including the second variable (i.e., amount of meals) as the quality measurement. Gal-Or [12] mainly aimed to consider the implications of entry upon quality levels and prices, assuming that the quality level is an observable variable. The author originally constructed a game model involving $n > 1$ firms who make products of different quality levels with the unit production cost as an increasing function of the quality level. Multiple symmetric equilibria were obtained if the distribution of consumers according to their willingness to pay is not uniform and more than one firm participates in the market. With a uniform distribution, the author showed that the impact of entry is to reduce average quality and to increase aggregate output.

In the note [28], Wauthy proposed a two-stage game where two firms "simultaneously" determine the quality level for their products in the first stage and then determine their prices to compete for consumers in the second stage. Wauthy showed that the quality differential is negatively related to the population dispersion, and found that the degree of heterogeneity in the population places an upper bound to the extent of product differentiation, which amends earlier results on the subject.

2.3 Quality Assurance in a Two-Echelon Supply Chain

We note that, for the quality problem in supply chain operations, a few publications appeared before one decade and most papers were published only during the past five years. This happens probably because the academic scholars in the management science/operations management field started to focus on supply chain-related research in the middle of 1990s. As an early paper on this subject, Chu and Chu [8] provided an example of renting the reputation of another agent to investigate a quality problem in which a manufacturer can signal quality by selling through a reputable retailers. As consumers' willingness to buy high quality products is increased, the manufacturer intends to make a high-quality good and signal its quality to consumers via a retailer. Using the signaling game, the author showed that, in a "maximally" separating equilibrium, the manufacturer of high-quality products distributes through retailers with a strong reputation, while the manufacturer of low-quality products distributes through retailers with no reputation.

Zhu, Zhang, and Tsung [31] considered a supply chain in which a buyer designs a product and owns the brand but outsources the production to a supplier. Both the buyer and the supplier incur quality-related costs, thus having an incentive to invest in quality-improvement efforts. The authors showed that the buyer's involvement for quality improvement has a significant impact on both the supplier's and the buyer's profits, and they also investigated how quality-improvement decisions interact with the buyer's order quantity and the supplier's production lot size. Saouma [22] analyzed a supply chain in which a manufacturer outsources an assembly (second-stage) task to a preestablished supplier. As the supplier invests in the quality liability under the constraint of warranty contracts, the manufacturer is shown to prefer more testing when she outsources assembly to her supplier as opposed to when she assembles products in house. The authors also found that the contracting frictions identified persist when the supplier's work is tested individually.

Chao, Iravani, and Savaskan [6] investigated a two-level supply chain involving a manufacturer and a supplier who may reach two contractual agreements by which they can share product recall costs to induce their quality improvement efforts. The two agreements includes (i) cost sharing based on selective root cause analysis and (ii) partial cost sharing based on complete root cause analysis. For each agreement, the authors used insights from supermodular game theory to characterize the levels of effort that the manufacturer and the supplier would exert in equilibrium to improve their component failure rate when their effort choices are subject to moral hazard. The authors also showed that the menu of contracts not only significantly decreases the manufacturer's cost due to information asymmetry, but also improves product quality. Hsieh and Liu [13] considered the quality improvement and inspection policy between a supplier and a manufacturer in a two-echelon supply chain, involving the supplier's and the manufacturer's quality investment and inspection strategies in four non-cooperative games with different degrees of information available. The authors investigated the effects of inspection-related information on both parties' equilibrium strategies and profits, and further discussed the rationality of the penalty on defective components.

From our above review, a large number of publications are concerned with the quality problem in a variety of operating systems. However, we note that very few papers considered a supply chain in which the manufacturer determines a wholesale price and a defective rate and the retailer serves as a quality gatekeeper to make a quality-related decision in addition to the retail pricing decision. The problem is actually important because, as discussed in Section 1, some practical retailers (e.g., Wal-Mart) have been assuring the quality as the gatekeepers. We thus accordingly investigate the problem in this paper.

3 The Game-Theoretic Model and Analysis when the Retailer Assures the Quality as a Gatekeeper

In this section, we investigate a two-echelon supply chain involving a manufacturer and a retailer. The manufacturer sells his products to the retailer at a wholesale price, who then serves consumers in a single market. To promise the product quality, the manufacturer announces a pre-determined defective rate to the retailer. In order to examine the manufacturer's quality commitment, the retailer adopts the "gatekeeping" approach to inspect the quality of incoming products. Similar to Balachandran and Radhakrishnan [1], we assume that the retailer may not find all defects in her sampling inspection process but instead identify a percentage of the defects, which depends on the retailer's inspection effort in terms of, e.g., sampling size. Hereafter, for simplicity, we name the percentage as "identification rate," which can be regarded as the retailer's decision variable, because of the following fact: Since the rate is dependent on the retailer's inspection effort, the retailer can determine her effort on the quality control and assure the identification rate.

In practice, the manufacturer should first make his decisions on the wholesale price and the committed defective rate and announce them to the retailer. Then, the retailer determines her retail price and identification rate. Accordingly, we can model the decision problem as a leader-follower (sequential) game, where the manufacturer and the retailer act as a "leader" and a "follower," respectively. As a result, these two supply chain members' optimal decisions can be characterized by using the Stackelberg equilibrium. To develop and solve the leader-follower game, we should use the backward approach to (i) analyze the retailer's best-response decisions given the manufacture's wholesale price and defective rate; (ii) investigate the manufacturer's optimal decisions using the retailer's best-response decisions; and then (iii) calculate the Stackelberg equilibrium.

Next, we start with the best-response analysis for the retailer.

3.1 The Retailer's Best-Response Analysis

We now assume that the manufacturer's wholesale price w and defective rate r are given, and maximize the retailer's profit to find her optimal retail price and identification rate. We begin by building the profit function for the retailer, who sells the manufacturer's products to meet consumers' deterministic demands. To simplify our analysis and find meaningful insights, we describe the aggregate price-dependent demand $D(p)$ in a linear form, i.e., $D(p) = a - bp$, where p denotes the retail price—which is the retailer's decision variable, and $a > 0$ and $b > 0$ represent the constant (price-*in*dependent) demand component and the sensitivity of demand to the retail price p, respectively. Such a linear price-dependent demand model has been widely used in the economics, marketing, and operations management fields; see Bertrand [2], Corbett and Karmarkar [10], Lim and Ho [17], etc.

Because the manufacturer's defective rate is $r \in [0, 1]$, the number of defects delivered by the manufacturer to the retailer is calculated as $r \times D(p)$. The retailer inspects the products, and identifies each defect only with the probability $\beta \in [0, 1]$, which represents the retailer's inspection effort. This means that the number of defects identified by the retailer is $\beta \times r \times D(p)$, and the number of defects that the retailer cannot find is $(1 - \beta) \times r \times D(p)$. For the identified and the unidentified defects, we have the following discussion:

1. When the retailer finds $\beta \times r \times D(p)$ units of the defects, she should immediately return the defects to the manufacturer, who then replaces the defects with $\beta \times r \times D(p)$ new, good-quality products. Meanwhile, the manufacturer should absorb the penalty cost $\alpha \times \beta \times r \times D(p)$, where α denotes the penalty cost that the manufacturer pays to the retailer for each unit of the

defect.

2. When the retailer cannot immediately find $(1-\beta)\times r\times D(p)$ units of the defects, the consumers will buy these defects and will eventually return them to the retailer, who thus incurs the penalty cost $\hat{\alpha}\times(1-\beta)\times r\times D(p)$ with $\hat{\alpha}$ representing the retailer's unit penalty cost. We note that, different from the manufacturer's unit penalty cost α, the retailer's unit penalty cost $\hat{\alpha}$ could be estimated as the handling and goodwill costs incurred by the retailer. But, the retailer needs to return $(1-\beta)\times r\times D(p)$ defects to the manufacturer, and charge the manufacturer the penalty cost $\alpha\times(1-\beta)\times r\times D(p)$. To assure the retailer's incentive on the quality control, we reasonably assume that $\hat{\alpha}\geq\alpha$.

From the above, we learn that $r\times D(p)$ units of defects will be replaced with good-quality products. To satisfy all consumers' demands, the retailer should order $D(p)$ units of the products from the manufacturer. It thus follows that the retailer's profit generated from the sale is calculated as $(p-w)D(p)$. In addition, the retailer absorbs the penalty cost $\hat{\alpha}\times(1-\beta)\times r\times D(p)$ for the unidentified defects. But, the retailer can receive the manufacturer's penalty $\alpha\times r\times D(p)$ while returning the identified and unidentified defects to the manufacturer. Since the retailer spends effort on the quality control to assure the identification rate β, she should incurs the $\beta-$dependent inspection cost $\hat{C}(\beta)\times D(p)$, where $\hat{C}(\beta)$ denote the unit inspection cost. Naturally, the retailer's effort and unit inspection cost $\hat{C}(\beta)$ should be increasing in β, i.e., $\hat{C}'(\beta)>0$. Moreover, when β is small (e.g., $\beta=0.1$), the retailer may not need to exert much effort to increase the identification rate, whereas, when β is large (e.g., $\beta=0.8$), the retailer may have to exert considerable effort for the improvement of the identification rate. Thus, it is reasonable to assume that $\hat{C}''(\beta)>0$.

Using the above, we can calculate the retailer's profit Π_R as her profit $(p-w)D(p)$ minus her inspection cost $\hat{C}(\beta)\times D(p)$ and her penalty cost $\hat{\alpha}\times(1-\beta)\times$

$r \times D(p)$ plus the manufacturer's penalty $\alpha \times r \times D(p)$. That is,

$$\Pi_R = (p - w)D(p) - \hat{C}(\beta) \times D(p) - \hat{\alpha} \times (1 - \beta) \times r \times D(p) + \alpha \times r \times D(p),$$

which, using $D(p) = a - bp$, can be re-written as,

$$\Pi_R = (a - bp)[(p - w) - \hat{C}(\beta) + \hat{\alpha} \times \beta \times r + (\alpha - \hat{\alpha}) \times r]. \qquad (1)$$

Theorem 1 Given the manufacturer's wholesale price w and promised defective rate r, we maximize her profit Π_R in (1) and find that the retailer's optimal identification rate β^* *uniquely* satisfies the equation that $\hat{C}'(\beta^*) = \hat{\alpha} r$. The retailer's optimal retail price p^* can be *uniquely* obtained as

$$p^* = \frac{1}{2}\left\{\frac{a}{b} + w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1 - \beta^*)]\right\}. \qquad (2)$$

Proof. The proof of this theorem and our proofs of all subsequent theorems are provided in Appendix B. ■

From the above theorem, we find that the retailer's optimal identification rate β^*—which reflects the retailer's effort on the quality assurance—is dependent on the retailer's quality control cost function $\hat{C}(\beta)$, the retailer's penalty cost $\hat{\alpha}$ for a unit of defect sold to consumers, and the manufacturer's promised defective rate r. This means that the manufacturer's wholesale price and the retailer's sale price do not affect the retailer's quality control decision. Moreover, we note that $\hat{\alpha} r$ means the retailer's *expected* penalty cost for each unit of the product bought from the manufacturer. As Theorem 1 indicates, the retailer's optimal identification rate β^* should be determined such that the retailer's marginal quality control cost is equal to her unit expected penalty cost. This result is justified as follows: If the retailer increases the identification rate (in examining each product) by one percent, then the retailer's quality assurance cost is increased by $\hat{C}'(\beta)$. For each unit of the product, the retailer incurs the expected penalty cost $\hat{\alpha} r$ for the quality problem. If $\hat{C}'(\beta^*) < \hat{\alpha}r$, then the retailer should spend more effort to increase the identification rate; otherwise, if $\hat{C}'(\beta^*) > \hat{\alpha}r$, then the retailer should decrease the rate because the quality assurance cost is higher than the expected penalty cost. Therefore, the optimal identification rate β^* should satisfy the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r$, which implies that the manufacturer can affect the retailer's optimal decision on her identification rate. More specifically, we differentiate the two sides of $\hat{C}'(\beta^*) = \hat{\alpha}r$ w.r.t. r, and find that $\hat{C}''(\beta^*)(\partial\beta^*/\partial r) = \hat{\alpha}$. As a result, $\partial\beta^*/\partial r > 0$; that is, if the manufacturer spends less effort on the quality control and his defective rate thus rises, then the retailer should respond by paying more attention to the quality gatekeeping and increasing her identification rate.

To discuss the retailer's optimal price p^*, we re-write the equation (2) that p^* satisfies (as given in Theorem 1) as,

$$\begin{aligned} p^* &= a/b - p^* + w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1-\beta^*)] \\ &= \{w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1-\beta^*)]\} + D(p^*)/b, \end{aligned}$$

where the expression in the bracket "{ }" represents the expected total cost incurred by the retailer when she sells one unit of the product. It thus follows that the retailer's expected profit generated from the sale of each unit of the product—i.e., $p^* - \{w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1-\beta^*)]\}$—is equal to the ratio of the demand to the price elasticity of demand (i.e., $D(p^*)/b$), which implies that the price sensitivity of the demand significantly impacts the retailer's pricing decision. More specifically, if the value of b—reflecting the demand's price sensitivity—increases, then $D(p^*)/b$ may decrease and the retailer's expected profit may be thus reduced. To gain a higher profit, the retailer would hope that the demand is not sensitivity to the retail price.

3.2 The Manufacturer's Optimal Decisions Given the Retailer's Best Response

We now use the retailer's best-response decisions given in Theorem 1 to compute the manufacturer's optimal decisions that maximize his profit. Note that the manufacturer incurs the acquisition cost c to make each unit of the product, and achieves the sale revenue w when a unit of the product is sold to the retailer. Since the aggregate demand for the product is $D(p) = a - bp$, as given in Section 3.1, the manufacturer's sale profit can be calculated as $D(p) \times (w - c)$, or, $(a - bp) \times (w - c)$.

In addition, as we discuss in Section 3.1, the manufacturer needs to pay the penalty cost α to the retailer for each returned defect. To assure the manufacturer's incentive on the quality control, we assume that his unit penalty cost α is greater than his unit sale profit $w - c$, i.e., $\alpha > w - c$. This assumption is reasonable because, if $\alpha \leq w - c$, then the manufacturer can profit from the sale of each defect. Because the number of defects is $r \times D(p) = r \times (a - bp)$, where r is the manufacturer's decision variable "defective rate," as defined previously. Thus, the manufacturer's total penalty cost is $\alpha \times r \times (a - bp)$. Moreover, the manufacturer needs to dispose all defects returned by the retailer. For example, the manufacturer may repair or salvage the returned defects. Without loss of generality, we, hereafter, simply consider the "disposing" cost for the manufacturer. Denoting the per defect disposing cost by s, we calculate the manufacturer's total cost of disposing the returned defects as $s \times r \times (a - bp)$.

In order to assure the manufacturer's incentive for the quality control, we should incorporate the manufacturer's cost of "promising" the defective rate r, which is denoted by $C(r)$. Naturally, in order to reduce the defective rate r, the manufacturer needs to exert more effort and incur a higher cost for the quality control. Furthermore, the manufacturer's marginal cost $C'(r)$ should be increasing in r, because it is more difficult for the manufacturer to further reduce a smaller defective rate r. That is, the cost $C(r)$ is a decreasing, convex function of the defective rate r, i.e., $C'(r) < 0$ and $C''(r) \geq 0$.

Using the above, we can construct the manufacturer's profit function as $\Pi_M = [(w-c)-(\alpha+s)r-C(r)](a-bp)$. In the non-cooperative game, the manufacturer and the retailer act as the "leader" and the "follower," respectively. That is, the manufacturer should consider the retailer's best-response decisions when he maximizes his profit to find optimal wholesale price and defective rate. Thus, for the maximization problem, we need to substitute the retailer's best-response decisions (as given in Theorem 1) into the manufacturer's profit function Π_M, which is re-written as,

$$\Pi_M = [(w-c) - (\alpha+s)r - C(r)](a - bp^*). \tag{3}$$

Theorem 2 Using the retailer's best-response decisions given in Theorem 1, we find that the manufacturer's profit Π_M in (3) is a unimodal function on (w, r) if and only if $C''(r^*)\hat{C}''(\beta^*) > \hat{\alpha}^2$ where r^* is the manufacturer's optimal defective rate satisfying the following equation:

$$s + \hat{\alpha}(1-\beta^*) = -C'(r^*); \tag{4}$$

and β^* satisfies the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r^*$, as given in Theorem 1.

More specifically, if the above necessary and sufficient condition is satisfied, then the manufacturer's optimal defective rate r^* can be *uniquely* determined by solving (4), and his optimal wholesale price w^* can be *uniquely* obtained as,

$$w^* = \left(\alpha + \frac{s}{2}\right) r^* + \frac{a}{2b} + \frac{c + C(r^*) - \hat{C}(\beta^*) - r^*\hat{\alpha}(1-\beta^*)}{2}. \tag{5}$$

Otherwise, if the above condition is not satisfied, then the manufacturer's optimal solutions may not be unique. ■

The above theorem indicates that the uniqueness of the optimal wholesale price and defective rate depends on the condition that $C''(r)\hat{C}''(\beta^*) > \hat{\alpha}^2$ when

r^* is attained by solving (25). The necessary and sufficient condition implies that, when the defective rate is increased from the point satisfying the first-order condition, the product of the increase in the slope of the manufacturer's quality assurance cost and the increase in the slope of the retailer's quality gatekeeping cost should be larger than $\hat{\alpha}^2$. Hereafter, we assume that the cost functions $C(r)$ and $\hat{C}(\beta)$ possess the property that $C''(r)\hat{C}''(\beta^*) > \hat{\alpha}^2$ at the point r^* satisfying (25). For our problem, this assumption is reasonable because of the following fact: the function $C(r)$ is decreasing and convex in $r \in [0, 1]$, and $\hat{C}(\beta)$ is increasing and convex in $\beta \in [0, 1]$. Therefore, if r is increased when it is small, then the manufacturer's effort on the quality assurance is reduced at a large rate; but, if β is increased when it is small, then the retailer's effort is increased at a small rate. Recalling from the proof of Theorem 2 that $\partial\beta^*/\partial r = \hat{C}''(\beta^*)/\hat{\alpha} > 0$, we find that β^*—satisfying the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r$—is increasing in r. When r is increased, the retailer should respond by increasing β^*. Note that, when the value of r is small, the value of β^* may not be small. Thus, when r is *properly* determined as a small value, $C''(r)\hat{C}''(\beta^*)$ could be greater than $\hat{\alpha}^2$. But, if the value of r is very small, then the value of β^* could be also very small and $\hat{C}''(\beta^*)$ may thus approach zero. As a result, even though $C''(r)$ is very large when r is very small, $C''(r)\hat{C}''(\beta^*)$ may be smaller than $\hat{\alpha}^2$. Hence, for our problem, the value of r should be determined as in a proper range such that $C''(r)\hat{C}''(\beta^*) > \hat{\alpha}^2$. In practice, the manufacturer cannot promise a very small defective rate (e.g., $r < 1\%$) to the retailer, because the corresponding quality assurance cost is very high; and, the manufacturer cannot determine a high defective rate (e.g., $r < 10\%$) because he would incur a high penalty cost and disposing cost for returned defects. Thus, we reasonably assume that the value of r is *properly* determined as in, e.g., $[1\%, 10\%]$.

From Theorem 2, we note that the manufacturer's optimal defective rate r^*—that characterizes his effort on the quality control—depends on the retailer's unit defective penalty cost $\hat{\alpha}$, the manufacturer's per defect disposing cost s, and the retailer's best-response identification rate β^*. According to our best-response analysis for the retailer in Section 3.1, we find that β^* is a function of the value of $\hat{\alpha}$ and the manufacturer's defective rate r. It thus follows that the manufacture's optimal defective rate satisfying the equation that $s + \hat{\alpha}(1 - \beta^*) = -C'(r^*)$ depends on the values of $\hat{\alpha}$ and s. The above reveals an important, and interesting, result as follows: when the retailer serves as the quality gatekeeper, the manufacturer's optimal defective rate is *independent* of his per defect penalty cost α but *dependent* on the retailer's unit penalty cost $\hat{\alpha}$ and his own cost of disposing each returned defect. The result occurs because of the following possible reasons: for each defect, the retailer's penalty cost is higher than or the same as the manufacturer's penalty cost; i.e., $\hat{\alpha} \geq \alpha$. The manufacturer should thus treat $\hat{\alpha}$ as an important factor in determining the optimal defective rate, in order to encourage the retailer to sell his products. Otherwise, the retailer may lose an incentive to provide the retailing service for the manufacturer.

