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First-order (conditional) risk aversion, background risk and risk diversification

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First-order (Conditional) Risk Aversion, Background Risk and Risk Diversification

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Abstract

Expected utility functions are limited to second-order (conditional) risk aversion, while non-expected utility functions can exhibit either first-order or second-order (conditional) risk aversion. We extend the concept of orders of conditional risk aversion to orders of conditional dependent risk aversion. We show that first-order conditional dependent risk aversion is consistent with the framework of the expected utility hypothesis. We relate our results to risk diversification, provide insights into their application in economic and finance examples, and discuss their relation with the stock market participation puzzle.

Keywords: Expected utility theory; first-order conditional dependent risk aversion; background risk; risk diversification; stock market participation puzzle

JEL classification: D81; G10

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1 Introduction

The concepts of second-order and first-order risk aversion were coined by Segal and Spivak (1990). For an actuarially fair random variable $\tilde{\varepsilon}$, second-order risk aversion means that the risk premium the agent is willing to pay to avoid $k\tilde{\varepsilon}$ is proportional to k^2 as $k \to 0$. Under first-order risk aversion, the risk premium is proportional to k. Loomes and Segal (1994) extend this notion to preferences about uninsured events, such as independent additive background risks. They introduce the concept of orders of conditional risk aversion. Define $\tilde{y}$ as an independent additive risk. The conditional risk premium is the amount of money the decision maker is willing to pay to avoid $\tilde{\varepsilon}$ in the presence of $\tilde{y}$. The preference relation satisfies first-order conditional risk aversion if the risk premium the agent is willing to pay to avoid $k\tilde{\varepsilon}$ is proportional to k as $k \to 0$. It satisfies second-order conditional risk aversion if the risk premium is proportional to k^2.

To the best of our knowledge, utility functions in the von Neumann-Morgenstern expected utility class exhibit only second-order conditional risk aversion, while non-expected utility functions can exhibit either first-order or second-order (conditional) risk aversion[1]. First-order (conditional) risk aversion implies that small risks matter. Because expected utility theory is limited to second-order (conditional) risk aversion, it cannot take into account many real world results. Epstein and Zin (1990) find that first-order risk aversion can help resolve the equity premium puzzle. Barberis et al. (2006) show that utility functions that combine first-order risk aversion with narrow framing can easily address the stock market participation puzzle. Schlesinger (1997) uses first-order risk aversion to explain why full insurance coverage may be optimal even when there is a positive premium loading. Further applications of first-order risk aversion appear in Schmidt (1999), Barberis et al. (2001), and Chapman and Polkovnichenko (2009), among others.

In this paper, we extend the concepts of orders of conditional risk aversion to orders of conditional dependent risk aversion, for which $\tilde{\varepsilon}$ and the background risk $\tilde{y}$ are dependent and $\tilde{y}$ may enter the agent's utility function arbitrarily. We investigate whether first-order conditional dependent risk aversion appears in the framework of the expected utility hypothesis; the general answer to this question is positive with some weak restrictions.

We propose conditions on the stochastic structure between $\tilde{\varepsilon}$ and $\tilde{y}$ that guarantee first-order conditional dependent risk aversion for expected utility agents with a certain type of risk prefer-

[1]See Eeckhoudt et al. (2005), Chapter 13, for more discussion.

ence, i.e., with correlation aversion. Eeckhoudt, Rey and Schlesinger (2007) provide an economic interpretation of correlation aversion: a higher level of the background variable mitigates the detrimental effect of a reduction in wealth. It turns out that the concept of expectation dependence, proposed by Wright (1987), is the key element to such a stochastic structure. Further, the more information that we have about the sign of higher cross derivatives of the utility function,[2] the weaker the dependence conditions on distribution we need. These weaker dependence conditions, which demonstrate the applicability of a weak version of expectation dependence (called N^{th}-order expectation dependence (Li, 2011)), induce weaker dependence conditions between $\tilde{\varepsilon}$ and $\tilde{y}$ to guarantee first-order conditional dependent risk aversion.

