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Essays on monetary policy and banking regulation

Jingyuan LI

ESSAYS ON MONETARY POLICY AND BANKING REGULATION

A Dissertation

by

JINGYUAN LI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2004

Major Subject: Economics

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ABSTRACT

Essays on Monetary Policy and Banking Regulation. (August 2004)

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A central bank is usually assigned two functions: the control of inflation and the maintenance of a safety-banking sector. What are the precise conditions under which trigger strategies from the private sector can solve the time inconsistency problem and induce the central bank to choose zero inflation under a nonstationary natural rate? Can an optimal contract be used together with reputation forces to implement a desired socially optimal monetary policy rule? How to design a truth-telling contract to control the risk taking behaviors of the bank? My dissertation attempts to deal with these issues using three primary methodologies: monetary economics, game theory and optimal stochastic control theory.

To my father, mother and wife

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CHAPTER I

INTRODUCTION

This dissertation studies several problems of monetary policy and banking regulation.

There are three essays in this dissertation. The first essay develops a model to examine the equilibrium behavior of the time inconsistency problem in a continuous time economy with stochastic nature rate and endogenized distortion. The second essay studies the time inconsistency problem on monetary policy for central banks using a unified approach that combines reputation forces and contracts. In the third essay, we study how to control the risk taking behaviors of the bank.

Chapter II develops a model to examine the equilibrium behavior of the time inconsistency problem in a continuous time economy with stochastic nature rate and endogenized distortion. First, we introduce the notion of sequentially rational equilibrium, and show that the time inconsistency problem may be solved with trigger reputation strategies for stochastic setting. We provide the conditions for the existence of sequentially rational equilibrium. Then the concept of sequentially rational stochastically stable equilibrium is introduced. We compare the relative stability between the cooperative behavior and uncooperative behavior and show that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this

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The journal model is Journal of Economic Theory.

monetary policy game is sequentially rational stochastically unstable equilibrium. In the long run, the zero inflation monetary policies are inherently more stable than the discretion rules, and once established, they tend to persist for longer periods of the time.

Chapter III studies the time inconsistency problem on monetary policy for central banks using a unified approach that combines reputation forces and contracts. We first characterize the conditions for reputation forces to eliminate the inflation bias of discretionary policy. We then propose an optimal contract that can be used with reputation forces to implement a desired socially optimal monetary policy rule when the reputation forces alone are not large enough to discourage a central bank to use a surprise inflation policy. In contrast to most of the existing contracts that are contingent on realized inflation rates which are in turn contingent on production shocks, like the standard reputation model, a central banker in our hybrid mechanism is punished only when she uses a surprise inflation rate. Since the penalty proposed is the lowest one that discourages the central bank from attempting to cheat and the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the loss at the socially optimal inflation rate, our hybrid mechanism is the most efficient and robust mechanism that implements the socially optimal monetary policy rule. We also provide an upper bound of the penalty that is lower than that of the existing contracts when realized inflation rate is greater than a certain level.

Chapter IV studies how to control the risk taking behaviors of the bank. First we get the expected bankruptcy time and conditional probability distribution of bankruptcy for a given time. We show that the risk-shifting behavior will increase the probability of bankruptcy during a given time. Then we use these results to analyze the regulation policies and show that capital requirements can not control the risk taking behaviors of bank at finite future point in time. We also prove that if we use the time horizon as an additional instrument, we can control the risk shifting problem. We give a theoretic explanation for the VaR regulation. Finally, we discuss the VaR contracts with asymmetric information and show that VaR contracts can induce the banker report the real risk of the project.

CHAPTER II

TIME INCONSISTENCY AND REPUTATION IN MONETARY POLICY: A STRATEGIC MODELLING IN CONTINUOUS TIME

This chapter develops a model to examine the equilibrium behavior of the time inconsistency problem in a continuous time economy with stochastic and endogenized distortion. First, we introduce the notion of sequentially rational equilibrium , and show that the time inconsistency problem may be solved with trigger reputation strategies for stochastic setting. We provide the conditions for the existence of sequentially rational equilibrium. Then the concept of sequentially rational stochastically stable equilibrium is introduced. We compare the relative stability between the cooperative behavior and uncooperative behavior and show that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this monetary policy game is sequentially rational stochastically unstable equilibrium. In the long run, the zero inflation monetary policies are inherently more stable than the discretion rules, and once established, they tend to persist for longer periods of the time.

A. Introduction

Time inconsistency is an interesting problem in macroeconomics in general, and monetary policy in particular. Although technologies, preferences, and information are the same at different time, the policymaker's optimal policy chosen at time t_1 differs from the optimal policy for t_1 chosen at $t_0 < t_1$. The study of time inconsistency is important. It not only provides positive theories that help us to understand the incentives faced by policy-makers and provides the natural starting point for attempts to explain the actual behavior of policymakers and actual policy outcomes, but also requires one to design policy-making institutions. Such a normative task can help one understand how institutional structures affect policy outcomes.

This problem was first noted by Kydland and Prescott [20]. Several solutions have been proposed to deal with this problem since then. Barro and Gordon [4] were the first to build a game model to analyze "reputation" of monetary policy.[1] A second solution is based on the incentive contracting approach to monetary policy. Persson and Tabellini [30], Walsh [41] and Svensson [37] developed models using this approach. A third solution is built on the legislative approach. The major academic contribution in this area was by Rogoff [33].

Among these approaches, the "reputation" problem is key. If reputation consideration discourages the monetary authorities from attempting surprise inflation, then legal or contracting constraints on monetary authorities are unnecessary and may be harmful.

The main questions on reputation are when and how the government chooses inflation optimally to minimize welfare loss, and, whether the pun-

[1]Backus and Driffill [2] extended the work of Barro and Gordon to a situation in which the public is uncertain about the preferences of the government. Persson and Tabellini [29] gave an excellent summarization of these models. Al-Nowaihi and Levine [1] discussed reputation equilibrium in the Barro-Gordon monetary policy game.

ishment can induce the government to choose zero inflation. The conclusions of Barro-Gorden models are: First, there exists a zero-inflation Nash equilibrium if the punishment for the government deviating from zero-inflation is large enough. However, this equilibrium is not sequentially rational over a finite time horizon. The only sequentially rational equilibrium is achieved if the government chooses discretionary inflation and the public expects it. Only over an infinite time horizon can one get a low-inflation equilibrium. Otherwise, the government would be sure in the last period to produce the discretionary outcome whatever the public's expectation were and, working backward, would be expected to do the same in the first period. Second, there are multiple Nash equilibria and there is no mechanism to choose between them.

This chapter develops a continuous times model of central bank at the spirit of Kydland and Prescott [20] and Barro and Gordon [4]. The main differences between our model and previous models are the following two assumptions:

(1) the natural rate is a Brownian motion;

(2) the distortion of the economy is correlated to the natural rate.

The reason we use assumption (1) is that the most recent literature shares the view that the natural rate changes over time and specifies the natural rate as a random walk without drift seems a plausible assumption for U.S. unemployment data [2].

The key aspect of this monetary time inconsistency problem is the dis-

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[2]See Staiger, Stock and Watson [39], and Salemi [34].

tortion which arises from the labor-market distortions and the political pressure on the central bank. Most often, some appeal is made to the presence of labor-market distortions, for example, a wage tax. So, it seems reasonable for us to assume that the distortion is an increasing function of the scale of the economy. We use an linear function to approximate this function in this chapter.

In this chapter, we use the optimal stopping theory in the stochastic differential equations literature to study the time inconsistency problem in monetary policy with the continuous finite or infinite time horizon model. The optimal stopping theory can cover many dynamic economic applications under uncertainty. The optimal stopping theory, though relatively complete in its theoretical development, has not yet been widely applied in economics. By using the optimal stopping theory and introducing the notions of sequentially rational equilibria, we give the conditions under which the time inconsistency problem may be solved with trigger reputation strategies. We provide the conditions for the existence of sequentially rational equilibrium.

We argue that the tradition concepts of equilibrium are not satisfactory as a predictor of long run behavior when the game is subjected persistent stochastic shocks. The concept of sequentially rational stochastically stable equilibrium is introduced. Loosely speaking, the sequentially rational stochastically stable equilibrium of a dynamic game are those equilibrium that the expected time to apart from them is infinite. Then we compare the relative stability between the cooperative behavior and uncooperative behavior and show that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this monetary policy game is a sequentially rational stochastically unstable equilibrium.

The results obtained in the chapter imply that, in the long run, the zero inflation monetary policies are inherently more stable than the discretion rules, and once established, they tend to persist for longer periods of the time.

Whether or not we can expect the monetary policy to have a tendency to become stable depends not only on the lifetime of the government, but also on the beliefs of the government and the public, ceteris paribus. If the time horizon is long enough, we may expect the monetary policy tends to stable beyond some point of time. Although the initial economy shocks and natural rate may not implement a stationary sequentially rational equilibrium at the beginning, under the sequentially strong rational strategy behavior assumption, the reputation trigger equilibrium have a tendency to reach a new stationary equilibrium beyond some point in time. If the life time of the government is not long enough to reach such a point, we may be able to use an incentive contract or a legislative approach to reach it.

The remainder of the chapter is organized as follows. Section B will set up the model and provides a solution for the optimal stopping problem faced by the government. In Section C, we study the equilibrium behavior. The stochastic stability of this monetary game is discussed in Section D. Section E gives the conclusion.

B. Model

1. The Setup

We consider a continuous time game theoretical model with two players: the government and the public. The government's strategy space is $R^+ \times L[0,T]$, from which the government is to choose an action $(\tau, \{\pi_t\}_{t\in T})$. Here τ is the time that the government changes its monetary policy from the zero-inflation rule to a discretion rule; π_t is the inflation rate chosen by the government at time t; T is the lifetime of the government which can be finite or infinite; and $L[0,T]$ is the class of Lebesgue integral functions defined on $[0,T]$. The public's strategy space is $L[0,T]$, from which the public is to choose an action $(\{\pi_t^e\}_{t\in T})$. Here π_t^e is the expected inflation rate formed by the public at time t.

Suppose that, at the beginning, the government commits an inflation rate $\pi_0 = 0$, and the public believes it so that $\pi_0^e = \pi_0 = 0$. The government has the right to switch from the zero-inflation to a discretion rule $\pi_t \neq 0$ at the time t between 0 and T. However, after he changes his policy, he loses his reputation.

The government's loss function is described by a quadratic discounted expected loss function of the form:

$$\Lambda = E \int_0^T e^{-\rho \cdot t} \left[\frac{1}{2}\theta \left(y_t - \bar{y}_t - k_t \right)^2 + \frac{1}{2}\pi_t^2 \right] dt \tag{II.1}$$

where ρ is the discount factor with $0 < \rho < 1$, y_t is aggregate output, $\bar{y}_t$ is the economy's natural rate of output, k_t is the distortion which is equal $\alpha \bar{y}_t$ and $\alpha > 0$. Some appeal is made to the presence of labor-market distortions, for example, a wage tax. Since a larger scale of a economy always implies a larger wage tax, so, it seems reasonable for us to assume that the distortion is an increasing function of the scale of the economy. We use an linear function to approximate this function in this chapter.

θ is a positive constant that represents the relative weight the government puts on output expansions relative to inflation stabilization. Here, the target inflation π is zero.[3] (II.1) is a typical marco welfare function that has played an important role in the literature, and means that the government desires to stabilize both output around $\bar{y}_t + k_t$, which exceeds the economy's equilibrium output of $\bar{y}_t$ by k_t, and inflation around zero.

Here we assume that $\bar{y}_t = X_t$ and

$$dX_t = \sigma dB_t, \quad X_0 = x,$$

which is a special case of the general Ito diffusion:

$$dX_t = b(X_t)dt + \sigma(X_t)dB_t$$

with $b(\bar{X}_t) = 0$ and $\sigma(\bar{X}_t) = \sigma$. Here, B_t is 1-dimensional Brownian motion and σ is the diffusion coefficient with $\sigma < \infty$.

The government's objective is to minimize this discounted expected loss function subject to the constraint imposed by a Lucas-type aggregate supply function, the so-called Phillips curve, which describes the relationship

[3]Without loss of generality, the target inflation rate is assumed to be zero. The results obtained in the chapter will continue to be true if the monetary authority has a target inflation that differs from zero.

between output and inflation in each period:

$$y_t - \bar{y}_t = a\left(\pi_t - \pi_t^e\right) + u_t, \tag{II.2}$$

where a is a positive constant that represents the effect of a money surprise on output, i.e., the rate of the output gain from the unanticipated inflation so that the larger is a, the greater is the central bank's incentive to inflate, and u_t is a bounded random variable with $E[u_t] = 0$, $Var[u_t] = \sigma_u^2$, $|u_t| \leq M_1$ for all t and $cov(u_s, u_t) = 0$, for $t \neq s$, which represents the shock at time t. And we assume that $\bar{y}_t$ and u_t are independent. We also assume that the government can observe u_t and X_t prior to setting π_t.

The public has complete information about the policymaker's objectives. It is assumed that the public forms his expectations rationally, and thus the assumption of rational expectation implicitly defines the loss function for the public as $E[\pi_t - \pi_t^e]^2$. The public's objective is to minimize this expected inflation error. Given the public's understanding of the government's decision problem, its choice of π^e is optimal.

We first examine the "one-shot" game. The single-period loss function ℓ_t for the government is:

$$\begin{aligned} \ell_t\left(\pi_t, \pi_t^e\right) &= \frac{1}{2}\theta\left(y_t - \bar{y}_t - k_t\right)^2 + \frac{1}{2}\pi_t^2 \\ &= \frac{1}{2}\theta[a(\pi_t - \pi_t^e) - \alpha X_t + u_t]^2 + \frac{1}{2}\pi_t^2. \end{aligned} \tag{II.3}$$

The equilibrium concept in this game is noncooperative Nash. Then the government minimizes ℓ_t by taking π_t^e as given, and thus we have the best response function for the policymaker:

$$\pi_t^D = \frac{a\theta}{1 + a^2\theta} \left(a\pi_t^e + \alpha X_t - u_t \right). \tag{II.4}$$

The public is assumed to understand the incentive facing the government so they use (II.4) in forming their expectations about inflation so that

$$\pi_t^e = E\pi_t^D = \frac{a\theta}{1 + a^2\theta} \left(a\pi_t^e + \alpha E X_t \right). \tag{II.5}$$

Solving (II.5) for π_t^e, we get the unique Nash equilibrium $\pi_t^{e*} = E\pi_t^{D*} = a\theta\alpha E X_t$. Thus, as long as $EX_t \neq 0$, the policymaker has incentives to use the discretion rule although the loss at $\pi_t^e = \pi_t = 0$ is lower than at $\pi_t^{e*} = E\pi_t^{D*}$.

A potential solution to the above time inconsistency problem is to force the government to bear some consequence penalties if it deviates from its announced policy of low inflation. One of such penalties that may take is a loss of reputation, and so, in this chapter, we will adopt the reputation approach that incorporates notions of reputation into a repeated-game framework to avoid this time consistency problem. If the government deviates from the low-inflation solution, credibility is lost and the public expects high inflation in the future. That is, the public expects zero-inflation as long as government has fulfilled the inflation expectation in the past. However, if actual inflation exceeds what was expected, the public anticipates that the policymaker will apply discretion in the future. So the public forms their expectation according to the trigger strategy: Observing "good" behavior induces the expectation of continued good behavior and a single observation of "bad" behavior triggers a revision of expectations.

2. The Optimal Stopping Problem for Government

In order to solve the time inconsistency problem by using the reputation approach, we first incorporate the government's loss minimization problem into a general optimal stopping time problem. During any time in $[0, T]$, the policymaker has the right to reveal his type (discretion or zero-inflation). Since he has the right but not the obligation to reveal his type, we can think it is an option for the policymaker. So the policymaker's decision problem is to choose a best time $\tau \in [0, T]$ to exercise this option.

The policymaker considers the following time-inhomogeneous optimal stopping problem: Find τ^* such that

$$L^*(x) = \inf_{\tau} E^x \left[\int_0^{\tau} f(t, X_t) dt + g(\tau, X_\tau) \right], \tag{II.6}$$

where

$$f(s, X_t) = \frac{1}{2} \theta e^{-\rho \cdot s} (\alpha X_t - u_t)^2 \tag{II.7}$$

is the instantaneous loss function for the policymaker when he uses the zero-inflation rate which is clearly Lipschits continuous, and

$$g(s, X_\tau) = e^{-\rho s} E^{X_\tau} \left[\int_s^T e^{-\rho(t-s)} \left[\frac{\theta}{2} [a(\pi_t^D - \pi_t^e) - \alpha X_t + u_t]^2 + \frac{\pi_t^{D^2}}{2} \right] dt \right] \tag{II.8}$$

is the expected loss function for policymaker in which he begin to use the discretion rule at time s. Note that $g(\cdot) \geq 0$ since the loss function $\ell_t \geq 0$. We assume that $g(\cdot)$ is a bounded function, i.e., $g(\cdot) \leq M$ for some constant number M.

Let $\{\mathcal{F}_t\}$ be a filtration, i.e., a nondecreasing family $\{\mathcal{F}_t : t \geq 0\}$ of sub-σ-fields of $\mathcal{F}$: $\mathcal{F}_s \subset \mathcal{F}_t \subset \mathcal{F}$ for $0 \leq s < t < \infty$, which is assumed to be generated by the process itself, i.e., $\mathcal{F}_t := \sigma(X_s : 0 \leq s \leq t)$. Then, $\mathcal{F}_t$ can be regarded the set of accumulated information up to time t.