Moreover, the unit disposing cost s certainly impacts the manufacturer's decision on the defective rate. If the manufacturer incurs a higher cost to dispose the defect, then he may have an incentive to spend more effort on the quality control and reduce the defective rate. Note that, as the equation in (4) implies, the manufacturer's unit disposing cost s should be smaller than or equal to his optimal marginal quality-control cost $-C'(r^*)$, i.e., $s \leq -C'(r^*)$. In fact, if $s \geq -C'(r^*)$, then the manufacturer may desire to reduce his defective rate to zero, because the disposal of defects is costly. For such a scenario, the retailer does not need to spend any effort on the quality gatekeeping; and, as a result, her optimal identification rate becomes zero (i.e., $\beta^* = 0$) and the manufacturer's optimal defective rate r^* should be determined such that $s = -C'(r^*)$. According to the above discussion, we find that $-C'(r^*) - s \geq 0$, which means that the difference between the manufacturer's quality control cost and his disposing cost. Such a difference can be treated as the manufacturer's "net quality control cost."

Corollary 1 The ratio of the manufacturer's optimal defective rate (i.e., r^*) to the retailer's optimal percentage of unidentified defects (i.e., $1-\beta^*$) is equal to the ratio of the retailer's marginal quality assurance cost [i.e., $\hat{C}'(\beta^*)$] to the manufacturer's net quality control cost (i.e., $-[s+C'(r^*)]$). That is,

$$\frac{r^*}{1-\beta^*} = \frac{\hat{C}'(\beta^*)}{-[s+C'(r^*)]}.$$

Proof. For a proof of this corollary and the proofs of all subsequent corollaries, see Appendix C. ■

The above corollary indicates an interesting result as follows: the manufacturer's and the retailer's optimal *relative* efforts on their quality assurance—i.e., $r^*/(1-\beta^*)$—should be *negatively correlated* with their *relative* marginal quality control costs—i.e., $\hat{C}'(\beta^*)/\{-[s+C'(r^*)]\}$. That is, if the retailer's marginal cost for her quality assurance—i.e., $\hat{C}'(\beta^*)$—is higher than the manufacturer's marginal cost—i.e., $-C'(r^*)-s$, then the percentage of the defects that cannot be identified by the retailer (i.e., $1-\beta^*$) is smaller than the defective rate announced by the manufacturer (i.e., r^*). Otherwise, $1-\beta^* \geq r^*$. This result implies that a supply chain member with a relatively higher marginal cost for quality assurance may exert an effort to assure fewer defects flowing to its downstream or consumers, which may be "surprising." We justify the result as follows: Recall from Section 3.1 that the retailer's quality-gatekeeping cost $\hat{C}(\beta)$ is an increasing, convex function of the identification rate β, i.e., $\hat{C}'(\beta), \hat{C}''(\beta) > 0$. This means that a high value of the marginal cost $\hat{C}'(\beta)$ implies a high value of β. Noting that the manufacturer's quality control cost $C(r)$ is a decreasing, convex function of its announced defective rate r, i.e., $C'(r) < 0$ and $C''(r) \geq 0$, we find that a low value of the marginal cost "$-[s+C'(r)]$" means a high value of r. Therefore, if $\hat{C}'(\beta^*) > -C'(r^*) - s$, then the retailer's optimal identification rate β^* could be significantly high and the manufacturer's optimal (announced) defective rate r^* could be significantly high. It thus follows that the percentage $1-\beta^*$ could be smaller than r^* when $\hat{C}'(\beta^*) > -C'(r^*) - s$.

3.3 Stackelberg Equilibrium

According to our best-response analysis for the retailer in Section 3.1 and the optimization analysis for the manufacturer in Section 3.2, we can easily obtain the Stackelberg equilibrium for the sequential game where the manufacturer and the retailer act as the leader and the follower, respectively. Specifically, we can find the Stackelberg equilibrium using the following three stages:

1. We first find the manufacturer's defective rate r^S and the retailer's identification rate β^S in Stackelberg equilibrium by solving the following equations:

$$\begin{cases} \hat{C}'(\beta^S) = \hat{\alpha} r^S, \\ s + \hat{\alpha}(1 - \beta^S) = -C'(r^S). \end{cases} \tag{6}$$

2. We replace r^* and β^* in (5) with r^S and β^S, respectively; and, we then calculate the Stackelberg equilibrium-characterized wholesale price w^S for the manufacturer.

3. We use r^S, β^S, and w^S to replace r, β^*, and w in (2), respectively, and we can find the retailer's price in Stackelberg equilibrium p^S.

Using r^S, β^S, w^S, and p^S, we can re-write (2) as,

$$\frac{a}{2b} = p^S + \frac{\alpha r^S}{2} - \frac{w^S}{2} - \frac{\hat{C}(\beta^S) + \hat{\alpha} r^S (1 - \beta^S)}{2}.$$

Using the above to replace a/b in (5), we find that

$$w^S - c - (\alpha + s) r^S - C(r^S) = 2[p^S - w^S - \hat{C}(\beta^S) + \hat{\alpha}\beta^S r^S + (\alpha - \hat{\alpha}) r^S] = 2\frac{D(p^S)}{b}, \tag{7}$$

where the second equation follows our discussion in Section 3.1. According to our analysis in Sections 3.1 and 3.2, the expression in the bracket "[ ]" of the RHS and the expression in the LHS of (7) mean the retailer's and the manufacturer's profit generated from the sale of a unit of the product, respectively. Therefore, noting that the sales are calculated as $D(p^S) = a - bp^S$, we find from (7) that, in the sequential game with the retailer as the quality gatekeeper, the manufacturer achieves his profit twice as much as the retailer's profit. This important result implies that the manufacturer benefits more from the retailer's quality gatekeeping than the retailer. In fact, some existing publications concerning supply chain analysis also found that, for some cases, the retailer's profit may be one half of the manufacturer's profit. For example, Burnetas and Ritchken [3] showed that, no matter whether or not option contracts are used, the retailer's profit always equals one-half of the manufacturer's profit. For other examples, see, e.g., Zhou et al. [30].

To illustrate the above game analysis, we provide the following numerical example.

Example 1 Consider a two-echelon supply chain in which a manufacturer and a retailer serve consumers whose aggregate demand for the manufacturer's product is measured in thousands of units, and is characterized by the linear function $D(p) = a - bp$ with $a = 150$ and $b = 1$. For each unit of product, the manufacturer absorbs the acquisition cost $c = \$80$, and spends an effort to control the quality of the product with a promised defective rate r to the retailer. If the manufacturer sells a unit of defect to the retailer who may or may not identify the defect, then the manufacturer will need to pay a penalty cost \$100 to the retailer, i.e., $\alpha = \$100$, and he will also have to dispose the defect (e.g., repair or salvage the defect) at the unit cost $s = \$10$. To assure the quality, the retailer serves as a gatekeeper to examine the manufacturer's product and achieve the defect identification rate β. If the retailer can identify a defect, then she will return it to the manufacturer and will not experience any loss; but, if the retailer cannot find the defect and sells it to a consumer, then the consumer will later return it to the retailer, who incurs the penalty cost $\hat{\alpha} = \$120$ even though she can charge the manufacturer the cost α.

As discussed in Section 3.2, the manufacturer incurs the quality control cost $C(r)$ in order to assure the defective rate r; and similarly, as discussed in Section 3.1, the retailer incurs the quality gatekeeping cost $\hat{C}(\beta)$ to identify each defect with the probability β. For our numerical study in this paper, we specify the above two quality control cost functions as,

$$C(r) = z_1 \exp(-z_2 r) \quad \text{and} \quad \hat{C}(\beta) = z_3 \exp(z_4 \beta), \text{ where } z_i > 0 \text{ for } i = 1, \ldots, 4,$$

which satisfy the following properties that are required for our problem: $C'(r) < 0$, $C''(r) > 0$; and $\hat{C}'(\beta) > 0$ and $\hat{C}''(\beta) > 0$. For this numerical example, we set $z_1 = 10$, $z_2 = 15$, $z_3 = 0.34$, and $z_4 = 3.2$. Noting from Theorem 2 that, to assure the uniqueness of the manufacturer's optimal solutions, we need to examine the necessary and sufficient condition that $C''(r^*)\hat{C}''(\beta^*) > \hat{\alpha}^2$ when r^* is attained by solving (25). Using the above parameters, we find that, if $2\% < r^* < 15\%$, then $C''(r^*)\hat{C}''(\beta^*) > \hat{\alpha}^2$ and the Stackelberg equilibrium must be unique.

Next, we compute Stackelberg equilibrium for the manufacturer and the retailer, following the above three-stage method. That is, in the first stage, we solve the equations in (6) to calculate the manufacturer's defective rate and the retailer's identification rate in Stackelberg equilibrium as $r^S = 6.67\%$ and $\beta^S = 62.34\%$, respectively. Since $r^S \in (2\%, 15\%)$, the Stackelberg equilibrium for the game is unique. In fact, in all subsequent numerical examples, we find that the necessary and sufficient condition is always satisfied and Stackelberg equilibrium—for the game *with* the retailer's quality gatekeeping effort—is thus unique. We will not again examine the uniqueness.

Then, in the second stage, we use (5) to find the manufacturer's Stackelberg equilibrium-characterized wholesale price as $w^S = \$121.08$. In the third stage, we use (2) to calculate the retailer's price in Stackelberg equilibrium as $p^S = \$134.96$.

The above result means that, in equilibrium, the manufacturer first announces his wholesale price $w^S = \$121.08$ and the defective rate $r^S = 6.67\%$ to the retailer, who then determines her identification rate and retail price as $\beta^S = 62.34\%$ and $p^S = \$134.96$.

It thus follows that the resulting aggregate demand is $D(p^S) = 1.504 \times 10^4$, and the manufacturer and the retailer achieve their profits as $\Pi_M = \$4.52 \times 10^5$ and $\Pi_R = \$2.26 \times 10^5$, respectively. Note that the manufacturer's profit is twice as much as the retailer's profit, as shown above. $\triangleleft$

4 Impacts of the Retailer's Quality Gatekeeping on the Supply Chain Operation

In this section, we investigate the impacts of the retailer's quality assurance on the manufacturer's and the retailer's equilibrium decisions and their profits. For such an analysis, we need to find the manufacturer's and the retailer's equilibrium decisions when the retailer does not act as a quality gatekeeper. The results are then compared with those obtained in Section 3—where the retailer is assumed to assure the quality as a gatekeeper—to draw managerial insights regarding the impacts of the retailer's quality control. In addition, we perform analytic and numerical sensitivity analysis to examine the effects of some important parameters (e.g., α and $\hat{\alpha}$) on the supply chain with the retailer as a quality gatekeeper. Next, we start with the supply chain analysis, assuming that the retailer does not take the quality gatekeeping role.

4.1 The Game-Theoretic Analysis with No Quality Gatekeeping of the Retailer

We now assume that the retailer does not exert any effort to assure the quality as a gatekeeper. Under the assumption, the retailer does not need to determine her identification rate as in Section 3. We calculate the manufacturer's optimal decision of wholesale price and defective rate, and compute the retailer's sale price. Similar to Section 3, we next analyze a sequential game, where the manufacturer, as a "leader," first announces his decisions on the wholesale price and the defective rate; and the retailer, as a "follower," then decides on her retail price.

4.1.1 The Best-Response Analysis for the Retailer who Does Not Serve as the Quality Gatekeeper

In Section 3.1, we particularly discuss all components in the retailer's profit function when she is involved into the quality assurance as a gatekeeper. Our analysis in this section differs in the following issue: When the retailer examines the quality, she can identify some defects with the rate β and thus reduce the defective rate from the manufacturer's promised rate r to $(1-\beta)\times r$. Recalling that the aggregate price-dependent demand is $D(p) = a - bp$, we find that, as a result of the retailer's quality gatekeeping, the number of defects that are sold to consumers is reduced from $r \times (a-bp)$ to $(1-\beta)\times r \times (a-bp)$ by $\beta \times r \times (a-bp)$. But, in this section, the retailer does not serve as the quality gatekeeper to assure the quality. As a consequence, the defective rate faced by consumers is r, which is determined by the manufacturer.

Therefore, when the retailer does not assure the quality as the gatekeeper, the retailer's resulting profit π_R as her profit $(p-w)\times D(p)$ minus her penalty cost $\hat{\alpha}\times r \times D(p)$ plus the manufacturer's penalty $\alpha \times r \times D$ (which is paid to the retailer); i.e., $\pi_R = (p-w)\times D(p) + (\alpha-\hat{\alpha})\times r \times D(p) = D(p)\times[(p-w)+(\alpha-\hat{\alpha})\times r]$, or,

$$\pi_R = (a-bp)\times[(p-w)+(\alpha-\hat{\alpha})\times r], \tag{8}$$

where the retailer only has one decision variable—retail price p. Note that, different from Section 3.1 where Π_R and Π_M denote the retailer's and the manufacturer's profits, respectively, we use π_R and π_M to represent the two supply chain members' profits.

Theorem 3 Suppose that, in the two-echelon supply chain, the retailer does not serve as the quality gatekeeper. When the manufacturer's wholesale price w and defective rate r are given, we find that the retailer's optimal retail price $\bar{p}^*$ maximizing π_R in (8) can be *uniquely* obtained as,

$$\bar{p}^* = \frac{1}{2}\left[\frac{a}{b} + w - r(\alpha - \hat{\alpha})\right]. \quad \blacksquare \tag{9}$$

In the above theorem, the bar symbol ($^-$) indicates the optimal retail price in the case that the retailer does not act as a quality gatekeeper. Similarly, we write all sequential optimal solutions with the bar symbol, in order to distinguish between them and those in Section 3.1. We re-write (9) as follows: $\bar{p}^* = [w - r(\alpha - \hat{\alpha})] + D(\bar{p}^*)/b$, where the expression in the bracket ([ ]) means the retailer's cost for each unit of the product. Therefore, the retailer's best-response retail price—given in (9)—is her cost plus the unit profit $D(\bar{p}^*)/b$, which is the ratio of the demand to the price elasticity of demand. The result is similar to that in Section 3.1 where, when the retailer acts as the quality gatekeeper, the retailer's unit profit is $D(p^*)/b$, where $p^* = \{a/b + w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1 - \beta^*)]\}/2$ as given in (2).

Corollary 2 Given the manufacturer's wholesale price w and defective rate r, we compare the retailer's unit profit when she serves as the quality gatekeeper with that when she does not implement the quality gatekeeping. We find that, if $\hat{C}(\beta^*) < \hat{\alpha} r \beta^*$, then the retailer's gatekeeping reduces the retail price (i.e., $p^* < \bar{p}^*$), and the retailer thus serves more demands [i.e., $D(p^*) > D(\bar{p}^*)$] and enjoys a higher unit profit [i.e., $D(p^*)/b > D(\bar{p}^*)/b$]. Otherwise, if $\hat{C}(\beta^*) \geq \hat{\alpha} r \beta^*$, then the retailer's quality assurance increases or does not change the retail price (i.e., $p^* \geq \bar{p}^*$), and thus, the retailer serves less or the same demands [i.e., $D(p^*) \leq D(\bar{p}^*)$] and enjoys a lower or the same unit profit [i.e., $D(p^*)/b \leq D(\bar{p}^*)/b$]. $\blacksquare$

As the above corollary indicates, whether or not the retailer should reduce her price as a result of serving as the quality gatekeeper is dependent on the comparison between her quality gatekeeping cost $\hat{C}(\beta^*)$—which is absorbed by the retailer during the inspection of each product to achieve the identification rate β^*—and the term $\hat{\alpha}r\beta^*$. Noting that $\hat{\alpha}$ is the unit penalty cost by the retailer and r is the manufacturer's defective rate, we find that $\hat{\alpha}r\beta^*$ represents the expected reduction in the retailer's penalty cost for each product. Therefore, $\hat{C}(\beta^*)$ and $\hat{\alpha}r\beta^*$ can be regarded as the retailer's "investment" on the quality gatekeeping and "gain" generated from the gatekeeping. If the retailer's investment is smaller than her gain (i.e., $\hat{C}(\beta^*) < \hat{\alpha}r\beta^*$), then the gatekeeping is worth implementing and the retailer is thus willing to reduce her price and attract consumers.

4.1.2 The Manufacturer's Optimal Decisions Given the Best Response of the Retailer with No Quality Gatekeeping

We now find the manufacturer's optimal wholesale price and defective rate that maximize his profit, using the retailer's best-response decision as given in Theorem 3. Note that, no matter whether or not the retailer acts as the quality gatekeeper, the manufacturer's profit is calculated as his sales revenue $(w-c)\times D(p)$ minus the sum of his penalty cost $\alpha\times r\times D(p)$, his defect-disposal cost $s\times r\times D(p)$, and his quality assurance cost $C(r)\times D(p)$. Therefore, when the retailer does not spend any effort on the quality assurance, the manufacturer's profit is obtained as, $\pi_M = [(w-c)-(\alpha+s)r - C(r)]\times D(p)$, which is the same as Π_M in (3) when the retailer serves as the quality gatekeeper.

Theorem 4 For the sequential game with no quality assurance by the retailer, we use the retailer's best-response retail price is $\bar{p}^*$ as given in (9) to find that the manufacturer's profit π_M is jointly concave on (w, r). The optimal defective rate $\bar{r}^*$ uniquely satisfies the following equation:

$$s + \hat{\alpha} = -C'(\bar{r}^*); \tag{10}$$

and the optimal wholesale price $\bar{w}^*$ can be uniquely computed as,

$$\bar{w}^* = \alpha\bar{r}^* + \frac{1}{2}\left[\frac{a}{b} + c + C(\bar{r}^*) + (s - \hat{\alpha})\bar{r}^*\right]. \quad \blacksquare \tag{11}$$

From the above theorem, we learn that, when the retailer does not serve as the quality gatekeeper, the manufacturer's optimal defective rate $\bar{r}^*$ and optimal $\bar{w}^*$ wholesale price can be both uniquely determined by solving the equations in (10) and (11), respectively. Moreover, $\bar{r}^*$ only is dependent on the retailer's per defect penalty cost $\hat{\alpha}$, the manufacturer's defect-disposing cost s, and the manufacturer's quality assurance cost function $C(\cdot)$. Similar to our analysis in Section 3.2, we find that, when the retailer does not serve as the quality gatekeeper, the manufacturer's optimal defective rate does not depend on his per defect penalty cost α but only depends on the retailer's unit penalty cost $\hat{\alpha}$ and the manufacturer's disposal cost s, possibly because $\hat{\alpha} \geq \alpha$.

Next, we examine the impacts of the retailer's gatekeeping on the manufacturer's optimal decisions. From (4) and (10), we note that, when the retailer serves as a gatekeeper, the manufacturer's optimal defective rate r^* uniquely satisfies the equation that $s + \hat{\alpha}(1 - \beta^*) = -C'(r^*)$; but, when the retailer does not assure the quality, the manufacturer's optimal defective rate is $\bar{r}^*$, which can be uniquely obtained by solving the equation that $s + \hat{\alpha} = -C'(\bar{r}^*)$. Since $0 \leq \beta^* \leq 1$, we find that $-C'(r^*) \leq -C'(\bar{r}^*)$, or, $C'(r^*) \geq C'(\bar{r}^*)$. Recall from Section 3.2 that the manufacturer's quality assurance cost $C(r)$ is decreasing and convex, i.e., $C'(r) < 0$ and $C''(r) \geq 0$. It thus follows that $r^* \geq \bar{r}^*$. That is, as a result of the retailer's quality gatekeeping, the manufacture raises his defective rate that is announced to the retailer. This result may be justified as follows: as the retailer serves as the quality gatekeeper, the rate of the defects sold to consumers is reduced. Thus, the manufacturer may respond by reducing his effort on the quality control.