Risk premium is an important concept in economics and finance. Intuition suggests that the risk premium for a diversified risk should relate to the number of trials n. We investigate a correlation-averse risk premium for a naive diversified risk in the presence of a dependent background risk. The naive diversified risk is defined as one in which a fraction $\frac{1}{n}$ of wealth is allocated to each of the n risks. In the absence of a dependent background risk, the population mean value of the naive diversified risk approximates the expected value. The Law of Large Numbers states that the risk premium converges to zero when n is large. This is often called the benefit of diversification. Given that in real life, an agent can diversify wealth only on a limited number of risks, a natural question is how small is the risk premium in the presence of a dependent background risk? In other words, what is the convergence rate or approximation error? Our results show that the convergence rate is at the order of $\frac{1}{n^2}$ in the presence of an independent background risk compared with $\frac{1}{n}$ in the presence of a dependent background risk. This difference is a quantitative statement on diversification which provides benefit information on how background risk affects the risk premium of a naively diversified risk. This result also provides additional insights into previous results on insurance supply, public investment decisions, naive diversified portfolio model, bank lending, and the lottery business in the presence of a dependent background risk.

One puzzle for economic theory is the non-participation in the stock market by households with significant financial wealth. Previous studies explain non-participation by the correlation between stock market returns and background risk (Heaton and Lucas, 1997, 2000; Vissing-Jorgensen, 2002; and Curcuru et al. 2005). Other contributions show that first-order risk

[2]Eeckhoudt et al. (2007) provide a context-free interpretation for the sign of higher cross-derivatives of the utility function.

aversion could provide an additional explanation (see Barberis et al., 2006, for a longer discussion). Our contribution offers a new explanation for the traditional expected utility framework by introducing first-order risk aversion along with a stronger definition of dependence between stock market risk and background risk exposure than covariance.

The paper proceeds as follows: Section 2 sets up the model. Section 3 discusses the concept of orders of conditional risk aversion. Section 4 investigates the orders of conditional dependent risk aversion. Section 5 proposes some weaker dependence conditions, and Section 6 applies the results to different economic and financial examples. Section 7 concludes the paper.

2 The model

We consider an agent whose preference for a random wealth, $\tilde{w}$, and a random outcome, $\tilde{y}$, can be represented by a bivariate expected utility function. Let $u(w, y)$ be the utility function, and let $u_1(w, y)$ denote $\frac{\partial u}{\partial w}$ and $u_2(w, y)$ denote $\frac{\partial u}{\partial y}$, and follow the same subscript convention for higher derivatives $u_{11}(w, y)$ and $u_{12}(w, y)$ and so on. We assume that all partial derivatives required for any definition exist. We make the standard assumption that $u_1 > 0$.

Let us assume that $\tilde{z} = k\tilde{\varepsilon}$ in the absence of a background risk. Parameter k can be interpreted as the size of the risk. One way to measure an agent's degree of risk aversion for $\tilde{z}$ is to ask her how much she is ready to pay to get rid of $\tilde{z}$. The answer to this question will be referred to as the risk premium $\pi(k)$ associated with that risk. For an agent with utility function u, $E\tilde{y}$, and non-random initial wealth w, the risk premium $\pi(k)$ must satisfy the following condition:

$$u(w + Ek\tilde{\varepsilon} - \pi(k), E\tilde{y}) = Eu(w + k\tilde{\varepsilon}, E\tilde{y}). \tag{1}$$

Segal and Spivak (1990) give the following definitions of first and second-order risk aversion:

Definition 2.1 *(Segal and Spivak, 1990) The agent's attitude towards risk at w is of first order if for every $\tilde{\varepsilon}$ with $E\tilde{\varepsilon} = 0$, $\pi'(0) \neq 0$.*

Definition 2.2 *(Segal and Spivak, 1990) The agent's attitude towards risk at w is of second order if for every $\tilde{\varepsilon}$ with $E\tilde{\varepsilon} = 0$, $\pi'(0) = 0$ but $\pi''(0) \neq 0$.*

They provide the following results linking properties of a utility function to its order of risk aversion given the level of wealth w_0:

(a) If a risk averse von Neumann-Morgenstern utility function u is not differentiable at w_0 but has well-defined and distinct left and right derivatives at w_0, then the agent exhibits first-order risk aversion at w_0.

(b) If a risk averse von Neumann-Morgenstern utility function u is twice differentiable at w_0 with $u_{11} \neq 0$, then the agent exhibits second-order risk aversion at w_0.