We assume that the public's strategy π_t^e for $t > \tau$ is $\{\mathcal{F}_\tau\}$-adapted. This means that when the public form their expectation at time t, they know the natural rate at τ.

To compute $g(\tau, X_\tau)$, putting (II.4) into (II.8), we have

$$g(\tau, X_\tau) = \frac{1}{2}\frac{\theta}{1+a^2\theta}e^{-\rho\tau}E^{X_\tau}\left[\int_\tau^T e^{-\rho(t-\tau)}\left(\alpha X_t - u_t + a\pi_t^e\right)^2 dt\right] \tag{II.9}$$

We now calculate the conditional expectation for X_t^2 and X_t. Let A be the of Ito diffusion $dX_t = b(X_t)dt + \sigma(X_t)dB$ (with $b = 0$). Then

$$\begin{aligned} Af &= \sum_i b_i \frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}\left(\sigma\sigma^T\right)_{i,j}\frac{\partial^2 f}{\partial x_i \partial x_j} \\ &= \frac{1}{2}\sum_{i,j}\left(\sigma\sigma^T\right)_{i,j}\frac{\partial^2 f}{\partial x_i \partial x_j} \end{aligned}$$

Then, by Dynkin's formula (Øksendal [27]), we have

$$E^{X_\tau}\left[X_t\right] = X_\tau + E^{X_\tau}\left[\int_\tau^t AX_s ds\right] = X_\tau \tag{II.10}$$

$$E^{X_\tau}\left[X_t^2\right] = X_\tau^2 + E^{X_\tau}\left[\int_\tau^t AX_s^2 ds\right] = X_\tau^2 + \sigma^2(t-\tau). \tag{II.11}$$

Substituting (II.10) and (II.11) into (II.9), we have

$$\begin{aligned} g(\tau, X_\tau) &= \frac{1}{2}\frac{\theta}{1+a^2\theta}\left\{\sigma^2\left[\frac{1}{\rho^2}\left(e^{-\rho\tau} - e^{-\rho T}\right) - \frac{1}{\rho}(T-\tau)e^{-\rho T}\right]\right. \\ &\quad \left. +[(\alpha X_\tau + a\pi_\tau^e)^2 + \sigma_u^2]\frac{1}{\rho}(e^{-\rho\tau} - e^{-\rho T})\right\}. \end{aligned} \tag{II.12}$$

Note that, if we define

$$\begin{aligned} f_1(s, X_t) &= -f(s, X_t), \\ g_1(s, X_\tau) &= -g(s, X_\tau) + M \geq 0, \end{aligned}$$

then the loss minimization problem in (II.6) can be reduced to the following maximization problem: Find τ^* such that

$$\begin{aligned} G_0^*(x) &= \sup_{\tau \in [0,T]} E^x \left[\int_0^\tau \left[-f(t, X_t) \right] dt - g(\tau, X_\tau) + M \right] \\ &= \sup_{\tau \in [0,T]} E^x \left[\int_0^\tau f_1(t, X_t) dt + g_1(\tau, X_\tau) \right]. \end{aligned} \tag{II.13}$$

In the following, we will use the optimal stopping approach to solve the optimization problem (II.13).

3. Solving the Optimal Stopping Problem

In order to solve the government's optimization problem (II.13) by using a standard framework of the optimal stopping problem involving an integral (cf. Øksendal [27]), we make the following transformations: Let

$$W_\tau = \int_0^\tau f_1(t, X_t) dt + w, \quad w \in R$$

and define the Ito diffusion $Z_t = Z_t^{(s,x,w)}$ in R^3 by

$$Z_t = \begin{bmatrix} s + t \\ X_t \\ W_t \end{bmatrix}$$

for $t \geq 0$. Then

$$dZ_t = \begin{bmatrix} dt \\ dX_t \\ dW_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{2}\theta e^{-\rho t}(X_t - k)^2 \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma \\ 0 \end{bmatrix} dB_t, \qquad Z_0 = (s, x, w).$$

So Z_t is an Ito diffusion starting at $z := Z_0 = (s, x, w)$. Let $R^z = R^{(s,x,w)}$ denote the probability law of $\{Z_t\}$ and let $E^z = E^{(s,x,w)}$ denote the expectation with respect to R^z. In terms of Z_t the problem (II.13) can be written

$$G_0^*(x) = G^*(0, x, 0) = \sup_\tau E^{(0,x,0)}[W_\tau + g_1(\tau, X_\tau)] = \sup E^{(0,x,0)}[G(Z_\tau)]$$

which is a special case of the problem

$$G^*(s, x, w) = \sup_\tau E^{(s,x,w)}[W_\tau + g_1(\tau, X_\tau)] = \sup E^{(s,x,w)}[G(Z_\tau)]$$

with

$$G(z) = G(s, x, w) := w + g_1(s, x).$$

Then, for

$$\begin{aligned} f_1(s,x) &= -\frac{1}{2}\theta e^{-\rho \cdot s}(\alpha x - u_s)^2 \\ g_1(s,x) &= -\frac{1}{2}\frac{\theta}{1 + a^2\theta}\left\{\sigma^2\left[\frac{1}{\rho^2}\left(e^{-\rho s} - e^{-\rho T}\right) - \frac{1}{\rho}(T - s)e^{-\rho T}\right]\right. \\ &\quad \left. + [(\alpha x + a\pi_s^e)^2 + \sigma_u^2]\frac{1}{\rho}\left(e^{-\rho s} - e^{-\rho T}\right)\right\} + M \end{aligned}$$

and

$$G(s, x, w) = w + g_1(s, x),$$

the A_Z of Z_t is given by

$$\begin{aligned} A_Z G &= \frac{\partial G}{\partial s} + \frac{1}{2}\sigma^2 \frac{\partial^2 G}{\partial x^2} - \frac{1}{2}\theta e^{-\rho s}(x-k)^2 \frac{\partial G}{\partial w} \\ &= \frac{1}{2}\frac{\theta}{1+a^2\theta}[(\alpha x + a\pi_s^e)^2 + \sigma_u^2]e^{-\rho s} - \frac{1}{2}\theta(\alpha x - u_s)^2 e^{-\rho s}. \qquad \text{(II.14)} \end{aligned}$$

Let

$$U = \{(s,x,w) : G(s,x,w) < G^*(s,x,w)\}$$

and

$$V = \{(s,x,w) : AG(x) > 0\}.$$

Then, by (II.14) we have

$$\begin{aligned} V &= \{(s,x,w) : A_Z G(s,x,w) > 0\} \qquad \text{(II.15)} \\ &= R \times \{x : (\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1+a^2\theta)(\alpha x - u_s)^2\} \times R. \end{aligned}$$

Remark B.1. *Øksendal [27] shows that: $V \subset U$, which means that it is never optimal to stop the process before it exits from V. If we choose a suitable $\pi^e(x)$ such that $(\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1+a^2\theta)(\alpha x - u_s)^2$, then we have $U = V = R^3$. Therefore, any stopping time less T will not be optimal for all $(s,x,w) \in V$, and thus $\tau^* = T$ is the optimal stopping time. We will use this fact to study the time inconsistency problem of the monetary policygame in the following sections.*

Remark B.2. *If $\rho \to \infty$, then $A_Z G \to 0$, it is optimal to use the cheating policyfor the government. If $\rho < \infty$, the term in equation (II.14), $\frac{1}{2}\frac{\theta}{1+a^2\theta}[(\alpha x + a\pi_s^e)^2 + \sigma_u^2]e^{-\rho s}$ can be regarded as the marginal benefit of not stopping zero inflation policy and the term, $\frac{1}{2}\theta(\alpha x - u_s)^2 e^{-\rho s}$, is the marginal* *cost. Since the government put the same weight($e^{-\rho s}$) on the marginal benefit and the marginal cost, so when he compares this two terms, the discount factor ρ will not effect his decision.*

Remark B.3. *In fact, we can verify directly that $\frac{dL(x)}{d\tau} < 0$ when π^e is bigger enough, where $L(x)$ is defined by*

$$L(x) = E^x \left[\int_0^\tau f(t, X_t)dt + g(\tau, X_\tau) \right] = E^x \left[\int_0^\tau f(t, X_t)dt + g(\tau X_\tau) \right].$$

Thus, $\tau^* = T$ is the time.

C. The Equilibrium Behavior of the Monetary Policy Game

In order to study the equilibrium behavior of the game, we first give the following lemma that shows that the government will keep the zero-inflation policy when the public uses trigger strategies and reputation penalties imposed by the public are large enough.

Lemma C.1. *Let $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Then, for all x, any trigger strategy of the public, $\{\pi_t^e(x)\}$, which has the form of*

$$\pi_t^e = \begin{cases} 0 & \textit{if } t = 0 \\ 0 & \textit{if } 0 < t < \tau \\ \pi^e(x) \in \{h : (\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha|x| + M)^2\} & \textit{if } t > s \textit{ and } t \geq \tau \end{cases},$$

discourages the policymaker from attempting surprise inflation.

Proof. For each $x \in R$, if we choose any $\pi^e \in \{h : (\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha x - u_s)^2\}$, we have

$$(\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha x - u_s)^2 \quad \text{for all } x \in R.$$

Then, V in (II.15) becomes $V = R^3$, and thus any stopping time less T is not optimal for the government. Hence, $\tau^* = T$. Thus, when the public applies this trigger strategy, it is never optimal for government to stop the zero-inflation policy. $\square$

Although there are (infinitely) many trigger strategies given in Lemma 1 that can discourage the policymaker from attempting surprise inflation, most of them are not optimal for the public in terms of minimizing the public's expected inflation error: $\pi_t - \pi_t^e$. To rule out the those non-optimal strategies, we have to impose some assumptions how the public form an expectation and what an equilibrium solution should be used to describe the public's self-interested behavior. Different assumptions on the public's behavior may result in different the optimal solutions. In the following, we introduce two types of sequentially rational equilibrium solution concepts.

Suppose the government knows the distribution of the natural rate, X_t, exactly, that is,

$$d\widetilde{P}^G = dP,$$

where $\widetilde{P}^G$ is the belief of the government for the movement of the shock, P is the measure of the natural rate.

We suppose that the public does not know the distribution of the natural rate, but it's belief $\widetilde{P}^P$ is absolutely continuous with respect to P [4], which means that if an event does not occur in probability, then the public will believe that this event will not happen.

Then, by Randon-Nikodym Theorem (Lipster & Shiryaev [22]), there

[4] $\widetilde{P}^P(A) = 0$ for each $A \in \mathcal{F}_t$, such that $P(A) = 0$.

exist Randon-Nikodym derivatives, $M(t)$, such that

$$d\widetilde{P}^P = M(t)dP, (a.s.),$$

and $M(t)$ is a martingale and bounded from above and below (i.e. $M_1 \leq M(t) \leq M_2$ for all $0 \leq t \leq T$). This means that, whenever new information becomes available, the belief of the public is adjusted. We can interpreter $M(t)$ is the information structure of the society, it is a measurement of how the public knows the real natural rate.

We suppose that $M(t)$ is P-square-integrable and X_t is $\widetilde{P}^P$-integrable. We also suppose that $\langle X_t, M(t)\rangle = 0$[5], heuristically, this assumption can be interpreted as: the history of the natural rate can't help the public to predict the movement of the future natural rate in generally.[6]

We denote by $\widetilde{E}$ the expectation operator with respect to $\widetilde{P}^P$.

A strategy $(\tau, \{\pi_t, \pi_t^e\})$ is said to be a *sequentially rational equilibrium strategy* for the dynamic model defined above if

(1) the belief of the public for the movement of the natural rate X_t, $\widetilde{P}^P$, satisfies Bayes' rule:

$$\widetilde{E}[X_t|\mathcal{F}_s] = \frac{1}{M(s)} E[X_t M(t)|\mathcal{F}_s] \qquad \text{(II.16)}$$

for all $s < t$;

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[5] $\langle X, Y\rangle$ is cross-variation, which is defined by $\langle X_t, Y_t\rangle = \lim_{||\Pi||\to 0} \sum_{1\leq k\leq m}(X_{t(k)} - X_{t(k-1)})(Y_{t(k)} - Y_{t(k-1)})$, where X_t and Y_t are square-integrable, and $\Pi = [t_0, t_1, ..., t_m]$ is a partition of [0,t].

[6] Note that, if one assumes that the public knows the distributions of the shocks, X_t, exactly, then $M(t) = 1$. This is a usual assumption made in the literature.

(2) The expectation of the public is rational: $\pi_t^e = E^{X_s}\pi_t^D := \widetilde{E}[\pi_t^D|\mathcal{F}_s]$ for all $s < t$;

(3) it optimizes the objectives of the public and the government.

Now we use this type of sequentially rational equilibria to study the time consistency problem in monetary policy. Proposition C.2 below shows the existence of such equilibria.

Proposition C.2. *Suppose the shocks $\{X_t\}$ satisfy the inequality:*

$$(\alpha x + a^2\theta\alpha X_t)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha|x| + M)^2 \textit{ for all } t \in [0, T] \textit{ and } x \in R\ . \tag{II.17}$$

Let $(\tau, \{\pi_s\})$ be the strategy of the government, where τ is the first time that the government changes its policy from zero-inflation to discretion rule, i.e., $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. Let the strategy of the public $\{(\pi_t^e)\}$ be given by

$$\pi_t^e = \begin{cases} 0 & \textit{if } t = 0 \\ 0 & \textit{if } 0 < t < \tau \\ a\theta\alpha X_\tau & \textit{if } t \geq \tau \end{cases}.$$

Then, $(\tau^*, \{\pi_t^*, \pi_t^{e*}\})$ with $\tau^* = T$, $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$ is a sequentially rational equilibrium strategy for the policymaker and the public.

Proof. To prove $(\tau, \{\pi_t, \pi_t^e\})$ defined above results in a sequentially rational equilibrium, $\tau^* = T$, $\pi_t^* = 0$ and $\pi_t^{e*} = 0$ for all $t \geq 0$, we need to show that (1) it satisfies Bayes' rule, (2) the rational expectation condition holds: $\pi_t^e = E^{X_\tau}\pi_t^D := \widetilde{E}[\pi_t^D|\mathcal{F}_\tau]$, (3) $\pi_t^e \in \{h : (\alpha x + ah)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha|x| + M)^2\}$, and (4) $(\tau^*, \{\pi_t^*, \pi_t^{e*}\})$ optimizes the objectives of the public and the government.

We first claim that the public updates its belief by Bayes' rule. Indeed, since $M(t)$ is a martingale and, for $s < t$, X_t is a $\widetilde{P}^P$-integrable random variable, then, by Lemma of Shiryaev & Kruzhilin [38], the Bayes' Rule holds:

$$\widetilde{E}[X_t|\mathcal{F}_s] = \frac{1}{M(s)}E[X_tM(t)|\mathcal{F}_s].$$

To show $\pi_t^e = E^{X_\tau}\pi_t^D$, first note that X_t and $M(t)$ are square-integrable martingale, using the fact that $X_tM(t) - \langle X_t, M(t)\rangle$ is a martingale (Karatzas & Shreve [17]) and the assumption $\langle X_t, M(t)\rangle = 0$, We can get that $X_tM(t)$ is a martingale, by Bayes' rule:

$$\widetilde{E}[X_t|\mathcal{F}_\tau] = \frac{1}{M(\tau)}E[X_tM(t)|\mathcal{F}_\tau] = \frac{1}{M(\tau)}X_\tau M(\tau) = X_\tau.$$

which means $\{X_t\}$ is also a martingale under $\widetilde{P}^P$. Since the policymaker's best response function is given by

$$\pi_t^D = \frac{a\theta}{1+a^2\theta}(a\pi_t^e + \alpha x - u_s),$$

$\{X_t\}$ is a martingale under $\widetilde{P}^P$, and $\pi_t^e = a\theta\alpha X_\tau$ is complete information at time t, we have

$$\begin{aligned} E^{X_\tau}\pi_t^D &= E^{X_\tau}\frac{a\theta}{1+a^2\theta}(a\pi_t^e + \alpha x - u_s) \\ &= \frac{a\theta}{1+a^2\theta}(a\pi_t^e + \alpha E^{X_\tau}X_t) \\ &= \frac{a\theta}{1+a^2\theta}(a\pi_t^e + \alpha X_\tau). \end{aligned} \tag{II.18}$$

Substituting $\pi_t^e = a\theta\alpha X_\tau$ into (II.18), we have $E^{X_\tau}\pi_t^D = \frac{a\theta}{1+a^2\theta}[a^2\theta\alpha X_\tau + \alpha X_\tau) = a\theta\alpha X_\tau = \pi_t^e$.

Now, if condition (II.17) is satisfied, we have $(\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1+$

$a^2\theta)(\alpha|x|+M)^2$ and thus $\pi_t^e \in \{h : (\alpha x + ah)^2 + \sigma_u^2 > (1+a^2\theta)(\alpha|x|+M)^2\}$ for all $x \in R$ with $x \neq k$. Then, by Lemma 1, and the time is $\tau^* = T$. Therefore, we must have $\pi_t^* = 0$ for all $t \in [0, T]$.