In addition, we learn from (5) and (11) that, if the retailer acts as the gatekeeper, then the manufacturer's optimal wholesale price w^* is uniquely obtained as $w^* = \alpha r^* + [a/b + c + C(r^*) - \hat{C}(\beta^*) - r^*\hat{\alpha}(1-\beta^*) + sr^*]/2$; otherwise, the manufacturer's optimal wholesale price $\bar{w}^*$ is uniquely found as $\bar{w}^* = \alpha\bar{r}^* + [a/b + c + C(\bar{r}^*) + (s - \hat{\alpha})\bar{r}^*]/2$. Then, we have,

$$w^* - \bar{w}^* = \kappa/2 = (\kappa_1 - \kappa_2)/2 \tag{12}$$

where

$$\begin{cases} \kappa_1 \equiv r^*\hat{\alpha}\beta^* - \hat{C}(\beta^*) - (\hat{\alpha} - \alpha)(r^* - \bar{r}^*), \\ \kappa_2 \equiv [(s+\alpha)\bar{r}^* + C(\bar{r}^*)] - [(s+\alpha)r^* + C(r^*)]. \end{cases} \tag{13}$$

In (13), $\hat{\alpha}r^*\beta^*$ and $\hat{C}(\beta^*)$ are the retailer's penalty cost reduction generated by identifying one unit of defect and the retailer's unit quality-gatekeeping cost, respectively. Moreover, in (13), $(\hat{\alpha} - \alpha)(r^* - \bar{r}^*)$ is explained as the *maximum* unit penalty cost incurred by the retailer due to the sale of a defect, because of the following fact: we learn from Section 3.1 that, if the retailer sells a unit of defect, then she incurs the penalty cost $\hat{\alpha}$ but charges the manufacturer the penalty cost α ($\alpha \le \hat{\alpha}$). Thus, the term $(\hat{\alpha} - \alpha)$ means the retailer's *net* penalty cost per defect. In addition, r^* and $\bar{r}^*$ ($r^* > \bar{r}^*$, as discussed previously) are the manufacturer's Stackelberg equilibrium defective rates with and without the retailer's gatekeeping effort. Noting that the retailer's defect identification rate is β^* (as the quality gatekeeper) and the defective rate faced by consumers is $r^*(1-\beta^*)$ where $0 \le \beta^* \le 1$, we find that the term $(r^S - \bar{r}^S)$ represents the retailer's maximum defective rate. It thus follows that $(\hat{\alpha} - \alpha)(r^S - \bar{r}^S)$ is the maximum unit penalty cost that the retailer incurs when she sells a defect.

In addition, we note that $(s+\alpha)\bar{r}^* + C(\bar{r}^*)$ $[(s+\alpha)r^* + C(r^*)]$ means the manufacturer's per unit total cost for quality assurance (i.e., the defective penalty cost and the quality control cost) when the retailer does not serve [serves] as the quality gatekeeper. Therefore, the terms κ_1 and κ_2 in (13) are explained as the retailer's and the manufacturer's per unit quality-related cost savings (reductions) generated by the retailer's quality gatekeeping, respectively. Note that each of κ_1 and κ_2 may be positive or may be negative. If $\kappa_1 < 0$, then $-\kappa_1$ is the retailer's unit cost increment; and, if $\kappa_2 < 0$, then κ_2 is the manufacturer's unit cost increment.

According to the above, we conclude that, when the retailer acts as the quality gatekeeper, the manufacturer may increase his wholesale price, which depends on the comparison between the manufacturer's and the retailer's cost savings resulting from the retailer's gatekeeping. More specifically, if the retailer's cost savings are higher than the reduction in the manufacturer's quality-related cost—i.e., $\kappa_1 > \kappa_2$, then the manufacturer should raise his wholesale price; otherwise, the manufacturer should maintain or decrease the wholesale price. The result indicates that the manufacturer's wholesale pricing decision is associated with his own and the retailer's benefits from the quality assurance, because the manufacturer with a low cost reduction may desire to increase his wholesale price to "share" the retailer's cost savings.

Furthermore, we find that κ in (12) actually denotes the different between the retailer's and the manufacturer's per unit cost savings as a result of the retailer's quality gatekeeping. Note that, if $\kappa > 0$, then the retailer's cost reduction is higher than the manufacturer's cost reduction; if $\kappa = 0$, then the retailer and the manufacturer enjoy the same cost reduction; and, if $\kappa < 0$, then the retailer has a lower cost reduction. It thus follows that the increase in the manufacturer's optimal wholesale price is a half of the difference between the retailer's and the manufacturer's unit cost savings. This implies that the manufacturer may intend to obtain a part of the retailer's cost savings through his wholesale pricing decision.

4.1.3 Stackelberg Equilibrium for the Game with No Quality Gatekeeping of the Retailer

We now use the retailer's best-response solution and the manufacturer's optimal decisions to find the two supply chain members' Stackelberg equilibrium decisions. Similar to Section 3.3 where the retailer assures the quality as the gatekeeper, we can find the equilibrium through the following three stages: In Stage 1, we solve the equation in (10) to find the manufacturer's defective rate $\bar{r}^S$ in Stackelberg equilibrium. In Stage 2, we substitute $\bar{r}^S$ into the equation in (11), and calculate the equilibrium wholesale price $\bar{w}^S$ for the manufacturer. In Stage 3, we replace r and w in (9) with $\bar{r}^S$ and $\bar{w}^S$, respectively, and obtain the retailer's Stackelberg equilibrium retail price $\bar{p}^S$.

Next, we compare the manufacturer's and the retailer's profits when the two supply chain members choose their decisions in Stackelberg equilibrium. Noting that each firm's profit is calculated as the aggregate demand $D(\bar{p}^S)$ times the firm's unit profit, we need to compare the manufacturer's unit profit [i.e., $(\bar{w}^S - c) - (s+\alpha)\bar{r}^S - C(\bar{r}^S)$] with the retailer's unit profit [i.e., $(\bar{p}^S - \bar{w}^S) + (\alpha - \hat{\alpha})\bar{r}^S$].

Substituting $\bar{r}^S$ and $\bar{w}^S$ into (9), we find that $\bar{p}^S - \bar{w}^S + \bar{r}^S(\alpha - \hat{\alpha}) = -\bar{p}^S + a/b = D(\bar{p}^S)/b$, which means that the retailer's unit profit is $D(\bar{p}^S)/b$, as discussed in Section 4.1.1. Then, using $\bar{r}^S$ and $\bar{w}^S$ to replace $\bar{r}^*$ and $\bar{w}^*$ in (10) and (11), respectively, we have the manufacturer's unit profit as,

$$\bar{w}^S - c - (s+\alpha)\bar{r}^S - C(\bar{r}^S) = \frac{a}{b} - \bar{w}^S - s\bar{r}^S + \bar{r}^S(\alpha - \hat{\alpha}),$$

which is greater than the retailer's unit profit $\bar{p}^S - \bar{w}^S + \bar{r}^S(\alpha - \hat{\alpha})$, if and only if $a/b - s\bar{r}^S > \bar{p}^S$ or $D(\bar{p}^S)/b > s\bar{r}^S$. The difference between the manufacturer's and the retailer's unit profits is computed as,

$$[\bar{w}^S - c - (s+\alpha)\bar{r}^S - C(\bar{r}^S)] - [\bar{p}^S - \bar{w}^S + \bar{r}^S(\alpha - \hat{\alpha})] = \frac{D(\bar{p}^S)}{b} - s\bar{r}^S. \quad (14)$$

Because the retailer's unit profit is $D(\bar{p}^S)/b$, we find that, if the retailer does not serve as the quality gatekeeper, then the manufacturer's unit profit is smaller than double of the retailer's unit profit, and thus the manufacturer's total profit is also smaller than double of the retailer's total profit. This result is different from that when the retailer acts as the quality gatekeeper.

We learn from the above result and the result in Section 3.3 that, when the retailer spends an effort on the quality gatekeeping, the ratio of the manufacturer's profit to the retailer's profit is increased. This means that the retailer's quality gatekeeping effort leads the manufacturer to make a higher profit compared with the retailer. Furthermore, comparing the above and our analysis in Section 3, we draw some important insights as given in the following theorem.

Theorem 5 The impacts of the retailer's quality gatekeeping on the manufacturer's and the retailer's equilibrium decisions and profits are found as follows:

1. The manufacturer's Stackelberg equilibrium defective rate r^S when the retailer serves as the quality gatekeeper is higher than the rate $\bar{r}^S$ when the retailer does not take the gatekeeper role.

2. If, as a result of the retailer's gatekeeping, the retailer's unit cost savings κ_1 [as given in (13)] are larger than the manufacturer's unit cost reduction κ_2 [also, as given in (13)], then the wholesale price w^S for the game with the retailer's gatekeeping is higher than the price $\bar{w}^S$ for the game with no gatekeeping by the retailer, i.e., $w^S > \bar{w}^S$. Otherwise, $w^S \leq \bar{w}^S$.

3. The impacts of the retailer's gatekeeping on the equilibrium retail price, and the resulting demand and profits of two supply chain members are dependent on the value of $\kappa_1^S + \kappa_2^S$, where

$$\begin{cases} \kappa_1^S \equiv r^S \hat{\alpha} \beta^S - \hat{C}(\beta^S) - (\hat{\alpha} - \alpha)(r^S - \bar{r}^S), \\ \kappa_2^S \equiv [(s + \alpha)\bar{r}^S + C(\bar{r}^S)] - [(s + \alpha) r^S + C(r^S)]. \end{cases} \tag{15}$$

(a) If $\kappa_1^S + \kappa_2^S > 0$, then the retailer's equilibrium price p^S when the retailer serves as the gatekeeper is smaller than her equilibrium price $\bar{p}^S$ when the retailer does not serve as the gatekeeper, i.e., $p^S < \bar{p}^S$. As a result, the demand $D(p^S)$ is greater than $D(\bar{p}^S)$, i.e., $D(p^S) > D(\bar{p}^S)$; and, the retailer's profit with her gatekeeping effort is higher than those with no gatekeeping.

(b) If $\kappa_1^S + \kappa_2^S = 0$, then the retailer's quality does not affect the manufacturer's and the retailer's equilibrium pricing decisions, and the resulting demands and profits of the two supply chain members.

(c) If $\kappa_1^S + \kappa_2^S < 0$, then $p^S > \bar{p}^S$, $D(p^S) < D(\bar{p}^S)$, and the retailer is worse off due to the retailer's gatekeeping effort. ■

We learn from Section 3.3 that, when the retailer serves as the quality gatekeeper, the manufacturer's and the retailer's unit profits are $2 \times D(p^S)/b$ and $D(p^S)/b$, respectively; thus, their total profits are calculated as $2 \times [D(p^S)]^2/b$ and $[D(p^S)]^2/b$. When the retailer does not exert any gatekeeping effort, we find that the retailer's total profit is $[D(\bar{p}^S)]^2/b$ but the manufacturer's total profit is smaller than $2 \times [D(\bar{p}^S)]^2/b$. Therefore, we find that, if $\kappa_1^S + \kappa_2^S > 0$, then the retailer enjoy higher profits from the retailer's gatekeeping but the manufacturer may not benefit. Otherwise, their profits may be lower than or the same as those when the retailer does not serve as the quality gatekeeper.

We find that $\kappa_1^S + \kappa_2^S$ in Theorem 5 means the supply chain-wide *unit* cost reduction that is induced by the retailer's quality gatekeeping effort. If the manufacturer and the retailer can jointly achieve a positive cost reduction as a result of the retailer's quality gatekeeping, then the retailer are better off than when the retailer does not serve as the quality gatekeeper, which happens possibly because of the following reason: When the supply chain-wide cost savings are positive, the retailer's quality gatekeeping effort results in a lower retail price, a larger demand, and a higher profit for the retailer. This reflects the retailer's "contribution" to the supply chain.

Note from our discussions in Section 3.3 and this section that, with the retailer's gatekeeping effort, the retailer's unit profit is $D(p^S)/b$; and, without the retailer's gatekeeping effort, the retailer's unit profit is $D(\bar{p}^S)/b$. Next, for the retailer, we compute the "Lerner index"—which is a well-known index reflecting a firm's market power; see Lerner [16]—the ratio of the retailer's unit profit to the corresponding Stackelberg equilibrium price. When the retailer assures the quality with her gatekeeping effort, her Lerner index is $a/(bp^S) - 1$, respectively. When the retailer is not involved into the quality assurance, the retailer's index is $a/(b\bar{p}^S) - 1$. As Theorem 5 indicates, if the supply chain can enjoy a higher profit as a result of the retailer's gatekeeping effort, i.e., $\kappa_1^S + \kappa_2^S > 0$, then $p^S < \bar{p}^S$ and the retailer's Lerner index is higher than that without the retailer's effort. However, since the manufacturer's profit may or may not be increased when the retailer is involved into the quality assurance, we cannot analytically determine if the manufacturer's Lerner index rises as a consequence of the retailer's quality assurance.

According to the above, we summarize our managerial insights as in the following remark.

Remark 1 When the retailer serves as the quality gatekeeper, the manufacturer would exert less effort on the quality control and his defective rate is thus increased. But, the manufacturer may or may not increase his wholesale price, which depends on whether or not his cost reduction is greater than the retailer's cost savings.

Even though the retailer's gatekeeping effort results in a higher defective rate, the retailer may still benefit, which depends on whether or not the supply chain can enjoy a positive cost reduction. If the supply chain-wide cost savings result from the retailer's gatekeeping effort, then the retailer reduces her retail price and the resulting demand is higher. As a consequence, the retailer's profit is higher than when the retailer does not serve as the quality gatekeeper. Moreover, the retailer's market power is stronger when the retailer implements the gatekeeping. Otherwise, if the supply chain cannot benefit from the retailer's quality assurance, then the retail price is increased; and, the retailer is worse off because her profit and market power are both reduced. The interesting result reveals that the retailer's pricing decision relies on the impact of the retailer's gatekeeping effort on the supply chain-wide profit rather than only the retailer's own profit. $\lhd$

To illustrate our above analysis, we provide a numerical example below.

Example 2 We re-consider Example 1 but assume that the retailer does not exert any effort to identify the defect. Using the three-stage approach, we solve the corresponding sequential game to find the manufacturer's and the retailer's decisions in Stackelberg equilibrium as follows: the manufacturer determines and announces his defective rate $\bar{r}^S = 0.95\%$ and wholesale price $\bar{w}^S = \$119.76$ to the retailer; and then, the retailer decides to choose her retail price $\bar{p}^S = \$134.98$. As a result, if the retailer does not act as the quality gatekeeper, then the manufacturer and the retailer can enjoy the profits $\pi_M = \$4.514 \times 10^5$ and $\pi_R = \$2.257 \times 10^5$, respectively; and the resulting aggregate demand is $D(\bar{p}^S) = 1.502 \times 10^4$.

When we compare the above results with those given in Example 1 (in which the retailer serves as the quality gatekeeper), we note that $p^S > \bar{p}^S$, $D(p^S) < D(\bar{p}^S)$, $\Pi_M < \pi_M$, and $\Pi_R < \pi_R$. This means that both the manufacturer and the retailer are worse off due to the retailer's quality gatekeeping effort. We calculate the retailer's unit cost reduction (induced by her gatekeeping) as $\kappa_1^S = \$1.34$ and the manufacturer's unit cost reduction as $\kappa_2^S = -\$1.29$; thus, the supply chain-wide unit cost reduction is $\kappa_1^S + \kappa_2^S = \$0.05$, which implies that the whole supply chain's cost is reduced by \$0.05 for each product when the retailer is involved into the quality assurance as the gatekeeper. It thus follows from Theorem 5 that the retail price decreases, the aggregate demand increases, and the manufacturer's and the retailer's profits are both increased.

Next, we present another example to illustrate the case that the manufacturer and the retailer cannot benefit from the retailer's gatekeeping effort. We still consider Example 1 but change the value of z_3—i.e., a parameter in the retailer's unit quality-gatekeeping cost function—from 0.34 to 0.38. Other parameter values in Example 1 remain. For the sequential games with and without the retailer's gatekeeping efforts, we compute the manufacturer's and the retailer's Stackelberg equilibrium decisions and corresponding profits, and calculate the aggregate demands and the supply chain-wide unit cost reductions $(\kappa_1^S + \kappa_2^S)$, which are given as in Table 1. We learn from Table 1 that, as the retailer serves the supply chain as the quality gatekeeper, she increases her retail price and the resulting demand is lower than when she does not acts as the gatekeeper. This occurs because the supply chain-wide unit cost reduction $(\kappa_1^S + \kappa_2^S)$ is negative. As a consequence, both the manufacturer and the retailer cannot enjoy higher profits and benefit from the retailer's gatekeeping effort. $\triangleleft$

	Stackelberg Equilibrium Decisions and Other Results	
Supply Chain Member	**Game with the Retailer's Gatekeeping**	**Game without the Retailer's Gatekeeping**
Manufacturer	Wholesale price $w^S = \$120.26$	Wholesale price $\bar{w}^S = \$119.76$
	Defective rate $r^S = 5.23\%$	Defective rate $\bar{r}^S = 0.95\%$
	Profit $\Pi_M = \$4.48 \times 10^5$	Profit $\pi_M = \$4.514 \times 10^5$
Retailer	Identification rate $\beta^S = 51.28\%$	—
	Retail price $p^S = \$135.03$	Retail price $\bar{p}^S = \$134.98$
	Profit $\Pi_R = \$2.24 \times 10^5$	Profit $\pi_R = \$2.257 \times 10^5$
Supply Chain Performance	Demand $D(p^S) = 1.497 \times 10^4$	Demand $D(\bar{p}^S) = 1.502 \times 10^4$
	Supply chain-wide unit quality-related cost reduction $\kappa_1^S + \kappa_2^S = -\$0.198$	

Table 1: The manufacturer's and the retailer's Stackelberg equilibrium decisions and corresponding profits, and the aggregate demands and the supply chain-wide unit cost reductions for the sequential games with and without the retailer's gatekeeping efforts.

4.2 Sensitivity Analysis: Analytic and Numerical Results

We now perform sensitivity analysis to investigate the impacts of all important parameters on the manufacturer's and the retailer's Stackelberg equilibrium decisions, the resulting aggregate demand, and two supply chain members' profits. If we cannot analytically find the impacts of a parameter, then we use the value of the parameter in Example 1 as the base values for our sensitivity analysis—that is, to examine the impact of the parameter, we vary its value but keep other parameters' base values unchanged. From our previous game analysis, we find that all parameters in the manufacturer's and the retailer's profit functions may significantly affect the Stackelberg equilibrium and the supply chain performance. Thus, we accordingly consider all parameters that are classified into the following four categories: (i) defect penalty costs (i.e., α and $\hat{\alpha}$); (ii) acquisition and disposing costs (i.e., c and s); (iii) demand parameters (i.e., a and b); and (iv) quality-assurance cost parameters (i.e., z_i, for $i = 1, \ldots, 4$).

Next, we present our results for the sensitivity analysis in each category, and draw managerial insights that expose the impacts of the retailer's quality gatekeeping effort on supply chain operations.

4.2.1 Defect Penalty Costs

We consider the sensitivity analysis of the manufacturer's unit defect penalty cost α and the retailer's unit defect penalty cost $\hat{\alpha}$. The analysis is important because, as indicated by our discussion in Sections 3 and 4.1, the two parameters significantly affect the manufacturer's and the retailer's decisions and their profits. Next, we begin by discussing the impacts of α on Stackelberg equilibrium-based decisions and the resulting profits for the supply chain, which shall be then followed by the discussion on the impacts of $\hat{\alpha}$ on the supply chain operations.

Impacts of the Manufacturer's Unit Defect Penalty Cost α We first investigate how the manufacturer's and the retailer's equilibrium decisions change when we vary the value of α. From Section 3.3 we learn that, when the retailer acts as a quality gatekeeper, the manufacturer's Stackelberg equilibrium defective rate r^S and the retailer's Stackelberg equilibrium identification rate β^S can be attained by solving the following two equations [that are given in (6)]: $\hat{C}'(\beta^S) = \hat{\alpha} r^S$ and $s + \hat{\alpha}(1 - \beta^S) = -C'(r^S)$. It follows that both r^S and β^S are independent of α. That is, any change in the value of α does not impact the manufacturer's and the retailer's equilibrium decisions for their quality assurance. In addition, we find from Section 4.1.3 that, when the retailer is not involved into the quality gatekeeping, we need to solve $s+\hat{\alpha} = -C'(\bar{r}^S)$ to find the manufacturer's defective rate $\bar{r}^S$ in Stackelberg equilibrium, which is also independent of α. Note that we consider the decision variable β for the retailer only when she serves as a gatekeeper.

The above indicates that, with and without the retailer's quality gatekeeping effort, the manufacturer's and the retailer's decisions for their quality assurance are independent of α. Next, we examine the impacts of α on Stackelberg equilibrium wholesale and retail pricing decisions.