Segal and Spivak (1997) point out that if the von Neumann-Morgenstern utility function is increasing, then it must be differentiable almost everywhere, and one may convincingly argue that non-differentiability is seldom observed in the expected utility model. Alternatively, concave utility functions must have a limited number of kink points. Therefore, this model cannot take first-order risk aversion into account.

3 Order of conditional risk aversion

Loomes and Segal (1994) introduced the order of conditional risk aversion by examining the characteristic of $\pi(k)$ in the presence of independent uninsured risks. For an agent with utility function u and initial wealth w, the conditional risk premium $\pi_c(k)$ must satisfy the following condition:

$$Eu(w + Ek\tilde{\varepsilon} - \pi_c(k), \tilde{y}) = Eu(w + k\tilde{\varepsilon}, \tilde{y}). \tag{2}$$

where $\tilde{\varepsilon}$ and $\tilde{y}$ are independent.

Definition 3.1 *(Loomes and Segal, 1994) The agent's attitude towards risk at w is first-order conditional risk aversion if for every $\tilde{\varepsilon}$ with $E\tilde{\varepsilon} = 0$, $\pi_c'(0) \neq 0$.*

Definition 3.2 *(Loomes and Segal, 1994) The agent's attitude towards risk at w is second-order conditional risk aversion if for every $\tilde{\varepsilon}$ with $E\tilde{\varepsilon} = 0$, $\pi_c'(0) = 0$ but $\pi_c''(0) \neq 0$.*

It is obvious that the definitions of first- and second-order conditional risk aversion are more general than the definitions of first- and second-order risk aversion

We can extend the above definitions to the case $E\tilde{\varepsilon} \neq 0$. Since u is differentiable, fully differentiating (2) with respect to k yields

$$E\{[E\tilde{\varepsilon} - \pi_c'(k)]u_1(w + Ek\tilde{\varepsilon} - \pi_c(k), \tilde{y})\} = E[\tilde{\varepsilon}u_1(w + k\tilde{\varepsilon}, \tilde{y})]. \tag{3}$$

Since $\tilde{\varepsilon}$ and $\tilde{y}$ are independent,

$$\pi_c'(0) = \frac{E\tilde{\varepsilon}Eu_1(w, \tilde{y}) - E[\tilde{\varepsilon}u_1(w, \tilde{y})]}{Eu_1(w, \tilde{y})} = 0. \tag{4}$$

Therefore, not only does $\pi_c(k)$ approach zero as k approaches zero, but also $\pi_c'(0) = 0$. This implies that at the margin, accepting a small zero-mean risk has no effect on the welfare of risk-averse agents. This is an important property of expected-utility theory: “in the small”, the expected-utility maximizers are risk-neutral.

Using a Taylor expansion of π_c around $k = 0$, we obtain that

$$\pi_c(k) = \pi_c(0) + \pi_c'(0)k + O(k^2) = O(k^2). \tag{5}$$

This result is the Arrow-Pratt approximation, which states that the conditional risk premium is approximately proportional to the square of the size of the risk.

Within the von Neumann-Morgenstern expected-utility model, if the random outcome and the background risk are independent, then second-order conditional risk aversion relies on the assumption that the utility function is differentiable. Hence, with an independent background risk, utility functions in the von Neumann-Morgenstern expected utility class can generically exhibit only second-order conditional risk aversion and cannot explain the rejection of a small, independent, and actuarially favorable gamble.

4 Order of conditional dependent risk aversion

We now introduce the concept of order of conditional dependent risk aversion. For an agent with utility function u and initial wealth w, the conditional dependent risk premium, $\pi_{cd}(k)$, must satisfy the following condition:

$$Eu(w + Ek\tilde{\varepsilon} - \pi_{cd}(k), \tilde{y}) = Eu(w + k\tilde{\varepsilon}, \tilde{y}). \tag{6}$$

where $\tilde{\varepsilon}$ and $\tilde{y}$ are not necessarily independent.[3]

Definition 4.1 *The agent's attitude towards risk at w is first-order conditional dependent risk aversion if for every $\tilde{\varepsilon}$, $\pi_{cd}(k) - \pi_c(k) = O(k)$.*

Definition 4.2 *The agent's attitude towards risk at w is second-order conditional dependent risk aversion if for every $\tilde{\varepsilon}$, $\pi_{cd}(k) - \pi_c(k) = O(k^2)$.*

[3]In the statistical literature, the sequence b_k is at most of order k^λ, denoted as $b_k = O(k^\lambda)$, if for some finite real number $\Delta > 0$, there exists a finite integer K such that for all $k > K$, $|k^\lambda b_k| < \Delta$ (see White 2000, p16).