Since the public only cares about his inflation prediction errors, so $\pi_t^e = a\theta\alpha X_t$ minimizes the public's expected loss when the policy change occurs at time t in this game. Hence, if both the policymaker and public believe that future shocks will grow enough to make the inequality (II.17) hold, the threat of the public is credible. Hence, we must have $\pi_t^{e*} = 0$ for all $t \in [0, T]$ since $\tau^* = T$. Thus, we have shown that the trigger strategies $(\tau, \{\pi_t, \pi_t^e\})$ result in a sequentially rational equilibrium, which is $\tau^* = T$, $\pi_t^* = 0$, and $\pi_t^{e*} = 0$ for all $t \geq 0$. □

Thus, Proposition C.2 implies that, as long as natural rate X_t is big enough, the public can use a trigger strategy to induce a zero-inflation sequentially rational equilibrium. Of course, the assumption that $(\alpha x + a\pi_s^e)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha|x| + M)^2$ for all $t \in [0, T]$ and $x \in R$ with $x \neq k$ seems very strong. Proposition D.1 in the next section shows that this is a reasonable assumption. As long as this inequality holds for the initial natural rate x, the public and the government will have a strong belief that it will be true for all $t \in (0, T]$ and $x \in R$.

D. Stochastically Stable Equilibrium

In this section we study the robustness of sequentially rational equilibrium. In order to get the sequentially rational equilibrium in Proposition C.2, we imposed the assumption that $B = \{(\alpha x + a^2\theta\alpha X_t)^2 + \sigma_u^2 >$

$(1 + a^2\theta)(\alpha|x| + M)^2\}$ for all $0 \leq t \leq T$ and $x \in R$. It seems that the concept of sequentially rational equilibrium is not satisfactory as a predictor of long-run behavior when the game is subjected to persistent stochastic shocks. So we introduce the concept of sequentially rational stochastically stable equilibrium[7].

Definition D.1. *Let* $\{A : (y, z \in R^2)\}$ *be the set of* sequentially rational equilibrium of a dynamic game under the shock X_t, we say A is a sequentially rational stochastically stable equilibrium if $E^x[\tau] = \infty$, where $\tau = \inf\{t : (y_t, z_t) \notin A\}$, and A is a *sequentially stochastically unstable rational equilibrium if* $E^x[\tau] < \infty$.

Loosely speaking, the sequentially rational stochastically stable equilibria of a dynamic game are those equilibria such that the expected time to depart from them is infinite.

Lemma D.1. *Let* $B = \{X_t : (\alpha x + a^2\theta\alpha X_t)^2 + \sigma_u^2 > (1 + a^2\theta)(\alpha|x| + M)^2$ *for* $t \geq 0\}$, *and let* $\eta = \inf\{t > 0 : X_t \notin B\}$ *be the first time* X_t *exits from* B. *Suppose that* $x \in B$. *Then, we have*

$$E^x[\eta] = \infty$$

for all $x \in R$.

[7]In determinate dynamic systems, in order to analyze the dynamic behavior, the concepts of Lyapunov stable and asymptotically stable are always used. For stochastic evolution system, Foster and Young [10] and Young [44] first introduce the concept of stochastic stability. But the concept in their paper is different from ours.

Proof. Solving $(\alpha x + a^2\theta\alpha X_t)^2 + \sigma_u^2 > (1+a^2\theta)(x-u_t)^2$ for X_t, we have

$$X_t > \frac{1}{a^2\theta\alpha}\left[-\sigma_u^2 - \alpha x + \sqrt{1+a^2\theta}(\alpha|x|+M)\right]$$

or

$$X_t < \frac{1}{a^2\theta\alpha}\left[(-\sigma_u^2 - \alpha x - \sqrt{1+a^2\theta}(\alpha|x|+M)\right].$$

Let $C = \frac{1}{a^2\theta\alpha}\left[u_t - \alpha x + \sqrt{1+a^2\theta}(\alpha|x|+M)\right]$

and $D = \frac{1}{a^2\theta\alpha}\left[(u_t - \alpha x - \sqrt{1+a^2\theta}(\alpha|x|+M)\right].$

Since $X_0 = x \in B$ for all $x \in R$, there are two cases to be considered: (1) $x > C$ and (2) $x < D$.

Case 1. $x > C$. Let $\eta_c = \inf\{t > 0\colon\ X_t \leq C\}$, and let η_n be the first exit time from the interval

$$\{X_t : C \leq X_t \leq n\}$$

for all integers n with $n > C$. We first show that $P^x(X_{\eta_n} = C) = \frac{n-x}{n-C}$ and $P^x(X_{\eta_n} = n) = \frac{x-C}{n-C}$. Consider function $h \in C_0^2(R)$ defined by $h(x) = x$ for $C \leq x \leq n$ ($C_0^2(R)$ means the functions in $C^2(R)$ with compact support in R). By Dynkin's formula,

$$E^x\left[h(X_{\eta_n})\right] = h(x) + E^x\left[\int_0^{\eta_n} Ah(X_s)ds\right] = h(x) = x, \tag{II.19}$$

we have

$$CP^x(X_{\eta_n} = C) + nP^x(X_{\eta_n} = n) = x.$$

Thus,

$$P^x(X_{\eta_n} = C) = \frac{n-x}{n-C}$$

and

$$P^x(X_{\eta_n} = n) = 1 - P^x(X_{\eta_n} = C) = \frac{x - C}{n - C}.$$

Now consider $h \in C_0^2(R)$ such that $h(x) = x^2$ for $C \leq x \leq n$. Applying Dynkin's formula again, we have

$$E^x\left[h(X_{\eta_n})\right] = h(x) + E^x\left[\int_0^{\eta_n} Ah(X_s)ds\right] = x^2 + \sigma^2 E^x\left[\eta_n\right], \qquad \text{(II.20)}$$

and thus

$$\sigma^2 E^x\left[\eta_n\right] = C^2 P^x(X_{\eta_n} = C) + n^2 P^x(X_{\eta_n} = n) - x^2.$$

Hence, we have

$$E^x\left[\eta_n\right] = \frac{1}{\sigma^2}\left[C^2\frac{n - x}{n - C} + n^2\frac{x - C}{n - C} - x^2\right].$$

Letting $n \to \infty$, we conclude that $P^x(X_{\eta_n} = n) = \frac{x-C}{n-C} \to 0$ and $\eta_c = \lim \eta_n < \infty$ a.s. Therefore, we have

$$E^x[\eta_c] = \lim_{n\to\infty} E^x\left[\eta_n\right] = \infty.$$

Case 2. $X_0 = x < D$. Define $\eta_D = \inf\{t > 0;\ X_t \geq D\}$. Let η_n be the first exit time from the interval

$$\{X_t : -n \leq X_t \leq D\}$$

for all integers n with $-n < D$. By the same method, we can prove that

$$E^x\left[\eta_n\right] = \frac{1}{\sigma^2}\left[D^2\frac{n + x}{n + D} + n^2\frac{D - x}{n + D} - x^2\right].$$

Letting $n \to \infty$, we conclude that $P^x(X_{\eta_n} = n) = \frac{D-x}{n+D} \to 0$ and

$\eta_D = \lim \eta_n < \infty$ a.s., and thus

$$E^x[\eta_D] = \lim_{n\to\infty} E^x\left[\eta_n\right] = \infty.$$

Thus, in either case, we have $E^x[\eta] = \infty$. $\square$

Lemma D.1 thus implies that, because the expected exit time from B is infinite since the expectation $E^x[\eta] = \infty$ for all $x \in R$ with $x \neq k$, the policymaker will have the belief that the future natural rate will stay in B forever, and consequently they will likely make decisions and behave according to this belief. As a result, the sequentially rational equilibrium will likely appear in the game when the public has the same belief as the government. So, in this sense, we can regard the class B as an absorbing class for X_t as long as $x \in B$.

What happens if the initial shock x is not in B? We have following proposition:

Lemma D.2. *Define* $\tau = \inf\{t > 0 : Z_t \in B\}$. *Then for* $x \notin B$*, i.e.,* $a(1-\theta) \geq 2$*, we have*

$$E^x[\tau] = \frac{a(1-\theta)-2}{\sigma^2 a\theta}(k-x)^2$$

for all $D \leq x \leq C$.

Proof. Since $x \notin B$, we have $D \leq x \leq C$. Define $\tau_C = \inf\{t > 0 : X_t \geq C\}$ and $\tau_D = \inf\{t > 0 : X_t \leq D\}$. Then $\tau = \tau_c \wedge \tau_D := \min\{\tau_c, \tau_D\}$. We first show that $P^x(X_\tau = C) = \frac{x-D}{C-D}$ and $P^x(X_\tau = D) = \frac{C-x}{C-D}$. Consider

$h \in C_0^2(R)$ such that $h(x) = x$ for $D \leq x \leq C$. By Dynkin's formula,

$$E^x\left[h(X_{\tau_c \wedge \tau_D})\right] = h(x) + E^x\left[\int_0^{\tau_c \wedge \tau_D} Ah(X_s)ds\right] = h(x) = x, \qquad \text{(II.21)}$$

we have

$$CP^x(X_\tau = C) + DP^x(X_\tau = D) = x.$$

Thus,

$$P^x(X_\tau = C) = \frac{x - D}{C - D}.$$

and thus

$$P^x(X_\tau = D) = 1 - P^x(X_\tau = C) = \frac{C - x}{C - D}.$$

Now consider $h \in C_0^2(R)$ such that $h(x) = x^2$ for $D \leq x \leq C$. By Dynkin's formula:

$$E^x[h(X_{\tau_c \wedge \tau_D})] = h(x) + E^x[\int_0^{\tau_c \Lambda \tau_D} Ah(X_s)ds] = h(x) + \sigma^2 E^x[\tau_c \wedge \tau_D], \quad \text{(II.22)}$$

we have

$$\sigma^2 E^x[\tau_c \wedge \tau_D] = C^2 P^x(X_\tau = C) + D^2 P^x(X_\tau = D) - x^2$$

and thus

$$E^x[\tau_c \wedge \tau_D] = \frac{a(1-\theta) - 2}{\sigma^2 a\theta}(k - x)^2 \geq 0. \qquad \text{(II.23)}$$

by noting that $a(1 - \theta) \geq 2$. □

Notice that, from (II.23), one can see that, the bigger the variance of the natural rate (measured by σ), the faster the convergence rate. In particular, if $\sigma^2 \to 0$, $E^x\left[\tau_c \wedge \tau_D\right] \to \infty$ and $\tilde{E}^x\left[\tau_c \wedge \tau_D\right] = E^x\left[M(t)(\tau_c \wedge \tau_D)\right] \geq$

$M_1 E^x [\tau_c \wedge \tau_D] \to \infty$. This means that, when the natural rate $\{X_t\}$ degenerates to a non-stochastic process, the government and the public will believe that $X_t \notin B$ for all $t \in [0, T]$, and thus a stationary sequentially strong rational equilibrium does not exist. On the other hand, if $\sigma^2 \to \infty$, then $E^x [\tau_c \wedge \tau_D] \to 0$ and $\tilde{E}^x [\tau_c \wedge \tau_D] = E^x [M(t)(\tau_c \wedge \tau_D)] \leq M_2 E^x [\tau_c \wedge \tau_D] \to 0$. This means that, when the variance of the natural rate $\{X_t\}$ becomes very large, the public and the government may believe that X_t will be in B for $t \in (0, T]$. As such, the policymaker and the public will likely have the beliefs that the natural rate X_t will be in B right after the initial natural rate x, and consequently, the stationary sequentially strong rational equilibrium will likely appear in the time horizon $(0, T]$.

When $0 < \sigma < \infty$, from Lemma D.2, the expected time of entering B, $E^x[\tau] = E^x [\tau_c \wedge \tau_D]$ is a finite number. Suppose the public has the same belief as the government. There are two cases to be considered: (1) $E^x[\tau] \geq T$. In this case, the government and the public likely believe that $X_t \notin B$ for all $t \in [0, T]$, and thus a stationary sequentially strong rational equilibrium will not likely exist. (2) $E^x[\tau] < T$. In this case, we should not expect the zero-inflation stationary monetary policy for the time period between $[0, E^x[\tau]]$ since $X_t \notin B$ for all $t \in [0, E^x[\tau]]$. However, once X_t enters B at the first time $E^x[\tau]$, we can regard X_τ as a new starting point. Then, by Lemma D.1, the policymaker and the public will believe X_t will stay in B for all $t \in [E^x[\tau], T]$, and thus we can expect to have a non-zero inflation stationary monetary policy on $[E^x[\tau], T]$. This implies that, although we do not have a time consistency policy on the whole time horizon $[0, T]$ when $x \notin B$, we could have a time consistency monetary policy beyond the point

$E^x[\tau]$. In other words, one will have an nonstationary policy period if the initial shock $x \notin B$, however, after a certain point τ, the monetary policy may become stationary. Thus, the time inconsistency can happen at most once.

Summarizing the above discussion, we can draw the following conclusions:

(i) If the initial natural rate x is in the class B, one can expect all future shocks X_t are in B and thus can expect a stationary zero-inflation outcome by the sequentially rational behavior.

(ii) If the initial natural rate x is not in the class B, whether or not we can expect the monetary policy to have a tendency to become stable depends on T, the lifetime of the government. If the expected first entry time to B, $E^x[\tau]$ is greater than the lifetime of the government, we do not expect a stationary monetary policy and thus we have the time inconsistency problem. If the first entering time into B, $E^x[\tau]$ is less than the lifetime of the government, we may expect a stationary monetary policy beyond the entry point $E^x[\tau]$, and monetary policy becomes stationary. Thus, the monetary policy can jump at most once.

Combine LemmaD.1 and LemmaD.2, we have following proposition:

Proposition D.3. *Let $(\tau, \{\pi_s\})$ be the strategy of the government, where τ is the first time that the government changes its policy from zero-inflation to* *discretion rule, i.e.,* $\tau = \inf\{s > 0 : \pi_s \neq 0\}$. *Let the strategy of the public* $\{(\pi_t^e)\}$ *be given by*

$$\pi_t^e = \begin{cases} 0 & \textit{if } t = 0 \\ 0 & \textit{if } 0 < t < \tau \\ a\theta(k - X_\tau) & \textit{if } t \geq \tau \end{cases} .$$

Then, $(\tau^*, \{\pi_t^*, \pi_t^{e*}\})$ *with* $\tau^* = T$, $\pi_t^* = 0$ *and* $\pi_t^{e*} = 0$ *for all* $t \geq 0$ *is a* sequentially rational stochastically stable equilibrium strategy for the policymaker and the public.

Then, we can see that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this monetary policy game is a sequentially rational stochastically unstable equilibrium. In the long run, the zero inflation monetary policies are inherently more stable than the discretionary rules, and once established, they tend to persist for longer periods of the time. Thus, for this continuous time dynamic stochastic game, sequentially strong rational stochastically stable equilibrium behavior can be predicted for any initial natural rate.

E. Conclusion

This chapter develops a model to examine the equilibrium behavior of monetary time inconsistency problem in a continuous time economy with stochastic natural rate and endogenized distortion. First, we introduce the notion of sequentially rational equilibrium, and show that the time inconsistency problem may be solved with trigger reputation strategies in a stochastic setting. We provide the conditions for the existence of sequentially rational equilibrium. Then the concept of sequentially rational stochastically stable equilibrium is introduced. We compare the relative stability between, of so=called the cooperative behavior, with so-called uncooperative behavior, and show that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this monetary policy game is sequentially rational stochastically stable equilibrium. In the long run, the zero inflation monetary policies are inherently more stable than the discretion rules, and once established, they tend to persist for longer periods of the time.

CHAPTER III

REPUTATION AND OPTIMAL CONTRACT FOR CENTRAL BANKERS: A UNIFIED APPROACH

This chapter studies the time inconsistency problem on monetary policy for central banks using a unified approach that combines reputation forces and contracts. We first characterize the conditions for reputation forces to eliminate the inflation bias of discretionary policy. We then propose an optimal contract that can be used with reputation forces to implement a desired socially optimal monetary policy rule when the reputation forces alone are not large enough to discourage a central bank to use a surprise inflation policy. In contrast to most of the existing contracts that are contingent on realized inflation rates which are in turn contingent on production shocks, like the standard reputation model, a central banker in our hybrid mechanism is punished only when she uses a surprise inflation rate. Since the penalty proposed is the lowest one that discourages the central bank from attempting to cheat and the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the loss at the socially optimal inflation rate, our hybrid mechanism is the most efficient and robust mechanism that implement the socially optimal monetary policy rule. We also provide a upper bound of the penalty that is be lower than that of the existing contracts when realized inflation rate is greater than a certain level.