Theorem 6 When the manufacturer's unit defect penalty cost α is increased by one dollar, the manufacturer will increase his wholesale price w^S by r^S dollars if the retailer serves as the quality gatekeeper but he will increase the wholesale price $\bar{w}^S$ by $\bar{r}^S$ dollars if the retailer does not serve as the quality gatekeeper. But, with and without the retailer's quality gatekeeping effort, the retailer's retail pricing decision is independent of α. As a result, the demand, and the two supply chain members' profits are also independent of α. ■

From the above theorem, we find that only wholesale price is dependent on the value of the manufacturer's unit penalty cost α. Recalling Theorem 5 that the retailer's gatekeeping effort results in the rise of the manufacturer's defective rate, i.e., $r^S > \bar{r}^S$, we find that, as the value of α is increased by one dollar, the manufacturer's Stackelberg equilibrium wholesale price w^S (when the retailer serves as the gatekeeper) increases at a higher rate than his wholesale price $\bar{w}^S$ (when the retailer does not serve as the gatekeeper). That is, when the retailer is involved into the quality assurance, the manufacturer should respond to the rise in his penalty cost by increasing the wholesale price at a high rate. This result may be justified as follows: if the manufacturer incurs a higher penalty cost for each defect returned by the retailer, then he could naturally increase his wholesale price. When the retailer spends her quality-gatekeeping effort, she will identify some defects that are not sold to consumers but are instead returned to the manufacturer. For these defects, the retailer does not need to absorb any penalty cost for the sale of defects; this means that the retailer's penalty cost is smaller compared with the case that the retailer does not spend any effort on the quality assurance. It may thus follow that, as the value of α rises, the wholesale price with the retailer's gatekeeping effort should be increased at a higher rate than that without the retailer's gatekeeping effort.

Impacts of the Retailer's Unit Defect Penalty Cost $\hat{\alpha}$ We now perform sensitivity analysis of the retailer's unit defect penalty cost $\hat{\alpha}$ to investigate how the manufacturer's and the retailer's equilibrium decisions and resulting profits vary when the retailer incurs a different penalty cost generated by selling one unit of defect to consumers. Note from our analysis of α (in the preceding section) that Stackelberg equilibrium-characterized wholesale price w^S is dependent on both α and $\hat{\alpha}$, and other Stackelberg equilibrium-based decisions (i.e., $r^S; \beta^S, p^S$) are independent of α but dependent on $\hat{\alpha}$. That is, the value of $\hat{\alpha}$ more significantly affects the supply chain than the value of α.

As discussed previously, when the retailer serves as the quality gatekeeper, the manufacturer's defective rate r^S and the retailer's identification rate β^S in Stackelberg equilibrium satisfy the following two equations: $\hat{C}'(\beta^S) = \hat{\alpha} r^S$ and $s + \hat{\alpha}(1 - \beta^S) = -C'(r^S)$, which can be solved to attain β^S and r^S that are two functions in terms of $\hat{\alpha}$. But, it would be intractable to find the impacts of $\hat{\alpha}$

on β^S and r^S, because of the following reason: we differentiate both the left- and the right-hand sides of the above two equations once w.r.t. $\hat{\alpha}$, and find that $\hat{C}''(\beta^S)\times(\partial\beta^S/\partial\hat{\alpha}) = r^S+\hat{\alpha}\times(\partial r^S/\partial\hat{\alpha})$ and $(1-\beta^S)-\hat{\alpha}\times(\partial\beta^S/\partial\hat{\alpha}) = -C''(r^S)\times(\partial r^S/\partial\hat{\alpha})$, which can be solved to attain $\partial\beta^S/\partial\hat{\alpha}$ and $\partial r^S/\partial\hat{\alpha}$. However, we cannot determine the signs of $\partial\beta^S/\partial\hat{\alpha}$ and $\partial r^S/\partial\hat{\alpha}$, and cannot analytically find the impacts of $\hat{\alpha}$ on β^S and r^S. Thus, similarly, we cannot analytically examine the impacts of $\hat{\alpha}$ on the Stackelberg equilibrium-based wholesale and retail prices, and the resulting profits of the manufacturer and the retailer.

Next, we conduct a numerical study to perform our sensitivity analysis in which we vary the value of $\hat{\alpha}$ around its base value given in Example 1 where $\hat{\alpha} =$ \$120 per defect; and, for each value, we compute the corresponding equilibrium decisions and profits for the manufacturer and the retailer. Since, in this paper, the retailer may or may not be involved into the quality assurance, we present our results for the case with the retailer's gatekeeping effort and the case without the retailer's gatekeeping effort. More specifically, noting that $\hat{\alpha} \geq \alpha = 100$, we, *ceteris paribus*, increase the value of $\hat{\alpha}$ from 100 to 145 in steps of 5; for each value of $\hat{\alpha}$, we calculate the Stackelberg equilibrium $(r^S, w^S; \beta^S, p^S)$ for the game in which the retailer serves as the quality gatekeeper and also compute the Stackelberg equilibrium $(\bar{r}^S, \bar{w}^S; \bar{p}^S)$ for the game in which the retailer does not serve as the quality gatekeeper. In addition, for each value of $\hat{\alpha}$, we compute the manufacturer's and the retailer's profits (Π_M, Π_R) for the game with the retailer's gatekeeping effort and their profits (π_M, π_R) for the game without the retailer's gatekeeping effort. For our numerical results, see Table 3 in Appendix D. To facilitate our discussion, we plot Figure 3 to show the impacts of $\hat{\alpha}$.

From Figure 3(a), we note that, when the retailer is involved into the quality assurance with a higher penalty cost per defect, she should determine a higher identification rate β^S and the manufacturer should determine a lower defective rate r^S. This result may be justified as follows: as the value of $\hat{\alpha}$ rises, the retailer incurs a higher penalty cost when she sells one unit of defect to con-

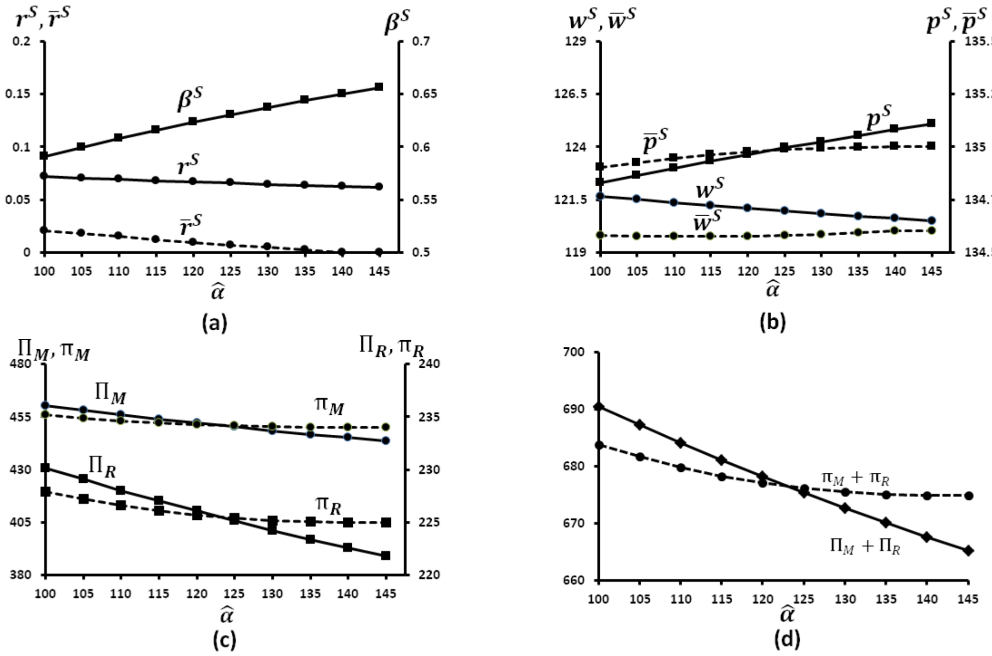

Figure 3: The impacts of the retailer's unit defect penalty cost $\hat{\alpha}$ on the Stackelberg equilibrium $(r^S, w^S; \beta^S, p^S)$ and the manufacturer's and the retailer's profits (Π_M, Π_R) for the game with the retailer's quality gatekeeping effort, and also on the Stackelberg equilibrium $(\bar{r}^S, \bar{w}^S; \bar{p}^S)$ and the two supply chain members' profits (π_M, π_R) for the game without the retailer's quality gatekeeping effort.

sumers. This may result in an increase in the retail price p^S—as indicated by Figure 3(b)—and a reduction in the demand. Moreover, to reduce the penalty cost payment, the retailer may spend more effort on the quality gatekeeping and assure a higher identification rate. As a result, before selling the products to consumers, the retailer returns more defects to the manufacturer and without absorbing any penalty cost for these returned defects. The manufacturer may respond to the retailer's increasing gatekeeping effort by reducing his defective rate and thus saving his penalty cost for the quality assurance. In addition, Figure 3(a) indicates that, if the retailer does not serve as the quality gatekeeper and her per defect penalty cost $\hat{\alpha}$ is increased, then the manufacturer reduces his defective rate, possibly because of the following reason: since the retailer does not spend any effort to reduce the number of defectives that are sold to consumers, a higher penalty cost for each defect may lead the retailer to increase his retail price $\bar{p}^S$, which is demonstrated by Figure 3(b). But, this may discourage consumers from buying from the retailer. To reduce the retailer's payment for the defect sales and mitigate the price increase, the manufacturer may intend to exert more effort on reducing the defective rate. Recall from Theorem 5 that the Stackelberg equilibrium-based defective rate with the retailer's gatekeeping effort (r^S) is higher than that without the retailer's gatekeeping effort ($\bar{r}^S$), which is demonstrated by Figure 3(a).

We also learn from Figure 3(b) that, as the value of $\hat{\alpha}$ increases, the retailer raises her retail price, no matter whether or not the retailer serves as the quality gatekeeper. Furthermore, when the retailer is involved into the quality assurance, the rate of the increase in the retail price is larger than that without the retailer's gatekeeping effort. This results reveals that the retailer intends to increase her retail price at a higher rate so as to offset her cost for the quality gatekeeping. In addition, we find that, when $\hat{\alpha}$ is sufficiently small (e.g., $\hat{\alpha} < 120$), the retail price without the retailer's gatekeeping effort (i.e., $\bar{p}^S$) is higher than that with the effort (i.e., p^S); but, when $\hat{\alpha}$ is sufficiently large (e.g., $\hat{\alpha} > 125$), $\bar{p}^S < p^S$. Recall from Theorem 5 that the comparison between $\bar{p}^S$ and p^S depends on the sign of $\kappa_1^S + \kappa_2^S$—which means the supply chain-wide *unit* cost reduction that is induced by the retailer's quality gatekeeping effort. That is, if the retailer's gatekeeping effort can reduce the supply chain-wide cost, then the retailer decides to reduce her price, i.e., $\bar{p}^S > p^S$. The above result as shown in Figure 3(b) implies that the entire supply chain benefits from a small value of $\hat{\alpha}$ but it is worse off from a high value of $\hat{\alpha}$. It thus follows that the retailer should assure the quality as a gatekeeper if her penalty cost per defect is small; otherwise, she should not be involved into the quality assurance.

Figure 3(b) indicates that, as the value of $\hat{\alpha}$ rises, the manufacturer reduces his wholesale price w^S if the retailer serves as the quality gatekeeper; but, he increases his wholesale price $\bar{w}^S$ if the retailer does not serve as the gatekeeper. As discussed above, when the retailer acts as the quality gatekeeper, an increase in the value of $\hat{\alpha}$ leads the retailer to raise her price at a large rate, which may reduce the sales of the manufacturer's products. To mitigate such an impact, the manufacturer would reduce his wholesale price to offset the retailer's cost increase and thus encourage the retailer to assure the quality as a gatekeeper. However, if the retailer does not act as the quality gatekeeper, then the manufacturer cannot gain any benefit from the retailer for the quality assurance. Therefore, when the retailer raises his price as a result of an increase in the value of $\hat{\alpha}$, the demand may be reduced and the manufacturer may respond by increasing his wholesale price to guarantee the profit. Moreover, we observe that the difference between w^S and $\bar{w}^S$ is reduced when the value of $\hat{\alpha}$ is increased, because of the following possible reason: the retailer's gatekeeping effort with a larger value of $\hat{\alpha}$ decrease the supply chain-wide benefit in terms of cost reduction.

We find from Figure 3(c) that, with and without the retailer's effort on the quality assurance, the manufacturer's profit is higher than the retailer's profit, as shown in Section 4.1.3. In addition, both the manufacturer's and the retailer's profits are decreasing in $\hat{\alpha}$, which demonstrates that a higher value of $\hat{\alpha}$ deteriorates the supply chain performance and discourages the retailer's participation in the quality assurance. Moreover, as discussed previously, the retailer's quality gatekeeping effort with a sufficiently small value of $\hat{\alpha}$ can result in the supply chain-wide cost reduction, whereas the retailer's effort with a sufficiently large value of $\hat{\alpha}$ cannot reduce the chain-wide cost. As a consequence, Figure 3(c) indicates that, when $\hat{\alpha} < 120$, both the manufacturer and the retailer benefit from the retailer's quality assurance by enjoying higher profits; but, when $\hat{\alpha} > 125$, the two supply chain members' profits when the retailer serves as the quality gatekeeper are smaller than those when the retailer does not serve as the gatekeeper. Furthermore, we find from Figure 3(d) that, when the value of $\hat{\alpha}$ is sufficiently small (i.e., $\hat{\alpha} < 120$), the supply chain-wide profit when the retailer spends effort on the quality assurance (i.e., $\Pi_M + \Pi_R$) is higher than that without the retailer's quality gatekeeping (i.e., $\pi_M + \pi_R$). This means that, if the retailer's unit penalty cost $\hat{\alpha}$ is small, then the quality gatekeeping at the retailing level will raise the supply chain-wide profit.

4.2.2 Acquisition and Disposing Costs

We now examine the impacts of the manufacturer's unit acquisition cost (i.e., c) and his cost of disposing a returned defect (i.e., s). Since c and s are two important cost components in the manufacturer's objective (profit) function, it is important to discuss how the two parameters affects the manufacturer's equilibrium decisions and the entire supply chain's performance. We start with the sensitivity analysis of the acquisition cost c.

Impacts of the Manufacturer's Unit Acquisition Cost c We now consider how the manufacturer and the retailer change their equilibrium decisions when the manufacturer incurs a different acquisition cost. According to Sections 3.3 and 4.1.3, we find that, when the retailer serves as the quality gatekeeper, the manufacturer's defective rate r^S and the retailer's identification rate β^S in Stackelberg equilibrium are dependent on $\hat{\alpha}$, s, and the cost functions $C(\cdot)$ and $\hat{C}(\cdot)$; and when the retailer does not act as the gatekeeper, the manufacturer's equilibrium defective rate $\bar{r}^S$ is similarly dependent on $\hat{\alpha}$, s, and the cost function $C(\cdot)$. Therefore, with and without the retailer's quality gatekeeping effort, the manufacturer's and the retailer's quality assurance-related decisions are independent of the manufacturer's unit acquisition cost c.

Next, we analyze the impacts of the value of c on the two supply chain members' equilibrium pricing decisions and resulting profits.

Theorem 7 When the manufacturer's acquisition cost c is increased by one dollar, we find that, no matter whether or not the retailer serves as the quality gatekeeper, the manufacturer's Stackelberg equilibrium-characterized wholesale price is always increased by a half of one dollar and the retailer's equilibrium price is always increased by a quarter of one dollar. Moreover, an increase in the acquisition cost c results in a reduction of the aggregate demand for both the game with and the game without the retailer's effort on the quality assurance.

When the value of c is increased for the game with or without the retailer's quality gatekeeping effort, the manufacturer's and the retailer's profits are both decreased. In addition, the reduction in the manufacturer's profit is greater than the reduction in the retailer's profit. ■

The above theorem indicates that the impacts of c on the Stackelberg equilibrium decisions do not differ between the game with and the game without the retailer's quality gatekeeping effort. This means that the involvement of the retailer into the quality assurance does not affect the supply chain operations and performance. In addition, as discussed previously, the value of c does not affect the quality assurance-related equilibrium decisions—i.e., the defective rate r^S and the identification rate β^S for the game with the retailer's gatekeeping effort and the defective rate $\bar{r}^S$ for the game without the retailer's gatekeeping.

However, we find that, as the value of c rises, the two supply chain members raise their prices to offset the cost increase. Note that the increase in the wholesale price is twice as much as the increase in the retail price, which implies that the retailer bears a half of the increase in the wholesale price, and the other half is included in the retail price and thus absorbed by consumers. If the value of c is higher, then both the manufacturer's and the retailer's profits are lower and they are worse off. Furthermore, the reduction in the manufacturer's profit is larger that the reduction in the retailer's profit. That is, a higher value of c deteriorates the two supply chain members' profits, and makes the manufacturer worse than the retailer.

Impacts of the Manufacturer's Unit Disposing Cost s We now conduct the sensitivity analysis of the unit cost s that the manufacturer incurs when he disposes each defect returned by the retailer. The impacts of the value of s are investigated to address the question of how the manufacturer and the retailer change their decisions if the manufacturer's disposing cost varies. Next, we begin by analyzing the impacts of s on the quality assurance-related decisions, which are the manufacturer's defective rate and the retailer's identification rate.

Theorem 8 As the manufacturer's unit defect-disposing cost s is increased, we find that, for both the game with and the game without the retailer's quality gatekeeping effort, the manufacturer's defective rate in Stackelberg equilibrium is always decreased. Moreover, when the retailer serves as the quality gatekeeper, her identification rate is also decreasing in s. ■

From the above theorem, we learn that, as the manufacturer incurs a higher cost to dispose the returned defects, he may respond by spending more effort on the quality assurance and reducing the defective rate, in order to dispose less defects. Since the manufacturer promises a low defective rate to the retailer, the retailer may have less incentive to assure the quality as a gatekeeper. As a consequence, the retailer's identification rate is reduced as the value of s increases.

Moreover, we find that whether or not the retailer is involved into the quality assurance does not affect the impacts of s on the two supply chain members' quality assurance-related decisions.

Next, we consider the impacts of s on the Stackelberg equilibrium pricing decisions and the resulting profits in the supply chain. Since it is intractable to analytically conduct such a sensitivity analysis with the retailer's gatekeeping effort, we have to perform a numerical study to investigate the dependence of w^S and p^S (and the corresponding profits) on the value of s. But, when the retailer does not serve as the quality gatekeeper, we can analyze the impacts of s on the Stackelberg equilibrium prices ($\bar{w}^S$ and $\bar{p}^S$) and the corresponding profits without doing any numerical study. We begin by providing our analytic results for the game without the retailer's gatekeeping effort, which is then followed by our numerical results regarding the sensitivity analysis for both the game with and the game without the retailer's gatekeeper role.

Theorem 9 For the game without the retailer's quality gatekeeping effort, we find that, as the manufacturer's cost of disposing each defect returned by the retailer is increased, both the manufacture's wholesale price and the retailer's retail price rise. In addition, the two supply chain members' profits are both decreasing in the value of s. ■

As the above theorem indicates, the manufacturer and the retailer respond to the increase in the value of s by raising their prices; as a result, their profits are decreased as the value of s rises. Due to intractable complexity of analyzing the impacts of s for the game with the retailer's gatekeeping effort, we now provide a numerical study where we vary the value of s around its base value (i.e., $s = \$10$) given in Example 1 and compute the corresponding wholesale and retail prices and resulting profits for the game with the retailer's gatekeeper role. Even though, from Theorem 9, we can learn the impacts of s when the retailer does not act as the gatekeeper, we still provide our numerical results for the case because we shall demonstrate our analytic results and compare the results with the retailer's gatekeeping effort with those without the gatekeeping effort to draw some insights.

We increase the value of s from \$6 to \$15 in increments of \$1; for each value of s, we compute corresponding wholesale and retail prices, and the two supply chain members' profits for both the game with and the game without the retailer's gatekeeping effort. We plot Figure 4 to describe our results that are provided in Table 3 (see Appendix D). As Figure 4(a) indicates, the wholesale price for the game with the retailer's gatekeeping effort is decreasing in the value of s whereas that without the effort is increasing in the value of s. That is, whether or not the retailer serves as the quality gatekeeper significantly affects the impact of s on the manufacturer's wholesale pricing decision, which may be justified as follows: if the retailer is involved into the quality assurance as a gatekeeper, then the retailer would not sell the identified defects to consumers but directly return them to the manufacturer. The manufacturer may benefit from the "early" return of defects, thereby reducing his wholesale price and encouraging the retailer to serve as the gatekeeper. Moreover, as the unit disposing cost s is increased, the manufacturer's unit profit is reduced. To assure the total profit, the manufacturer may intend to reduce his wholesale price, prevent the retailer from choosing a high retail price, and thus achieve a high demand.

