Interpersonal bundling

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Abstract. Sellers sometimes offer goods for sale under both a regular price and a discount for group purchase if the consumer group reaches some minimum size. This selling practice, which we term *interpersonal bundling*, is analyzed in a general framework of product bundling. We characterize the optimal prices and bundle size by a monopoly seller, and explain why interpersonal bundling is a profitable strategy in response to demand uncertainty. We also discuss other strategic considerations in formulating this selling strategy. Our analysis provides sufficient conditions for interpersonal bundling to dominate separate selling, and offers insights on how to enhance its profit advantage.

Keywords: Interpersonal bundling, bundling, group purchase, group discount, demand uncertainty

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1. INTRODUCTION

This paper studies a form of product bundling where a good is offered for sale under both a regular price and a discount for group purchase if the consumer group reaches some minimum size—the bundle size. The defining characteristic of this selling format is that the purchase of the bundle is made by different consumers—and hence we term it *interpersonal bundling*—rather than by a single consumer as under traditional mixed bundling.[1]

Interpersonal bundling is a widely observed selling practice. In many markets and for many goods, multiple consumers may form a purchase group to qualify for a group discount, as, for example, when buying tickets for a concert, purchasing a tour, or dining at a restaurant.[2] In recent years, many Internet sites have emerged that allow sellers to offer interpersonal bundling, where consumers purchasing with group coupons receive substantial discounts when the minimum group size is reached. Launched in November 2008, Groupon was a pioneer in this selling format on the Internet, and it exceeded a billion dollars in revenue in just its third year of operation (Levin, 2012).[3] Despite its popularity, the pricing and profitability of interpersonal bundling have not been studied in a general bundling framework. How should a seller optimally choose prices and bundle size under interpersonal bundling? When will interpersonal bundling be more profitable than separate selling?[4] What determine the size of its profit advantage? How does this selling format

[1] Mixed bundling refers to offering goods for sale both as a package and as individual components.

[2] Miller Farms, a local family farm in Colorado, runs the Fall Harvest Festival each year. In 2012, a customer is charged $15 to participate in the Festival and pick up vegetables to take home. For a group of 10 or more, the price per person is lowered to $13.

[3] Many other group buying websites offer variants of interpersonal bundling, including Livingsocial, where a consumer receives a free deal if she gets three people buy the product. There are numerous interpersonal bundling sites around the global, such as uBuyiBuy, Gaopeng, and Lashou in Asia, MyCityDeal in Europe, Downtown Colombia in South America, and Spreets in Australia.

[4] Here, separate selling means offering a good for sale under a single unit price to all consumers, whereas a pure bundle would consist of multiple units of the same good under a unit price for group purchase. The recent economics literature has investigated product bundling that is different from traditional mixed bundling. See, for example, the study of bundle size pricing by Chu, Leslie, and Sorensen (2011), and of inter-firm bundling by Gans and King (2006) and Armstrong (2012).

affect consumers and welfare? We provide some answers to these questions in this study.

The literature on product bundling has found that mixed bundling often is more profitable than separate selling through two main mechanisms: segmenting consumer population to facilitate price discrimination and reducing the dispersion of consumer values to extract consumer surplus (e.g., Adams and Yellen 1976; Schmalensee, 1984; Long, 1984; McAfee, McMillan, and Whinston 1989; and Chen and Riordan, 2011).[5] This paper will explore an alternative motive for bundling: as a profitable strategy in response to demand uncertainty. While this motive can also arise when each bundle is purchased by an individual consumer,[6] it is especially relevant and important for interpersonal bundling.

We start with a stylized model where a monopolist sells to a consumer population with an uncertain number of low-value consumers and possibly also an uncertain number of high-value consumers. Under separate selling, the seller may optimally pursue either a high-price strategy targeting only the high-value consumers, or a low-price strategy that will also attract low-value consumers. The low price will be profitable only if it results in a sufficiently high sales volume—if the number of low-value consumers is sufficiently large. However, because price is set before the uncertainty is resolved, setting a single price is generally not optimal. By offering the good for sale under interpersonal bundling, the low price will become effective only if it will indeed lead to a sufficiently high increase in sales, while the high price will prevail when the number of low-value consumers turns out to be relatively small. Thus, interpersonal bundling potentially enables the seller to use optimal option pricing under uncertain demand, leading to higher profit than separate selling.

Our analysis of the basic model leads to several interesting results. After characterizing the optimal prices and bundle size, we provide a sufficient condition for interpersonal

[5] In a standard model of two goods, some consumers may value one good highly but another very little, while others may value two goods together relatively highly, and values for the bundle may be less dispersed than values for individual goods. By charging the former (who purchase only a single unit) a higher price and the latter a bundle discount, mixed bundling generally leads to higher profit than separate selling.

[6] Under standard mixed bundling with two goods, there can be uncertainties on each individual consumer's valuation for the two goods, and mixed bundling can thus be viewed as a form of option pricing, where a consumer will obtain the bundle discount only if she has sufficiently high demand for both goods.

bundling to dominate separate selling. Remarkably, this condition is invariant to the functional forms of the distributions of consumer numbers. We also show how the profit difference between interpersonal bundling and separate selling may vary with parameter values of the market environment, and provide simple conditions for the former to have higher or lower welfare than the latter. Moreover, in the simple setting of the basic model, we find that interpersonal bundling is in fact an optimal selling scheme among all selling mechanisms that may depend on realized aggregate demand. Our analysis also explores alternative assumptions on the distributions of consumer values.

We further present two variants of the basic model to explore how a seller may incorporate additional strategic considerations in designing a bundle, with explicit modeling of the decision process of individual consumers. In the first variant, we allow the possibility that some consumers are initially uninformed about the existence of the seller's product. Then, in order to qualify for the discount available only when the minimum bundle size is reached, informed consumers may take (costly) actions to transmit product information to the uninformed, and the seller can exploit this incentive when strategically setting the bundle size. Interpersonal bundling can thus increase the seller's profit by facilitating the dissemination of product information. While this informational role of group buying has also been identified and explored in Jing and Xie (2011), their model focuses on exogenously fixed group size and known demand. By contrast, bundle size is a key decision variable in our analysis of interpersonal bundling, and demand uncertainty is a central feature of our model that interacts with the consideration for information transmission.

Our second variant of the basic model brings in the possibility that high-value consumers need to incur transaction costs to sign up for group purchase. The seller may then be able to partially segment the consumer population when practicing interpersonal bundling, pushing high-value consumers with high sign-up costs to purchase at the regular price while attracting low-value consumers with the bundle discount. To allow for a richer modeling of consumers' decision process, we consider two forms of interpersonal bundling in a two-period setting, in parallel to Hu, Shi and Wu (2013)'s study of two group-buying mechanisms: a simultaneous format where the seller does not inform period-2 consumers how many buyers signed up in period 1, and a sequential format where the seller does. Hu, Shi and Wu (2013) find that the seller prefers the sequential mechanism, because it encourages consumer participation by removing their uncertainty in period 2, which leads to higher group formation rates. Interestingly, in our model the seller, who aims to maximize profit, may instead prefer the simultaneous format. This is because the simultaneous format does *not* remove uncertainty to the high-value consumers, which facilitates price discrimination by discouraging the high-value consumers from obtaining the bundle discount.[7]

In addition to offering a new perspective on product bundling, this paper is also closely related to the literature on pricing under demand uncertainty. Dana (2001), for example, studies a model in which demand can be *either* high or low. He finds that a monopolist optimally offers two prices, with only a limited quantity offered under a low price, which is set for the low demand state. A high price then allows the firm to extract additional consumer surplus when demand turns out to be high, in which case the limited quantity available at the low price will sell out so that some high-priced units will be purchased. Anand and Aron (2003), in an early study of web-based group buying, also consider a model with either a high or a low demand regime, represented by two linear demand functions. They demonstrate that group buying may enable the seller to set price-quantity schedules that optimize revenue under each demand regime, and that the profitability of group buying relative to posted pricing depends on whether the two linear demand functions are parallel or intersecting.[8] Our paper differs by adopting an entirely different analytical approach, capturing the group buying problem in a general bundling framework. Additionally, we are concerned with the uncertainty of a different nature: there are *both* high- and low-value consumers, and the uncertainty is over their respective numbers. We believe that

[7] In advertising group purchase deals, Groupon informs potential buyers how many consumers have already signed up for a deal, consistent with the strategy of maximizing deal success rates (Hu, Shi, Wu, 2013). As we shall further discuss later, some of the practices by Groupon, in its role as an intermediary offering interpersonal bundling, may not be in the best interest of sellers.

[8] Also related are Gale and Holmes (1992, 1993), who study how a monopolist may use advance purchase discounts to allocate capacity more efficiently in the presence of demand uncertainty. See also Dana (1998) for a related analysis in competitive markets.

this formulation is a plausible, albeit simplified, description of the market environment faced by many firms, and it is especially relevant for interpersonal bundling. Furthermore, as discussed above, our analysis leads to interesting new results on the profitability and optimal design of interpersonal bundling. Our paper thus contributes to the literatures on product bundling, on pricing under demand uncertainty, and, more generally, on the economics and management of marketing.