A. Introduction

The time inconsistency problem is one of the most common problems that plague economic policy. Even though technologies, preferences, and information are the same at different times, the policymaker's optimal policy chosen at time t_1 differs from the optimal policy for t_1 chosen at t_0. One can see such a time inconsistency problem exists almost everywhere. For instance, politicians quite often announce that they will carry out a specific policy in the future, but then do something else when the time comes.[1] It is well known from Kydland and Prescott [20] and Barro and Gordon [3] that the time inconsistency of optimal monetary policy may appear when a central bank faces an incentive to expand output above its equilibrium level, and the monetary policy games between the central bank and the public may result in inflation bias as a bad Nash equilibrium outcome. The society experiences a positive average inflation with no systematic improvement in output performance. Indeed, when the marginal benefit of inflation exceeds the marginal cost at a low inflation, the central bank will have an incentive to use a discretionary policy of inflationary bias, and since the public understands that it will do so, the central bank's announcement of a low inflation policy will not be credible. The public will expect a positive rate of inflation, and the central bank cannot do better than to fulfill those expectations. Thus, in order to induce the set of equilibria that lead to desired outcomes, some methods that increase the marginal cost of the central bank must be

[1] In fact, the time consistency problem can be regarded a special case of the general incentive compatibility problem in the incentive mechanism design literature.

used to change the central banker's incentives.

Since the time inconsistency problem was first noted by Kydland and Prescott [20], several solutions have been proposed to deal with this problem in monetary policy. Barro and Gordon [3, 4] were the first to build a game theoretical model to analyze "reputation" of monetary policy. Backus and Driffill [2] extended the work of Barro and Gordon to a situation in which the public is uncertain about the preferences of the government. Persson and Tabellini [29] gave an excellent summarization of these models. Al-Nowaihi and Levine [1] discussed reputation equilibrium in the Barro-Gordon monetary policy game. Li and Tian [21] developed a reputation strategic model of monetary policy with a continuous time horizon. The second solution is built on the legislative approach. The major contribution in this area was made by Rogoff [33]. Following the legislative approach of Canzoneri [8] and Garfinkle and Oh [13], Lohmann [23] showed how the welfare effects of Rogoff's conservative bankers can be improved by adding an escape clause. The third solution is based on the incentive contracting approach to monetary policy. Persson and Tabellini [30], Walsh [41, 42, 43], Svensson [37], Jensen [16], and Huang and Padilla [15] among many others use this approach.

The basic idea of these three approaches is that if the incentives faced by a central bank in choosing how much to inflate can be affected by some means, the inflation bias may be eliminated while still leaving the central bank free to respond to aggregate output shocks. The insight is that since the inflation bias reflects the monetary authority underestimating the equilibrium cost of inflation, the bias can be eliminated if it can internalize an additional penalty to high realized inflation.

Walsh [41] was the first to use the contracting approach to investigate the central banker's incentive problem. He showed that by tying reward of the central bank to realized inflation through a simple linear incentive contract, the inflation bias of discretionary policy is eliminated and an optimal response is achieved. Walsh's model has then been extended in several directions. Persson and Tabellini [30] showed how the credibility problem may be resolved by a simple performance contract that imposes a linear penalty for inflation on the central bank, and argued that this kind of contract has some resemblance to real-world institutions (also see Beestma and Jensen [5], Herrendorf and Lockwood [14], and Svensson [37]).

However, a main drawback of the contracting approach in the literature is that it completely ignores the equilibrium cost of using discretion inflation rule. The existing contracting models fail to capture the fact that reputation forces can restore credibility or at least reduce the cost of contracts to some extent. Also, the existing contracting schemes are costly for use than necessary. For instance, in Walsh's setting, the government will punish the central banker by an amount that is proportional to the realized socially optimal monetary policy π_t^R. Even though a realized inflation rate does not comes from a surprise inflation, but from a production shock, the central bank will then be nevertheless penalized. Since a production shock can be arbitrary large when it is an unbounded random variable, a huge contract cost will be required for implementing the socially optimal monetary policy rule although the central bank has no incentive to use a cheating monetary policy. A central bank is normally financed by the public and making them pay a pecuniary fine would simply be a reshuffling of tax money.

In this chapter, we study the time inconsistency for monetary policy by using a unified approach that combines the reputation effect and contract effect. The government designs an incentive optimal contract for the central bank, and simultaneously the public may also be able to punish the central bank by reputation forces. Each game may involve more than one period dependent on whether the central banker will cheat, and the game will be played repeatedly. We assume that the government has complete ability to commit to the contract he proposes to the central banker. In order to focus on the nature of the incentives with which the monetary authority should be faced, like Walsh, we assume that, *ex ante*, both the government and the central banker share the same preference over inflation and output fluctuations at each period. This may reflect the outcome of some appointment process that ensures a similarity of views between the government and the monetary authority. As in the standard model of the time-inconsistency monetary policy, both the government and the central bank prefer to have a low-inflation policy. When the reputation force is not big enough, the government then needs to design an additional incentive compatible contract for the central bank to ensure that, *ex post*, the central bank implements a low inflation policy.

To compare the total loss when the central bank cheats with the social loss when the central does not cheats, we assume that the central bank's objective is to minimize the average expected loss conditional upon the realization of information up to the present. We provide the necessary and sufficient condition for reputation forces to eliminate the discretionary policy of inflation bias. If the reputation force from the public is large enough to discourage central bankers to use a surprise inflation policy, no contract should be imposed to the central bank. Because the reputation approach does not make any transfer payment and a central bank is punished (by losing the credibility and thus increasing the social loss) only when she uses a surprise inflation rate, it is the most efficient way to implement a socially optimal monetary policy rule, and thus should be the first choice to be used. However, when reputation forces from the public alone cannot restrain the central bank from using a cheating rule, then one must impose additional cost to the central bank such as penalty determined by a contract.

We then show how the government may present an optimal contract that can be used with reputation forces to give the central bank incentives to induce a socially optimal policy as a desired equilibrium outcome when the reputation force from the public alone cannot restrain the central bank from using a cheating rule. We present a hybrid mechanism that combines reputation forces and penalty threats. Our approach unifies the reputational approach and the contracting approach. It suggests a simple optimal incentive scheme or institution that discourages the central bank from surprise inflation and gives her enough flexibility to respond to aggregate output shocks. The length of reputation impact can reduce the penalty cost imposed to the central bank. As it will be seen, the longer the reputation effect lasts, the smaller the penalty will be required for discouraging the central bank from surprise inflation.

Our hybrid mechanism approach differs from the pure contracting approach in the following main aspects. First, the contract part in the hybrid mechanism will be used only when the reputation mechanism does not work.

It is well known that the reputation mechanism is the best choice when it works since no explicit transfer payment is needed, and thus the cost of implementing a socially optimal monetary policy rule is the lowest. In this case, no contract is needed. If the reputation mechanism does not work, then one should consider other approaches.

Secondly, even if the reputation forces alone cannot eliminates the central bank's inflationary bias, it nevertheless can reduce to some extent the temptation for the central bank to cheat, and thus a contract scheme may be used together with the reputation forces to solve the central bank's incentive problem so that the contract cost will be lower. To have such a unified approach of reputation and contract, the way for the public to form their expectations in our setting is assumed to be different from the way assumed in the existing contracting approach. Like the reputation model, we assume that the public responds the central bank's cheating to the expected discretion inflation with a lag of one period while the existing contracting approach assumes that the public responds the central bank's cheating to the expected discretion inflation immediately (without any lag of time). This difference of timing in forming expectations makes our contract differs from the existing contracts.

Thirdly, unlike many existing contracts such as Walsh's contract in which the central banker will be punished as long as the realized inflation rate is not zero even though it fully results from production shocks and is out of the central bank's control, the penalty imposed to the central bank in our hybrid mechanism is independent of realized production shocks and the penalty depends only on whether or not she will use a surprise inflation rate, i.e., whether or not expected inflation rate is positive. Thus, just like the existing reputation models, the penalty determined by the both parts of reputation and contract in the hybrid mechanism depends on surprise rate, but not on the realized inflation rate that may result from some uncontrollable shocks by the central bank such as production shocks.

Fourthly, the penalty in our hybrid mechanism is bounded for preventing the central bank from cheating. While most of the existing contract mechanisms are linear in inflation rate and thus the penalty is unbounded, the penalty function in our approach is quadratic, concave, and continuous in surprise inflation rate so that the maximum penalty exists. The central bank can be punished by this upper bound of penalty if she uses a surprise inflation rate in the cheating set we will specify, and yet, this upper bound of penalty may give a lower penalty than the existing contracts. For instance, even if the reputation forces are not taken into account, the penalty is lower than the penalty determined by Walsh's contract when a realized inflation rate is greater than half of the expected optimal cheating rate $\bar{\pi}^C$ specified in (III.6).

Fifthly, the penalty is just the difference between temptation and enforcement for any inflation rate that makes the difference positive, and thus it the lowest penalty that just discourages the central bank from cheating, and so it reaches a lower bound of penalty payment. Thus, the penalty payment function specifies the minimum required payment that implements the socially optimal monetary policy rule, and therefore, the contract is the most efficient way to implement a socially optimal monetary policy. Hence, our hybrid mechanism provides both lower bound and upper bounded of the penalty which can discourage the central bank from using a surprise inflation monetary policy rule. Finally, since the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the social loss at the optimally socially optimal inflation rate even if there are production shocks, our optimal contract is a robust mechanism that implements the socially optimal monetary policy rule.

The remainder of the chapter is organized as follows. Section B sets up the model and the time-inconstancy problem faced by the central banker. Section C considers the dynamic incentives and reputation forces faced by the central bank, and provides a necessary and sufficient condition for reputation forces to eliminate the discretionary policy of inflation bias. Section D presents an optimal contract that can be used with reputation forces to give the central bank incentives to induce a socially optimal policy as a desired equilibrium outcome when the reputation force from the public alone cannot restrain the central bank from using a cheating rule. Section E gives the conclusion.

B. The Setup

1. Economy

As a standard framework in the literature, we consider an economy characterized by the aggregate supply function:

$$y_t - \bar{y} = a(\pi_t - w_t) + x_t \quad t = 1, 2, 3, ... \tag{III.1}$$

where y_t is aggregate output at time t, $\bar{y}$ is the equilibrium level of output in absence of supply shocks or unanticipated inflation, π_t is the inflation rate, $w_t = \pi_t^e$ is the rate of growth of nominal wage which is equal to the public's inflationary expectations, $\{x_t\}$ are aggregate supply shocks which are assumed to be identically and independently distributed, with $E[x_t] = 0$, $var[x_t] = \sigma_x^2 < \infty$ where E is the expectations operator, and a is a positive constant that represents the effect of a money surprise on output, i.e., the rate of the output gain from the unanticipated inflation so that the larger is a, the greater is the central bank's incentives to inflate.

In order to focus on the nature of the incentives with which the monetary authority should be faced, we assume that both the government and the central bank share the same *ex ante* preference over inflation and output fluctuations at each period, which is described by a quadratic loss function of the form:

$$\pounds_t = \frac{1}{2}[\pi_t^2 + \theta(y_t - \bar{y} - k)^2], \qquad \text{(III.2)}$$

where θ is a positive constant that represents the weight the central bank puts on output expansions relative to inflation stabilization, k is a constant that can be considered as the amounts of output that excesses the equilibrium output, and thus $\bar{y} + k$ is interpreted as the target level of output that the government and the central bank want to reach. As it will be seen below, in order to provide an incentive for the policymaker to attempt to create inflation surprises, k must be positive. Notice that the we have implicitly assumed that the central bank has a zero target inflation rate. The inflation term in (III.2) will be replaced by $\frac{1}{2}(\pi_t - \pi^*)^2$ if the central bank has a target inflation π^* that differs from zero inflation rate.

Substituting (III.1) into (III.2), the loss function becomes

$$\pounds(\pi_t, \pi_t^e) = \frac{1}{2}\{\pi_t^2 + \theta[a(\pi_t - \pi_t^e) + x_t - k]^2\} \quad t = 1, 2, 3, ... \tag{III.3}$$

We assume that the government, the central banker and the public all know the distribution of the output shocks, but only the central banker knows the current shock exactly. The government and the public only know the shocks in previous periods exactly. Thus we can think of the current output shock x_t as private information for the central banker.

2. Three Types of Monetary Policy Rules

First note that, since the objective function is quadratic in inflation rate π_t and output y_t that in turn are linear in x_t, an optimal inflation policy rule must be a linear function in x_t.[2] That is, it belongs to the class

$$\pi_t = \bar{\pi} + a_1 x_t,$$

where $\bar{\pi}$ and a_1 are constants to be determined. When $\bar{\pi} > 0$, we say the central bank has a positive inflation bias or uses a surprise inflation rate. a_1 represents the effect of production shock on the inflation rate π_t. Since the

[2] In general, it is useful to distinguish between inflation and the central bank's policy instrument, the latter taken to be the rate of growth of a monetary aggregate directly controlled by the central bank so that inflation rate is given by $\pi_t = m_t + \nu_t + \mu_t$, where m_t is the money growth rate, ν_t is a demand (or velocity) shock, and μ_t is a "control error" in monetary policy. In this paper, we simplify the stochastic structure by setting $\nu_t = \mu_t = 0$ without affecting the results in any essential way. With these simplifications, we have $\pi_t = m_t$ and thus we have assumed that the central bank controls π_t directly.

central banker knows the shock x_t exactly, to determine an optimal inflation policy rule, the central bank only needs to determine the optimal surprise inflation rate $\bar{\pi}$ and the parameter a_1 under various behavior assumptions. As in Barro and Gordon (1983a), there are three types of monetary policy rules a central banker may choose.

a. Ideal Rule: Socially Optimal Monetary Policy Rule

In this case, the central bank is assumed to be able to commit herself in advance to a linear contingent inflation rule subject to the condition that $E\pi_t = \pi_t^e$. That is, it is assumed that the public believes that the central bank follows this contingent inflation rule and the central bank does not cheat by letting actual inflation deviate from the announcement of the inflation rate. Then $\pi_t^e = E(\pi_t) = E(\bar{\pi} + a_1 x_t) = \bar{\pi}$. Substituting π_t and π_t^e into the loss function (III.3) and solving this unconditional expected minimization problem by choosing $\bar{\pi}$ and a_1, we get $\bar{\pi} = 0$ and $a_1 = -\frac{a\theta}{1+a^2\theta}$. Then, $\pi_t^e = 0$, and thus the optimal contingent inflation policy rule, denoted by π_t^R, which minimizes the value of social loss conditional on the realization of x_t and the constraint that $\pi_t^e = E(\pi_t) = 0$ is given by

$$\pi_t^R = -\frac{a\theta}{1 + a^2\theta} x_t, \tag{III.4}$$

and the corresponding expected social loss is given by

$$E[\pounds(\pi_t^R, E\pi_t^R)] = \frac{1}{2}\theta k^2 + \frac{1}{2}\frac{\theta}{1 + a^2\theta}\sigma_x^2, \tag{III.5}$$

which is constant for all t by the i.i.d. assumption on x_t. This is the benchmark case where the society reaches its desired socially optimal rule π_t^R. The rule, π_t^R, in this benchmark case was called an ideal rule by Barro and Gordon. (III.5) gives the lowest social loss for implementing this socially optimal monetary policy rule π_t^R.

b. Cheating Rule

The monetary policy rule given by (III.4), is not credible if implemented either directly by the government or by a monetary authority whose objective function is given by (III.3). When the public expects that the central bank will use the contingent rule π_t^R so that the expected inflation rate by the public is $\pi_t^e = E\pi_t^R = 0$, then the central bank would like to implement a positive surprise inflation rate in order to secure some benefits from a surprise inflation. Indeed, when the central bank chooses an optimal inflation policy π_t^C to minimize the value of the loss function (III.3) conditional on the realization of x_t and $\pi_t^e = E\pi_t^R = 0$ as given, we have

$$\pi_t^C = \frac{a\theta}{1+a^2\theta}(k - x_t), \tag{III.6}$$

and the corresponding expected social loss is:

$$E[\pounds(\pi_t^C, E\pi_t^R)] = \frac{1}{2}\frac{\theta}{1+a^2\theta}k^2 + \frac{1}{2}\frac{\theta}{1+a^2\theta}\sigma_x^2, \tag{III.7}$$

which is lower than that given by (III.5), and thus the central banker has an incentive to adopt a cheating monetary policy rule with a surprise inflation rate given by $\bar{\pi}^C = E(\pi_t^C) = \frac{a\theta}{1+a^2\theta}k$.

c. Discretion Rule

The discretionary rule is defined in the present context as a Nash equilibrium of a non-cooperative game between the central bank and the public. Under the assumption of rational expectations, the public will not believe that the central bank will use the contingent rule π_t^R so that the expected inflation by the public is $\pi_t^e \neq 0$.[3] Thus, the central bank will choose the optimal discretionary $\pi_t = \pi_t^D$ to minimize the value of the loss function (III.3) conditional on the realization of x_t and taking π_t^e as given. The equilibrium level of inflation is $\pi_t^D = \frac{a^2\theta\pi^e + a\theta(k - x_t)}{1+a^2\theta}$ and thus the expected inflation is $\pi_t^e = E(\pi_t^D) = a\theta k > 0$ which means that there exists an inflation bias on average. Thus, the discretionary inflation rule is given by

$$\pi_t^D = a\theta k - \frac{a\theta}{1 + a^2\theta} x_t, \tag{III.8}$$

and the corresponding expected social loss is given by

$$E[\pounds(\pi_t^D, E\pi_t^D)] = \frac{1}{2}\theta(1 + a^2\theta)k^2 + \frac{1}{2}\frac{\theta}{1 + a^2\theta}\sigma_x^2, \tag{III.9}$$

which reaches a higher expected social loss than the benchmark case given by (III.5).