Moreover, we learn from Figure 4(a) that, if s is sufficiently small (e.g., $s < 11$), then the retail price with the retailer's gatekeeping effort (i.e., p^S) is lower than that without the effort (i.e., $\bar{p}^S$); but, otherwise, $p^S > \bar{p}^S$. Recall from Theorem 5 that, if the supply chain-wide cost is reduced as a result of the retailer's gatekeeping effort (i.e., $\kappa_1^S + \kappa_2^S > 0$), then $p^S < \bar{p}^S$. It thus follows that the supply chain benefits from the retailer's gatekeeping effort when the value of s is sufficiently small. If the value of s is large, then the retailer's quality assurance cannot generate any benefit to the supply chain. We can also observe from Figure 4(b) that the retailer's gatekeeping effort with a sufficiently high value of s (e.g.,

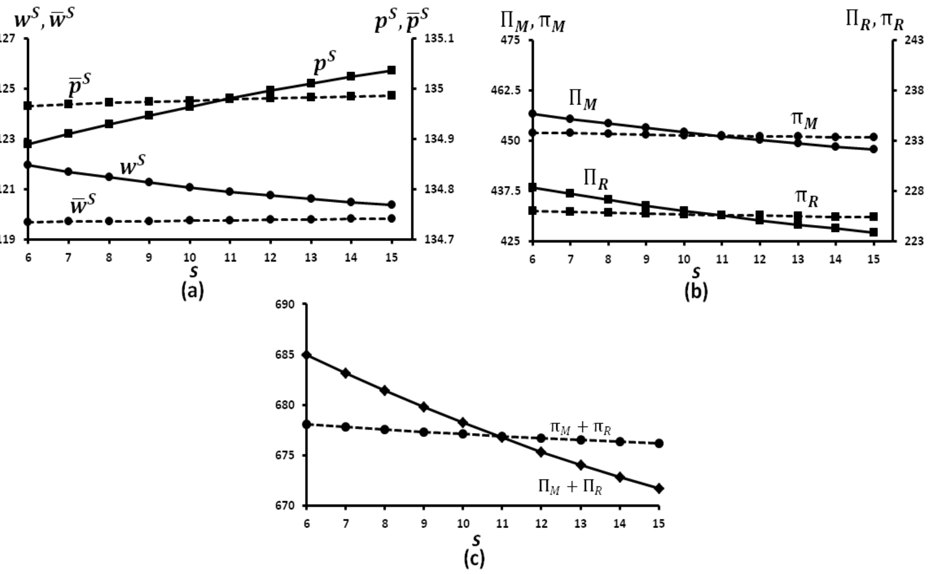

Figure 4: The impacts of the manufacturer's unit defect disposing cost s on the wholesale and retail prices and the resulting profits.

$s > 11$) deteriorates the supply chain, because both the manufacturer's and the retailer's profits when the retailer serves as the gatekeeper are lower than those when the retailer does not serve as the gatekeeper. Therefore, the supply chain-wide profit is improved by the retailer's quality gatekeeping when the value of s is sufficiently small, as shown in Figure 4(c).

Furthermore, we find that, when the value of s rises, both p^S and $\bar{p}^S$ rise; but, p^S is increased with a larger slope. This means that the impact of s on the retail price is higher when the retailer serves as the quality gatekeeper. We also note from Figure 4(a) that, when the retailer serves as the gatekeeper, the retailer's sale profit margin (i.e., $p^S - w^S$) is increased as the value of s rises. However, this does not imply that the retailer's total profit (i.e., Π_M) is higher when the value of s is increased, as shown in Figure 4(b).

4.2.3 Parameters in the Price-Dependent Demand Function

We investigate the effects of the parameters a and b in the aggregate price-dependent demand function $D(p) = a - bp$. The analysis is necessary because the two parameters represent the sensitivity of the demand to the retail price. More specifically, the parameter a is the constant demand that is independent of the price p. If the value of a is increased but the value of b is unchanged, then the demand is less sensitive to the price p. Similarly, if the value of a does not vary, the price p affects the demand more significantly when the value of b is higher. Next, we first consider the impacts of the constant parameter a.

Impacts of the Constant Parameter a We consider the impacts of a on the pricing decisions and profits as shown in the following theorem.

Theorem 10 When the constant (price-independent) demand a is increased, we find that the manufacturer's defective rate (with or without the retailer's quality gatekeeping effort) and the retailer's identification rate (with the retailer's quality gatekeeping effort) are independent of the value of a. Moreover, no matter whether or not the retailer is involved into the quality assurance, the wholesale and retail prices, and the two supply chain members' profits are all increasing in a. Furthermore, the increase in the wholesale price is greater than that in the retail price, and the increase in the manufacturer's profit is also greater than that in the retailer's profit. ■

From the above theorem we learn that the manufacturer's and the retailer's quality assurance-related decisions (i.e., r^S, β^S; $\bar{r}^S$) are independent of the constant demand component a. Note from our previous discussion that the parameter a reflects the sensitivity of demand to the retail price. As the value of a rises, the demand is less sensitive to the retail price if the value of b is unchanged. Therefore, the supply chain members' efforts on their quality control are not associated with the sensitivity of demand to the retail price. Moreover, we find from Theorem 10 that, as the demand is less sensitive to the retail price, the manufacturer and the retailer shall raise their prices and also benefit by enjoying higher profits. This may be justified as follows: since the sensitivity of the demand is reduced, any increase in the retail price has a smaller impact on the demand. Therefore, the manufacturer and the retailer may be likely to increase their prices without losing their profits. This means that the supply chain benefits from the decrease in the demand sensitivity.

Impacts of the Variable Parameter b Our result for the sensitivity analysis of the parameter b is presented as in the following theorem.

Theorem 11 When the demand is more sensitive to the retail price—that is, the value of b is increased, the quality assurance-related decisions (i.e., r^S, β^S; $\bar{r}^S$) are not changed. Moreover, with and without the retailer's quality gatekeeping effort, the wholesale and retail prices are both decreasing in b; and, the manufacturer's profit is always decreasing in b. But, the resulting demand and profit of the retailer may or may not be decreasing in b. More specifically, for the game with (without) the retailer's gatekeeping effort, we find that, if $3a < 4bp^S$ ($3a < 4b\bar{p}^S$), then the demand and the retailer's profit is decreasing in b. Otherwise, they may not decrease as the value of b rises. ■

From the above theorem we find that the demand sensitivity does not impact the manufacturer's and the retailer's quality assurance-related decisions but affects their pricing decisions and profits. As the value of b increases, the demand is more sensitivity to the retail price. the wholesale and retail prices should be both reduced to entice consumers to buy from the retailer, which, but, reduces the manufacturer's and the retailer's sale profit margins. Therefore, the manufacturer's total profit is decreasing in the value of b. However, the demand and the retailer's profit may not be decreased as a result of increasing the value of b, depending on the degree of demand sensitivity to the price. Specifically, if the value of b is sufficiently high and the value of a is sufficiently low, then the demand is highly sensitive to the retail price, the condition that $3a < 4bp^S$ ($3a < 4b\bar{p}^S$) may be satisfied, and the demand and the retailer's profit are both reduced. Otherwise, the demand and the retailer's profit are both increased. Such a result has been demonstrated by a large number of our numerical examples. The above indicates that the demand sensitivity may deteriorate the supply chain performance. Thus, the supply chain can benefit more from the retailer's quality assurance when the demand is less sensitive to the retail price, probably because the supply chain members can increase their prices and thus profit margins without reducing the demand.

4.2.4 Quality-Assurance Cost Parameters

Suppose that the manufacturer's and the retailer's quality-assurance unit cost functions are given as $C(r) = z_1 \times \exp(-z_2 \times r)$ and $\hat{C}(\beta) = z_3 \times \exp(z_4 \times \beta)$, respectively, which are used in Examples 1 and 2 to illustrate our analysis in Sections 3 and 4.1. We now discuss the impacts of the parameters z_1 and z_2 in the manufacturer's quality-assurance unit cost function $C(r)$ and those of the parameters z_3 and z_4 in the retailer's quality-assurance unit cost function $\hat{C}(\beta)$. Such an analysis is important because, as indicated by Theorems 1, 2, and 4, the parameter values in the functions $C(r)$ and $\hat{C}(\beta)$ have significant impacts on the manufacturer's and the retailer's decisions. Next, we first conduct the sensitivity analysis of z_1 and z_2 in the function $C(r)$ and then perform our analysis for the parameters z_3 and z_4 in the function $\hat{C}(\beta)$.

Impacts of the Parameters in the Manufacturer's Quality-Assurance Unit Cost Function $C(r)$ We start with the analysis of the impacts of z_1 and z_2 on the manufacturer's and the retailer's equilibrium decisions. Note that, when the retailer serves as the quality gatekeeper, the defective rate r^S and the identification rate β^S in Stackelberg equilibrium are determined by the equations in (6). Since it would be intractable to analytically find the impacts of z_1 and z_2 on r^S and β^S, we perform a numerical study by varying the value of z_1 from 9 to 13.5 in increments of 0.5 and increasing the value of z_2 from 13 to 22 in steps of 1. For the sensitivity analysis of z_1, we use the numerical results—given in

Table 4; see Appendix D—to plot Figure 5 that indicates impacts of z_1 on the manufacturer's and the retailer's equilibrium decisions and their profits.

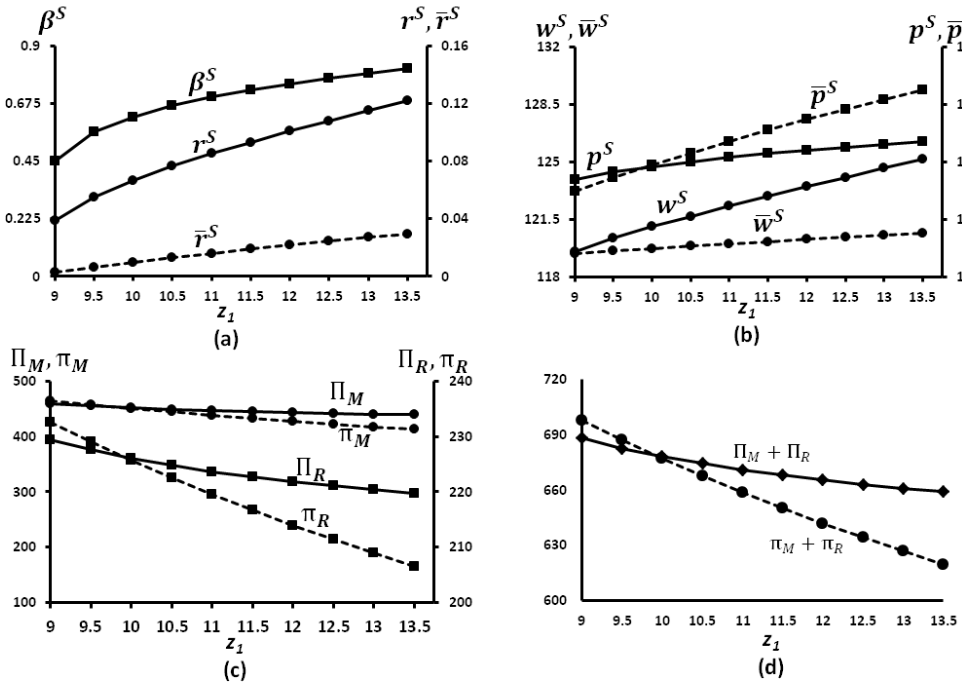

Figure 5: The impacts of the parameter z_1 in the manufacturer's quality-assurance unit cost function $C(r) = z_1 \times \exp(-z_2 \times r)$.

We note from Figure 5(a) that, as the value of z_1 is increased, the manufacturer's cost for quality assurance is higher and the manufacture naturally reduces his effort on the promised defective rate. Therefore, as Figure 5(a) indicates, any increase in the value of z_1 leads the manufacturer to raise his defective rates r^S (for the game with the retailer's gatekeeping) and $\bar{r}^S$ (for the game without the retailer's gatekeeping). Since the retailer faces a higher defective rate as a consequence of an increase in z_1, she should exert more effort on the quality assurance if she acts as the gatekeeper. Otherwise, the retailer have to incur a higher penalty cost for the defects returned by consumers. It thus follows that β^S is increasing in z_1, as shown by Figure 5(a).

Figure 5(b) presents the impacts of z_1 on the wholesale and the retail prices in Stackelberg equilibrium. More specifically, if the value of z_1 is increased, then the manufacturer should raise his wholesale price no matter whether or not the retailer serves as the quality gatekeeper. Furthermore, the increase in the wholesale price w^S for the game with the retailer's gatekeeping is higher than that in the wholesale price $\bar{w}^S$ for the game without the retailer's gatekeeping. We also find that, when z_1 is sufficiently high (e.g., $z_1 > 10$), $p^S < \bar{p}^S$. This implies that, for a sufficiently-high value of z_1, the supply chain-wide cost is reduced as a result of the retailer's quality gatekeeping effort, as indicated by Theorem 5. In addition, as we increase the value of z_1, the difference between p^S and $\bar{p}^S$ is increased. However, even though the supply chain benefits form the retailer's quality gatekeeping by enjoying the cost reduction, we still find that both the manufacturer's and the retailer's profits are decreasing in z_1 as shown in Figure 5(c). As a result, the supply chain-wide profit is decreasing in z_1 and the profit without the retailer's gatekeeping is higher than that with the retailer's quality assurance effort unless the value of s is sufficiently high, as shown in Figure 5(d). This means that any increase in the value of z_1 deteriorates the supply chain performance. But, the manufacturer and the retailer should be both willing to operate with the retailer's quality gatekeeping effort, because they can achieve the cost reductions. Moreover, when the manufacturer's quality assurance-related cost is sufficiently high, the supply-chain-wide profit is improved by the retailer's quality gatekeeping activity.

Next, we consider the impacts of z_2 in the cost function $C(r)$ by conducting a numerical study where the value of z_2 is increased from 13 to 22 in steps of 1. Our results for the sensitivity analysis are presented in Table 4 in Appendix D and are also depicted as in Figure 6. From Figure 6(a) we find that, when the value of z_2 increases, the manufacturer's defective rate r^S rises when z_2 is smaller than a cutoff level (e.g., $z_2 < 15$) but decreases when z_2 is larger than the cutoff level (e.g., $z_2 > 15$). This result happens because of the following possible reason: if the value of z_2 is small, then the manufacturer's quality-assurance unit cost $C(r) = z_1 \times \exp(-z_2 \times r)$ is high. Even though an increase in the value of z_2 results in an reduction in $C(r)$, the manufacturer still absorbs a high cost for assuring the quality. To reduce such a cost, the manufacturer may decide to spend less effort on the quality control. As a result, the defective rate is increasing in z_2 when the value of z_2 is sufficiently small. On the other hand, if the value of z_2 is sufficiently high, then increasing its value significantly decreases the cost $C(r)$, thereby encouraging the manufacturer to promise a lower defective rate. Because $\partial\beta^S/\partial r^S > 0$ as discussed previously, the retailer's identification rate β^S varies in the same manner as r^S. But, when the retailer does not serve as the quality gatekeeper, we find that the defective rate $\bar{r}^S$ is increasing in the value of z_2, which may reflect the fact that, without the retailer's involvement into the quality assurance, the manufacturer does not reduce the defective rate but increase the rate in order to achieve a high profit. Noting that, $\bar{r}^S < r^S$, as shown in Theorem 5, we find that increasing such a low rate does not result in a large number of defects.

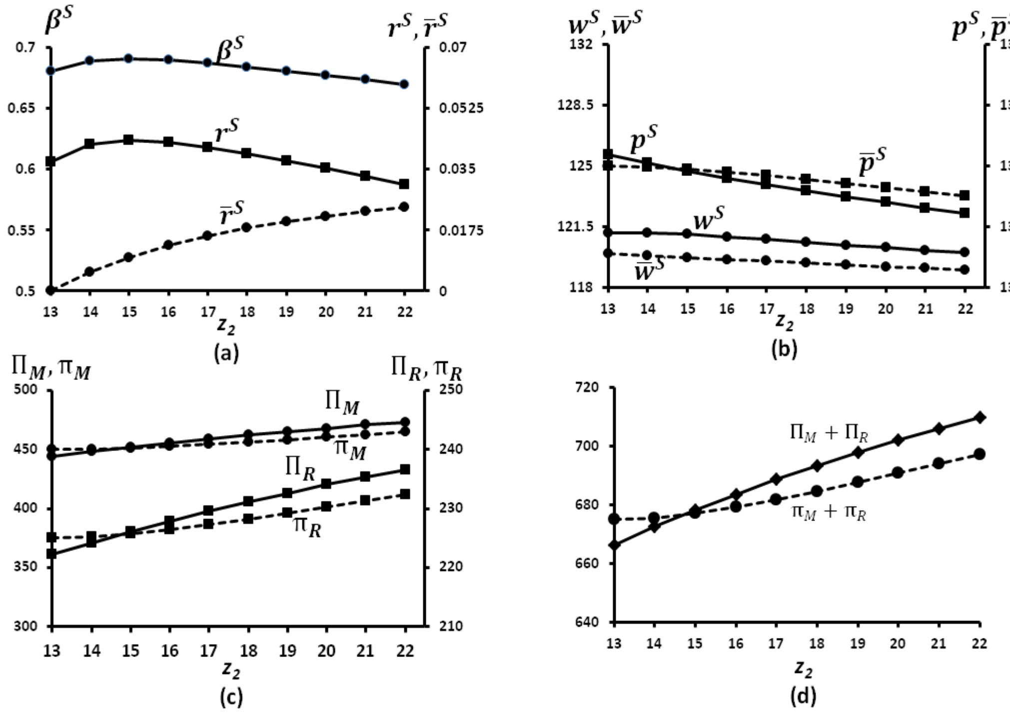

Figure 6: The impacts of the parameter z_2 in the manufacturer's quality-assurance unit cost function $C(r) = z_1 \times \exp(-z_2 \times r)$.

We learn from Figure 6(b) that, no matter whether or not the retailer takes the quality gatekeeper role, the manufacturer's wholesale price and the retailer's retail price are always decreasing in the value of z_2. This results from the fact that, as a result of an increase in the value of z_2, the manufacturer incurs a lower cost for quality assurance and he could decrease his wholesale price without reducing his profit margin. The retailer may also decide to reduce her price, which may lead to a higher demand and thus a higher profit for each supply chain member, as shown in Figure 6(c). Moreover, we find from Figure 6(b) and (c) that, when the value of z_2 is sufficiently high (e.g., $z_2 > 15$), the retail price p^S (both the manufacturer's and the retailer's profits) for the game with the retailer's gatekeeping is smaller (are higher) than the retail price $\bar{p}^S$ (the two supply chain member's profits) for the game without the retailer's gatekeeping, as indicated by Theorem 5. Furthermore, as shown in Figure 6(d), the supply chain-wide profit resulting from the retailer's quality assurance effort is higher than that without the retailer's quality gatekeeping, when the value of z_2 is sufficiently high (e.g., $z_2 > 15$). That is, the retailer should be involved in the quality assurance when the manufacturer's quality assurance-related cost is sufficiently small.

Impacts of the Parameters in the Retailer's Quality-Assurance Unit Cost Function $\hat{C}(\beta)$ We now consider the sensitivity analysis of the parameters z_3 and z_4 in the function $\hat{C}(\beta) = z_3 \exp(z_4\beta)$. Similar to our above discussion on the cost function $C(r)$, we cannot analytically investigate the impacts of z_3 and z_4 on the Stackelberg equilibrium decisions and the manufacturer's and the retailer's profits. Note that the retailer does not need to decide on the identification rate β and incur the quality assurance cost $\hat{C}(\beta)$, if she does not serve as the quality gatekeeper. It thus follows that, for the game without the retailer's gatekeeping effort, the manufacturer's and the retailer's decisions in Stackelberg equilibrium and their profits are independent of the parameters in the function $\hat{C}(\beta)$. Therefore, we next only perform a numerical study to investigate the impacts of z_3 and z_4 on the supply chain with the retailer as the quality gatekeeper. Using our results, we plot Figure 7 and 8—that corresponds to Table 5 in Appendix D—to describe the impacts.