In the rest of the paper, we conduct our core analysis with respect to the basic model in section 2, study the two variants of the basic model in section 3, and conclude in section 4.

2. DEMAND UNCERTAINTY AND INTERPERSONAL BUNDLING

2.1 Basic Model

A monopolist offers a product for sale. There are two types of consumers, high-value and low-value, whose product valuations are respectively H and L, with $H > L > 0$. A consumer's type is her private information, and each consumer desires to purchase at most one unit. The numbers of low- and high-value consumers are respectively x and y, which are realizations of random variables X and Y that have joint distribution function $G(x, y)$ on support $[a_x, b_x] \times [a_y, b_y]$, where $0 \leq a_x < b_x$ and $0 \leq a_y \leq b_y$. The marginal distribution function of X is $F(x)$, with density function $f(x) > 0$ on $[a_x, b_x]$. Production cost is normalized to zero, and the firm maximizes expected profit.

Let $\bar{x}$ and $\bar{y}$ be the expected number of low- and high-value consumers, respectively. Then

$$\bar{x} = \int_{a_x}^{b_x} x dF(x); \qquad \bar{y} = \int_{a_x}^{b_x} \int_{a_y}^{b_y} y dG(x, y). \tag{1}$$

We allow the possibility that $y = \bar{y}$ is a constant, in which case $G(x, y)$ degenerates to $F(x)$.[9]

As a benchmark, consider the case of separate selling where the firm posts a single unit price to all consumers. Then, profit is higher under $p = H$ if $H\bar{y} > L(\bar{x} + \bar{y})$ and it is higher under $p = L$ if $H\bar{y} < L(\bar{x} + \bar{y})$. It follows that the optimal price and the corresponding

[9] The uncertainty in x is essential for our analysis and is maintained throughout the paper.

profit are, respectively:[10]

$$p^s = \begin{cases} H & if \quad \bar{x} \leq \left(\frac{H}{L} - 1\right)\bar{y} \\ L & if \quad \bar{x} > \left(\frac{H}{L} - 1\right)\bar{y} \end{cases}, \qquad \pi^s = \begin{cases} H\bar{y} & if \quad \bar{x} \leq \left(\frac{H}{L} - 1\right)\bar{y} \\ L\left(\bar{x} + \bar{y}\right) & if \quad \bar{x} > \left(\frac{H}{L} - 1\right)\bar{y} \end{cases}. \tag{2}$$

Therefore, if the expected number of low-value consumers ($\bar{x}$) is small, the firm will only sell to the high-value consumers at $p^s = H$; otherwise, it will sell to all consumers at $p^s = L$.

Under interpersonal bundling, the firm sets a stand-alone unit price p, a discounted unit price under group purchase $q \leq p$, and a minimum group size m for the discounted price to take effect (i.e., for the deal to be on). Each consumer can separately purchase the good at price p, but consumers who sign up for group purchase can buy at the discounted price q if and only if there are M ($\geq m$) consumers in the group. Notice that if $q = p$, then bundling is equivalent to separate selling under price p. In our basic model, we assume that there is no transaction cost for a consumer to use the group coupon, which implies that if $q < p$, all consumers will attempt to purchase at the discounted bundle price. Thus, bundled selling with (p, q, m) is equivalent to separate selling with price q if $m \leq a_x + a_y$, and to separate selling with price p if $m \geq b_x + b_y$.

2.2 Profitability of Interpersonal Bundling

Under bundling, with (p, q, m), all consumers will purchase at price q if $x + y \geq m$ and $q \leq L$, whereas when $x + y < m$ only high-value consumers will purchase at price p if $L < p \leq H$. The firm's problem is to maximize (expected) profit:

$$\max_{q \leq L < p \leq H, m} \pi\left(p, q, m\right) = q \int\int_{x+y \geq m} \left(x + y\right) dG\left(x, y\right) + p \int\int_{x+y<m} y dG\left(x, y\right). \tag{3}$$

Since $\pi\left(p, q, m\right)$ weakly increases in p and q for any m, the optimal p and q that maximize $\pi\left(p, q, m\right)$ are $p^* = H$ and $q^* = L$. Hence the firm's maximum profit under bundling and the optimal (minimum) bundle size are

$$\pi^* \equiv \max_m \pi\left(H, L, m\right); \quad m^* = \arg\max_m \pi\left(H, L, m\right). \tag{4}$$

[10] For ease of exposition, when profit is the same under $p = H$ and $p = L$, we assume $p^s = H$.

Notice that m can be set low enough so that it is not a constraint, or high enough so that the minimum group size can never be reached. In particular, $\pi(H, L, a_x + a_y) = L(\bar{x} + \bar{y})$ and $\pi(H, L, b_x + b_y) = H\bar{y}$, which implies $\pi^* \geq \pi^s$. Thus, same as mixed bundling, interpersonal bundling will always be at least as profitable as separate selling. We are, however, interested in when interpersonal bundling is more profitable than separate selling, and how large its profit advantage is. Condition (A1) below provides a sufficient condition for $\pi^* > \pi^s$:

$$\left(1 + \frac{a_x}{a_y}\right) < \frac{H}{L} < \left(1 + \frac{b_x}{b_y}\right). \tag{A1}$$

Proposition 1 *Interpersonal bundling is always at least as profitable as separate pricing, and it is more profitable than separate selling if condition (A1) holds.*

Proof. We show that under (A1) interpersonal bundling is more profitable than separate selling whether $L(\bar{x} + \bar{y}) \leq H\bar{y}$ or $L(\bar{x} + \bar{y}) > H\bar{y}$.

(i) If $L(\bar{x} + \bar{y}) \leq H\bar{y}$, $\pi^s = H\bar{y}$ under separate selling; and, if in addition $Hb_y < L(b_x + b_y)$, then for $\varepsilon \equiv \frac{1}{2}\left(b_x + b_y - \frac{H}{L}b_y\right) > 0$,

$$\begin{aligned}
\pi^* &\geq \pi(H, L, b_x + b_y - \varepsilon) = \int\int_{x+y \geq b_x+b_y-\varepsilon} [L(x+y) - Hy]\, dG(x,y) + H\bar{y} \\
&\geq \int\int_{x+y \geq b_x+b_y-\varepsilon} [L(b_x + b_y - \varepsilon) - Hy]\, dG(x,y) + H\bar{y} > H\bar{y} = \pi^s.
\end{aligned}$$

(The first inequality above is due to revealed preference, the second to $x+y \geq b_x+b_y-\varepsilon$, and the last to $L(b_x + b_y - \varepsilon) - Hy = \frac{1}{2}(L(b_x + b_y + Hb_y)) - Hy > \frac{1}{2}L(b_x + b_y - Hb_y) > 0$.)

(ii) If $L(\bar{x} + \bar{y}) > H\bar{y}$, $\pi^s = L(\bar{x} + \bar{y})$ under separate selling; and, if in addition $Ha_y > L(a_x + a_y)$, then for $\varepsilon \equiv \frac{1}{2}\left[\frac{H}{L}a_y - (a_x + a_y)\right] > 0$,

$$\begin{aligned}
\pi^* &\geq \pi(H, L, a_x + a_y + \varepsilon) = L(\bar{x} + \bar{y}) + \int\int_{x+y<a_x+a_y+\varepsilon} [Hy - L(x+y)]\, dG(x,y) \\
&> L(\bar{x} + \bar{y}) + \int\int_{x+y<a_x+a_y+\varepsilon} [Hy - L(a_x + a_y + \varepsilon)]\, dG(x,y) > L(\bar{x} + \bar{y}) = \pi^s.
\end{aligned}$$

■

The proof of Proposition 1 uses simple arguments that start from the optimal prices under separate selling: if $p^s = H$, profit can be increased by keeping the regular price but adding a group bundle with unit price L and a minimum size that is slightly lower than $b_x + b_y$ (the maximum possible total number of consumers); if $p^s = L$, profit can be increased by rasing the regular price to H and adding a bundle with unit price L and a minimum size that is slightly higher than $a_x + a_y$ (the minimum possible total number of consumers). Condition (A1), which requires $b_y H < (b_x + b_y) L$ and $(a_x + a_y) L < a_y H$, ensures that these changes starting from separate selling will indeed strictly increase profit. This condition is thus invariant to the functional form of the joint distribution of X and Y, depending only on the upper and lower limits of the support for the distribution. It holds if the H/L ratio is relatively large compared to a_x/a_y but small compared to b_x/b_y. Intuitively, when (A1) holds, profit can be higher either with high price (H) or with low price (L), depending on the demand realization. Interpersonal bundling allows the firm to sell at the low price only if profit is higher under the low price—otherwise the high price will prevail, thereby assuring a higher profit than separate selling.[11]

In many situations where group coupons are issued by sellers such as restaurants and hair salons, H could be considered as the regular price at which the seller has less uncertainty about the number of consumers. Thus the difference between a_y and b_y tends to be relatively small. On the other hand, there might be more uncertainty about the number of consumers who will purchase at the sale price L, so the difference between a_x and b_x tends to be relatively large. In such situations, condition (A1) is likely satisfied.[12]

To illustrate our result and to make explicit profit comparisons, consider the example below:

Example 1 *Suppose that X and Y are independently and uniformly distributed on $[0, 3]$ and $[1, 2]$, respectively. Then, $\bar{x} = \bar{y} = \frac{3}{2}$, $p^s = H$ if $H \geq 2L$, $p^s = L$ if $H < 2L$, and (A1)*

[11] If H/L is too small, it may be optimal always to sell at $p^s = L$, so the option to sell at alternative prices under interpersonal bundling has no value. Likewise, if H/L is too large, it could be optimal always to sell at $p^s = H$, which would then achieve the same profit as interpersonal bundling. Notice that if $a_x = 0$, then (A1) becomes $H < L(1 + b_x/b_y)$ and bundling is always more profitable than charging $p^s = L$.