As will be seen below, since

$$E[\pounds(\pi_t^C, E\pi_t^R)] < E[\pounds(\pi_t^R, E\pi_t^R)] < E[\pounds(\pi_t^D, E\pi_t^D)], \tag{III.10}$$

the central bank may have the incentives to cheat or deviate from the optimal

[3] The assumption of rational expectations implicitly defines the expected loss function for the public as $\pounds_p = E[\pi_t - \pi_t^e]^2$; given the public's understanding of the central bank's choice problem, their choice of π_t^e is optimal.

policy if the time discount factor is small and as a result, she reaches a worse noncooperative Nash equilibrium outcome than the benchmark case if no additional cost is imposed on the central bank. Then, the economy suffers from a positive bias inflation with an even higher expected social loss. Thus, we have the time-inconsistency problem, which leads to an non-socially-optimal monetary policy.

In this case, the central banker needs to be given additional incentives to implement the desired socially optimal monetary policy (III.4) if the reputation force alone is not large enough to prevent the central bank from cheating. The main purpose of this paper is to solve this problem by giving an optimal hybrid mechanism that combines the reputation effect and contract enforcement. To do so, in what follows, we first characterize the conditions under which the reputation mechanism alone can solve the central bank's incentive problem. We then present an optimal incentive compatible hybrid mechanism that eliminates the inflationary bias and, at the same time, has a lowest contract cost.

C. Reputation Mechanism and Enforcement of Ideal Rule

The idea of the reputation model is that a credible rule comes with some enforcement power that can reduce an central bank's temptation to cheat. If the central bank adopts a higher rate of inflation than people expect, then they will raise their expectations of future inflation and it results in a higher inflation in the future and a higher social loss.

The timing of the monetary policy game can be described as follows.

At the beginning of each period, the central bank announces her inflation policy rule, and then the public form expectations (or write wage contracts) based on its belief or lack of belief in the central bank's announcement. The output shock is then realized, and finally the central bank chooses an inflation policy rate that obeys or does not obey her announcement. This process, the interaction between the central bank and the public, will then be repeated over time. Thus, this iterative process incorporates notions of reputation into a repeated-game version of the basic framework.

The public is assumed to use the following behavior strategy in this repeated monetary policy game to form their expectations. If the central bank uses the socially optimal monetary policy rule in the previous period, the public trusts the central bank will continue to use the rule in the current period, and forms their expectations by this belief which equals $\pi_t^e = E\pi_t^R = 0$. But, if the central bank departs from the socially optimal monetary rule π_{t-1}^R last period by using a cheating rule π_{t-1} so that $E\pi_{t-1} \neq \pi_{t-1}^e = E\pi_{t-1}^R$, the public then loses their trust and does not expect the central bank to follow her rule. With a lag of one period (the contract length), the public expects the central bank to pursue the discretionary policy π_{t+i}^D, and responds to a deviation $\pi_{t+i}^e = E\pi_{t+i}^D = \theta ak$ for the next P periods where $i = 1, \ldots, P$ and P may be interpreted as the punishment length for the cheating or the negotiation power over wages of a monopoly union. The punishment length P is assumed to be exogenously given and fixed.[4] The credibility is restored

[4]If P is an endogenous variable, there exists a multiplicity of reputational equilibria that can be supported as subgame-perfect equilibria on the part of the public sector. Al-Nowaihi and Levine [1] considered this problem with solution.

as the P periods punishment. That is, it is assumed that the public has the following form of expectation mechanism:

$$\pi_t^e = E\pi_t^R \quad \text{if } E\pi_{t-1} = \pi_{t-1}^e = E\pi_{t-1}^R \tag{III.11}$$

$$\pi_{t+i}^e = E\pi_{t+i}^D \quad \text{if } E\pi_{t-1} \neq \pi_{t-1}^e = E\pi_{t-1}^R \tag{III.12}$$

for $i = 0, 1, \ldots, P-1$. Notice that the form of expectations above is different from the form of expectations used in the contracting literature in which the public is assumed to respond the central bank's cheating to the expected discretion inflation without any lag of time, i.e., $\pi_t^e = E\pi_t$ for any period of time t so that the cheating monetary policy rule π_t^Cdoes not appear in the contracting model. This difference of timing in forming expectations makes our contract differs from the existing contracts.

Accordingly, the central bank can maintains its reputation or credibility in each period if she wants. On the other hand, if the central bank cheats during period t, the expectations are the ones associated with the discretionary rule π_{t+i}^D for next P periods. Notice that the assumption of rational expectations implies that there is at least one period punishment to the central banker if she cheats.

The government wants to eliminate the inflation bias of discretionary policy while still preserving the ability of the central bank to respond to aggregate output shocks. Thus, he wants the central bank to implement the socially optimal contingent rule π_t^R for any length of periods. While the reputation game can be repeated independently as many times as desired, it is assumed that the future is discounted by $0 < \beta < 1$ so that the government's total loss for $1+P$ periods has present values $\sum_{i=1}^{P} \beta^i E[\pounds(\pi_{t+i}^R, E\pi_{t+i}^R)]$ under the socially optimal monetary policy rule π_t^R. For the convenience of discussion, we may consider the average loss, and may also want to discount the time, so the total time having this total loss has the present value $\sum_{i=0}^{P} \beta^i$. Thus the government's average expected social loss for the length of $P+1$ periods at π_t^R is given by

$$\pounds_G = \sum_{i=0}^{P} \frac{\beta^i E[\pounds(\pi_{t+i}^R, E\pi_{t+i}^R)]}{\sum_{i=0}^{P} \beta^i} = \frac{1}{2}\theta k^2 + \frac{1}{2}\frac{\theta}{1+a^2\theta}\sigma_x^2. \tag{III.13}$$

However, the central bank may have a different *ex-post* average objective function. Since the central bank's decision may cheat at any time t, the central banker's average expected loss under the cheating is given by

$$\lambda(\pi_t) = \frac{E_t[\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D)]}{\sum_{i=0}^{P} \beta^i} \tag{III.14}$$

where E_t denotes the expectations conditional upon the realization of all information up to and including period t. This objective function shows that the central bank has an option to use a discrete monetary policy in period t when she thinks it is necessary. We now investigate under which conditions, reputation forces will prevent the central bank from using a cheating monetary policy.

Note that $E_t(\pounds(\pi_t^C, E\pi_t^R)) \leq E_t(\pounds(\pi_t, E\pi_t^R))$ for any linear contingent inflation rate π_t, and thus the central bank does not have any incentive to cheat at π_t if she does not have an incentive to cheat at π_t^C. So we only need to consider the loss at the optimal cheating policy π_t^C.

Note that

$$\lambda(\pi_t^C) - \lambda(\pi_t^R) = \frac{a^2\theta^2k^2}{2\sum_{i=0}^P \beta^i}(\sum_{i=1}^P \beta^i - \frac{1}{1+a^2\theta}). \tag{III.15}$$

Thus, if $\sum_{i=1}^P \beta^i \geq \frac{1}{1+a^2\theta}$, then the central bank does not have the incentives to cheat or deviate from the socially optimal monetary policy. This result can be regarded as a version of the folk theorem for infinite-horizon repeated games, which suggests that the central bank has an incentive to cheat when the discount factor is small. The smaller is β, the larger is the reputation force P needed. However, when β is too small so that $\sum_{i=1}^P \beta^i < \frac{1}{1+a^2\theta}$, the reputation force alone cannot solve the central bank's cheating problem. Indeed, when $\beta < \frac{1}{2+a^2\theta}$, $\sum_{i=1}^P \beta^i = \beta\frac{1-\beta^P}{1-\beta} < \frac{1}{1+a^2\theta}$ for any P, and thus the minimum discount factor for the central bank to keep the socially optimal monetary policy rule is $\beta = \frac{1}{2+a^2\theta}$.

The intuition behind this is that: if the central bank uses a cheating policy, she will receive a benefit or gain of $E_t[\pounds(\pi_t^R, E\pi_t^R)) - \pounds(\pi_t, E\pi_t^R))]$ from reneging arising from the one period before the public can retaliate, but the public will then retaliate the central bank for P periods and the penalty will be $\sum_{i=1}^P \beta^i E_t[\pounds(\pi_{t+1}^D, E\pi_{t+1}^D) - \pounds(\pi_{t+1}^R, E\pi_{t+i}^R))]$ which arises from the P periods of punishment. As in Barro and Gordon, we may call the benefit of cheating as temptation and the cost of cheating as enforcement to renege on the socially optimal monetary policy rule π_t^R. When the enforcement is greater than the temptation, i.e., $\sum_{i=1}^P \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] > E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))]$, the reputation mechanism implements the socially optimal contingent monetary policy π_t^R.

Thus, we have the following proposition.

Proposition C.1. *For the reputation repeated game of monetary policy between the central bank and the public, there is no inflation bias for any period of time if and only if* $\sum_{i=1}^{P} \beta^i \geq \frac{1}{1+a^2\theta}$*, and further the socially optimal monetary policy rule* $\{\pi_t\}$ *cannot be supportable for any length of reputation forces if* $\beta < \frac{1}{2+a^2\theta}$*.*

Note that, when β is large, we only need a small P to keep this inequality held. In particular, when β is close to one, we have $\sum_{i=1}^{P} \beta^i > \frac{1}{1+a^2\theta}$ and thus the central bank does not have an incentive to cheat or deviate from the optimal policy in any period t for any $P \geq 1$. Then, we have the following corollary.

Corollary C.2. *For the reputation repeated game of monetary policy between the central bank and the public, when* β *is close to one, there is no inflation bias for any positive length of reputation.*

Hence, reputation forces discourage the central bank from attempting to cheat and the legal, institution, or contracting constraint on the central bank is unnecessary and will impose unnecessary contract cost. In particular, all contracts such those in Walsh are unnecessary for use.

Remark C.1. *The conclusion of Corollary C.2 is based on the quadratic specification of objective function in (III.2). However, as shown in Barro and Gordon, when the objective function is replaced by the quadratic-linear objective function*

$$\pounds_t = \frac{1}{2}[\pi_t^2 - \theta(y_t - \bar{y} - k)], \tag{III.16}$$

the zero average inflation policy rule is no longer supportable by the reputation mechanism for any discount factor $0 \leq \beta \leq 1$ *although a non-zero but less than the discretionary inflation rate may be sustainable. In this case, we need to adopt other means such as the contracting approach we will discuss below to solve the central bank's incentive problem.*

D. A Hybrid Mechanism of Optimal Contracts and Reputation

The above proposition shows that, when $\sum_{i=1}^{P} \beta^i < \frac{1}{1+a^2\theta}$, the reputation force alone is not large enough to discourage the central bank from surprise inflation. The repeated monetary policy game between the central bank and the public yields inflation bias as a bad noncooperative Nash equilibrium outcome so that the society experiences a positive average inflation without systematic improvement in output performance and suffering a higher social loss. The government then needs to step in and may play an important role of providing an incentive compatible mechanism that induces a desired socially optimal monetary policy rule. In this section, we use the principal-agent framework (a simple case of general mechanism design) to determine how the optimal contract is designed and combined with the reputation punishment together to solve the central bank's incentive compatibility problem. In this framework, the principal is the government whose goal is to implement the socially optimal monetary policy rule π_t^R, and the agent is the central bank, to which the government delegates the task of implementing the goal. In this section, we show that by combining the reputation pressure with an additional incentive contract, one can induce the central bank to eliminate the inflation bias of discretionary policy, and give the central bank the right incentives to induce the socially optimal monetary policy π_t^R as a desired equilibrium outcome. In addition, the hybrid mechanism has the lowest contract cost in the sense that the transfer payment is the lowest to implement the socially optimal monetary policy for any rate of surprise inflation.

Our hybrid mechanism approach differs from the pure contracting approach in the following main aspects. First, a contract part in the hybrid mechanism will be used only when the reputation mechanism does not work, i.e., only when $\sum_{i=1}^{P} \beta^i < \frac{1}{1+a^2\theta}$. It is well known that the reputation mechanism is the best choice when it works since no explicit transfer payment is needed, and thus the cost of implementing a socially optimal monetary policy rule is the lowest. In this case, no contract is needed. If the reputation mechanism does not work, then one should consider other approaches. Secondly, even if the reputation forces alone cannot eliminates the central bank's inflationary bias, it nevertheless can reduce in some extent the temptation for the central bank to cheating, and thus a contract scheme may be used together with the reputation forces to solve the central bank's incentive problem so that the contract cost will be lower. Thirdly, unlike many existing contracts such as Walsh's contract in which the central banker will be punished as long as the realized inflation rate is not zero even though it fully results from production shocks, the penalty function for the central bank in our hybrid mechanism is independent of realized production shocks x_t and the penalty depends only on whether or not she will use a surprise inflation rate, i.e., whether or not expected inflation rate $\bar{\pi} > 0$. Fourthly, while most existing contract mechanisms are linear in inflation rate and thus the penalty is unbounded, the penalty function in our approach is quadratic, concave, and continuous in surprise inflation rate so that the maximum penalty exists. Finally, the penalty is just the difference between temptation and enforcement: $E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))] - \sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))]$, for any π_t that makes the difference positive. Thus, the penalty payment function specifies the minimum required payment that implements the socially optimal monetary policy rule $\{\pi_t^R\}$.

Now, we formally present the optimal hybrid mechanism below. In the hybrid mechanism, the central bank receives a penalty payment (which may be zero) from the government when the central bank uses a surprise inflation rate $\bar{\pi}$ at time t. The payment could be considered as a direct cost of the central bank or more broadly as legal constraints for the central bank, denoted by $W(\bar{\pi})$. Notice that, the penalty function is the function of surprise inflation, but not a function of the actual inflation rate π_t that is given by $\pi_t = \bar{\pi} + a_1 x_x$ which depends on both surprise inflation rate $\bar{\pi}$ and production shock x_t. This specification makes our contract be significantly different from one in Walsh's model that is linear and depends on the inflation rate π_t which in turn depends on production shock x_t.

The problem faced by the government (principal) is to design a penalty function $W(\bar{\pi})$ that makes the central bank have no incentives to cheat and thus induces the central bank to choose the socially optimal monetary policy $\{\pi_t^R\}$, and further minimizes the expected value of the loss

$$\lambda(\pi_t, W) = \frac{\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D) + W(\bar{\pi})}{\sum_{i=0}^{P} \beta^i} \tag{III.17}$$

conditional on the realization of x_t.

Like the first term in (III.14), the loss function $\pounds(\cdot)$ is valued at $(\pi_t, E\pi_t^R) = (\pi_t, 0)$, but not at $(\pi_t, E\pi_t)$ when the central bank cheats at the current period t. Notice that this is the main difference between our unified approach and the existing contracting approach. In the existing contract models, the public is assumed to respond the central bank's cheating to the expected discretion inflation $E\pi_{t+i}^D = a\theta k$ without any lag of time so that the cheating monetary policy rule π_t^C does not appear in the central bank's objective function.

When

$$\pounds(\pi_t, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^D, E\pi_{t+i}^D) > \pounds(\pi_t^R, E\pi_t^R) + \sum_{i=1}^{P} \beta^i \pounds(\pi_{t+i}^R, E\pi_{t+i}^R) \tag{III.18}$$

or equivalently, when enforcement is greater than temptation:

$$\sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R)] > E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R)],$$

it is more costly for the central bank to use the discretionary policy π_t, and thus she does not have an incentive to cheat. Thus, the reputation mechanism alone cannot solve the central bank's incentive problem. So we only need to consider the case where $\sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] < E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))]$. Define the set of inflation rates in which the central bank has an incentive to deviate from the socially optimal monetary policy rule π_t^R by

$$\Pi_t = \{\pi_t : A(\pi_t) < 0\} \tag{III.19}$$

where

$$
\begin{aligned}
A(\pi_t) \;\; = \;\; & \sum_{i=1}^{P} \beta^i E_t[\pounds(\pi_{t+i}^D, E\pi_{t+i}^D) - \pounds(\pi_{t+i}^R, E\pi_{t+i}^R))] \\
& -E_t[\pounds(\pi_t^R, E\pi_t^R) - \pounds(\pi_t, E\pi_t^R))]
\end{aligned} \tag{III.20}
$$

which may be called the cheating set at time t. Note that $E_t[\lambda(\pi_t^R, 0)] = E_t[\pounds(\pi_t^R, E\pi_t^R)]$. The contract $W(\bar{\pi})$ implements the optimal policy $\{\pi_t^R\}$ if $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R, E\pi_t^R)]$ for all $\pi_t \in \Pi_t$ and for all t. It is clear that there are many such contracts which implement the optimal policy $\{\pi_t^R\}$. For instance, any $W(\bar{\pi})$ that makes $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R)]$ for all t can be used as such a contract. Hence, our interest here is to find an optimal one which has the lowest penalty cost.

Then, we have the following definition about the optimal contract.

Definition D.1. *A contract $W(\bar{\pi})$ is said to be an optimal contract which implements the optimal policy $\{\pi_t^R\}$ if it satisfies the following three conditions.*

(1) (Incentive Compatibility): $E_t[\lambda(\pi_t, W)] \geq E_t[\pounds(\pi_t^R, E\pi_t^R)]$ for any $\pi_t \in \Pi_t$.

(2) (Optimal Choice): π_t minimizes $E_t[\lambda(\pi_t, W)]$ for all $\pi_t \in \Pi_t$.