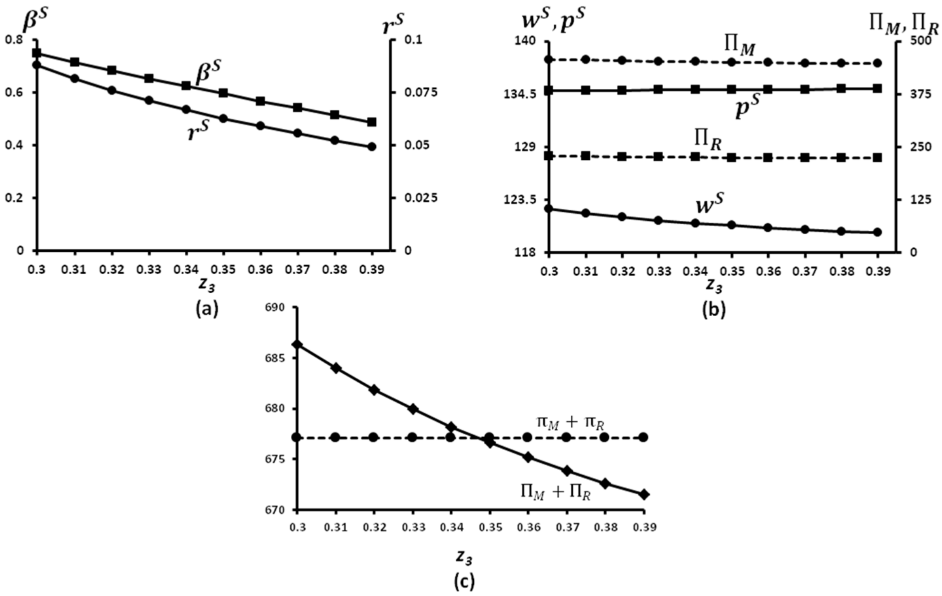

Figure 7: The impacts of the parameters z_3 in the retailer's quality-assurance unit cost function $\hat{C}(\beta) = z_3 \times \exp(z_4 \times \beta)$.

Figure 7(a) indicates that the manufacturer's defective rate and the retailer's identification rate are both decreasing in the value of z_3, which is justified as follows: as the value of z_3 increases, the retailer incurs a higher cost for the quality assurance. For the sake of the profitability, the retailer may have to reduce her effort on the quality gatekeeping, thus decreasing the identification rate. Since $\partial r^S / \partial \beta^S > 0$, the manufacturer should respond by reducing the defective rate. This may reflect the following fact: the manufacturer and the retailer jointly assure the quality. When the retailer does not participate in the quality assurance or only spends a small effort on such an issue, the manufacturer may spend more efforts to promise a small defective rate.

Because the retailer's quality assurance cost rises as a result of an increase in the value of z_3, the retailer would increase her retail price to offset her cost increase. But, since the retailer decides to exert less effort for the quality gatekeeping, she does not incur a significantly increasing cost. Therefore, the retailer slightly raise her retail price as a response to the increase in z_3, as indicated by Figure 7(b). Different from the retail price, the manufacturer's wholesale price is decreasing in z_3, possibly because the manufacturer intends to prevent the retailer with a high quality assurance cost from giving up the gatekeeper role. Furthermore, we learn from Figure 7(b) that both the manufacturer's and the retailer's profits are slightly decreasing in the value of z_3, as shown in Table 5. That is, any increase in z_3 deteriorates the manufacturer and the retailer's benefits. However, we find from Figure 7(c) that, when the value of z_3 is sufficiently small (e.g., $z_3 < 0.35$), the supply chain-wide profit with the retailer's quality gatekeeping effort is higher than that without the retailer's quality assurance. This means that the whole supply chain benefits from the retailer's quality gatekeeping if the retailer's quality assurance-related cost is sufficiently small. Otherwise, the retailer's quality gatekeeping effort will make the supply chain worse off.

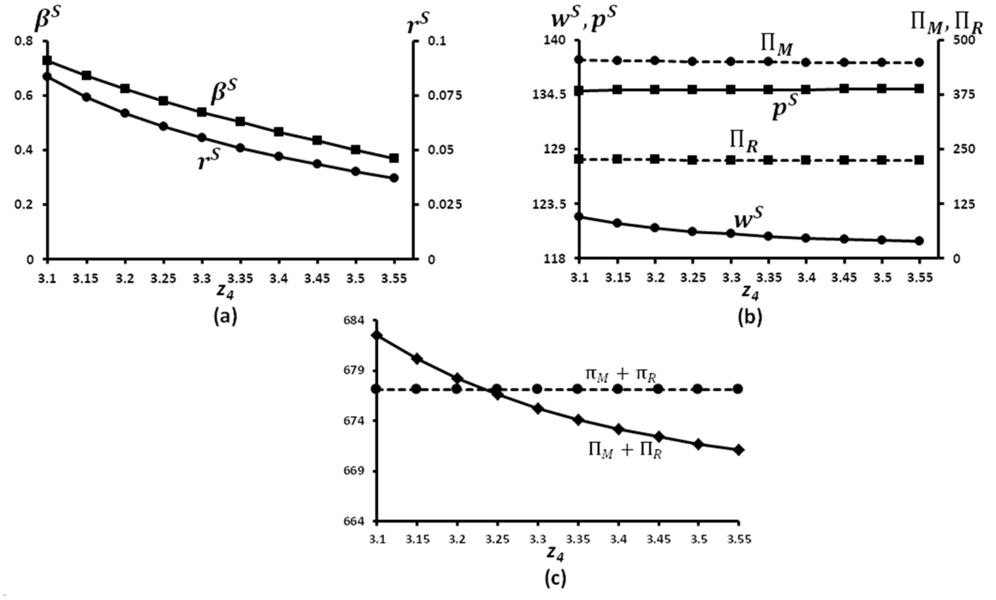

Figure 8: The impacts of the parameters z_4 in the retailer's quality-assurance unit cost function $\hat{C}(\beta) = z_3 \times \exp(z_4 \times \beta)$.

Figure 8(a) indicates the impacts of z_4 on r^S and β^S and Figure 8(b) presents the impacts of z_4 on the Stackelberg equilibrium-characterized pricing decisions and the resulting profits of the supply chain members. Comparing Figure 8(a) with 7(a), Figure 8(c) with 7(c), and also Figure 8(b) and 7(b), we find that the impacts of z_4 are very similar to those of z_3, which results from the fact that $\hat{C}(\beta) = z_3 \times \exp(z_4 \times \beta) = z_3 \times \exp(z_4) \times \exp(\beta)$. Thus, our discussion regarding the sensitivity analysis of z_4 follows similar lines as that of z_3.

5 Summary and Concluding Remarks

In this paper, we consider a quality-assurance problem in a two-echelon supply chain, where a manufacturer promises a defective rate to a retailer who identifies the defects as a quality gatekeeper. Specifically, the manufacturer determines his wholesale price and defective rate—i.e., the percentage of the defects in all products—and announces the two decisions to the retailer. The retailer then responds by deciding on her retail price and the identification rate—i.e., the ratio of the defects identified by the retailer to all defects sold by the manufacturer to the retailer. Assuming a deterministic, price-dependent demand faced by the retailer, we characterize the pricing and quality-assurance decision problem as a leader-follower (sequential) game where the manufacturer and the retailer act as the "leader" and "follower," respectively.

We solve the leader-follower game and obtain Stackelberg equilibrium. We find that the retailer's optimal identification rate should be determined such that the retailer's marginal quality-assurance cost is equal to her expected penalty cost for each defect sold to consumers. Moreover, the price sensitivity of the demand significantly affects the retailer's pricing decision; and, the retailer would profit more when the demand is not sensitivity to the retail price. Our analytic results also reveal that the manufacturer's optimal defective rate is *independent* of his per defect penalty cost but *dependent* on the retailer's per defect penalty cost and his own cost of disposing each returned defect. That is, the value of the manufacturer's penalty cost for each defect does not affect his decision on the defective rate, when the retailer takes the gatekeeping role. We also learn from our analysis that, if the manufacturer's marginal cost for quality assurance is higher than the retailer's, then he spends an effort to assure fewer defects flowing to the retailer. We show that, in the game with the retailer as the quality gatekeeper, the manufacturer can achieve his profit twice as much as the retailer's profit.

Next, we analyze the impacts of the retailer's quality gatekeeping effort on the manufacturer's and the retailer's Stackelberg equilibrium-based decisions and their profits. For such an analysis, we first formulate a leader-follower game in which the manufacturer, as the leader, still announces his wholesale pricing and defective rate decisions to the retailer and the retailer does *not* serve as the quality gatekeeper and only determines her retail price. Solving the game, we obtain the corresponding Stackelberg equilibrium and compare it with that for the game with the retailer's quality gatekeeping effort. We find that the manufacturer's Stackelberg equilibrium-based defective rate when the retailer serves as the quality gatekeeper is higher than the rate $\bar{r}^S$ when the retailer does not take the gatekeeper role. This means that, as a result of the retailer's gatekeeping effort, the manufacturer spends less effort on the quality assurance, thus increasing the defective rate.

Moreover, we find that, if the retailer's gatekeeping effort can generate a larger reduction in her own cost per product compared with manufacturer's cost per product, then the wholesale price for the game with the retailer's gatekeeping is higher than that for the game without the retailer's gatekeeping. We also show that, if the entire supply chain can benefit by achieving a cost reduction from the retailer's gatekeeping effort, then the retailer's equilibrium price when the retailer serves as the gatekeeper is smaller than that when the retailer does not serve as the gatekeeper, and the retailer's market power in terms of Lerner index is thus stronger. Our analytic results indicate that, without the retailer's gatekeeping, the manufacturer's profit is less than twice as much as the retailer's profit. Recalling that, when the retailer serves as the quality gatekeeper, the manufacturer's profit is double of the retailer's profit, we conclude that the retailer's gatekeeping effort increase the ratio of the manufacturer' profit to the retailer's profit. But, the result does *not* mean that the retailer is worse off from her gatekeeping.

In order to investigate the impacts of the parameters in our game model, we perform sensitivity analysis of each parameter analytically and numerically, and draw a number of managerial insights, which are summarized as follows:

1. We analytically show that the retailer's per defect penalty cost $\hat{\alpha}$ more significantly affects the manufacturer's and the retailer's equilibrium decisions and profits than the manufacturer's per defect penalty cost α. Specifically, the two supply chain members' quality assurance-related decisions (i.e., defective rate and identification rate) are independent of the value of α but dependent on the value of $\hat{\alpha}$. Moreover, the retail price, the demand, and the two supply chain members' profits do not depend on α but depend on $\hat{\alpha}$.

   We show that the manufacturer's wholesale price is increasing in his per defect penalty cost α, no matter whether or not the retailer serves as the quality gatekeeper. As the retailer's per defect penalty cost $\hat{\alpha}$ increases, the manufacturer reduces his wholesale price when the retailer serves as the quality gatekeeper; but, he increases his wholesale price when the retailer does not serve as the gatekeeper.

2. When the retailer is involved into the quality assurance, the retailer's identification rate is increasing in her per defect penalty cost $\hat{\alpha}$ whereas the manufacturer's defective rate is decreasing in $\hat{\alpha}$. When the retailer does not serve as the quality gatekeeper, the manufacturer's defective rate is also decreasing in $\hat{\alpha}$.

   As the value of $\hat{\alpha}$ increases, the retailer raises her retail price, for both the game with and the game without the retailer's gatekeeping effort. Furthermore, if $\hat{\alpha}$ is sufficiently small, then the retailer should assure the quality as the gatekeeper, because the supply chain-wide cost is reduced as a result of the retailer's gatekeeping effort. Our results also demonstrate that both the manufacturer's and the retailer's profits are decreasing in $\hat{\alpha}$; that is, a higher value of $\hat{\alpha}$ deteriorates the supply chain performance and discourages the retailer's participation in the quality assurance.

In this thesis, we assume that the retailer's unit penalty cost $\hat{\alpha}$ is larger than manufacturer's unit penalty cost α to assure the incentive of the retailer on the quality gatekeeping. However, one may note from some practices that $\hat{\alpha}$ may be smaller than α. For example, the manufacturer (e.g., Toyota) may possess a higher reputation than the retailer (e.g., Toyota's dealers). Therefore, the return of a defect bought by a customer may greatly deteriorate the manufacturer's reputation but may not significantly impact the retailer's profitability. For such cases, the value of α should be higher than the value of $\hat{\alpha}$; thus, the retailer may benefit from each sold defect by attaining the penalty difference $\alpha - \hat{\alpha}$. This may lead the retailer to lose an incentive for assuring the quality as a gatekeeper in the supply chain. According to the above discussion, we find that both a significantly high value of $\hat{\alpha}$ and a significantly small value of $\hat{\alpha}$ (i.e., $\hat{\alpha} < \alpha$) discourage the retailer from participating in the quality assurance. That is, a proper value of the unit penalty cost $\hat{\alpha}$ is important to enticing the retailer to serve its supply chain as a quality gatekeeper.

3. When the manufacturer's acquisition cost c is increased, we analytically find that, no matter whether or not the retailer serves as the quality gatekeeper, the manufacturer and the retailer should increase their wholesale and retail prices, respectively. Moreover, an increase in the acquisition cost c results in a reduction of the demand for both the game with and the game without the retailer's gatekeeping effort.

   We analytically show that the manufacturer's and the retailer's profits are both decreasing in the value of c, no matter whether or not the retailer is involved into the quality gatekeeping. In addition, the reduction in the manufacturer's profit is greater than the reduction in the retailer's profit.

4. Our analytic results indicate that, as the manufacturer incurs a higher cost in disposing each unit of defect (i.e. the value of unit disposing cost s increases), the manufacturer's defective rate is always decreased for both the game with and the game without the retailer's gatekeeping. Moreover, when the retailer serves as the quality gatekeeper, her identification rate is also decreasing in s.

   For the game without the retailer's quality gatekeeping effort, we find that, as the value of s rises, both the manufacture's wholesale price and the retailer's retail price are increased, and their profits are both decreasing. But, for the game with the retailer's gatekeeping, the wholesale price is decreasing in s whereas the retail price is increasing in s. Moreover, if s is sufficiently small, then the retail price with the retailer's gatekeeping effort is lower than that without the effort, which reflects that a small value of s results in a reduction in the supply chain-wide cost and thus induces the retailer to reduce her price. But, as a result of increasing the value of s, both the manufacturer's and the retailer's profits are reduced.

5. We investigate the impacts of the constant term a and the variable term b in our linear, deterministic price-dependent demand function $D(p) = a - bp$. We show that the manufacturer's defective rate and the retailer's identification rate are independent of both a and b, for both the game with and the game without the retailer's gatekeeping. Moreover, no matter whether or not the retailer is involved into the quality assurance, the wholesale and retail prices, and the two supply chain members' profits are all increasing in a. Furthermore, the increase in the wholesale price is greater than that in the retail price, and the increase in the manufacturer's profit is also greater than that in the retailer's profit.

However, When the demand is more sensitive to the retail price—that is, the value of b is increased, the wholesale and retail prices are both decreasing in b, and the manufacturer's profit is always decreasing in b, with and without the retailer's quality gatekeeping effort. But, the demand and the retailer's profit may or may not be decreasing in b, which depends on a specific condition.

6. We conduct a numerical study to examine the impacts of the parameters z_1 and z_2 in the manufacturer's quality-assurance cost function $C(r) = z_1 \times \exp(-z_2 \times r)$. Our results demonstrate that any increase in the value of z_1 results in a higher defective rate and identification rate. Both the wholesale price and the retail price are increasing in the value of z_1; and, for a sufficiently-high value of z_1, the supply chain-wide cost is reduced as a result of the retailer's quality gatekeeping effort.

   When the value of z_2 increases with the retailer's gatekeeping, the manufacturer's defective rate and the retailer's identification rise when z_2 is smaller than a cutoff level but decrease when z_2 is larger than the cutoff level. But, when the retailer does not serve as the quality gatekeeper, we find that the defective rate is increasing in z_2. No matter whether or not the retailer takes the quality gatekeeper role, the wholesale price and the retail price are both decreasing in z_2, and the manufacturer's and the retailer's profits are both increasing in z_2.

7. We also conduct a numerical study to discuss the impacts of the parameters z_3 and z_4 in the retailer's quality-assurance cost function $\hat{C}(\beta) = z_3 \times \exp(z_4 \times \beta)$. We find that the manufacturer's defective rate and the retailer's identification rate are both decreasing in the value of z_3 and also in the value of z_4. Moreover, the retail price (the manufacturer's wholesale price) is increasing (decreasing) in both z_3 and z_4. Both the manufacturer's and the retailer's profits are slightly decreasing in the values of z_3 and z_4; that

is, any increase in z_3 or z_4 deteriorates the supply chain performance.

In conclusion, we find that the retailer's quality gatekeeping effort significantly affects the manufacturer's and the retailer's decisions and profits. As the retailer serves a quality gatekeeper, the manufacturer would reduce his effort on the quality assurance but both the manufacturer and the retailer could achieve their cost reductions, which depends on their quality-assurance costs, their penalty costs per defect, and the manufacturer's cost of disposing each unit of returned defect. This paper discloses the importance of the retailer's gatekeeper role in supply chain operations. In future, we may extend this paper to, e.g., address the following research problem: two supply chains—including one with the retailer's gatekeeping effort and one without the retailer's gatekeeping effort—may compete for consumers in a market. For such a case, we should obtain Stackelberg equilibrium-based decisions for each supply chain under the competition between two supply chains.

Appendices

Appendix A Notations

We provide a summary of major mathematical notations for our game analysis in Table 2.

Supply Chain Member	**Definition**	**Notations**	
		The Game With Retailer's Gatekeeping	**The Game Without Retailer's Gatekeeping**
Manufacturer	Defective rate	r	$\bar{r}$
	Wholesale price	w	$\bar{w}$
	Unit quality-assurance cost	$C(r) = z_1 \exp(-z_2 r)$, where $z_1, z_2 > 0$	$C(\bar{r}) = z_1 \exp(-z_2 \bar{r})$, where $z_1, z_2 > 0$
	Profit	Π_M	π_M
	Cost reduction	κ_1	
	Penalty cost per defect	α	
	Unit acquisition cost	c	
	Disposing cost per defect	s	
Retailer	Identification rate	β	—
	Unit quality-assurance cost	$\hat{C}(\beta) = z_3 \exp(z_4 \beta)$, where $z_3, z_4 > 0$	—
	Retail price	p	$\bar{p}$
	Profit	Π_R	π_R
	Cost reduction	κ_2	
	Penalty cost per defect	$\hat{\alpha}$	
The linear, deterministic, retail price-dependent demand function		$D(p) = a - bp$, where $a, b > 0$ are the constant term and the sensitivity of demand, respectively	

Table 2: A summary of major mathematical notations in our game analysis.

Appendix B Proofs of Theorems

Proof of Theorem 1. We find from (1) that the retailer's profit Π_R is dependent on two decision variables p and β. Taking the first- and second-order derivatives of Π_R w.r.t. p, we have,

$$\frac{\partial \Pi_R}{\partial p} = a - b\{2p - w - \hat{C}(\beta) + r[\alpha - \hat{\alpha}(1-\beta)]\} \quad \text{and} \quad \frac{\partial^2 \Pi_R}{\partial p^2} = -2b < 0. \tag{16}$$

Moreover, we differentiate Π_R once and twice w.r.t. β, and find that

$$\frac{\partial \Pi_R}{\partial \beta} = (a - bp)[\hat{\alpha}r - \hat{C}'(\beta)] \quad \text{and} \quad \frac{\partial^2 \Pi_R}{\partial \beta^2} = -(a - bp)\hat{C}''(\beta) < 0. \tag{17}$$

Next, we compute the second-order cross-partial derivative of Π_R as

$$\frac{\partial^2 \Pi_R}{\partial p \partial \beta} = -b[\hat{\alpha}r - \hat{C}'(\beta)].$$

It thus follows that, in the Hessian matrix for the maximization problem, the first leading principal minors (i.e., $\partial^2\Pi_R/\partial p^2$ and $\partial^2\Pi_R/\partial\beta^2$) are negative; and the second leading principal minor is calculated as,

$$\frac{\partial^2 \Pi_R}{\partial p^2} \times \frac{\partial^2 \Pi_R}{\partial \beta^2} - \left(\frac{\partial^2 \Pi_R}{\partial p \partial \beta}\right)^2 = 2b(a - bp)\hat{C}''(\beta) - b^2[\hat{\alpha}r - \hat{C}'(\beta)]^2, \tag{18}$$

which cannot be immediately determined to be positive or negative. But, we find from the above that, at the point satisfying the first-order condition that $\partial\Pi_R/\partial\beta = 0$, $\hat{C}'(\beta) = \hat{\alpha}r$ and the second leading principal minor in (18) is reduced to $2b(a - bp)\hat{C}''(\beta) > 0$.