[12] We may view interpersonal bundling as allowing the seller to experiment with a lower price that will prevail only when the number of purchasing consumers reaches a minimum size, or only when it is more profitable than the regular price.

holds if $H < \frac{5}{2}L$. *Under interpersonal bundling,*

$$\pi(H,L,m) \equiv L \int_{\max\{1,m-3\}}^{2} \int_{\max\{m-y,0\}}^{3} (x+y)\frac{1}{3}dxdy + H \int_{1}^{\min\{m,2\}} \int_{0}^{\min\{m-y,3\}} y\frac{1}{3}dxdy.$$

Setting $\partial\pi(H,L,m)/\partial m = 0$, *we find the optimal (minimum) bundle size as*

$$m^* = \begin{cases} \frac{H}{2L-H}, \text{ with } \pi^* > \pi^s & if \quad H \leq \frac{4}{3}L \\ \frac{3}{2}\frac{H}{L}, \text{ with } \pi^* > \pi^s & if \quad \frac{4}{3}L < H < 2.6L \\ \geq 5, \text{ with } \pi^* = \pi^s & if \quad 2.6L \leq H \end{cases}.$$

For instance, if $L = 1$ *and* $H = 2$, *then* $m^* = 3$ *and* $\pi^* = 3.3333 > \pi^s = 3$, *so interpersonal bundling increases (expected) profit by about 11%.*

Several observations can be made in Example 1. First, condition (A1) is sufficient, but not necessary, for the profitability of interpersonal bundling. In Example 1, while (A1) holds for $H < 2.5L$, bundling is also profitable when $H \in [2.5L, 2.6L)$.

Second, (A1) is fairly tight as a sufficient condition. When $H \geq 2.6L$, interpersonal bundling is no longer profitable. In this case, $\frac{3}{2}\frac{H}{L} \geq \frac{3}{2}(2.6) = 3.9$. However, for any $m \in [3.9, 5)$, the expected profit under $x+y \geq m$, in which case all sales will occur at the discounted price L, is lower than the expected profit under separate selling. Therefore, it is optimal for the seller not to offer the bundle, which is equivalent to setting a sufficiently large bundle size ($m^* \geq 5$).

Third, when interpersonal bundling is profitable, m^* increases in H but decreases in L. A marginal increase in m reduces the probability that the sale will occur at the low price (with a large volume) and raises the probability that the sale will occur at the high price. Thus, m^*, which balances these two effects, increases with the high price and decreases with the low price. Put differently, a higher H (or a lower L) makes sales under the stand-alone price H relatively more profitable, reducing the benefit of selling at the bundle discount. Consequently the optimal (minimum) bundle size with which the discount price will become effective is larger.

We now turn to the question of how the advantage of interpersonal bundling, relative to separate selling, may vary with the market environment. We first consider how the ratio

H/L, or the difference between the reservation prices of the high- and low-value consumers, affects the relative profitability of bundling.

Corollary 1 *Suppose that (A1) holds and L is fixed. Then, $\pi^* - \pi^s$ exhibits an inverted-U shape with respect to changes in H, first increasing and then decreasing, reaching maximum at $H = \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$.*

Proof. When $H < \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$, $\pi^* - \pi^s = \max_m \pi(H, L, m) - L(\bar{x} + \bar{y})$. From (3), $\pi(H, L, m)$ increases in H for all interior m. Thus, if (A1) holds so that $\pi^* > \pi^s$, $\max_m \pi(H, L, m)$ is also increasing in H, and so is $\pi^* - \pi^s$. Similarly, when $H \geq \left(1 + \frac{\bar{x}}{\bar{y}}\right) L$, $\pi^* - \pi^s = \max_m \int\int_{x+y \geq m} [L(x+y) - Hy]\, dG(x, y)$, which decreases in H. ■

When H/L is low (or high), the profit advantage of bundling is low relative to separate selling, because selling at price L (or H) is often more profitable than at price H (or L), which implies that the option to sell at one of the two prices contingent on the realizations of $X + Y$ under bundling has very limited value. This option becomes more valuable when H/L is at some intermediate level, implying more profound profit advantage of bundling.

We next consider how the dispersion of X affects the profits under interpersonal bundling. Intuitively, when X is more dispersed, demand is more uncertain and the advantage of interpersonal bundling is larger. The result below shows that this is indeed the case under some conditions, assuming that X and Y are independent with the (marginal) distribution of Y being $J(y)$, and comparing profits under two different distributions of X.

Following Johnson and Myatt (2006), we say that distribution $\hat{F}(x)$ is more dispersed than $F(x)$ if $\hat{F}(x)$ is a rotation of $F(x)$ such that $x \gtreqless \hat{x} \Longleftrightarrow \hat{F}(x) \lesseqgtr F(x)$ for some rotation point $\hat{x}$. Under $\hat{F}(x)$ and $F(x)$, respectively, let $\bar{x}_{\hat{F}}$ and $\bar{x}_F$ be the expected values of X, $\hat{b}_x$ and b_x the upper limits of $\hat{F}$ and F, and $\hat{m}^*$ and m^* the optimal bundle sizes, where $\hat{b}_x \geq b_x$ and $\bar{x}_{\hat{F}} \geq \bar{x}_F$. Let the corresponding profits be $\hat{\pi}^*$ and π^* under bundling, and $\hat{\pi}^s$ and π^s under separate selling.

Corollary 2 *Suppose (A1) holds and $\hat{F}$ is a rotation of F such that: (i) $H\bar{y} \geq L\left(\bar{y} + \bar{x}_{\hat{F}}\right)$, (ii) $\hat{x} \leq m^* - b_y$, and (iii) $\int_{a_y}^{b_y} [Lm^* - Hy] \left[\hat{F}(m^* - y) - F(m^* - y)\right] dJ(y) \leq 0$. Then,*

$\hat{\pi}^* - \hat{\pi}^s > \pi^* - \pi^s$; *that is, the profit advantage of bundling relative to separate selling is larger if X is more dispersed.*

Proof. See the appendix. ■

Although the result seems intuitive, the comparison of profits under $\hat{F}(x)$ and $F(x)$ turns out to be subtle. Condition (i) ensures that $p^s = H$ under separate selling for both $\hat{F}(x)$ and $F(x)$. Under condition (ii), $\hat{F}(x) < F(x)$ for $x \geq m^* - y$, so that more dispersion under $\hat{F}$ leads to higher probabilities for higher realizations of x; and under (iii) this similarly holds on average weighted by the density of Y. Together, conditions (i) and (iii) ensure that unambiguous comparisons can be made. All three conditions can be easy to verify. For instance, in Example 1, where $F(x) = \frac{x}{3}$, these conditions are satisfied for any rotation $\hat{F}(x) = \frac{x}{\alpha}$ with $\alpha > 3$ and $H/L \in [(3+\alpha)/3,\ 2.6)$, where $m^* = \frac{3}{2}\frac{H}{L} > 3 > b_y = 2$, and $\hat{x} = 0$.

Finally, comparing consumer and social welfare under interpersonal bundling and separate selling is straightforward in our simple setting. When $p^s = L$, interpersonal bundling raises expected price and lowers expected output, whereas the opposite is true when $p^s = H$. From Proposition 1, we can therefore state the following sufficient conditions for the welfare effects of interpersonal bundling:

Corollary 3 *Suppose that (A1) holds. Interpersonal bundling increases consumer and social welfare if $H \geq L\left(1 + \frac{\bar{x}}{\bar{y}}\right)$, but it reduces consumer and total welfare if $H < L\left(1 + \frac{\bar{x}}{\bar{y}}\right)$.*

Intuitively, interpersonal bundling has two effects on consumer welfare. First, it may facilitate more effective extraction of the high-value consumers' surplus, which negatively impacts consumer welfare. Second, it may expand output when demand from low-value consumers is high, which positively impacts consumer welfare. This intuition is consistent with what is in the classic bundling literature, but here we obtain transparent conditions on when each effect dominates (and total welfare happens to change in the same direction as consumer welfare due to the same underlying output change): When H/L is relatively high, low-value consumers will not be served under separate selling but may be served under interpersonal bundling, and hence the latter leads to higher (expected) consumer and total welfare; and the converse is true when H/L is relatively low.