(3) (Efficient Contract): $E[\lambda(\pi_t, W)] = E[\pounds(\pi_t^R, E\pi_t^R)]$ for all $\pi_t \in \Pi_t$.

In the above definition, Condition 1 is known as the incentive compatibility requirement that discourages the central bank deviating from the socially optimal monetary policy $\{\pi_t^R\}$ so that the central bank's interest is compatible with the government's interest. Condition 2 is known as the central banker's rational (optimal) choice condition on monetary policy rule. Condition 3 is known as the efficient contract condition under which the penalty determined by the contract is the lowest penalty that just discourages the central bank from cheating, i.e., the sum of the loss at any discretionary monetary policy rule, reputation forces, and the penalty is exactly equal to the loss at the socially optimal monetary policy π_t^R. Notice that Condition 2 is not the same as Condition 3. For instance, when a big constant term is added into the penalty function W, the central banker's original optimal choice on π_i remains optimal, but the $\lambda(\pi_t, W)$ becomes larger. Also notice that expectations in the first two conditions are taken conditional on the realization of x_t since, by assumption, it is known by the central bank.

As we mentioned earlier, any optimal monetary policy rule of the central bank belongs to the class of linear contingent function in x_t: $\pi_t = \bar{\pi} + a_1 x_t$, the central bank wants to choose the optimal $\bar{\pi}$ and a_1 so that π_t minimizes $E_t[\lambda(\pi_t, W)]$ for all $\pi_t \in \Pi_t$. Then the first order conditions for the central bank's problem are obtained by differentiating (III.17) with respect to a_0 and $\bar{\pi}$, respectively

$$(1 + \theta a^2)\pi_t x_t + \theta a(x_t - k)x_t = 0 \tag{III.21}$$

and

$$(1 + \theta a^2)\pi_t + \theta a(x_t - k) + \frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = 0. \tag{III.22}$$

Taking the unconditional expectation for equation (III.21) and solving for a_1, the optimal a_1 is given by $a_1 = -\frac{a\theta}{1+a^2\theta}$. Thus, in the hybrid mechanism, the optimal monetary policy rule of the central bank has the form of $\pi_t = \bar{\pi} - \frac{a\theta}{1+a^2\theta} x_t = \bar{\pi} + \pi_t^R$, where $\bar{\pi}$ is a surprise inflation bias deviating from the socially optimal inflation policy rule π_t^R by the central banker.

To find out the penalty function $W(\pi_t)$, substituting $\pi_t = \bar{\pi} - \frac{a\theta}{1+a^2\theta}x_t = \bar{\pi} + \pi_t^R$ into (III.22) and solving for $\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}}$, we have

$$\frac{\partial W(\bar{\pi})}{\partial \bar{\pi}} = \theta ak - (1+\theta a^2)\bar{\pi}. \tag{III.23}$$

Thus, we have

$$W(\bar{\pi}) = W^0 + \theta ak\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2. \tag{III.24}$$

To make $W(\bar{\pi})$ be the optimal incentive compatible contract, we need to determine the constant term W^0 so that $E_t[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] \geq 0$ and $E[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] = 0$.

Note that, by substituting $W(\bar{\pi})$ into (III.17), we have

$$\begin{aligned} \lambda(\pi_t, W) - \lambda(\pi_t^R) &= \frac{[\frac{1}{2}(1+\theta a^2)\bar{\pi} - \theta ak)]\bar{\pi} + \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i + W(\bar{\pi})}{\sum_{i=0}^{P}\beta^i} \\ &= \frac{\frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i + W^0}{\sum_{i=0}^{P}\beta^i}. \end{aligned} \tag{III.25}$$

Then, when $W^0 = -\frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i$, we have $\lambda(\pi_t, W)) = \pounds(\pi_t^R, E\pi_t^R)$. Therefore, we have $E_t[\lambda(\pi_t, W) - \pounds(\pi_t^R, E\pi_t^R)] \geq 0$, which means that $W(\bar{\pi})$ is the optimal contract under which the cost of using a cheating monetary policy rule π_t is the same as the cost for the central bank to use the socially optimal monetary policy rule. Thus, the central bank cannot benefit from the cheating, although it is not worse off either.

Thus, the optimal incentive compatible contract that is contingent on the surprise inflation rate $\bar{\pi}$ and discourages the central bank from using the discretionary monetary policy rule π_t is given by

$$W(\bar{\pi}) = \theta ak\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i \quad \forall\, \pi_t \in \Pi_t. \qquad \text{(III.26)}$$

The central bank will be penalized by the amount of $W(\bar{\pi})$ if she uses a surprise inflation rate $\pi_t \in \Pi_t$ at time t, and will not be penalized if she uses a socially optimal monetary policy rule π_t^R or uses a surprise inflation rate π_t which is not in the cheating set Π_t.[5] Thus, the optimal contract penalty payment $W(\bar{\pi})$ is solely based on whether the central bank uses the surprise inflation rate $\bar{\pi}$, but not based on the inflation rate which in turn depends on the magnitude of production shocks x_t.

To determine the cheating set Π_t, note that $W(\bar{\pi}) \geq 0$ if and only if $g_1 < \bar{\pi} < g_2$, where

$$g_1 \equiv \frac{a\theta k\left[1-\sqrt{1-(1+\theta a^2)\sum_{i=1}^{P}\beta^i}\right]}{1+\theta a^2}$$

and

$$g_2 \equiv \frac{a\theta k\left[1+\sqrt{1-(1+\theta a^2)\sum_{i=1}^{P}\beta^i}\right]}{1+\theta a^2}.$$

Thus the central banker does not have an incentive to cheat at the cheating set $\Pi_t = [g_1, g_2]$ and the optimal contract that implements the socially optimal monetary policy rule $\{\pi_t^R\}$ can be written as

$$W(\bar{\pi}) = \begin{cases} \theta ak\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i & \text{if } g_1 < \bar{\pi} < g_2 \\ 0 & \text{otherwise} \end{cases}. \qquad \text{(III.27)}$$

Remark D.1. *The term* $\frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i$ *in the penalty function* $W(\bar{\pi})$ *in*

[5]Since the central bank is assumed to be rational, she will never have an incentive to choose an inflation rate not in Π_t, otherwise she will be worse off.

(III.27) is the reputation effect. Thus, this hybrid mechanism is the combination of the reputation mechanism and the contract $W(\bar{\pi})$. When $1 < (1+\theta a^2)\sum_{i=1}^{P}\beta^i$, the reputation force is enough to solve the central bank's incentive problem by Proposition C.1, and thus no penalty is needed so that the contract cost is zero.

Remark D.2. *The penalty function $W(\bar{\pi})$ depends not only on the discount factor but also the length of reputation P. Furthermore, $W(\bar{\pi})$ is a decreasing function in P, which means the length of the reputation impact from the public can reduce the contract cost.*

Remark D.3. *When future is perfectly discounted, i.e., β approaches to zero, there is no reputation forces imposed on the central bank. In this case, the cheating set has a simple form which is given by $\Pi_t = (0, \frac{2\theta ak}{1+\theta a^2})$, and the reputation term disappears in the above penalty function. Thus, our hybrid mechanism becomes a pure contract mechanism that is given by*

$$W(\bar{\pi}) = \begin{cases} \theta ak\bar{\pi} - \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 & \textit{if } \bar{\pi} \in (0, \frac{2\theta ak}{1+\theta a^2}) \\ 0 & \textit{otherwise} \end{cases}, \tag{III.28}$$

which implements the socially optimal monetary policy rule $\{\pi_t^R\}$. This is a new contract scheme and is different from the existing contract schemes. The hybrid mechanism defined in (III.27) is the sum of the reputation forces given by $\sum_{i=1}^{P}\beta^i[\pounds(\pi_{t+i}^D) - \pounds(\pi_{t+i}^R)] = \frac{1}{2}a^2\theta^2k^2\sum_{i=1}^{P}\beta^i$ and the pure contract mechanism defined by (III.28).

Remark D.4. *Notice that, if the central bank chooses a cheating inflation rule according to the rule specified by (III.6): $\pi_t = \pi_t^C = \frac{1}{1+\theta a^2}\theta a(k - x_t)$,* *then* $\bar{\pi}^C = \frac{1}{1+\theta a^2}\theta ak$ *and thus* $W(\bar{\pi}^C)$ *is given by*

$$W(\bar{\pi}^C) = \frac{1}{2}\frac{\theta^2 a^2 k^2}{1+\theta a^2} - \frac{1}{2}a^2\theta^2 k^2 \sum_{i=1}^{P} \beta^i. \tag{III.29}$$

In fact, since the penalty function is linear-quadratic (i.e., the first term in the penalty function is linear and the second term is quadratic), continuous, and concave in $\bar{\pi}$*, the maximum penalty will be reached at* $\bar{\pi}^C = E\pi_t^C = \frac{1}{1+\theta a^2}\theta ak$*. The intuition behind this is that, since we are looking for the minimal penalty for the central bank to keep the socially optimal monetary policy* π_t^R *for any level of surprise inflation and the bank's loss will be minimized at* π_t^C*, the maximum penalty needs to be imposed to make the sum of the loss, the reputation forces, and the penalty as big as the loss at the socially optimal policy* π_t^R *in order for the central bank to have no incentives to cheat. Thus, the penalty function is an increasing function when* $\bar{\pi} < \frac{1}{1+\theta a^2}\theta ak$ *and a decreasing function when* $\bar{\pi} > \frac{1}{1+\theta a^2}\theta ak$*. Note that the penalty will be zero when* $\bar{\pi} \leq g_1$ *or* $\bar{\pi} \geq g_2$*. However, since the sum of the loss and the penalty equals the loss at the socially optimal monetary policy* π_t^R *over the interval* (g_1, g_2)*, the central bank does not have the incentives to use a discretionary monetary policy and thus the optimal contract is robust so that we can allow the price fluctuations due to measurement error or uncontrollable production shocks.*

Remark D.5. *The maximum penalty specified in (III.29) gives a upper bound of penalty when the central bank adopts a surprise inflation rate in the cheating set. One may simply use this upper bound of penalty to punish the central bank when she cheats. That is, the penalty will be equal to this upper bound and so it is constant for any* $\bar{\pi} = (g_1, g_2)$ *and zero otherwise. Yet,* *this penalty may be lower than the one determined by Walsh's penalty function since his payment is continent on observed inflation rate which in turn is implicitly dependent on production x_t that can be arbitrarily larger. Indeed, since Walsh's penalty is linear which is given by $\theta a k \pi_t$ and our penalty $W(\bar{\pi}) < \frac{\theta^2 a^2 k^2}{2(1+\theta a^2)}$. Then, when $\pi_t > \bar{\pi}^C/2$, $\frac{\theta^2 a^2 k^2}{2(1+\theta a^2)} = \frac{1}{2}\theta a k \bar{\pi}^C < \theta a k \pi_t$ by noting that $\bar{\pi}^C = \frac{\theta a k}{1+\theta a^2}$. Thus, even if we do not consider the reputation effect, the penalty from the pure mechanism specified in (III.28) is lower the penalty in Walsh's contract when $\pi_t > \bar{\pi}^C/2$.*

Thus, the optimal contract $W(\bar{\pi})$ combined with the reputation enforcement implements the socially optimal monetary policy rule $\{\pi_t^R\}$ given by (III.4) and has the lowest cost for preventing the central bank from using an inflation bias of discretionary policy. Also, the total cost for using a cheating monetary policy under this hybrid mechanism is the same as the social cost at π_t^R, and thus the hybrid mechanism is the most efficient way to implement the socially optimal monetary policy rule π_t^R. Hence, when the reputation enforcement alone cannot restrain the central bank from using a cheating rule, this simple hybrid contingent mechanism may be used.

Summarizing the above discussion we have the following proposition.

Proposition D.1. *In the monetary policy repeated game among the public, the central bank and the government, suppose the reputation enforcement lasts for P periods so that $\sum_{i=1}^{P} \beta^i < \frac{1}{1+a^2\theta}$, and suppose the penalty function $W(\bar{\pi})$ is given by (III.27). Then, the hybrid mechanism is an optimal mechanism that implements the socially optimal monetary policy rule π_t^R.*

The above results on the optimal contract with the can be illustrated by the quadratic and convex curve equation

$$L_\beta(\bar{\pi}) \;=\; \frac{1}{2}(1+\theta a^2)\bar{\pi}^2 - \theta a k \bar{\pi} + \frac{1}{2}a^2\theta^2 k^2 \sum_{i=1}^{P} \beta^i < 0, \qquad \text{(III.30)}$$

which is the difference between the reputation enforcement and the temptation to cheat for $\pi_t \in \Pi_t$. The penalty function is given by the quadratic and concave curve which is the mirror image (reflection mapping) of $L_\beta(\bar{\pi})$ for $g_1 < \bar{\pi} < g_2$ and equals zero otherwise. The penalty reaches the maximum while $L_\beta(\bar{\pi})$ reaches its minimum at $\bar{\pi} = \frac{\theta a k}{1+a^2\theta}$. Since $W(\bar{\pi}) + L_\beta(\bar{\pi}) = 0$ for all $g_1 < \bar{\pi} < g_2$ due to their mirror image each other, the penalty for the central bank to cheat just equals the difference between the temptation to cheat and the reputation enforcement. When $\bar{\pi} \geq g_2$ or $\bar{\pi} \leq g_1$, the central banker does not have an incentive to cheat since $\lambda(\bar{\pi}, 0)$ is greater than $\pounds(\pi_t^R)$. Thus, no penalty is necessarily imposed to the central bank.

Thus, our optimal contract has some advantages that the existing contracts do not share. Our approach answers some criticisms for the contracting approach. One criticism is that the incentive contract is costly to use. In Walsh's model, since only the socially optimal monetary policy rule π_t^R satisfies the first order condition and the linear contract is contingent on π_t^R which in turn is contingent on production shocks, the central bank will be nevertheless penalized even though the central bank has no incentive to use a cheating monetary policy. Since the production shock x_t can be arbitrary large when x_t is an unbounded random variable, a huge contract cost may be required for implementing a socially optimal monetary policy rule for a very large production shock. A central bank is normally financed by the public and making them pay a pecuniary fine would simply be a reshuffling of tax money. In our approach, however, a contract penalty will be imposed only when the reputation forces alone are not big enough to prevent the central bank from cheating and further she uses an inflation bias of monetary policy $\pi_t \in \Pi_t$. The magnitude of the penalty is in fact the difference between the temptation to cheat and the reputation enforcement, and thus it is the lowest penalty that just discourages the central bank from cheating. Since the central bank has no incentive to cheat, the penalty is actually zero when $\pi_t = \pi_t^R$. In addition, since the sum of the social loss, reputation cost, and contract penalty for the central bank to cheat is the same for any rate of surprise inflation and just equal the social cost at the socially optimal monetary policy rule π_t^R, our contract is robust. Thus, our optimal contract is the most efficient and stable mechanism that implements the socially optimal monetary policy rule, and at the same time it freely responds to production shocks.

It may be remarked that, just like an agent in the usual principal-agent model that satisfies the binding participation constraint, the central banker, as an agent, is indifferent for both the socially optimal monetary policy rule π_t^R and a cheating monetary policy. This, however, may not be a problem. Since the central bank in any case cannot benefit from using a surprise inflation monetary policy, but more likely she will be hurt by cheating due to the loss of credibility or pressure of reputation, influence on future promotion, the central bank will choose the socially optimal monetary policy. Even if the credibility, reputation pressure, or negative influence on future promotion does not work, one may give the central bank additional (any positive) amounts of penalty when she cheats, and then the socially optimal monetary policy π_t^R becomes the unique optimal choice to the central banker.

Thus our optimal hybrid mechanism approach suggests that there may be a simple optimal incentive scheme that can solve the time inconsistency problem in monetary policy. When the reputation forces alone do not work, our hybrid mechanism approach presents an optimal contract that has some nice properties that an existing pure contract mechanism may not share. In any case, we provide a lower bound of the penalty for the central bank to implement a socially optimal monetary policy rule. The contract is an optimal contract that has the lowest cost of implementing the socially optimal monetary policy rule. The penalty function W depends solely on surprise inflation rate.

We know that among the approaches that solve the time inconsistency problem, the "reputation" problem is key. If reputation consideration discourages the central bank from attempting surprise inflation, then legal or contracting constraints on central bankers are unnecessary and may be harmful. The result in this section, however, suggests that one can reduce the contract cost of maintaining the stationary inflation policy by combining the reputation impact with the contracting penalty when the reputation enforcement alone cannot solve the time inconstancy problem.

E. Conclusion

In this chapter, we have studied the time-inconsistency problem for central bankers. When the reputation enforcement from the public is not large enough to discourage the central bank to use a surprise inflation policy, a contract can be used together with reputation forces to implement the socially optimal monetary policy rule. We presented an optimal hybrid mechanism that combine the reputation approach and contracting approach. This hybrid mechanism discourages the central bank from surprise inflation and gives her full flexibility to respond to output shocks.