$\pi_{cd}(k) - \pi_c(k)$ measures how dependence between risks affects risk premium. Second-order conditional dependent risk aversion implies that, in the presence of a dependent background risk, small risk has no effect on risk premium, while first-order conditional dependent risk aversion implies that, in the presence of a dependent background risk, small risk affects risk premium.

We denote by $F(\varepsilon, y)$ and $f(\varepsilon, y)$ the joint distribution and density functions of $(\tilde{\varepsilon}, \tilde{y})$, respectively. $F_\varepsilon(\varepsilon)$ and $F_y(y)$ are the marginal distributions.

Wright (1987) introduced the concept of expectation dependence in the economics literature. In the following definition, we use a weaker definition of expectation dependence ($ED(y)$).

Definition 4.3 *If*

$$ED(y) = [E\tilde{\varepsilon} - E(\tilde{\varepsilon}|\tilde{y} \leq y)] \geq 0 \; for \; all \; y, \tag{7}$$

$$and \; there \; is \; at \; least \; some \; y_0 \; for \; which \; a \; strong \; inequality \; holds,$$

then $\tilde{\varepsilon}$ is positive expectation dependent on $\tilde{y}$. Similarly, $\tilde{\varepsilon}$ is negative expectation dependent on $\tilde{y}$ if (7) holds with the inequality sign reversed. The family of all distributions F satisfying (7) will be denoted by $\mathcal{H}_1$ and the family of all negative expectation dependent distributions will be denoted by $\mathcal{I}_1$.

Wright (1987, p115) interprets negative first-degree expectation dependence as follows: "when we discover $\tilde{y}$ is small, in the precise sense that we are given the truncation $\tilde{y} \leq y$, our expectation of $\tilde{\varepsilon}$ is revised upward." Definition 4.3 is useful for deriving an explicit value of $\pi_{cd}(k)$.

Lemma 4.4

$$\pi_{cd}(k) = -k \frac{\int_{-\infty}^{\infty} ED(y) u_{12}(w, y) F_y(y) dy}{E u_1(w, \tilde{y})} + O(k^2). \tag{8}$$

Proof See Appendix.

Lemma 4.4 shows the general condition for first-order risk aversion. The condition involves two important concepts u_{12}, the cross-derivative of the utility function, and $ED(y)$, the expectation dependence between two risks. The sign of u_{12} indicates how this first element acts on utility u. Eeckhoudt et al. (2007) provide a context-free interpretation of the sign of u_{12}. They show that $u_{12} < 0$ is necessary and sufficient for "correlation aversion," meaning that a higher level of the background variable mitigates the detrimental effect of a reduction in wealth. The condition also captures the welfare interaction between the two risks. The sign of expectation dependence indicates whether the movements on background risk tend to reinforce the movements on wealth (positive expectation dependence) or to counteract them (negative expectation dependence). Lemma (4.4) allows a quantitative treatment of the direction and size of the effect of expectation dependence on first order risk aversion. To clarify this, consider the following cases: (1) Assume the agent is correlation neutral ($u_{12} = 0$) or the background risk is independent ($ED(y) = 0$), then the agent's attitude towards risk is second-order conditional dependent risk aversion; (2) Assume $u_{12} < 0$ and $ED(y) > 0$ ($ED(y) < 0$), then the agent's attitude towards risk is first-order conditional dependent risk aversion and her marginal risk premium for a small risk is positive (negative) (i.e., $\lim_{k\to0^+} \pi'_{cd}(k) > (<)0$).