2.3 Interpersonal Bundling as an Optimal Selling Scheme

We now further argue that, in our simple setting, interpersonal bundling is an optimal selling scheme. Since all consumers are *ex ante* the same, we can consider mechanisms for a representative consumer. From the revelation principle, we can limit our search for an optimal selling scheme to direct mechanisms where the consumer is asked to report her type $\theta \in \{H,\ L\}$, who will receive a unit of the good with probability $\lambda(\cdot)$ by paying $p(\cdot)$,[13] and truth reporting is optimal for the consumer. Given that there is a continuum of consumers, $\lambda(\cdot)$ and $p(\cdot)$ will depend on θ and on some aggregate measure(s) of consumers. We assume that a mechanism may depend on the realized aggregate demand, $x+y$, but not on individual values of x and y. One possible motivation for this assumption is that x and y are not separately verifiable while $x+y$ potentially is. Under this assumption, which we shall call the verifiability restriction, a mechanism specifies $\{\lambda(\theta, x+y),\ p(\theta, x+y)\}$.

The seller chooses $\{\lambda(\theta, x+y),\ p(\theta, x+y)\}$ to maximize

$$\pi = \int\int [xp(L, x+y)\lambda(L, x+y) + yp(H, x+y)\lambda(H, x+y)]\, dG(x,y), \tag{5}$$

subject to individual rationality constraints

$$\begin{aligned} (L - p(L, x+y))\lambda(L, x+y) &\geq 0, & (6)\\ (H - p(H, x+y))\lambda(H, x+y) &\geq 0; & (7) \end{aligned}$$

and incentive compatibility constraints

$$\begin{aligned} (L - p(L, x+y))\lambda(L, x+y) &\geq (L - p(H, x+y))\lambda(H, x+y), & (8)\\ (H - p(H, x+y))\lambda(H, x+y) &\geq (H - p(L, x+y))\lambda(L, x+y). & (9) \end{aligned}$$

[13] We can also allow a transfer payment when the consumer does not receive the good, but it would be optimal for the seller to set this payment to zero.

From standard arguments, $p(L, x+y) = L$ so that the low-value type receives no information rents, and (8) holds with $p(H, x+y) \geq L$. From (9), which holds in equality at the optimum, and with $p(L, x+y) = L$, we have

$$p(H, x+y)\lambda(H, x+y) = H\lambda(H, x+y) - (H-L)\lambda(L, x+y). \tag{10}$$

Thus (7) and (10) are the two remaining constraints. Substituting (10) into (5), with $p(L, x+y) = L$, we obtain

$$\pi = \int\int \{[xL - y(H-L)]\lambda(L, x+y) + yH\lambda(H, x+y)\} dG(x,y),$$

which increases in $\lambda(H, x+y)$. Since constraint (7) is not less likely satisfied with an increase in λ, it follows that $\lambda(H, x+y) = 1$ at the optimum. Then, subject to (7), the seller chooses $\lambda(L, x+y)$ to maximize

$$\pi = \int\int \{(x+y)L\lambda(L, x+y) + yH[1 - \lambda(L, x+y)]\} dG(x,y).$$

Hence, the optimal solution must involve a cut-off value for $x+y$, m^*, such that $\lambda(L, x+y) = 1$ when $x+y \geq m^*$ and $\lambda(L, x+y) = 0$ when $x+y < m^*$,[14] where

$$m^* = \arg\max_m \int\int_{x+y\geq m} L(x+y)\, dG(x,y) + \int\int_{x+y<m} HydG(x,y),$$

with $p(H, x+y) = L$ if $\lambda(L, x+y) = 1$ and $p(H, x+y) = H$ if $\lambda(L, x+y) = 0$. But this is exactly optimal interpersonal bundling under (4). We have thus shown:

Proposition 2 *Interpersonal bundling is an optimal selling scheme among all mechanisms satisfying the verifiability restriction.*

Note that if Y is a constant and takes the value y, then $m^* = \frac{H}{L}y$ and interpersonal bundling is an optimal selling scheme among all selling mechanisms, with no need for the verifiability restriction.

[14] If mechanisms could depend on the realizations of x and y separately, then the optimal mechanism would set $\lambda(L, x+y) = 1$ when $(x+y)L > yH$ and $\lambda(L, x+y) = 0$ when $(x+y)L \leq yH$. Notice that with a continuum of consumers, a single consumer cannot change the realizations of x or y by reporting or not reporting her type.

2.4 Continuous Distributions of Consumer Values

Our basic model assumes that the high- and low-value consumers have constant reservation prices H and L, respectively. This allows us to illustrate our ideas in a most transparent setting. Our analysis can be extended to situations where the product values of these two types of consumers are v_H and v_L, which are realizations of continuous random variables. To illustrate this, we assume that the number of low–value consumers, x, again follows distribution $F(x)$, and the number of high-value consumers is a given parameter $a > 1$. Furthermore, v_L, v_H, and x are independently and uniformly distributed on $[0,1]$, $[0,a]$, and $[0,2]$, respectively.[15] Thus a higher a indicates a higher product valuation or higher demand from the high-value consumers.

First, under separate selling, the firm's expected profit is:

$$\pi(p) = \begin{cases} p(a-p) & if \quad 1 < p \leq a \\ \int_0^2 p(x(1-p)+a-p)\frac{1}{2}dx & if \quad 0 \leq p \leq 1 \end{cases},$$

which is maximized if either $p = \frac{a}{2}$ or $p = \frac{a+1}{4}$. Since $\pi\left(\frac{a}{2}\right) - \pi\left(\frac{a+1}{4}\right) \gtreqless 0$ if $a \gtreqless \sqrt{2}+1$, we have $p^s = \frac{a}{2}$ if $a > \sqrt{2}+1$ and $p^s = \frac{a+1}{4}$ if $1 < a \leq \sqrt{2}+1$. It follows that

$$\pi^s = \begin{cases} \frac{a^2}{4} & if \quad a > \sqrt{2}+1 \\ \frac{(a+1)^2}{8} & if \quad 1 < a \leq \sqrt{2}+1 \end{cases}.$$

Next, under interpersonal bundling, given (p,q,m) with $q < p$ and $m \geq 0$, consumers whose value is at least q will purchase at price q if $x(1-q)+a-q \geq m$, or

$$x \geq \frac{m-a+q}{1-q}. \tag{11}$$

If inequality (11) does not hold, then no group purchase will occur and consumers can only

[15] Equivalently, we can relax the unit demand assumption and allow each of these two types of consumers to have a downward-sloping demand curve. In particular, each low–value consumer has demand $q_L = 1-p$, and each high-value consumer has demand $q_H = a-p$. Note that the number of each type of consumers is a continuum.

purchase at the "regular" price p. Thus, the seller chooses (p, q, m) to maximize

$$\pi(p,q,m) =$$
$$q\int_{\min\left\{\frac{m-a+q}{1-q},2\right\}}^{2}\frac{x(1-q)+a-q}{2}dx+p\int_{0}^{\min\left\{\frac{m-a+q}{1-q},2\right\}}\frac{\max\{x(1-p),0\}+a-p}{2}dx.$$

We can now establish:

Proposition 3 *For the variant of the basic model with continuous distributions of v_H and v_L, interpersonal bundling dominates separate selling (i.e., $\pi^* > \pi^s$) if and only if $1 < a < \sqrt{3}+1$.*

The proof of Proposition 3, formally presented in the appendix, starts with two observations linking profits under interpersonal bundling and under separate selling: (1) $\pi(p, p^s, 0) = \pi^s$ for $p > p^s$ and (2) $\pi(p^s, q, m) = \pi^s$ for $q < 1$ and $m = a + 2 - 3q$, where, under interpersonal bundling, all consumers purchase with bundle discount in case (1) and no consumer qualifies for the bundle discount in case (2). Differentiating $\pi(\cdot,\cdot,m)$ with respect to m, we can then show that, if $1 < a < \sqrt{3}+1$, π is increasing in m at $m = 0$ in case (1) and decreasing in m at $m = a+2-3q$ in case (2), so that interpersonal bundling achieves higher profit than separate selling. Furthermore, $\pi(p,q,m)$ is concave in m when $a > \sqrt{2}+1$, which, together with π increasing at $m = a+2-3q$ if $a \geq \sqrt{3}+1$, leads to the conclusion that $\pi^* = \pi^s$ if $a \geq \sqrt{3}+1$.

Table 1 below contains some comparisons for three values of a:

Table 1

	p^*	q^*	m^*	π^*	π^s	$\frac{\pi^*-\pi^s}{\pi^s}$
$a = 2$	1	0.728 71	1.372 3	1.130 9	1.125	0.5%
$a = 2.4$	1.2	0.778 30	1.850 2	1.480 5	1.44	2.8%
$a = 2.7$	1.35	0.788 60	2.311 1	1.823	1.822 5	0.03%

Apparently, in this simple variant of the basic model, interpersonal bundling achieves higher profit than separate selling if the value of a is in an intermediate range, and the profit advantage, $\pi^* - \pi^s$, exhibits an inverted-U shape with respect to changes in a. These findings are analogous to those in Proposition 1 and Corollary 1 of the basic model.