Our unified approach of reputation and contract has some nice properties that the existing mechanisms may not share. Our results answer the concern that using the incentive contract is very costly. The results obtained in the paper suggest that one can reduce the contract cost of implementing a socially optimal inflation policy by combining the reputational approach with the contracting approach if the reputation enforcement alone cannot solve the time inconstancy problem. Also, unlike the existing optimal contracts, our contract is only contingent on surprise inflation rate, but not on production shocks. The central bank will be punished only when she has an incentive to use an inflation bias of monetary policy. The magnitude of the penalty is in fact the difference between the temptation to cheat and the lowest penalty which just discourages the central bank from cheating, and so it reaches a lower bound of penalty payment. Thus, our hybrid mechanism is an optimal mechanism that has the lowest cost of implementing the socially optimal monetary policy rule, and it therefore is the most efficient way to implement a socially optimal monetary policy. In addition, since the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the social loss at the optimally socially optimal inflation rate even if there are production shocks, our optimal contract is a robust mechanism that implements the socially optimal monetary policy rule. We have also provided a upper bound of penalty that may be lower than the linear contract when inflation rate is greater than $\bar{\pi}^C/2$.

Of course, like the reputation approach, a weakness of our hybrid mechanism is that the penalty is the function of surprise inflation rate, but not contingent on realized inflation rates. Thus, it imposes a stronger information requirement to the government than the existing contracts in the literature since the surprise inflation rate may be hard to be verifiable exactly although one can estimate easily by various existing econometric methods in the literature. In any case, we can similarly give a hybrid mechanism that combines reputation forces and a contract that is contingent on realized inflation rates, but not on surprise rates if one is willing to increase the contract cost.

CHAPTER IV

BANK CAPITAL REGULATION AND REGULATION HORIZON

In this chapter, we study how to control the risk taking behavior of banks. First we derive the expected bankruptcy time and the conditional probability distribution of bankruptcy for a given time. We show that risk-shifting behavior will increase the probability of bankruptcy during a given time. Then we use these results to analyze certain regulation policies and show that capital requirements alone cannot control the risk-taking behavior of banks at finite future point in time. We also prove that if we use the time horizon as an additional instrument, we can control the risk shifting problem. We give a theoretic explanation for the VaR regulation. Finally, we discuss the VaR contracts with asymmetric information and show that VaR contracts can induce the banker to report the real risk of the project.

A. Introduction

As a consequence of the observed instability of banking systems in the last two decades, various regulation policies have been proposed and enacted. The Basel Accord (1978) represents a landmark financial agreement for regulation of commercial bank. Since 1994, more flexible methods of measuring the risk, Value at Risk (VaR), have been developed and implemented by the major banks. In an important regulatory innovation, the Basel Committee (1996) has proposed that such models be used in the determination of capital that banks must hold to back their securities trading.

At the same time, the theory of prudential regulation of banking has received much attention. Rochet [31] studies the consequences of capital regulations on the portfolio choices of commercial bank. Thakor [40], Santos [36] discuss Basel Accord with a "Credit Crunch". Blum [7] analyzes the consequences of more stringent capital requirements in a dynamic framework. On the second approach, Merton [25, 26] was the first to use a diffusion model for studying the behavior of commercial bank. Since the diffusion model can capture the new characters in the last twenty years[1] and be tractable, many economists study the behavior of commercial bank following Merton's approach. Mella-Barral, Fries and Perraudin [24] derive the optimal closure rule and bailout policy. Rochet, Decamps and Roger [32] develop of continuous time model of commercial bank's behavior and analyze three instruments adopted by Basel Accord. Pages and Santos [28] analyze the impact of depositor-preference laws on the supervisors. Bhattacharya, Plank, Strobl and Zechner [6] consider a model of optimal bank closure rules with Possion-distributed audit. The third approach is using asymmetric information theory to understand financial markets and the regulation of these markets. Risk taking and observability are two forms of moral hazard exist in financial contracting. As a consequence of shareholders limited liability, bank shareholders might try to influence the return distribution of their loans to increase their expected payoff at the expense of the depositors. In a situation of hidden information, the shareholders typically are the persons

[1]During the last two decades, in order to make trading profits and hedge exposure elsewhere in their banking portfolios, banks have greatly increased their holdings of trading asset, such as bonds, equities, interest rate and equity derivatives.

that can observe the risk of their loans at no cost. To the extent that their payment depends on the risk related regulation policy, they might have an incentive to understate the risk. This is called observability problem. Merton [25], Furlong and Keeley [12] first formalize moral hazard in banking. Frexias and Santomero [11] give an excellent summarization of these models. But all above model do not consider the time horizon's affects on risk regulation.

A different approach to bank regulation has its roots in the theory of gambler's ruin. Santomero and Vinso [35] use it for measuring the soundness of banking system and they claim that in order to capture the time dimensionality of the risk we should abandon the simple capital ratio regulation. This is the original ideal of Value at Risk regulation we are using now. Koehn and Santomero [19] study how to reduce the probability of failure by reduce the riskiness of the bank portfolio. Kim and Santomero [18] develop a mean-variance model to investigate the role of bank capital regulation in risk control. But these models do not allow for the possibilities of moral hazard and do not consider the incentives for the bank shareholders.

The present chapter is related to the gambler's ruin problem approach in a continuous time dynamic context but unlike above models we study the regulation policy in an asymmetric information framework, and show that Value at Risk regulation contracts are truthful signaling contracts.

The time at which the bank become insolvent is of obvious important to the regulator whose goal is a "safe and sound" banking system. We assume that regulator has regulation goals in mind that he/she wants to attain within a stipulated planning period. Two important goals need to be considered:

-The time horizon before the bank assets fall to their insolvent level;

-The probability of the bank assets fall to their insolvent level, given a fixed time horizon.

It is important to acknowledge that such goals may not minimize social loss. However, these goals are reasonable under the situation of banking regulation. We give the following two reasons for using the probabilistic model. First, the cost of bank failure is very big and the negative externalities generated by a bank failure are very difficult to calculate exactly, because the failure may spread throughout the banking system, amplifying the negative effects on unrelated intermediaries. So, if we approx the externalities of a bank failure as a big constant, then minimizing the probability of the bank failure implies minimizing the social loss. Second, in some situations (such as economic crisis), keeping the bank from failure may be an overriding concern to the public regulator. In practice, VaR represents these ideas.

We try to examine the relationship between regulation policies (capital requirement and VaR), risk-taking and the two regulation goals of the bank.

We study how to control the risk taking behaviors of the bank. First we get the expected bankruptcy time and conditional probability distribution of bankruptcy for a given time. We show that the risk-shifting behavior will increase the probability of bankruptcy during a given time. The higher risk investments will make higher profit for the shareholders, but they hurt the depositors.

Then we use these results to analyze the regulation policies and show that capital requirements alone cannot control the risk taking behaviors of bank at finite future point in time. We then prove that if one uses the time horizon as an additional instrument, one can control the risk-shifting problem. Thus, we give a theoretic explanation for the VaR regulation.

Finally, we discuss the VaR contracts with asymmetric information and show that VaR contracts can induce the banker report the real risk of the project.

We study the model without regulation in section B. In section C we analyze the two regulation policies: capital requirements and Value at Risk regulation. The asymmetric information cases are discussed in section D. In section E, we conclude.

B. The Model without Regulation

1. Bank Shareholder's Risk-Taking Problem

In our economy, uncertainty is represented by a filtered space $(\Omega, \mathcal{F}, P)$, on which is defined on 1-dimensional Brownian motion $B(t)$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t\}$, the filtration generation by $B(t)$. At the beginning, the bank accepts deposits D and has input of initial capital, x. Then the bank undertakes risky investments. The evolution of asset values of the investments X_t follows diffusion processes:

$$dX_{ti} = \mu X_{ti} dt + \sigma_i X_{ti} dB_t, \quad X_0 = x, \quad with \quad 0 < x < \infty \quad i = 1, 2, ..., N \tag{IV.1}$$

where μ is the drift and σ_i is the instantaneous standard deviation of the process with $\sigma_1 < \sigma_2 < ... < \sigma_N$, dB_t denotes the increment of a standard Wiener process. We assume that $\mu < \frac{\sigma_i^2}{2}$ for $i = 1, 2, .., N$[2]. This implies

[2]It can be shown that for any $x > 0$, $X_t \to 0$ a.s., as $t \to \infty$

that the investment of the bank is risky, because, in a competitive banking industry, bank closure will always happen in the long run[3].

We assume that the bank manager wants to maximize the shareholders' profit. So, if he chooses project i as the investment, his objective function can be described by the following:

$$B(x,\sigma_i) = E^x[\int_0^\tau e^{-\rho t}(X_{ti} - rD)dt] \tag{IV.2}$$

where ρ is the discount factor with $\mu < \rho < 1$, r is the interests earned by depositors and $x > rD$ and $D > 0$[4]. $\tau = \inf\{t \geq 0, X_{ti} \leq x_i^*\}$, x_i^* is the value for which the shareholders choose to declare bankruptcy. Here and in the following E^x denotes the expectation w.r.t. the probability measure Q^x. We define the value of bank equity is the present conditional expected profit in the future. Then we have following lemma

Lemma B.1. *The value of bank equity is*

$$B(x,\sigma_i) = (\frac{x}{\rho-\mu} - \frac{rD}{\rho}) + [\frac{rD}{\rho} - \frac{x^*}{\rho-\mu}](\frac{x}{x_i^*})^{m_{2i}}$$

where, m_{2i} is the negative root of $m^2 + (\frac{2\mu}{\sigma_i^2} - 1)m - \frac{2\rho}{\sigma_i^2} = 0$, and $x_i^* = \frac{-m_{2i}}{1-m_{2i}}\frac{\rho-\mu}{\rho}rD$.

Proof. By Dirichlet-Poission Theorem (Øksendal Theorem 9.3.3 [27]), we know that $B(x,\sigma_i)$ is the boundary solution of the [5]

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[3]It can be shown that $Prob^x\{X_{\eta i} = a\} = 1$, where a is an arbitrary constant and $\eta = \inf\{t \geq 0, X_{ti} = a\}$.

[4]This is the preliminary condition for the existence of banks

[5]Note that we use the Remark in Øksendal [27]

$$\begin{cases} -\rho B + \mu x \frac{\partial B}{\partial x} + \frac{1}{2}\sigma_i^2 x^2 \frac{\partial^2 B}{\partial x^2} = -(x - rD) \quad for \quad x^0 < x < \infty \\ B(x_i^*) = 0 \end{cases}$$

The general solution is

$$B(x) = \frac{x}{\rho - \mu} - \frac{rD}{\rho} + A_1 x^{m_{1i}} + A_2 x^{m_{2i}}$$

where A_1, A_2 are arbitrary constants and

$$m_{ji} = \frac{1}{2} - \frac{\mu}{\sigma_i^2} \pm \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma_i^2})^2 + \frac{2\rho}{\sigma_i^2}} \qquad (j = 1, 2), \qquad m_{2i} < 0 < m_{1i}.$$

First, the boundary condition require a bounded derivative as $X_t \to \infty$, so we have $A_1 = 0$. Then by the boundary condition $B(x_i^*) = 0$, we conclude that the solution of B is

$$B(x) = (\frac{x}{\rho - \mu} - \frac{rD}{\rho}) + [\frac{rD}{\rho} - \frac{x_i^*}{\rho - \mu}](\frac{x}{x_i^*})^{m_{2i}}$$

Now the shareholders are entitled to choose the bankruptcy-triggering point x_i^*. By the first order condition for $B(x)$, we get

$$x_i^* = \frac{-m_{2i}}{1 - m_{2i}} \frac{\rho - \mu}{\rho} rD$$

□

Since

$$\frac{dm_{2i}}{d\sigma_i^2} = \frac{\frac{2\mu}{\sigma_i^4} m_{2i} - \frac{2\rho}{\sigma_i^4}}{2m_{2i} + \frac{2\mu}{\sigma_i^2} - 1} > 0, \tag{IV.3}$$

and

$$\frac{dx_i^*}{d\sigma_i^2} = \frac{-1}{(1 - m_{2i})^2} \frac{\rho - \mu}{\rho} rD \frac{dm_{2i}}{d\sigma_i^2} < 0, \tag{IV.4}$$

then we have

$$\frac{dB}{d\sigma_i^2} = \frac{rD}{\rho}\frac{1}{1-m_{2i}} ln(\frac{x}{x_i*})(\frac{x}{x_i^*})^{m_{2i}} > 0. \tag{IV.5}$$

$\frac{dx_i^*}{d\sigma_i^2} < 0$ implies that as the risk of the invest increasing, the bank's default-triggering value x^* will decrease. Since default for the deposit is an irreversible action for the bank, instead of close the bank, waiting for good movements of the investment is a better choice under high risk situation. This is consistent with the results in real option literature.

$\frac{dB}{d\sigma_i^2} > 0$ implies that bank manager has the incentives to increase the shareholders' payoff at the expense of the depositor. So, at initial time, the manager will choose project N to invest. This is so-called risk-taking behavior. We will show that this behavior will make the expected bankruptcy time horizon of the bank shorter and the probability for the bankruptcy bigger under certain conditions.

2. The Consequences of the Risk-Taking Problem

In this section, we show that the risk-shifting problem will make the bank more unstable. First we have the following Lemma

Lemma B.2. *If the manager chooses the project i to invest, then the expected bankruptcy time is*

$$E^x(\tau) = \frac{2ln(\frac{x}{x_i^*})}{\sigma_i^2 - 2\mu} \qquad where \qquad \tau = \inf\{t > 0, X_t \notin (x_i^*, \infty)\} \tag{IV.6}$$

Proof. From Øksendal (exercise 7.18 (a)[27]), we know for $x \in (x_i^*, R)$, $\tau_0 =$

$\inf\{t > 0, X_{ti} = x_i^*\}$ and $\tau_1 = \inf\{t > 0, X_t = R\}$

$$Prob(\tau_0 < \tau_1) = \frac{R^\gamma - x^\gamma}{R^\gamma - {x_i^*}^\gamma}, \qquad where \qquad \gamma = 1 - \frac{2\mu}{{\sigma_i}^2} \tag{IV.7}$$

Let $f \in C_0^2(R)$, such that $f(x) = \frac{2ln(x)}{\gamma{\sigma_i}^2} + x^\gamma$. Applying Dynkin's formula (Øksendal, Theorem 7.4.1[27]) for $\tau_R = inf\{t > 0, X_t \notin (x_i^*, R)\}$,

$$E^x[f(X_{\tau_R})] = f(x) + E^x[\int_0^{\tau_R} \mathcal{A}f(X_s)ds] = \frac{2lnx}{\gamma{\sigma_i}^2} + x^\gamma - E^X[\tau_R]$$

Then, we have

$$E^x[\tau_R] = \frac{2lnx}{\gamma{\sigma_i}^2} + x^\gamma - E^x[\frac{2lnX_{\tau_R}}{\gamma{\sigma_i}^2} + X_{\tau_R}^\gamma]$$

Using (IV.7), we get

$$E^x[\tau_R] = \frac{2lnx}{\gamma{\sigma_i}^2} + x^\gamma - \frac{R^\gamma - x^\gamma}{R^\gamma - {x_i^*}^\gamma}(\frac{2lnx_i^0}{\gamma{\sigma_i}^2} + {x_i^*}^\gamma) - \frac{x^\gamma - {x_i^*}^\gamma}{R^\gamma - {x_i^*}^\gamma}(\frac{2lnR}{\gamma{\sigma_i}^2} + R^\gamma)$$

Let $R \to \infty$ and use $\tau_R \to \tau$ as $R \to \infty$,we get

$$E^x(\tau) = \frac{2ln(\frac{x}{x_i^*})}{{\sigma_i}^2 - 2\mu}$$

□

Since

$$\frac{dE^x[\tau]}{d\sigma_i^2} = \frac{-2ln(\frac{x}{x^*})}{(\sigma^2 - 2\mu)^2} - \frac{2}{(\sigma^2 - 2\mu)x^*}\frac{dx^*}{d\sigma^2} \tag{IV.8}$$

The risk-taking behavior have two sides affects on the expected operation time of the bank. On the one hand higher risk investments will make the variance increase and decrease the operation time of the bank; on the other hand, higher risk investment will make the shareholders choose a lower bankruptcy point, the extend the operation time of the bank.

Consider now the conditional probability distribution of the bankruptcy level attained by the bank's assets in a finite time horizon.

$$P(x,t) = Prob\{inf_{s\in[0,t]} X_{si} \le rD | X_0 = x\}$$

Now, we give the following Lemma

Lemma B.3. *The Probability that the bank's assets fall to the bankruptcy level during time t is*

$$\begin{aligned} & P(x,t) \\ = \quad & 1 - N(\frac{\ln(\frac{x}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}) + e^{-\frac{2(\mu-\frac{\sigma_i^2}{2})\ln(\frac{x}{x^*})}{\sigma_i^2}} N(\frac{-\ln(\frac{x}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}). \end{aligned} \quad \text{(IV.9)}$$

where

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt. \quad \text{(IV.10)}$$

Proof. Since X_t is a geometric Brownian motion, we can write $\ln(\frac{X_t}{x})$ as

$$\ln(\frac{X_t}{x}) = (\mu - \frac{\sigma^2}{2})t + \sigma dB_t \quad \text{(IV.11)}$$

Then from Dana and Jeanblanc [9], we have

$$P(x,t) = 1 - N(\frac{\ln(\frac{x}{x^*}) + (\mu - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}) + e^{-\frac{2(\mu-\frac{\sigma^2}{2})\ln(\frac{x}{x^*})}{\sigma^2}} N(\frac{-\ln(\frac{x}{x^*}) + (\mu - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}). \quad \text{(IV.12)}$$

□

Then, we have

$$\frac{dP(t,x)}{d\sigma_i} = \frac{2ln(\frac{x}{x^*})}{\sigma_i^2\sqrt{t}} \exp\{-\frac{1}{2}(\frac{\ln(\frac{x}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}})^2\} > 0. \quad \text{(IV.13)}$$

So, the risk-taking behavior will increase the probability of bankruptcy during a given time. The higher risk investments will give higher profit for shareholders, but hurt depositors. So when the banks capital is unregulated, its level reflects only the shareholders' welfare. As a consequence of shareholder limited liability, bank shareholders gain from upside risk but are protected from downside risk. This is a typ of market externality. This risk-taking behavior necessitates a regulatory response if the safety net is to remain viable, and the response is normally some form of public regulation of bank risk-taking behavior.