As a result, the Hessian matrix for the maximization problem is negative definite for all possible values of (p, β). This means that the retailer's profit Π_R in (1) is quasi-concave on (p, β). Thus, equating both $\partial\Pi_R/\partial p$ in (16) and $\partial\Pi_R/\partial\beta$ in (17) to zero and solve the resulting equations for p and β, we prove the theorem. ■

Proof of Theorem 2. We learn from Theorem 1 that $p^* = \{a/b + w + \hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1-\beta^*)]\}/2$. Using this we re-write the manufacturer's profit Π_M in (3) as,

$$\Pi_M = [(w-c)-(\alpha+s)r-C(r)]\left\{\frac{a}{2} + \frac{br}{2}[\alpha - \hat{\alpha}(1-\beta^*)] - \frac{bw}{2} - \frac{b\hat{C}(\beta^*)}{2}\right\}, \quad (19)$$

where β^* satisfies the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r$, as given in Theorem 1. That is, β^* is dependent on the manufacturer's decision variable r but is independent of the manufacturer's wholesale price w, i.e., $\partial\beta^*/\partial w = 0$. Differentiating both sides of the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r$ w.r.t. r, we have, $\hat{C}''(\beta^*) \times (\partial\beta^*/\partial r) = \hat{\alpha}$, or, $\partial\beta^*/\partial r = \hat{\alpha}/\hat{C}''(\beta^*)$.

Taking the first- and second-order derivatives of Π_M w.r.t. w gives,

$$\begin{aligned}\frac{\partial\Pi_M}{\partial w} &= \left\{\frac{a}{2} + \frac{br}{2}[\alpha - \hat{\alpha}(1-\beta^*)] - \frac{bw}{2} - \frac{b\hat{C}(\beta^*)}{2}\right\} \\ &\quad -\frac{b}{2}[(w-c) - (\alpha+s)r - C(r)], \quad (20)\end{aligned}$$

and $\partial^2\Pi_M/\partial w^2 = -b < 0$. This means that, for any given value of r, Π_M is strictly concave in w and the optimal $r-$dependent wholesale price $w^*(r)$ uniquely satisfies the equation that $\partial\Pi_M/\partial w = 0$ where $\partial\Pi_M/\partial w$ is given as in (20); that is,

$$w^*(r) = \left(\alpha + \frac{s}{2}\right)r + \frac{a}{2b} + \frac{c + C(r) - \hat{C}(\beta^*) - r\hat{\alpha}(1-\beta^*)}{2}. \quad (21)$$

Substituting $w^*(r)$ into Π_M in (19) and simplifying it, we have,

$$\Pi_M = \frac{b}{2}[(w^*(r) - c) - (\alpha+s)r - C(r)]^2.$$

We then differentiate the above once w.r.t. r, and find that

$$\frac{\partial\Pi_M}{\partial r} = b[(w^*(r) - c) - (\alpha+s)r - C(r)]\left[\frac{\partial w^*(r)}{\partial r} - (\alpha+s) - C'(r)\right]. \quad (22)$$

Using (21) we calculate $\partial w^*(r)/\partial r$ as,

$$\frac{\partial w^*(r)}{\partial r} = \alpha + \frac{s + C'(r) - \hat{\alpha}(1-\beta^*)}{2};$$

and as a result, $\partial \Pi_M/\partial r$ in (22) can be re-written as,

$$\frac{\partial \Pi_M}{\partial r} = -\frac{b}{2}[(w^*(r) - c) - (\alpha + s)r - C(r)][s + C'(r) + \hat{\alpha}(1-\beta^*)]. \quad (23)$$

Differentiating the above once w.r.t. r yields,$\hat{C}'(\beta^*) = \hat{\alpha}r$

$$\begin{aligned}\frac{\partial^2 \Pi_M}{\partial r^2} &= \frac{b}{4}[s + C'(r) + \hat{\alpha}(1-\beta^*)]^2 \\ &\quad -\frac{b}{2}[(w^*(r) - c) - (\alpha + s)r - C(r)][C''(r) - \hat{\alpha}^2/\hat{C}''(\beta^*)], \quad (24)\end{aligned}$$

which cannot be immediately determined as a positive or negative value.

To prove the uniqueness of the optimal solutions, we next analyze the sign of $\partial^2 \Pi_M/\partial r^2$ at the points satisfying the first-order condition that $\partial \Pi_M/\partial r = 0$. Equating $\partial \Pi_M/\partial r$ in (23) to zero, we have,

$$s + \hat{\alpha}(1-\beta^*) = -C'(r), \quad (25)$$

because, in (23), the term $[(w^*(r) - c) - (\alpha + s)r - C(r)]$ represents the manufacturer's unit profit that is positive. Using (25), we reduce $\partial^2 \Pi_M/\partial r^2$ in (24) as,

$$\left.\frac{\partial^2 \Pi_M}{\partial r^2}\right|_{\partial \Pi_M/\partial r = 0} = -\frac{b}{2}[(w^*(r) - c) - (\alpha + s)r - C(r)][C''(r) - \hat{\alpha}^2/\hat{C}''(\beta^*)],$$

which is negative if and only if $C''(r)\hat{C}''(\beta^*) > \hat{\alpha}^2$ when the defective rate satisfies (25).

From our above analysis, we learn that the manufacturer's profit Π_M in (3) is a unimodal function with unique wholesale price and defective rate if and only if

$C''(r)\hat{C}''(\beta^*) > \hat{\alpha}^2$ at the point satisfying (25). Thus, we prove the theorem. ■

Proof of Theorem 3. We find from (8) that the retailer's profit π_R is dependent on only one decision variable p. Taking the first- and second-order derivatives of π_R w.r.t. p yields,

$$\frac{\partial \pi_R}{\partial p} = a - 2bp + bw - br(\alpha - \hat{\alpha}) \quad \text{and} \quad \frac{\partial^2 \pi_R}{\partial p^2} = -2b < 0, \tag{26}$$

which means that the profit function π_R is strictly concave in p and the optimal retail price $\bar{p}^*$ uniquely exists. Equating $\partial \pi_R/\partial p$ in (26) to zero and solving it for p, we have the optimal solution $\bar{p}^*$ as in (9). ■

Proof of Theorem 4. Noting from (9) that $p^* = [a/b + w - r(\alpha - \hat{\alpha})]/2$, we re-write the manufacturer's profit π_M—which is the same as Π_M in (3)—as,

$$\pi_M = [(w - c) - (\alpha + s)r - C(r)] \left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right]. \tag{27}$$

The first- and second-order derivatives of π_M w.r.t. w are computed as,

$$\frac{\partial \pi_M}{\partial w} = \left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] - \frac{b}{2}[(w - c) - (\alpha + s)r - C(r)], \tag{28}$$

and $\partial^2 \pi_M/\partial w^2 = -b < 0$. We next differentiate π_M once and twice w.r.t. r, which are given as,

$$\begin{aligned} \frac{\partial \pi_M}{\partial r} &= -[\alpha + s + C'(r)] \left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] \\ &\quad + \frac{b}{2}(\alpha - \hat{\alpha})[(w - c) - (\alpha + s)r - C(r)], \end{aligned} \tag{29}$$

$$\begin{aligned} \frac{\partial^2 \pi_M}{\partial r^2} &= -C''(r) \left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] \\ &\quad - b(\alpha - \hat{\alpha})[\alpha + s + C'(r)]. \end{aligned} \tag{30}$$

Note that we cannot immediately determine the sign of $\partial^2 \pi_M/\partial r^2$ in (30). In order to show that the optimal solutions are unique, we analyze $\partial^2 \pi_M/\partial r^2$ at the points (w, r) satisfying the first-order conditions that $\partial \pi_M / \partial w = 0$. Equating $\partial \pi_M / \partial w$ in (29) to zero yields,

$$\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2} = \frac{b}{2}[(w - c) - (\alpha + s)r - C(r)]; \tag{31}$$

and then, setting $\partial \pi_M / \partial r$ in (29) to zero and simplifying it using (31), we find that $s + \hat{\alpha} = -C'(r)$. It thus follows that the second-order derivative $\partial^2 \pi_M / \partial r^2$ in (30) can be re-written as,

$$\frac{\partial^2 \pi_M}{\partial r^2} = -C''(r)\left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] - b[\alpha + s + C'(r)]^2 < 0.$$

In order to show the quasi-concavity of the function π_M on (w, r), we need to find whether or not the Hessian matrix is negative semidefinite. Our above analysis indicates that the first leading principal minors (i.e., $\partial \pi_M / \partial w$ and $\partial \pi_M / \partial r$) are both negative at the points satisfying the first-order conditions. Next, we calculate the second leading principal minor. The second-order cross-partial derivative of π_M as $\partial^2 \pi_M / \partial w \partial r = b[2\alpha + s - \hat{\alpha} + C'(r)]/2$. Furthermore, we find that, at the points (w, r) satisfying the first-order conditions, $\partial^2 \pi_M / \partial w \partial r = b(\alpha + s + C'(r)) < 0$ and the second leading principle minor is computed as,

$$\begin{aligned}
\frac{\partial^2 \Pi_M}{\partial w^2} \times \frac{\partial^2 \Pi_M}{\partial r^2} - \left(\frac{\partial^2 \Pi_M}{\partial w \partial r}\right)^2 &= b \times C''(r)\left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] \\
&\quad + b^2[\alpha + s + C'(r)]^2 - b^2[\alpha + s + C'(r)]^2 \\
&= b \times C''(r)\left[\frac{a}{2} + \frac{br}{2}(\alpha - \hat{\alpha}) - \frac{bw}{2}\right] > 0,
\end{aligned}$$

because $s + \hat{\alpha} = -C'(r)$. Thus, the Hessian matrix is negative definite at the points (w, r) under the first-order conditions, which means that the retailer's profit π_M is strictly quasi-concave on (w, r). Equating both $\partial \pi_M / \partial w$ in (28) and $\partial \pi_M / \partial r$ in (29) to zero and solve the resulting equations for w and r, we prove the theorem. ■

Proof of Theorem 5. The comparison between w^S and $\bar{w}^S$ follows along the lines similar to those we presented for the comparison between w^* and $\bar{w}^*$ in Section 4.1.2. Next, we consider the comparison between p^S and $\bar{p}^S$. Substituting r^S, $\bar{r}^S$, w^S, $\bar{w}^S$, p^S, $\bar{p}^S$, and β^S into the optimal (best-response) retail prices in Theorems 1 and 3, we find that $\bar{p}^S = [a/b + \bar{w}^S - \bar{r}^S(\alpha - \hat{\alpha})]/2$ and $p^S = [a/b + w^S - r^S(\alpha - \hat{\alpha}) + \hat{C}(\beta^S) - r^S\hat{\alpha}\beta^S]/2$. As a result,

$$p^S - \bar{p}^S = \frac{1}{2}\left[(\hat{\alpha} - \alpha)(r^S - \bar{r}^S) - \bar{\kappa}\right], \quad \text{where } \bar{\kappa} \equiv \bar{w}^S - w^S + \hat{\alpha} r^S \beta^S - \hat{C}(\beta^S), \quad (32)$$

where $\hat{\alpha} \geq \alpha$; and $r^S \geq \bar{r}^S$, resulting from our argument on Theorem 4 in Section 4.1.2.

Using r^S, $\bar{r}^S$, w^S, $\bar{w}^S$, and β^S to replace r^*, $\bar{r}^*$, w^*, $\bar{w}^*$, and β^* in (12), we find that $w^S - \bar{w}^S = (\kappa_1^S - \kappa_2^S)/2$, where κ_1^S and κ_2^S are defined as in (15). Then, we have,

$$p^S - \bar{p}^S = -\frac{\kappa_1^S + \kappa_2^S}{4}.$$

It thereby follows that, if $\kappa_1^S + \kappa_2^S > 0$, then $p^S < \bar{p}^S$ and $D(p^S) > D(\bar{p}^S)$; if $\kappa_1^S + \kappa_2^S = 0$, then $p^S = \bar{p}^S$ and $D(p^S) = D(\bar{p}^S)$; otherwise, if $\kappa_1^S + \kappa_2^S < 0$, then $p^S > \bar{p}^S$ and $D(p^S) < D(\bar{p}^S)$. This theorem is thus proved. ■

Proof of Theorem 6. We learn from Section 3.3 that the Stackelberg equilibrium wholesale price w^S is computed as,

$$w^S = \left(\alpha + \frac{s}{2}\right) r^S + \frac{a}{2b} + \frac{c + C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1 - \beta^S)}{2}, \quad (33)$$

where r^S and β^S are independent of α, as discussed above. Differentiating w^S once w.r.t. α yields that $\partial w^S/\partial\alpha = r^S$, which means that, as the manufacturer's unit penalty cost α is increased by one dollar, the manufacturer should increase his wholesale price by r^S dollars.

In addition, we find that the retailer's Stackelberg equilibrium retail price p^S

is,

$$p^S = \frac{1}{2}\left\{\frac{a}{b} + w^S + \hat{C}(\beta^S) - r^S[\alpha - \hat{\alpha}(1-\beta^S)]\right\}. \tag{34}$$

The first-order derivative of p^S w.r.t. α is computed as, $\partial p^S/\partial\alpha = (\partial w^S/\partial\alpha - r^S)/2 = 0$, which means that, when the retailer does not exert any effort on the quality assurance, the retail price in Stackelberg equilibrium is independent of the retailer's penalty cost per unit of defect.

The resulting demand $D(p^S)$ does not change. According to (3) we have the manufacturer's profit in terms of Stackelberg equilibrium as $\Pi_M = [(w^S - c) - (\alpha + s)r^S - C(r^S)]D(p^S)$, which can, using (33), be rewritten as,

$$\Pi_M = \frac{1}{2}\left[\frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)\right] D(p^S), \tag{35}$$

which is independent of α. Using (1) and (33), we compute the retailer's profit in terms of Stackelberg equilibrium as,

$$\Pi_R = \frac{1}{2}\left[2p^S - \frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)\right] D(p^S), \tag{36}$$

which is also independent of α.

We next analyze the impacts of α when the retailer does not serve as the quality gatekeeper. From (11), we find that, for the case, the Stackelberg equilibrium-based wholesale price is $\bar{w}^S = \alpha\bar{r}^S + [a/b + c + C(\bar{r}^S) + (s - \hat{\alpha})\bar{r}^S]/2$, which is increased by $\bar{r}^S$ dollars if the value of α rises by one dollar. Using (9) we have,

$$\bar{p}^S = [a/b + \bar{w}^S - \bar{r}^S(\alpha - \hat{\alpha})]/2 = [3a/b + c + C(\bar{r}^S) + (s + \hat{\alpha})\bar{r}^S]/4,$$

which does not depend on the value of α. Therefore, the aggregate demand $D(\bar{p}^S)$ is independent of α. Similar to our above analysis for the case that the retailer serves as a gatekeeper, we can show that the manufacturer's and the retailer's profits are independent of α. This theorem is thus proved. ■

Proof of Theorem 7. From the proof of Theorem 6, we find that, for the game with the retailer's quality gatekeeping effort, the manufacturer's wholesale price in Stackelberg equilibrium is attained as in (33). The first-order derivative of w^S w.r.t. c is thus computed as $\partial w^S/\partial c = 1/2$, which means that, as the value of c is increased by one dollar and other parameter values are not changed, the manufacturer should raise his wholesale price by a half of dollar.

Moreover, we learn from the proof of Theorem 6 that the Stackelberg equilibrium-characterized retail price with the retailer's gatekeeping effort is given as in (34); thus, differentiating p^S once w.r.t. c yields, $\partial p^S/\partial c = (\partial w^S/\partial c)/2 = 1/4$. This means that the retailer should respond to an one-dollar increase in the value of c by raising her retail price by 25 cents, which is a half of the increase in the manufacturer's wholesale price. As a result, the aggregate demand $D(p^S) = a - bp^S$ is decreasing in c. Specifically, since $\partial D(p^S)/\partial c = -b(\partial p^S/\partial c) = -b/4$, we find that, as c is increased by one dollar, the aggregate demand is decreased by $b/4$ units.

The proof of Theorem 6 also indicates that, when the manufacturer and the retailer adopt their Stackelberg equilibrium decisions for the game with the retailer's gatekeeping effort, the two supply chain members' profits (i.e., Π_M and Π_R) are attained as in (35) and (36). We take the first-order derivatives of Π_M and Π_R w.r.t. c, and have,

$$\begin{aligned}
\frac{\partial \Pi_M}{\partial c} &= -\frac{1}{2}D(p^S) - \frac{b}{8}\left[\frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)\right] < 0, \\
\frac{\partial \Pi_R}{\partial c} &= -\frac{1}{4}D(p^S) - \frac{b}{8}\left[2p^S - \frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)\right] \\
&< 0.
\end{aligned}$$

The above shows that, as the value of c rises in the game with the retailer's gatekeeping effort, both the manufacturer's and the retailer's profits are decreased. Furthermore, the decrease in the retailer's profit is smaller than that in the manufacturer's profit, because $a/b > 2p^S - a/b$ and $\partial \Pi_M/\partial c < \partial \Pi_R/\partial c$.

Next, we analyze the impacts of c on the performance of the two-echelon supply chain without the retailer's quality gatekeeping effort. As given in the proof of Theorem 6, the corresponding Stackelberg equilibrium-based wholesale and retail prices are attained as $\bar{w}^S = \alpha\bar{r}^S + [a/b + c + C(\bar{r}^S) + (s - \hat{\alpha})\bar{r}^S]/2$ and $\bar{p}^S = [3a/b + c + C(\bar{r}^S) + (s + \hat{\alpha})\bar{r}^S]/4$, respectively. Therefore, $\partial\bar{w}^S/\partial c = 1/2$ and $\partial\bar{p}^S/\partial c = 1/4$, which are the same as those in the game with the retailer's quality gatekeeping effort. It also follows that the resulting demand $D(\bar{p}^S)$ is decreased at the rate $\partial D(\bar{p}^S)/\partial c = -b/4$, which is the same as for the game involving the retailer's gatekeeper role.

Using the above, we find that, when the retailer does not participate in the quality assurance, the manufacturer's and the retailer's profits can be written as,

$$\begin{cases} \pi_M = D(\bar{p}^S) \times [(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)], \\ \pi_R = D(\bar{p}^S) \times [(\bar{p}^S - \bar{w}^S) + (\alpha - \hat{\alpha}) \times \bar{r}^S]. \end{cases} \tag{37}$$

Differentiating π_M and π_R once w.r.t. c gives,

$$\begin{aligned} \frac{\partial \pi_M}{\partial c} &= -\frac{1}{2}D(\bar{p}^S) - \frac{b}{4}[(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)] < 0, \\ \frac{\partial \pi_R}{\partial c} &= -\frac{1}{4}D(\bar{p}^S) - \frac{b}{4}[(\bar{p}^S - \bar{w}^S) + (\alpha - \hat{\alpha}) \times \bar{r}^S] < 0, \end{aligned}$$

which means that any increase in the value of c results in a decrease in the manufacture's profit and also a reduction in the retailer's profit. According to (14), we find that the manufacturer's unit profit [i.e., $(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)$] is higher than the retailer's unit profit [i.e., $\bar{p}^S - \bar{w}^S + \bar{r}^S(\alpha - \hat{\alpha})$]. Thus, $\partial\pi_M/\partial c < \partial\pi_R/\partial c$, which is similar to our above analysis for the game with the retailer's quality gatekeeping effort. This theorem is thus proved. ■

Proof of Theorem 8. From Section 3.3, we find that, in the game with the retailer as the quality gatekeeper, the manufacturer's defective rate r^S and the retailer's identification rate β^S satisfy the equations that $\hat{C}'(\beta^S) = \hat{\alpha} r^S$ and

$s + \hat{\alpha}(1 - \beta^S) = -C'(r^S)$, which imply that β^S and r^S are dependent on s. Differentiating the two sides of the former equation—i.e., $\hat{C}'(\beta^S) = \hat{\alpha} r^S$—once w.r.t. r^S gives $\hat{C}''(\beta^S)(\partial\beta^S/\partial r^S) = \hat{\alpha}$, or, $\partial\beta^S/\partial r^S = \hat{\alpha}/\hat{C}''(\beta^S) > 0$. Taking the first-order derivative of two sides of the equation that $s + \hat{\alpha}(1 - \beta^S) = -C'(r^S)$ w.r.t. s, we have,

$$1 - \hat{\alpha}\frac{\partial\beta^S}{\partial r^S}\frac{\partial r^S}{\partial s} = -C''(r^S)\frac{\partial r^S}{\partial s},$$

which can be re-written as,

$$\frac{\partial r^S}{\partial s} = -\frac{\hat{C}''(\beta^S)}{\hat{C}''(\beta^S)C''(r^S) - \hat{\alpha}^2} < 0,$$

because, as discussed in Section 3.3, we reasonably assume that $\hat{C}''(\beta^S)C''(r^S) > \hat{\alpha}^2$. Since $\partial\beta^S/\partial r^S > 0$, we find that β^S is also decreasing in s.