3. INFORMATION DISSEMINATION AND PRICE DISCRIMINATION

Our basic model has focused on the role of uncertain demand in the profitability of interpersonal bundling. Demand uncertainty is a common phenomenon in many markets, and our analysis demonstrates how firms can use this selling strategy to increase profit in such market environments. In this section, we discuss how a seller may incorporate two additional strategic considerations in the design of interpersonal bundling to enhance its profitability, in two variants of the basic model.

3.1 Dissemination of Product Information

The existence of a seller's product may be known to some consumers but unknown to others. In order to achieve the group size to qualify for the low (bundle) price, an informed potential buyer may have the incentive to transmit the information about the sale to other consumers. A seller should take this incentive into account in its optimal design of the bundle. To formalize this idea in a simple setting, we consider a variant of the basic model by assuming that the number of high-value consumers is initially a given number $n \geq 1$, and each of them ($i = 1, ..., n$) can make an effort in order to inform a set of $k > 0$ high-value consumers who are initially unaware of the seller's product and prices.[16] Define set $N \equiv \{i : i = 1, ..., n\}$. Each $i \in N$ succeeds in transmitting the information to the k uninformed consumers with probability β_i at a personal cost $C(\beta_i)$, where $C'(\cdot) > 0$ with $C'(0) \rightarrow 0$, $C''(\cdot) \geq 0$, and the k uninformed consumers become informed if at least one

[16] Unlike in Section 2, the number of initial high-value consumers is now an integer. This avoids the problem that no consumer is willing to incur the information transmission cost when the number is a continuum. For convenience, we assume that the initially uninformed consumers also are all of the high-value type.

$i \in N$ succeeds. Thus, the number of high-value consumers is potentially

$$y = \begin{cases} n+k & \text{with probability} \quad 1 - \Pi_{i=1}^{n} (1-\beta_i) \\ n & \text{with probability} \quad \Pi_{i=1}^{n} (1-\beta_i) \end{cases}.$$

Other aspects of the model are the same as the basic model in Section 2. In particular, all low-value consumers are informed about the seller's product and price(s), and their number, x, is the realization of random variable X that has distribution $F(x)$. The reservation prices of the high- and low-value consumers are again H and L, respectively. Under separate selling, informed consumers have no incentive to incur the cost to transmit product information to uninformed consumers. Hence $p^s = L$ and $\pi^s = L(n+\bar{x})$ if $L(n+\bar{x}) > Hn$, whereas $p^s = H$ and $\pi^s = Hn$ if $L(n+\bar{x}) \leq Hn$.

Under interpersonal bundling, the seller first posts (p, q, m), after which all $i \in N$ simultaneously choose β_i. Both x and y are then realized, and possible purchases are made. For convenience, we again treat m as a continuous number, and without loss of generality, we can confine our search for the optimal (p, q, m) to $q \leq L < p \leq H$.

We consider a symmetric equilibrium where each $i \in N$ chooses the same β. Given (p, q, m), and all other high-value consumers' choice $\tilde{\beta}$, consumer i chooses her β_i to maximize her expected surplus:

$$U\left(\beta | m, \tilde{\beta}\right) = (H-q) \Pr(X+Y \geq m) + (H-p) \Pr(X+Y < m) - C(\beta),$$

where $\Pr(X+Y \geq m) =$

$$[1-F(m-n-k)]\left[1-\left(1-\tilde{\beta}\right)^{n-1}(1-\beta)\right] + (1-F(m-n))\left(1-\tilde{\beta}\right)^{n-1}(1-\beta).$$

In equilibrium, $\tilde{\beta} = \beta^*$, where β^* is the equilibrium choice of all consumers, and we denote the equilibrium bundle by (p^*, q^*, m^*).

Notice that interpersonal bundling now can increase profit for two distinct reasons. First, as a profitable pricing strategy under uncertainty, it increases profit even if $\beta_i = 0$ for all i (in which case uninformed consumers do not learn about the product information). From Proposition 1 and (A1), this is ensured if

$$\left(1 + \frac{a_x}{n}\right) < \frac{H}{L} < \left(1 + \frac{b_x}{n}\right). \tag{A1'}$$

Second, interpersonal bundling can motivate consumers to transmit product information to the uninformed, or to choose $\beta_i > 0$ at a personal cost, in hope of reaching the minimum bundle size so that the discount will be effective. Our next result, which provides a sufficient condition for higher profits under interpersonal bundling with the additional channel of encouraging information transmission to expand demand (i.e., in equilibrium $\beta_i = \beta^* > 0$), refers to the following condition

$$\left(1 + \frac{a_x}{n}\right) < \frac{H}{L} \leq \left(1 + \frac{b_x}{n+k}\right). \tag{A2}$$

Note that (A2), which implies the weaker condition (A1'), similarly holds if H/L is in an intermediate range.

Proposition 4 *Suppose that (A2) holds. Then, interpersonal bundling has higher profit than separate selling with* $\beta^* > 0$, $p^* = H$, *and* $m^* \in (n + a_x, n + k + b_x)$.

Proof. See the appendix. ■

Since the discount price can be valid only if the minimum bundle size is reached, the informed consumers have the incentive to transmit costly product information to the uninformed, hoping that more consumers will join the group purchase. As is shown in the proof for the symmetric equilibrium contained in the appendix, it is indeed optimal for each informed consumer to choose β^*, given that other informed consumers will do the same. Notice that the optimal bundle size, m^*, is now chosen also to provide the incentive for β^*, in addition to responding optimally to demand uncertainty. Therefore, interpersonal bundling also provides a mechanism to expand market demand.

To illustrate, consider the next example:

Example 2 *Suppose that* $n = 2$, $k = 1$, $C(\beta) = \frac{1}{2}\beta^2$, $F(x) = \frac{x}{3}$ *for* $x \in [0, 3]$, *and* $L < H < \frac{5}{2}L$. *Then, condition (A1') is satisfied, which is sufficient for interpersonal bundling to increase profit. With* $L = 1$, *Table 2 below lists the equilibrium interpersonal bundle and the profit comparisons with separate selling.*

Table 2

	p^*	q^*	m^*	β^*	π^*	π^s	$\frac{\pi^*-\pi^s}{\pi^s}$
$H=2$	2	1	4.875	0.25	4.805	4	10%
$H=1.8$	1.8	1	4.278	0.21	4.377	3.6	22%
$H=1.5$	1.5	1	3.398	0.143	3.914	3.5	12%

As in Example 1, given L, m^* is higher for higher H. Furthermore, β^* is also higher for higher H, directly because of the larger bundle discount $(H-L)$, and indirectly because of the higher bundle size (m^*).

3.2 Price Discrimination

To obtain the bundle discount under interpersonal bundling, a consumer may need to incur transaction costs to sign up for group purchase. If high-value consumers have higher time costs, they are less likely to participate. Interpersonal bundling can thus be a device for price discrimination, as in the textbook example of price discrimination through coupons. With bundling, however, there is an additional instrument to screen the buyers: Through the choice of the (minimum) bundle size that may not be reached due to uncertainty, the seller can further discourage high-value consumers from attempting to receive the bundle discount.

To illustrate how a seller can incorporate the possibility of price discrimination in bundle design, consider another variant of the basic model, where the low-value consumers have no cost to participate in group purchase, but the high-value consumers incur a transaction cost t to do so. Assume that t is distributed on $[\underline{t}, \bar{t}]$ with p.d.f. $\phi(t) > 0$, c.d.f. $\Phi(t)$, and $0 \leq \underline{t} < \bar{t}$. The number of low-value consumers is again x with cumulative distribution function $F(x)$, while the mass of high-value consumers is normalized to 1. As in the basic model, these two types of consumers value the product respectively at L and H. Thus, under separate selling, $p^s = H = \pi^s$ if $H \geq L(\bar{x}+1)$, whereas $p^s = L$ and $\pi^s = L(\bar{x}+1)$ if $H < L(\bar{x}+1)$.

As in the basic model, the game under interpersonal bundling proceeds as follows: First, the seller offers (p, q, m). Second, the number of low-value consumers and the private t for each high-value consumer are realized. Third, consumers choose whether to sign up for group purchase. Fourth, the total number of consumers who sign up becomes known. If this number exceeds m, each group member pays q while consumers who have not signed up will pay p; otherwise, all consumers are charged regular price p.

In order to analyze price discrimination under alternative forms of interpersonal bundling, we further assume that consumers can sign up for group purchase possibly in two periods, 1, or 2. (Neither the seller nor consumers discount time.) Under the *simultaneous* format, at the beginning of period 2 the seller does not reveal how many consumers signed up in the first period, whereas under the *sequential* format the firm does. Hence, with the former all consumers effectively make sign-up decisions simultaneously, whereas with the latter they make sign-up decisions sequentially.