C. Regulation Policies with Complete Information

1. Regulation with Capital Requirements

The effect of capital requirements have been extensively analyzed in previous literature[6]. In this section, we study the effects of capital requirements on a bank's expected closure time and the probability of failure during a given time.

Since a flat Cooke ratio implies $z = \frac{x}{D}$ are constants, equation (IV.6) can be written as

$$E^x(\tau) = \frac{ln(z\frac{1-m_{2i}}{-m_{2i}}\frac{\rho}{r(\rho-\mu)})}{\sigma_i^2 - 2\mu} \qquad where \qquad \tau = inf\{t > 0, X_t \notin ((r+p)D, \infty)\}$$

So, we can see that a flat Cooke ratio z cannot eliminate the risk-taking behavior of the bank.

Assume a regulator sets $z(\sigma_i^2) = \frac{-m_{2i}}{1-m_{2i}}\frac{r(\rho-\mu)}{\rho}e^{(\sigma_i^2-2\mu)K}$, where K is the expected time of the operation of the bank on which the regulator wants to

[6] Furlong and Keely [12], Koehn and Santomero [19], Blum [7].

implement. Then $E^x[\tau] = K$ and the expected time of closure of the bank will be free from the affects due to the risk-taking behavior of the bank.

The probability that the bank's assets fall to the bankruptcy level during time t can be written as

$$P(x,t) = 1 - N\left(\frac{\ln(z\frac{D}{x^*})+(\mu-\frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}\right) + e^{-\frac{2(\mu-\frac{\sigma_i^2}{2})\ln(z\frac{D}{x^*})}{\sigma_i^2}} N\left(\frac{-\ln(z\frac{D}{x^*})+(\mu-\frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}\right).$$

In order to avoid risk-shifting behavior, the regulator may chose z that the bank's behavior cannot influence the probability of bankruptcy during time t. This can be achieved by making the probability $P(x,t)$ independent of investment risk σ_i.

So, if z satisfies

$$-\frac{2(\mu-\frac{\sigma_i^2}{2})\ln(z\frac{D}{x^*})}{\sigma_i^2} = A_1 \tag{IV.14}$$

$$\frac{-\ln(z\frac{D}{x^*})-(\mu-\frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}} = A_2 \tag{IV.15}$$

and

$$\frac{-\ln(z\frac{D}{x^*})+(\mu-\frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}} = A_3 \tag{IV.16}$$

where A_1, A_2 and A_3 are constants, the regulator can use the capital requirement z to control the risk taking behavior of the bank during time t. But it is very difficult to set a capital requirement z that satisfies these three equations at the same time. Since the capital ratios fail to adjust for the dynamic relationship between book and market values of bank equity, so setting the Cooke ratios cannot eliminate the effect of risk-taking behavior on the probability of bankruptcy for a given time horizon. Thus our model suggests that relying only on a capital requirement cannot rule out the affections the bank's risk-taking behavior. This is consistent with the fact that from most of U.S. history, and certainly for the half-century following the Great Depression, capital requirements tended to be ineffective. More efficient regulatory rules are needed.

2. Regulation with VaR

In an important regulatory innovation, the Basel Committee has proposed use of the that Value-at-Risk (VaR) model to determine capital requirements that banks must need to back their securities trading.

VaR can be defined as the minimal loss under extraordinary market circumstance. From its definition, VaR is evaluated under a probabilistic framework. Let $\triangle X(\triangle t)$ be the change in value of assets of a bank over a time period of length $\triangle t$. Denote the cumulative distribution function (CDF) of $\triangle X(\triangle t)$ by $P_{\triangle X}(x)$. Then the VaR of the asset over a time period $\triangle t$ with probability α is defined as:

$$\alpha = Prob[\triangle X(\triangle t) \geq VaR]$$

This definition states that the probability of loss greater than or equal to VaR over the time horizon $\triangle t$ is α. VaR regulation demands that, in an audit, the bank's safety assets $x - x^*$ must be at least as high as the $\alpha\%$ VaR for a time horizon of t. So in VaR regulation, both the probability of failure and the relevant time period are laid down by the regulator.

From the previous section, we know that in our model we can set

$$VaR = x - x^*.$$

Then we can write equation (IV.9) as:

$$\begin{aligned} P(x,t) &= 1 - N\left(\frac{\ln(1+\frac{VaR}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}\right) \\ &\quad + e^{-\frac{2(\mu-\frac{\sigma_i^2}{2})\ln(1+\frac{VaR}{x^*})}{\sigma_i^2}} N\left(\frac{-\ln(1+\frac{VaR}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}}\right). \end{aligned}$$

Given a bankruptcy probability α, we can set VaR such that

$$-\frac{2(\mu - \frac{\sigma_i^2}{2})\ln(1+\frac{VaR}{x^*})}{\sigma_i^2} = A_1 \tag{IV.17}$$

where A_1 is constant. And using the time horizon as another instrument, set t so that

$$\frac{\ln(1+\frac{VaR}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i\sqrt{t}} = A_2 \tag{IV.18}$$

where A_2 is a constant. Then we can derive[7]

$$P(x,t) \leq 1 - N(A_2) + e^{A_1} N(-A_2).$$

Then, given α, the regulator can use the pair (VaR, t) as instruments to control the risk taking behavior of the bank.

Combine (IV.17) and (IV.18), we can get the following lemma:

Lemma C.1. *The audit frequency for VaR regulation contracts, t, should satisfy*

$$t = \left(\frac{A_2 \pm \sqrt{A_2^2 + 2A_1}}{2(\frac{\mu}{\sigma_i} - \frac{\sigma_i}{2})}\right)^2 \tag{IV.22}$$

Proof. Combining (IV.17) and (IV.18), we get

$$-\frac{\sigma_i^2 A_1}{2(\mu - \frac{\sigma_i^2}{2})} + (\mu - \frac{\sigma_i^2}{2})t = \sigma_i \sqrt{t} A_2, \tag{IV.23}$$

and simple algebraic calculations shows that

$$t = \left(\frac{A_2 \pm \sqrt{A_2^2 + 2A_1}}{2(\frac{\mu}{\sigma_i} - \frac{\sigma_i}{2})}\right)^2. \tag{IV.24}$$

□

[7]Note that

$$\frac{\ln(1 + \frac{VaR}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i \sqrt{t}} = A_2 \tag{IV.19}$$

implies

$$\frac{-\ln(1 + \frac{VaR}{x^*}) - (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i \sqrt{t}} = -A_2, \tag{IV.20}$$

Then

$$\frac{-\ln(1 + \frac{VaR}{x^*}) + (\mu - \frac{\sigma_i^2}{2})t}{\sigma_i \sqrt{t}} = -A_2 + \frac{2(\mu - \frac{\sigma_i^2}{2})t}{\sigma_i \sqrt{t}} \leq -A_2. \tag{IV.21}$$

Proposition C.2. *The VaR contract* $\{VaR,\ t\}$ *should satisfy*

$$\ln(1+\frac{VaR}{x})(\frac{\mu}{\sigma_i^2}-\frac{1}{2})=\frac{A_1}{2}. \tag{IV.25}$$

and

$$\sqrt{t}(\frac{\mu}{\sigma_i}-\frac{\sigma_i}{2})=\frac{A_2-\sqrt{A_2^2+2A_1}}{2}. \tag{IV.26}$$

Next we examine the properties of $E^x[\tau]$. Since

$$\{inf_{s\in[0,t]}X_s \le x^*|X_0=x\}=\{\tau\le u|X_0=x\}$$

We have

$$P\{inf_{s\in[0,t]}X_s \le x^*|X_0=x\}=P\{\tau\le u|X_0=x\}=1-P\{\tau>u|X_0=x\}$$

By conditional Markov Inequality,

$$P\{\tau>u|X_0=x\}\le\frac{E^x[\tau]}{u}$$

we get

$$\begin{aligned} E^x[\tau] &\ge uP\{\tau>u|X_0=x\} \\ &\ge u(1-P\{inf_{s\in[0,t]}X_s\le x^*|X_0=x\}) \\ &= u(1-P(x,u)) \end{aligned}$$

So, given u, if we can control $P(x,t)$, we can also control $E^x[\tau]$.

D. Value at Risk Regulation with Asymmetric Information

From the previous section we know that if the regulator is able to observe the bank's characteristics $(\frac{x}{D},\mu,\sigma)$, then it is possible to control the bank's risk-taking problem. However, in practice it is very difficult to get information on σ. The regulator must try to induce the bank to accurately report their risk. Otherwise, the bank might misrepresent their risk exposure. In this section, we study the case of asymmetric information.

The timing of this game can be described by two situations:

Situation 1: first, the banker chooses the project to invest; then he reports the risk of this project to the regulator; finally, the regulator chooses the VaR contract.

Situation 2: first the bank reports the risk of this project to the regulator, then the regulator chooses the VaR contract, finally, the banker chooses the project to invest.

We give the following definition:

Definition: A VaR regulation contract is a *truthful signaling contract* if the bank's choice of risk to report is accurately the risk of the project under consideration. If the bank does not report the true risk of his investment to the regulator, we have a *pooling contract*.

Then, we get the following Proposition.

Proposition D.1. *VaR regulation Contracts are* truthful signaling contract *for **Situation 1** and **Situation 2**.*

Proof. Under the VaR regulation, the shareholders' objective function will change to

$$B(x) = (\frac{x}{\rho - \mu} - \frac{rD}{\rho}) + [\frac{rD}{\rho} - \frac{x^*}{\rho - \mu}](1 + \frac{VaR}{x^*})^{m_{2i}} \tag{IV.27}$$

and

$$-\frac{2(\mu-\frac{\sigma^2}{2})\ln(1+\frac{VaR}{x^*})}{\sigma^2}=A_1 \qquad \text{(IV.28)}$$

Combine these two equations we get

$$B(x)=(\frac{x}{\rho-\mu}-\frac{rD}{\rho})+[\frac{rD}{\rho}-\frac{x^*}{\rho-\mu}]e^{\frac{A_1 m_{2i}}{\frac{\mu}{\sigma^2}-2}} \qquad \text{(IV.29)}$$

We write the risk of the project the banker chooses and the risk he reports as σ_c^2 and σ_r^2 respectively.

First, we consider **Situation 1**:

We use backward induction and consider the final stage first. If the banker has chosen his investment project, before he reports the risk of this project to the regulator, his indirect utility function can be written as:

$$\max_{\sigma_r^2} B(x,\sigma_c^2,\sigma_r^2)=(\frac{x}{\rho-\mu}-\frac{rD}{\rho})+[\frac{rD}{\rho}-\frac{x^*(\sigma_c^2)}{\rho-\mu}]e^{\frac{A_1 m_{2i}(\sigma_c^2)}{\frac{\mu}{\sigma_r^2}-2}} \qquad \text{(IV.30)}$$

Since $\frac{dB(x,\sigma_c^2,\sigma_r^2)}{d\sigma_r^2}>0$, the bank will choose to report σ_N, so $\sigma_r=\sigma_N$.

Then, we consider the first stage. Before the bank chooses the project to invest, his indirectly utility function can be written as

$$\max_{\sigma_c^2} B(x,\sigma_c^2,\sigma_N^2)=(\frac{x}{\rho-\mu}-\frac{rD}{\rho})+[\frac{rD}{\rho}-\frac{x^*(\sigma_c^2)}{\rho-\mu}]e^{\frac{A_1 m_{2i}(\sigma_c^2)}{\frac{\mu}{\sigma_N^2}-2}} \qquad \text{(IV.31)}$$

Since $\frac{dB(x)}{d\sigma_c^2}>0$, the bank will choose σ_N to invest, so $\sigma_c=\sigma_N$.

Then we have $\sigma_c=\sigma_r$. By definition, we know a VaR regulation contract is *truthful signaling contract.*

Next, consider **Situation 2**:

We consider the final stage first. If he bank has reported the risk of this project to the regulator and the regulator has chosen the VaR contract, then his indirect utility function can be written as:

$$\max_{\sigma_c^2} B(x,\sigma_c^2,\sigma_r^2) = (\frac{x}{\rho-\mu} - \frac{rD}{\rho}) + [\frac{rD}{\rho} - \frac{x^*(\sigma_c^2)}{\rho-\mu}]e^{\frac{A_1 m_{2i}(\sigma_c^2)}{\frac{\mu}{\sigma_r^2}-2}} \tag{IV.32}$$

Since $\frac{dB(x,\sigma_c^2,\sigma_r^2)}{d\sigma_c^2} > 0$, the bank will choose to report σ_N, so $\sigma_c = \sigma_N$.

Then, we consider the first stage. Before the bank chooses the project to report, his indirect utility function can be written as

$$\max_{\sigma_r^2} B(x,\sigma_c^2,\sigma_N^2) = (\frac{x}{\rho-\mu} - \frac{rD}{\rho}) + [\frac{rD}{\rho} - \frac{x^*(\sigma_N^2)}{\rho-\mu}]e^{\frac{A_1 m_{2i}(\sigma_N^2)}{\frac{\mu}{\sigma_r^2}-2}} \tag{IV.33}$$

Since $\frac{dB(x)}{d\sigma_r^2} > 0$, the bank will choose σ_N to invest, so $\sigma_r = \sigma_N$.

Then we have $\sigma_c = \sigma_r$. By definition, we know the VaR regulation contract is a *truthful signaling contract*. □

So, the VaR contracts can induce the banker to report the true characters of the project he invests.

E. Conclusion

In this chapter, we study how to control the risk-taking behavior of a bank. First we drive the expected bankruptcy time and conditional probability distribution of bankruptcy for a given time. We show that the risk-shifting behavior will increase the probability of bankruptcy during a given time. The higher risk investments will make higher profit for the shareholders, but they hurt the depositors.

Then we use these results to analyze the regulation policies and show that capital requirements alone can not control the risk taking behaviors of bank at finite future point in time. We also prove that if we use the time horizon as an additional instrument, we can control the risk shifting problem. So, we give a theoretic explanation for the VaR regulation.

Finally, we discuss the VaR contracts with asymmetric information and show that VaR contracts can induce the banker report the real risk of the project.

CHAPTER V

CONCLUSION

This dissertation studies several problems of monetary policy and banking regulation.

Chapter II develops a model to examine the equilibrium behavior of the time inconsistency problem in a continuous time economy with stochastic nature rate and endogenized distortion. First, we introduce the notion of sequentially rational equilibrium, and show that the time inconsistency problem may be solved with trigger reputation strategies for stochastic setting. We provide the conditions for the existence of sequentially rational equilibrium. Then the concept of sequentially rational stochastically stable equilibrium is introduced. We compare the relative stability between the cooperative behavior and uncooperative behavior and show that the cooperative equilibrium in this monetary policy game is a sequentially rational stochastically stable equilibrium and the uncooperative equilibrium in this monetary policy game is sequentially rational stochastically unstable equilibrium. In the long run, the zero inflation monetary policies are inherently more stable than the discretion rules, and once established, they tend to persist for longer periods of the time.

Chapter III studies the time inconsistency problem on monetary policy for central banks using a unified approach that combines reputation forces and contracts. We first characterize the conditions for reputation forces to eliminate the inflation bias of discretionary policy. We then propose an optimal contract that can be used with reputation forces to implement a desired socially optimal monetary policy rule when the reputation forces alone are not large enough to discourage a central bank to use a surprise inflation policy. In contrast to most of the existing contracts that are contingent on realized inflation rates which are in turn contingent on production shocks, like the standard reputation model, a central banker in our hybrid mechanism is punished only when she uses a surprise inflation rate. Since the penalty proposed is the lowest one that discourages the central bank from attempting to cheat and the sum of the loss, reputation forces, and the penalty for the central bank to cheat is the same as the loss at the socially optimal inflation rate, our hybrid mechanism is the most efficient and robust mechanism that implements the socially optimal monetary policy rule. We also provide an upper bound of the penalty that is lower than that of the existing contracts when realized inflation rate is greater than a certain level.

Chapter IV studies how to control the risk taking behaviors of the bank. First we get the expected bankruptcy time and conditional probability distribution of bankruptcy for a given time. We show that the risk-shifting behavior will increase the probability of bankruptcy during a given time. Then we use these results to analyze the regulation policies and show that capital requirements can not control the risk taking behaviors of bank at finite future point in time. We also prove that if we use the time horizon as an additional instrument, we can control the risk shifting problem. We give a theoretic explanation for the VaR regulation. Finally, we discuss the VaR contracts with asymmetric information and show that VaR contracts can induce the banker report the real risk of the project.

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