When the retailer does not serve as the quality gatekeeper, we learn from Theorem 4 that the manufacturer's Stackelberg equilibrium-characterized defective rate $\bar{r}^S$ uniquely satisfies the equation that $s+\hat{\alpha} = -C'(\bar{r}^S)$. Taking the first-order derivatives of both sides of the equation w.r.t. s gives that $1 = -C''(\bar{r}^S)(\partial\bar{r}^S/\partial s)$, or, $\partial\bar{r}^S/\partial s = -1/C''(\bar{r}^S) < 0$, which means that, for the game without the retailer's quality gatekeeping effort, $\bar{r}^S$ is decreasing in s. This theorem is thus proved. ■

Proof of Theorem 9. According to Section 4.1.3, we find that, without the retailer's gatekeeping effort, the manufacturer's Stackelberg equilibrium wholesale price is $\bar{w}^S = \alpha\bar{r}^S + [a/b + c + C(\bar{r}^S) + (s - \hat{\alpha})\bar{r}^S]/2$. Differentiating $\bar{w}^S$ once w.r.t. s yields,

$$\frac{\partial\bar{w}^S}{\partial s} = (\alpha - \hat{\alpha})\frac{\partial\bar{r}^S}{\partial s} + \frac{\bar{r}^S}{2},$$

which is positive because $\alpha \leq \hat{\alpha}$ and $\partial\bar{r}^S/\partial s < 0$ as shown in Theorem 8. In addition, we learn from Section 4.1.3 that the retail price in Stackelberg equilibrium is $\bar{p}^S = [3a/b + c + C(\bar{r}^S) + (s + \hat{\alpha})\bar{r}^S]/4$. Taking the first-order derivative of $\bar{p}^S$ w.r.t. s gives that $\partial\bar{p}^S/\partial s = \bar{r}^S/4 > 0$.

Differentiating the manufacturer's profit π_M and the retailer's profit π_R—which are given as in (37); see the proof of Theorem 7—once w.r.t. s, we have,

$$\begin{aligned}\frac{\partial \pi_M}{\partial s} &= -b\frac{\partial \bar{p}^S}{\partial s} \times [(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)] - D(\bar{p}^S) \times \frac{\bar{r}^S}{2} < 0,\\ \frac{\partial \pi_R}{\partial s} &= -b\frac{\partial \bar{p}^S}{\partial s} \times [(\bar{p}^S - \bar{w}^S) + (\alpha - \hat{\alpha}) \times \bar{r}^S] - D(\bar{p}^S) \times \frac{\bar{r}^S}{4} < 0.\end{aligned}$$

This theorem is thus proved. ■

Proof of Theorem 10. We first analyze the impacts when the retailer serves as the quality gatekeeper. As discussed in Section 3.3, r^S and β^S can be attained by solving the equations that $\hat{C}'(\beta^S) = \hat{\alpha} r^S$ and $s + \hat{\alpha}(1 - \beta^S) = -C'(r^S)$, which do not involve the parameter a. Thus, r^S and β^S are both independent of a.

We also learn from (33) [given in the proof of Theorem 6] that the Stackelberg equilibrium wholesale price is $w^S = (\alpha + s/2)r^S + a/(2b) + [c + C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)]/2$. Differentiating w^S once w.r.t. a yields that $\partial w^S/\partial a = 1/(2b) > 0$, which means that, as the constant parameter a is increased, the manufacturer should respond by increasing his wholesale price. In addition, we compute the first-order derivative of the retailer's Stackelberg equilibrium retail price p^S—that is given as in (34)—w.r.t. a, and find that $\partial p^S/\partial a = 3/(4b) > \partial w^S/\partial a > 0$. That is, when the value of a is increased, the retailer should raise her retail price. Furthermore, the increase in the retail price is larger than the increase in the manufacturer's wholesale price. As a consequence, the demand for the manufacturer's product is increasing in a because $\partial D(p^S)/\partial a = 1/4$.

To analyze the impacts of a on the manufacturer's and the retailer's profits, we differentiate Π_M in (35) and Π_R in (36) once w.r.t. a, and have,

$$\begin{aligned}\frac{\partial \Pi_M}{\partial a} &= \frac{1}{2b}D(p^S) + \frac{1}{8}\left[\frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1 - \beta^S)\right] > 0,\\ \frac{\partial \Pi_R}{\partial a} &= \frac{1}{4b}D(p^S) + \frac{1}{8}\left[2p^S - \frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1 - \beta^S)\right] > 0,\end{aligned}$$

which means that the two supply chain members benefit from the increase in the value of a. Noting that the manufacturer's unit profit [i.e., $a/b - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)$] is greater than the retailer's unit profit [i.e., $2p^S - a/b - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)$], as discussed in Section 3.3, we find that $\partial\Pi_M/\partial a > \partial\Pi_R/\partial a$.

Next, we discuss the impacts of a when the retailer does not participate in the quality assurance. Similar to the above, we find that the manufacturer's defective rate $\bar{r}^S$ does not depend on the value of a, because, as Theorem 4 indicates, the equation that $s + \hat{\alpha} = -C'(\bar{r}^S)$ does not involve the parameter a. In addition, we differentiate $\bar{w}^S$ and $\bar{p}^S$ once w.r.t. a, and find that $\partial\bar{p}^S/\partial a = 3/(4b) > \partial\bar{w}^S/\partial a = 1/(2b)$, which is the same as the case in which the retailer serves as the gatekeeper. It then follows that $\partial D(\bar{p}^S)/\partial a = 1/4$.

We calculate the first-order derivatives of π_M and π_R [as given in (37)] w.r.t. a, and find,

$$\begin{aligned}\frac{\partial\pi_M}{\partial a} &= \frac{1}{4}[(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)] + \frac{D(\bar{p}^S)}{2b} > 0,\\ \frac{\partial\pi_R}{\partial a} &= \frac{1}{4}[(\bar{p}^S - \bar{w}^S) + (\alpha - \hat{\alpha}) \times \bar{r}^S] + \frac{D(\bar{p}^S)}{4b} > 0,\end{aligned}$$

which imply that both π_M and π_R are increasing in a. Similar to the above, we also find that $\partial\pi_M/\partial a > \partial\pi_R/\partial a$. This theorem is thus proved. ■

Proof of Theorem 11. Similar to the proof of Theorem 10, we can find that Stackelberg equilibrium decisions r^S and β^S (for the game with the retailer's gatekeeping effort) and $\bar{r}^S$ (for the game without the effort) are all independent of the variable demand parameter b.

When the retailer acts as the quality gatekeeper, we differentiate w^S in (33) once w.r.t. b, and find that $\partial w^S/\partial b = -a/(2b^2) < 0$, which means that, as the parameter b is increased, the manufacturer should reduce his wholesale price. In addition, we compute the first-order derivative of p^S in (34) w.r.t. b, and have, $\partial p^S/\partial b = -3a/(4b^2) < \partial w^S/\partial a < 0$. That is, when the value of b is increased, the retailer should decrease her retail price. Furthermore, the reduction in the retail price is smaller than the reduction in the manufacturer's wholesale price. However, since $\partial D(p^S)/\partial b = -p^S + 3a/(4b) = (3a - 4bp^S)/(4b)$, we find that the demand for the manufacturer's product may or may not be increasing in b, which depends on the sign of the term $3a - 4bp^S$.

We then calculate the first-order derivative of Π_M w.r.t. b, and find that

$$\begin{aligned}\frac{\partial \Pi_M}{\partial b} &= -\frac{a}{8b}\left[\frac{a}{b} - sr^S - c - C(r^S) - \hat{C}(\beta^S) - r^S\hat{\alpha}(1-\beta^S)\right] \\ &\quad -\frac{D(p^S)}{2b}[sr^S + c + C(r^S) + \hat{C}(\beta^S) + r^S\hat{\alpha}(1-\beta^S)] \\ &< 0,\end{aligned}$$

which means that the manufacturer's profit Π_M is always decreasing in b. We also compute the first-order derivative of Π_R w.r.t. b, and have $\partial\Pi_R/\partial b = D(p^S)(3a - 4bp^S)/(2b^2) - [D(p^S)]^2/b^2$, which is negative if $3a < 4bp^S$.

Next, we examine the impacts of b when the retailer does not serve as the gatekeeper. The first-order derivatives of $\bar{w}^S$ and $\bar{p}^S$ w.r.t. b are given as follows: $\partial\bar{p}^S/\partial b = -3a/(4b^2) < \partial\bar{w}^S/\partial a = -a/(2b^2) < 0$. Similarly, we find that $\partial D(\bar{p}^S)/\partial b = (3a - 4b\bar{p}^S)/(4b)$. We then differentiate π_M and π_R once w.r.t. b, and find that

$$\begin{aligned}\frac{\partial \pi_M}{\partial b} &= -aD(\bar{p}^S)/(8b^2) - 3D(\bar{p}^S) \times [c + C(\bar{r}^S) + (s+\hat{\alpha})\bar{r}^S]/(8b) \\ &\quad -\bar{p}^S[(\bar{w}^S - c) - (\alpha + s)\bar{r}^S - C(\bar{r}^S)]/4 \\ &< 0,\end{aligned}$$

and $\partial\pi_R/\partial b = D(\bar{p}^S)(3a - 4b\bar{p}^S)/2b^2 - [D(\bar{p}^S)]^2/b^2$, which is negative if $3a < 4b\bar{p}^S$, similar to the case in which the retailer serves as the quality gatekeeper. The theorem is thus proved. ■

Appendix C Proofs of Corollaries

Proof of Corollary 1. From (4), we find that the manufacturer should determine his optimal defective rate r^* such that $s + \hat{\alpha}(1-\beta^*) = -C'(r^*)$, which can be re-written as,

$$\hat{\alpha}r^* = -[s + C'(r^*)]\frac{r^*}{1-\beta^*}. \tag{38}$$

From Theorem 1, we note that, when the manufacturer's defective rate is r^*, the retailer's optimal defective identification rate β^* satisfies the equation that $\hat{C}'(\beta^*) = \hat{\alpha}r^*$. Substituting this into (38), we find that $-[s+C'(r^*)]r^*/(1-\beta^*) = \hat{C}'(\beta^*)$, which proves this corollary. ■

Proof of Corollary 2. We find that, when the retailer serves as the quality gatekeeper, her unit profit is $D(p^*)/b = a/b - p^*$, where p^* is given as in (2); but, when the retailer is not involved into the quality assurance, her unit profit is $D(\bar{p}^*)/b = a/b - \bar{p}^*$, where $\bar{p}^*$ is given as in (9). In order to determine in which setting the retailer's unit profit is higher, we need to compare p^* and $\bar{p}^*$. Using (2) and (9) we find that, if $\hat{C}(\beta^*) < \hat{\alpha}r\beta^*$, then $\hat{C}(\beta^*) - r[\alpha - \hat{\alpha}(1-\beta^*)] < -r(\alpha - \hat{\alpha})$, which means that $p^* < \bar{p}^*$. It then follows that $D(p^*) > D(\bar{p}^*)$ and $D(p^*)/b > D(\bar{p}^*)/b$. The corollary is thus proved. ■

Appendix D Numerical Results for the Sensitivity Analysis in Section 4.2

In this appendix, we provide our numerical results for the sensitivity analysis of some parameters in Section 4.2. Specifically, Table 3 indicates the impacts of the retailer's unit defect penalty cost $\hat{\alpha}$ and the manufacturer's unit defect disposing cost s, which are depicted as in Figures 3 and 4, respectively. In Table 4, we present the data indicating the impacts of the parameters z_1 and z_2 in the manufacturer's unit quality-assurance cost function $C(r) = z_1 \exp(-z_2 r)$, which are used to plot Figures 5 and 6. Table 5 involves the data that characterize the impacts of the parameters z_3 and z_4 in the retailer's unit quality-assurance cost function $\hat{C}(\beta) = z_3 \exp(z_4 \beta)$, which correspond to Figure 7 and 8.

Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
$\hat{\alpha}$	r^S (%)	β^S (%)	w^S (\$)	p^S (\$)	Π_M (\$ $\times 10^3$)	Π_R (\$ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ (\$)	$\bar{p}^S$ (\$)	π_M (\$ $\times 10^3$)	π_R (\$ $\times 10^3$)
100	7.19	59.05	121.66	134.83	460.35	230.17	2.07	119.80	134.90	455.90	227.95
105	7.05	59.94	121.50	134.86	458.16	229.08	1.77	119.76	134.93	454.46	227.23
110	6.92	60.79	121.35	124.90	456.07	228.03	1.49	119.74	134.95	453.23	226.61
115	6.79	61.59	121.22	134.93	454.06	227.03	1.22	119.74	134.96	452.21	226.11
120	6.67	62.34	121.08	134.96	452.14	226.07	0.95	119.76	134.98	451.40	225.70
125	6.55	63.06	120.96	135.00	450.29	225.14	0.70	119.80	134.99	450.78	225.39
130	6.43	63.74	120.84	135.02	448.51	224.25	0.46	119.85	135.00	450.34	225.17
135	6.33	64.39	120.72	135.05	446.79	223.40	0.22	119.92	135.00	450.08	225.04
140	6.22	65.00	120.61	135.08	445.14	222.57	0.00	120.00	135.00	450.00	225.00
145	6.12	65.58	120.51	135.11	443.54	221.77	0.00	120.00	135.00	450.00	225.00

Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
s	r^S (%)	β^S (%)	w^S (\$)	p^S (\$)	Π_M (\$ $\times 10^3$)	Π_R (\$ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ (\$)	$\bar{p}^S$ (\$)	π_M (\$ $\times 10^3$)	π_R (\$ $\times 10^3$)
6	8.42	69.63	121.97	134.89	456.65	228.32	1.16	119.70	134.97	452.03	226.02
7	7.92	67.73	121.70	134.91	455.42	227.71	1.11	119.72	134.97	451.86	225.93
8	7.47	65.90	121.47	134.93	454.26	227.13	1.06	119.73	134.97	451.70	225.85
9	7.05	64.11	121.27	134.95	453.17	226.58	1.01	119.75	134.97	451.55	225.77
10	6.67	62.34	121.08	134.96	452.14	226.07	0.95	119.76	134.98	451.40	225.70
11	6.30	60.57	120.92	134.98	451.16	225.58	0.90	119.78	134.98	451.26	225.62
12	5.95	58.79	120.77	135.00	450.24	225.12	0.85	119.79	134.98	451.13	225.56
13	5.61	56.96	120.63	135.01	449.38	224.69	0.80	119.81	134.98	451.00	225.50
14	5.28	55.08	120.50	135.02	448.56	224.28	0.75	119.82	134.99	450.89	225.44
15	4.96	53.12	120.38	135.04	447.79	223.90	0.70	119.83	134.99	450.78	225.39

Table 3: The numerical results for the sensitivity analysis of the retailer's unit defect penalty cost and that of the manufacturer's unit defect disposing cost.

Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
z_1	r^S (%)	β^S (%)	w^S ($)	p^S ($)	Π_M ($ $\times 10^3$)	Π_R ($ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ ($)	$\bar{p}^S$ ($)	π_M ($ $\times 10^3$)	π_R ($ $\times 10^3$)
9	3.85	45.20	119.58	134.85	458.85	229.42	0.25	119.45	134.75	465.22	232.61
9.5	5.52	56.45	120.39	134.91	455.15	227.58	0.61	119.61	134.87	458.10	229.05
10	6.67	62.34	121.08	134.96	452.14	226.07	0.95	119.76	134.98	451.40	225.70
10.5	7.64	66.60	121.73	135.01	449.57	224.79	1.28	119.91	135.08	445.07	222.53
11	8.51	70.00	122.34	135.04	447.34	223.67	1.59	120.05	135.18	439.07	219.54
11.5	9.32	72.82	122.94	135.08	445.38	222.69	1.89	120.18	135.28	433.38	216.69
12	10.08	75.27	123.52	135.11	443.64	221.82	2.17	120.31	135.37	427.97	213.99
12.5	10.81	77.44	124.09	135.13	442.09	221.04	2.44	120.43	135.46	422.81	211.41
13	11.50	79.39	124.66	135.16	440.69	220.35	2.70	120.55	135.55	417.88	208.94
13.5	12.18	81.18	125.22	135.18	439.44	219.72	2.95	120.66	135.63	413.17	206.58
Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
z_2	r^S (%)	β^S (%)	w^S ($)	p^S ($)	Π_M ($ $\times 10^3$)	Π_R ($ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ ($)	$\bar{p}^S$ ($)	π_M ($ $\times 10^3$)	π_R ($ $\times 10^3$)
13	6.31	60.61	121.15	135.10	444.30	222.15	0	120.00	135.00	450.00	225.00
14	6.60	62.05	121.18	135.03	448.33	224.17	0.53	119.88	134.99	450.39	225.20
15	6.67	62.34	121.08	134.96	452.14	226.07	0.96	119.76	134.98	451.40	225.70
16	6.63	62.17	120.95	134.91	455.71	227.86	1.30	119.65	134.95	452.82	226.41
17	6.55	61.77	120.79	134.85	459.07	229.54	1.58	119.53	134.92	454.53	227.27
18	6.44	61.26	120.63	134.80	462.24	231.12	1.81	119.42	134.89	456.44	228.22
19	6.32	60.67	120.46	134.75	465.22	232.61	2.00	119.32	134.86	458.46	229.23
20	6.19	60.05	120.31	134.70	468.04	234.02	2.15	119.22	134.83	460.56	230.28
21	6.07	59.42	120.15	134.66	470.72	235.36	2.28	119.12	134.79	462.70	231.35
22	5.95	58.77	120.01	134.62	473.25	236.63	2.39	119.03	134.75	464.85	232.43

Table 4: The numerical results for the sensitivity analysis of the parameters in the manufacturer's quality-assurance unit cost function.

Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
z_3	r^S (%)	β^S (%)	w^S (\$)	p^S (\$)	Π_M (\$ $\times 10^3$)	Π_R (\$ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ (\$)	$\bar{p}^S$ (\$)	π_M (\$ $\times 10^3$)	π_R (\$ $\times 10^3$)
0.30	8.80	74.92	122.60	134.87	457.56	228.78	0.95	119.76	134.98	451.40	225.70
0.31	8.14	71.49	122.10	134.90	456.00	228.00	0.95	119.76	134.98	451.40	225.70
0.32	7.59	68.29	121.70	134.92	454.57	227.29	0.95	119.76	134.98	451.40	225.70
0.33	7.10	65.26	121.37	134.95	453.30	226.65	0.95	119.76	134.98	451.40	225.70
0.34	6.67	62.34	121.08	134.96	452.14	226.07	0.95	119.76	134.98	451.40	225.70
0.35	6.63	59.51	120.84	134.98	451.08	225.54	0.95	119.76	134.98	451.40	225.70
0.36	5.90	56.74	120.62	135.00	450.12	225.06	0.95	119.76	134.98	451.40	225.70
0.37	5.56	54.00	120.43	135.01	449.23	224.62	0.95	119.76	134.98	451.40	225.70
0.38	5.23	51.28	120.26	135.03	448.43	224.21	0.95	119.76	134.98	451.40	225.70
0.39	4.92	48.54	120.11	135.04	447.69	223.84	0.95	119.76	134.98	451.40	225.70
Parameter	**The Game with the Retailer's Gatekeeping**						**The Game without the Retailer's Gatekeeping**				
z_4	r^S (%)	β^S (%)	w^S (\$)	p^S (\$)	Π_M (\$ $\times 10^3$)	Π_R (\$ $\times 10^3$)	$\bar{r}^S$ (%)	$\bar{w}^S$ (\$)	$\bar{p}^S$ (\$)	π_M (\$ $\times 10^3$)	π_R (\$ $\times 10^3$)
3.10	8.37	72.74	122.23	134.92	455.02	227.51	0.95	119.76	134.98	451.40	225.70
3.15	7.41	67.17	121.55	134.94	453.43	226.72	0.95	119.76	134.98	451.40	225.70
3.20	6.67	63.34	121.08	134.96	452.14	226.07	0.95	119.76	134.98	451.40	225.70
3.25	6.06	57.99	120.73	134.98	451.07	225.53	0.95	119.76	134.98	451.40	225.70
3.30	5.55	53.98	120.46	135.00	450.17	225.09	0.95	119.76	134.98	451.40	225.70
3.35	5.11	50.24	120.25	135.01	449.42	224.71	0.95	119.76	134.98	451.40	225.70
3.40	4.72	46.71	120.08	135.02	448.79	224.39	0.95	119.76	134.98	451.40	225.70
3.45	4.36	43.34	119.94	135.03	448.25	224.13	0.95	119.76	134.98	451.40	225.70
3.50	4.04	40.11	119.82	135.04	447.80	223.90	0.95	119.76	134.98	451.40	225.70
3.55	3.74	36.97	119.73	135.04	447.42	223.71	0.95	119.76	134.98	451.40	225.70

Table 5: The numerical results for the sensitivity analysis of the parameters in the retailer's quality-assurance unit cost function.

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