Simultaneous Format

In this case, a high-value consumer, if she wishes to participate in the group purchase, needs to incur t before it becomes known how many low-value consumers have signed up for group purchase, or what the realization of x is (it is optimal for all low-value consumers to sign up for group coupon since they incur no sign-up cost). Suppose that there is some $t^* \in [0, \bar{t}]$ that solves

$$H - p = \int_{x+\Phi(t^*)\geq m} (H - q) f(x) \, dx + \int_{x+\Phi(t^*)<m} (H - p) f(x) \, dx - t^*. \tag{12}$$

Then, there will be an equilibrium where all low-value consumers sign up for group purchase, and a high-value consumer will sign up if and only if $t \leq t^*$.[17] We shall focus on this equilibrium.[18] Rearranging (12), we obtain

$$t^* = (p - q) \left[1 - F(m - \Phi(t^*))\right]. \tag{13}$$

[17] Equation (12) says that the marginal high-value consumer with t^* will just be willing to sign up, given (p, q, m) and given the equilibrium behavior of all other consumers.

[18] There can also be a trivial equilibrium where no one signs up for the group coupon, due to there being a continuum of consumers.

The seller's problem is, with $t^* = t^*(p, q, m)$, to maximize

$$\pi(p, q, m) = \int_{m-\Phi(t^*)}^{b_x} [q(x + \Phi(t^*)) + p(1 - \Phi(t^*))] f(x)\,dx + p\int_{a_x}^{m-\Phi(t^*)} f(x)\,dx \quad (14)$$

subject to $q \leq L$, $L \leq p \leq H$, $a_x \leq m - \Phi(t^*) \leq b_x$. The solution to (14) defines the equilibrium (p^*, q^*, m^*).

The seller can increase its profit by charging a lower price to the low-value consumers (a price no higher than L) and a higher price to the high-value consumers (as high as H). With regular price p and discounted bundle price q, a high-value consumer may nevertheless prefer to purchase at p, because she incurs sign-up cost t for group purchase and she may lose t without receiving the bundle discount if the minimum bundle size is not reached. Hence, a higher m will reduce the incentive of a high-value consumer to engage in group purchase. Interpersonal bundling may thus price discriminate more effectively both than traditional coupons and than usual mixed bundling.

A higher m, however, can hurt the seller if the sales to the low-value consumers do not materialize. This should also be taken into account when the seller chooses its optimal m. Notice that any q below L will lower the seller's profit when the good is sold at a discount and will also make participating in group purchase more attractive to the high–value consumers. Thus it is optimal for the seller to set $q^* = L$. On the other hand, a higher p may have the opposing effects of increasing the profit from the high-value consumers purchasing at the regular price but also making purchasing at the bundle discount more attractive. Consequently, the optimal value of p is determined jointly with m. Because $\pi(H, L, a_x) \geq \pi(L, L, m)$ for any m, to search for the optimal $(p,\ m)$ we can limit our attention to situations where $p > L$.

Again denote the seller's equilibrium profit under interpersonal bundling by π^*. To derive a sufficient condition under which $\pi^* > \pi^s$, we utilize the condition below

$$\text{(i) } \bar{t} > H - L; \quad \text{(ii) } L\bar{x} > (H - L)\,\Phi(H - L). \quad \text{(A3)}$$

Since $p^* \leq H$, part (i) in (A3) ensures that some high-value consumers will not incur t for the bundle discount, and, from (14),

$$\pi(H, L, a_x) = L[\bar{x} + \Phi(H-L)] + H[1-\Phi(H-L)] > L(\bar{x}+1) = \pi^s|_{p^s=L},$$

so that bundling with $(p, q, m) = (H, L, a_x)$ is always more profitable than separate selling with $p^s = L$. Moreover, condition (ii) in (A3) ensures that

$$\pi(H, L, a_x) = L[\bar{x} + \Phi(H-L)] - H\Phi(H-L) + H > H = \pi^s|_{p^s=H},$$

so that bundling with $(p, q, m) = (H, L, a_x)$ is also always more profitable than separate selling with $p^s = H$. Therefore, since $p^s = L$ or H, under condition (A3) it must be true that $\pi^* \geq \pi(H, L, a_x) > \pi^s$ and $p^* > L = q^*$. We have therefore established:

Proposition 5 *Suppose that condition (A3) is satisfied. Then, the seller's profit is higher under interpersonal bundling than under separate selling with $p^* > L = q^*$.*

We illustrate the result with the following example:

Example 3 *Assume that $\phi(t) = 1$ on $[0,1]$, $f(x) = \frac{1}{2}$ on $[0,2]$, and $L = 1$. For different values of H, Table 3 below lists the equilibrium interpersonal bundle and the profit comparisons with separate selling.*

Table 3

	p^*	q^*	m^*	t^*	π^*	π^s	$\pi^* - \pi^s$	$\frac{\pi^*-\pi^s}{\pi^s}$
$H = 2.5$	2.5	1	1.846	0.462	2.808	2.5	0.308	12%
$H = 2$	2	1	1.5	0.5	2.5	2	0.5	25%
$H = 1.5$	1.5	1	0.8	0.4	2.3	2	0.3	15%

Several observations can be made from Table 3. First, $\pi^* > \pi^s$ for all $H \in \{1.5,\ 2,\ 2.5\}$, and for profitable interpersonal bundling, (A3) is sufficient but not necessary. For instance, when $H = 2.5$, (A3) is not satisfied but $m^* > a_x$, $t^* < \bar{t}$, and $\pi^* > \pi^s$. Second, m^* increases in p^*, so as to discourage the high-value consumers from using the group coupon. Third, similarly as in the basic model, $\pi^* - \pi^s$ varies non-monotonically as H changes, reaching maximum when H is some intermediate value.

Sequential Format

Now consider the sequential format, where at the beginning of period 2 it becomes public information how many consumers signed up in the first period. Recall that the seller first offers (p, q, m). Since a low-value consumer has no cost to sign up, it is optimal for her to do so in the first period. Therefore in equilibrium all low-value consumers sign up in period 1 and the number of low-value consumers is then publicly known.[19]

Next consider the sign-up decision of high-value consumers, for whom it is optimal to wait until the beginning of period 2 to make the choice, after learning the realization of the number of low-value consumers. Suppose for a moment that, in equilibrium, depending on the realization of x, there exists a cutoff value $t^{**}(x)$ such that only high-value consumers with $t \leq t^{**}$ will sign up for group purchase. Given such a strategy by other consumers, consider the incentive of a high-value consumer with sign-up cost t. She chooses to sign up only if this leads to a (weakly) higher surplus for her and if a group discount is expected to be offered:

$$H - q - t \geq H - p \qquad \text{and} \qquad x + \Phi(t^{**}) \geq m.$$

Hence the sign-up cost of the marginal high-value consumer is $t = p - q$. It follows that, if $x \geq \hat{x}$, it is optimal for any high-value consumer with $t \leq t^{**}$ to sign up given that the others will do the same, where

$$t^{**} = p - q \qquad \text{and} \qquad \hat{x} = m - \Phi(p - q), \tag{15}$$

and the group size will be reached. Therefore, under the sequential format, there is indeed an equilibrium, where the seller chooses (p, q, m) optimally, low-value consumers will sign up in the first period, and: (i) if $x \geq \hat{x}$, then high-value consumers with $t \leq t^{**}$ will sign up in the second period and m will be reached, so that group participants will pay discounted

[19] Again, with a continuum of low-value consumers, this may not be the only equilibrium, but it is the natural one to focus on, because low-value consumers cannot lose from signing up early, and may possibly gain if their action encourages high-value consumers to sign up in period 2 so that the group discount will more likely be available.

price q while non-participants (high-value consumers with $t > t^{**}$) will pay regular price p; (ii) if $x < \hat{x}$, no high-value consumers will sign up and only regular price p prevails.

Comparing (15) with (13), we have $t^{**} > t^{*}$. That is, more high-value consumers will sign up for group purchase under the sequential than under the simultaneous format of interpersonal bundling. This implies that, for the same bundle, group purchases will occur more often under the sequential format. The intuition behind this finding, as in Hu, Shi, and Wu (2013), is that the sequential format removes the uncertainty faced by period-2 consumers about the number of participating consumers in period 1, which makes period-2 consumers more willing to sign up. Although our model and analysis differ from those in Hu, Shi, and Wu (2013),[20] our finding supports their conclusion that the sequential group-buying mechanism will lead to higher deal success rates. While this implies that a seller would prefer the sequential format if, as they assume, it aims to maximize the deal success rates, in our model the seller, whose objective is to maximize profit, may actually prefer the simultaneous format.

To see that profit can be higher under simultaneous than under sequential interpersonal bundling, we notice that the seller's profit function for the sequential format can be obtained by using the profit expression for the simultaneous format in (14) but replacing t^{*} with t^{**}:

$$\pi\left(p,q,m\right)=\int_{m-\Phi(t^{**})}^{b_x}\left[q\left(x+\Phi\left(t^{**}\right)\right)+p\left(1-\Phi\left(t^{**}\right)\right)\right]f\left(x\right)dx+p\int_{a_x}^{m-\Phi(t^{**})}f\left(x\right)dx. \tag{16}$$

While a complete comparison of profits under the two formats is rather complicated and beyond the scope of our paper, we demonstrate that profit can be higher in the simultaneous format with the following example, which has the same assumptions as example 3:

Example 4 *Assume that $\phi(t) = 1$ on $[0,1]$, $f(x) = \frac{1}{2}$ on $[0,2]$, and $L = 1$. For different values of H,*[21] *Table 4 compares equilibrium simultaneous and sequential interpersonal*

[20] Among other differences, in their group-buying mechanisms consumers have heterogenous valuations but identical participation costs, whereas in our model high-value consumers differ in participation costs but have identical valuation.

[21] Under these values of H there exist interior solutions of optimal m under both formats, while if $H = 2.5$ or $H = 2$ this is not ensured.

bundles, denoted with superscripts * *and* **, *respectively.*

Table 4

	Simultaneous Format						Sequential Format				
H	p^*	q^*	m^*	t^*	π^*		p^{**}	q^{**}	m^{**}	t^{**}	π^{**}
1.6	1.6	1	0.97	0.44	2.335		1.6	1	0.96	0.6	2. 272
1.5	1.5	1	0.80	0.40	2. 300		1.5	1	0.75	0.5	2. 26 6
1.4	1.4	1	0.62	0.35	2. 262		1.4	1	0.56	0.4	2. 246

Example 4 makes it clear that a profit-maximizing seller may prefer the simultaneous over the sequential format. This is because the seller wishes to price discriminate when using interpersonal bundling, and, unlike the sequential format, the simultaneous format does not remove uncertainty for the high-value consumers, thereby discouraging them from signing up to obtain the group discount.

5. CONCLUDING REMARKS

This paper has conducted a strategic analysis of interpersonal bundling. As a form of option pricing under demand uncertainty, this selling strategy can be optimal among all selling mechanisms that may depend on realized aggregate demand, and its profitability is illustrated neatly in a general bundling framework. The profit advantage of interpersonal bundling (relative to separate selling) tends to be an inverted-U function of the valuation ratio of high vs. low-value consumers (H/L), maximized when the ratio is some intermediate value; and the profit advantage is more profound when the number of low-value consumers has a more dispersed distribution. Moreover, the profitability of this selling strategy will be enhanced if the incentive to qualify for group purchase motivates buyers to disseminate product information, and if more high-value consumers can be induced to pay the regular instead of the discounted price.

While interpersonal bundling is often a profitable selling strategy, our analysis also reveals that it is not always more profitable than separate selling, even if no additional selling cost is required. Moreover, like other selling formats, interpersonal bundling can achieve its potential benefits for the seller only if it is properly implemented. In particular, losses may occur if the bundle discount under group purchase is too big. For example, when a restaurant offers a group coupon for 70% off its regular price, it could be unwisely pricing below marginal cost.[22] While many businesses have profited from offering interpersonal bundling on the Internet, there have also been media reports about how a merchant is hurt by its deep group discount through Groupon and other "social buying" intermediaries.[23] Part of the problem is a potential conflict in incentives: even though the seller should use the advertised deal to maximize its profit, an intermediary like Groupon benefits from a higher deal success rate. However, it need not be in the best interests of the sellers (and, in the long run, also their Internet intermediaries such as Groupon) to focus only on deal success rates. As our theory suggests, the seller's profit is sometimes higher when the deal is off —– if the realized number of low-value consumers is not high.[24] And, it would be even worse for sellers if below-cost group sale prices are used to boost deal success rates.[25]

We have studied monopoly interpersonal bundling in this paper. It would be desirable for

[22] The restaurant may want to attract repeat customers by taking a one-time loss, but is the loss necessary? Our analysis suggests that interpersonal bundling can be profitable without the repeat-business effect, and a seller need not incur losses in order to generate repeat businesses.

[23] See, for example, "Groupon demand almost finishes cupcake-maker" (November 22, 2011, *The Telegraph),* which tells the story of a British cakemaker who offered her product at 75% off its regular price through Groupon and had to produce at costs substantially above price in order to meet a huge demand increase. See also Byers, Mitzenmacher and Zervas (2012) for discussions about negative side effects for merchants using Groupon.

[24] As a form of advertising, interpersonal bundling on the Internet can also serve as a promotional device that encourages consumers to try the product and become repeat customers. While we do not model such roles, they can also be important. Indeed, some sellers may have used Groupon as an advertising platform to attract repeat customers, or to fill up their off-peak capacity.

[25] According to a survey reported in "Groupon hurt by lack of repeat biz" (January 4, 2012, *The New York Post),* although 8 out of 10 merchants who ran a daily group coupon deal were satisfied with the results, 52 percent of those surveyed were not planning to run a daily deal in the next six months. The article states that "[Groupon] has been accused of coercing businesses to basically give away goods and services while it takes its up to 50 percent cut."

future research to analyze interpersonal bundling by competing firms. The profitability of this selling strategy, and its potential adoption by a firm, may then depend on competitive conditions, possibly also including considerations such as product differentiation. It could also be interesting to extend our analysis to markets with more complex uncertain demand, where firms may use more general nonlinear pricing schemes.

APPENDIX

The appendix contains proofs for Corollary 2, Proposition 3, and Proposition 4.

Proof of Corollary 2. From (i), $H\bar{y} \geq L\left(\bar{y} + \bar{x}_{\hat{F}}\right)$. Hence under separate selling the optimal price is H for either $\hat{F}$ or F. It follows that

$$\begin{aligned}\hat{\pi}^* - \hat{\pi}^s &= \int\int_{x+y\geq\hat{m}^*} \left[L\left(x+y\right) - Hy\right] d\hat{F}\left(x\right) dJ\left(y\right) + H\bar{y} - H\bar{y} \\ &\geq \int_{a_y}^{b_y} \left\{\int_{m^*-y}^{\hat{b}_x} \left[Lx - \left(H-L\right)y\right] d\hat{F}\left(x\right)\right\} dJ\left(y\right),\end{aligned}$$

where the inequality is due to revealed preference. Since $\hat{F}\left(x\right) < F\left(x\right)$ for $x \geq m^* - y$ from (ii), we have

$$\begin{aligned}&\int_{m^*-y}^{\hat{b}_x} \left[Lx - \left(H-L\right)y\right] d\hat{F}\left(x\right) \\ = \ &\left[L\hat{b}_x - \left(H-L\right)y\right] - \left[Lm^* - Hy\right]\hat{F}\left(m^*-y\right) - \int_{m^*-y}^{b_x} L\hat{F}\left(x\right)dx - \int_{b_x}^{\hat{b}_x} L\hat{F}\left(x\right)dx \\ > \ &\left[Lb_x - \left(H-L\right)y\right] - \left[Lm^* - Hy\right]\hat{F}\left(m^*-y\right) - \int_{m^*-y}^{b_x} LF\left(x\right)dx.\end{aligned}$$

Thus

$$\begin{aligned}
\hat{\pi}^* - \hat{\pi}^s &> \int_{a_y}^{b_y} [Lb_x - (H-L)\,y]\,dJ(y) - \int_{a_y}^{b_y} [Lm^* - Hy]\,\tilde{F}(m^* - y)\,dJ(y) \\
&\quad - \int_{a_y}^{b_y} \int_{m^*-y}^{b_x} LF(x)\,dx\,dJ(y)\,. \\
\text{(from (iii))} \quad &\geq \int_{a_y}^{b_y} [Lb_x - (H-L)\,y]\,dJ(y) - \int_{a_y}^{b_y} [Lm^* - Hy]\,F(m^* - y)\,dJ(y) \\
&\quad - \int_{a_y}^{b_y} \int_{m^*-y}^{b_x} LF(x)\,dx\,dJ(y) \\
&= \int_{m^*-y}^{b_x} [Lx - (H-L)\,y]\,dF(x) = \pi^* - \pi^s.
\end{aligned}$$

■

Proof of Proposition 3. We consider in turn two cases:

Case 1: $1 < a \leq \sqrt{2}+1$. Then $\pi^s = \frac{(a+1)^2}{8} = \pi(p,q,0)$ with $q = \frac{a+1}{4} < 1 < p < a$. Notice that

$$\left.\frac{\partial \pi(p,q,m)}{\partial m}\right|_{m=0,\ q=\frac{a+1}{4},\ 1<p<a} = \frac{1}{2(1-q)}(a-p)\,p > 0.$$

Therefore starting from separate selling at $p^s = \frac{a+1}{4}$, introducing interpersonal bundling with $q = \frac{a+1}{4} < 1 < p < a$ and $m > 0$ leads to a higher profit than π^s.

Case 2: $a > \sqrt{2}+1$. We argue that in this case $\pi^* > \pi^s$ if $\sqrt{2}+1 < a < \sqrt{3}+1$, and $\pi^* = \pi^s$ if $a \geq \sqrt{3}+1$.

First, notice that $\pi^s = \frac{a^2}{4} = \pi(p,q,m)$ with $p = \frac{a}{2}$, $q < 1$ and $m = a+2-3q$; and, for interpersonal bundling to have a higher profit than separate selling, it is necessary that $q < 1 < p$ and $a - q < m < a + 2 - 3q$. Next, since

$$\frac{\partial \pi(p,q,m)}{\partial p} = -\frac{(2p-a)(m-a+q)}{2(1-q)},$$

the optimal p satisfies $p^* = a/2$. Furthermore, since $\frac{\partial \pi(p,q,m)}{\partial m} = -\frac{mq-ap+p^2}{2(1-q)}$ and $\frac{\partial^2 \pi(p,q,m)}{\partial m^2} < 0$, $\pi^* > \pi^s$ if and only if $\pi(a/2,q,m)$ is decreasing in m at $m = a+2-3q$ for some $q \in (0,1)$. Finally,

$$-\left.\frac{\left(mq - ap + p^2\right)}{2(1-q)}\right|_{m=a+2-3q,\ p=\frac{a}{2}} = \frac{a^2 - 4aq - 8q + 12q^2}{8(1-q)} < 0$$

if and only if both $a < \sqrt{3}+1$ and

$$q \in (\frac{1}{3} + \frac{a}{6} - \frac{\sqrt{2}}{6}\sqrt{2a - a^2 + 2},\ \frac{1}{3} + \frac{a}{6} + \frac{\sqrt{2}}{6}\sqrt{2a - a^2 + 2}) \equiv \Omega\left(a\right),$$

where $\Omega(a)$ is an interval on $[0.5,\ 1)$ when $a < \sqrt{3}+1$. We conclude that $\pi^* > \pi^s$ if and only if $1 < a < \sqrt{3}+1$. ■

Proof of Proposition 4. First, in equilibrium, $\tilde{\beta} \equiv \tilde{\beta}(p,q,m)$ satisfies $\partial U\left(\beta \middle| m, \tilde{\beta}\right)/\partial\beta\Big|_{\beta=\tilde{\beta}} = 0$, or

$$(p-q)\left[F(m-n) - F(m-n-k)\right]\left(1-\tilde{\beta}\right)^{n-1} - C'\left(\tilde{\beta}\right) = 0. \tag{17}$$

The firm's problem is:

$$\begin{aligned} & \max_{q \le L < p \le H, m} \pi(p,q,m) \qquad (18) \\ = \;& q\left[\left[1-\left(1-\tilde{\beta}\right)^n\right]\int_{x \ge m-n-k}(x+n+k)\,dF(x) + \left(1-\tilde{\beta}\right)^n \int_{x\ge m-n}(x+n)\,dF(x)\right] \\ & +p\left[\left[1-\left(1-\tilde{\beta}\right)^n\right](n+k)F(m-n-k) + \left(1-\tilde{\beta}\right)^n nF(m-n)\right]. \end{aligned}$$

Next, from (17) and with $C'' \ge 0$, we have $\tilde{\beta} \equiv \tilde{\beta}(p,q,m)$ increasing in p and decreasing in q; and furthermore

$$\frac{\partial\tilde{\beta}(p,q,m)}{\partial m} = \frac{(p-q)\left[f(m-n) - f(m-n-k)\right]\left(1-\tilde{\beta}\right)^{n-1}}{(n-1)(p-q)\left[F(m-n)-F(m-n-k)\right]\left(1-\tilde{\beta}\right)^{n-2} + C''}.$$

Thus $\tilde{\beta}(p,q,m)$ is increasing in m at $m = n + a_x$ but decreasing in m at $m = n+k+b_x$. At the optimum, $\pi(p,q,m)$ must increase in $\tilde{\beta}$. Thus, since $\pi(p,q,m)$ and $\tilde{\beta}(p,q,m)$ both increase in p, the solution to problem (18) must have $p = H$, so that problem (18) becomes $\max_{q\le L,m} \pi(H,q,m)$.

Next,

$$\begin{aligned} \frac{\partial\pi(H,q,m)}{\partial\tilde{\beta}} = \;& qn\left(1-\tilde{\beta}\right)^{n-1}\left[\int_{x\ge m-n-k}(x+n+k)\,dF(x) - \int_{x\ge m-n}(x+n)\,dF(x)\right] \\ & +Hn\left(1-\tilde{\beta}\right)^{n-1}\left[(n+k)F(m-n-k) - nF(m-n)\right], \end{aligned}$$

with

$$\left.\frac{\partial\pi\left(p,q,m\right)}{\partial\tilde{\beta}}\right|_{m=n+a_x} = qn\left(1-\tilde{\beta}\right)^{n-1}k>0,$$
$$\left.\frac{\partial\pi\left(p,q,m\right)}{\partial\tilde{\beta}}\right|_{m=n+k+b_x} = Hn\left(1-\tilde{\beta}\right)^{n-1}k>0.$$

Next, since $Hn\geq L\left(n+a_x\right)$ by assumption (A2),

$$\begin{aligned}
&\left.\frac{\partial\pi\left(H,q,m\right)}{\partial m}\right|_{m=n+a_x}\\
=\ &\left[1-\left(1-\tilde{\beta}\right)^{n}\right]\left[H\left(n+k\right)-qm\right]f\left(m-n-k\right)\Big|_{m=n+a_x}\\
&+\left(1-\tilde{\beta}\right)^{n}\left(Hn-qm\right)f\left(m-n\right)\Big|_{m=n+a_x}+\left.\frac{\partial\pi\left(p,q,m\right)}{\partial\tilde{\beta}}\frac{\partial\tilde{\beta}\left(p,q,m\right)}{\partial m}\right|_{m=n+a_x}\\
\geq\ &\left.\frac{\partial\pi\left(p,q,m\right)}{\partial\tilde{\beta}}\right|_{m=n+a_x}\left.\frac{\partial\pi\left(p,q,m\right)}{\partial\tilde{\beta}}\right|_{m=n+k+b_x}>0.
\end{aligned}$$

On the other hand, at $m=n+k+b_x$, $\frac{\partial\pi(p,q,m)}{\partial\tilde{\beta}}\frac{\partial\tilde{\beta}(p,q,m)}{\partial m}<0$, $f\left(m-n\right)=0$, $f\left(m-n-k\right)>0$, $\tilde{\beta}$ is not affected by q from (17), but $\pi\left(H,q,m\right)$ increases in q, which implies that $q^*=L$ at $m=n+k+b_x$. And since $H\left(n+k\right)\leq L\left(n+k+b_x\right)$ by assumption (A2), we have

$$\left.\frac{\partial\pi\left(H,q,m\right)}{\partial m}\right|_{m=n+k+b_x}<0.$$

Therefore, the equilibrium m is interior: $m^*\in\left(n+a_x,n+k+b_x\right)$. It follows from (17) that $\beta^*>0$. ■

REFERENCES

[1] Adams, W. and J. Yellen (1976). "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics* 90: 475-498.

[2] Anand, K. and R. Aron (2003). "Group Buying on the Web: A Comparison of Price-Discovery Mechanisms," *Management Science* 49: 1546–1562.

[3] Armstrong, M. (2012). "A More General Theory of Commodity Bundling," *Journal of Economic Theory*, forthcoming.

[4] Byers, J., M. Mitzenmacher and G. Zervas (2012). "The Groupon Effect on Yelp Ratings: A Root Cause Analysis," *Proceedings of the 13th ACM Conference on Electronic Commerce*, 248-256.

[5] Chen, Y. and M. Riordan (2013). "Profitability of Product Bundling," *International Economic Review*, 54: 35-57.

[6] Chu, S., P. Leslie, and A. Sorensen (2011). "Bundle-Size Pricing as an Approximation to Mixed Bundling," *American Economic Review* 101: 263–303.

[7] Dana, J. (1998). "Advanced Purchase Discounts and Price Discrimination in Competitive Markets," *Journal of Political Economy* 106: 395-422.

[8] Dana, J. (2001). "Monopoly price dispersion under demand uncertainty," *International Economic Review* 42: 649–670.

[9] Gale, I. and T. Holmes (1992). "The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty," *International Journal of Industrial Organization* 10: 413–437.

[10] Gale, I. and T. Holmes (1993). "Advance-purchase discounts and monopoly allocation of capacity," *American Economic Review* 83: 135–146.

[11] Gans, J. and S. King (2006). "Paying for Loyalty: Product Bundling in Oligopoly," *Journal of Industrial Economics*, 54: 43-62.

[12] Hu, M., M. Shi and J. Wu (2013). "Simultaneous vs. Sequential Group-Buying Mechanisms," *Management Science* mnsc.2013.1740.

[13] Jing, X. and J. Xie (2011). "Group Buying: A New Mechanism for Selling Through Social Interactions," *Management Science* 57: 1354–1372.

[14] Johnson, J. and D. Myatt (2006). "On the Simple Economics of Advertising, Marketing, and Product Design," *American Economic Review* 96: 756-784.

[15] Levin, J. (2012). “The Economics of Internet Markets,” *Advances in Economics and Econometrics,* Acemoglu, D., M. Arellano and E. Dekel ed.

[16] Long, J. (1984). "Comments on 'Gaussian Demand and Commodity Bundling," *Journal of Business* 57: S235-S246.

[17] McAfee, R.P., J. McMillan, and M. Whinston (1989). "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics* 104: 371-383.

[18] Schmalensee, R. (1984), "Gaussian Demand and Commodity Bundling," *Journal of Business* 57: S211-S230.