From Lemma (4.4) and Equation (5), we obtain

Proposition 4.5 *(i) If $\tilde{\varepsilon}$ is positive expectation dependent on $\tilde{y}$ and $u_{12} < 0$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) - \pi_c(k) = |O(k)|$;*

(ii) If $\tilde{\varepsilon}$ is negative expectation dependent on $\tilde{y}$ and $u_{12} > 0$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) - \pi_c(k) = |O(k)|$;

(iii) If $\tilde{\varepsilon}$ is positive expectation dependent on $\tilde{y}$ and $u_{12} > 0$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) - \pi_c(k) = -|O(k)|$;

(iv) If $\tilde{\varepsilon}$ is negative expectation dependent on $\tilde{y}$ and $u_{12} < 0$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) - \pi_c(k) = -|O(k)|$.

We consider two examples to illustrate Proposition 4.5.

Example 1. Consider the additive background risk case $u(x, y) = U(x+y)$. Here x may be the random wealth of an agent and y may be a random income risk which cannot be insured. Since $u_{12} < 0 \Leftrightarrow U'' < 0$, part (i) and (iv) of Proposition 4.5 imply that if the agent is risk averse and $\tilde{\varepsilon}$ is positive (negative) expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) > (<)\pi_c(k)$.

Example 2. Consider the multiplicative background risk case $u(x, y) = U(xy)$. Here x may be the random wealth of an agent and y may be a random interest rate risk which cannot be hedged. Since $u_{12} < 0 \Leftrightarrow -xy\frac{U''(xy)}{U'(xy)} > 1$ (relative risk aversion greater than 1), Proposition 4.5 implies that (i) if $-xy\frac{U''(xy)}{U'(xy)} > 1$ and $\tilde{\varepsilon}$ is positive (negative) expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) > (<)\pi_c(k)$; (ii) if $-xy\frac{U''(xy)}{U'(xy)} < 1$ and $\tilde{\varepsilon}$ is positive (negative) expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) < (>)\pi_c(k)$.

5 First-order conditional dependent risk aversion and N^{th}-order expectation dependent background risk

Li (2011) considers the following weaker dependence definition. Suppose $\tilde{y} \in [c, d]$, where c and d are finite. Rewriting $1^{th}ED(\tilde{x}|y) = FED(\tilde{x}|y)$, $2^{th}ED(\tilde{x}|y) = SED(\tilde{x}|y) = \int_c^y FED(\tilde{x}|t)F_y(t)dt$, and repeating integrals defined by

$$N^{th}ED(\tilde{x}|y) = \int_c^y (N-1)^{th}ED(\tilde{x}|t)dt,\ for\ N \geq 3, \tag{9}$$

we obtain:

Definition 5.1 *(Li 2011) If $m^{th}ED(\tilde{x}|d) \geq 0$, for $m = 2, ..., N-1$ and*

$$N^{th}ED(\tilde{x}|y) \geq 0\ for\ all\ y \in [c, d], \tag{10}$$

then $\tilde{x}$ is positive N^{th}-order expectation dependent (NED) on $\tilde{y}$. The family of all distributions F satisfying (10) will be denoted by $\mathcal{H}_N$. Similarly, $\tilde{x}$ is negative N^{th}-order expectation dependent on $\tilde{y}$ if (10) holds with the inequality sign reversed, and the family of all negative N^{th}-order expectation dependent distributions will be denoted by $\mathcal{I}_N$.

From this definition, we know that $\mathcal{H}_N \supset \mathcal{H}_{N-1}$ and $\mathcal{I}_N \supset \mathcal{I}_{N-1}$. In the following lemma, we obtain the risk premium in the presence of an N^{th}-order expectation dependent background risk.

Lemma 5.2

$$\begin{aligned} & \pi_{cd}(k) \\ = \quad & -k\frac{\sum_{m=2}^{N}(-1)^m u_{12^{(m-1)}}(w,d)m^{th}ED(\tilde{x}|d) + \int_c^d (-1)^{N+1}u_{12^{(N)}}(w,y)N^{th}ED(\tilde{x}|y)dy}{Eu_1(w,\tilde{y})} \\ & +O(k^2). \end{aligned} \tag{11}$$

Proof See Appendix.

From Lemma (5.2) and Equation (5), we obtain

Proposition 5.3 *(i) If* $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{H}_N$ *and* $(-1)^m u_{12(m-1)} \leq 0$ *for* $m = 1, 2, ..., N+1,$ *then the agent's attitude towards risk is first-order conditional dependent risk aversion and* $\pi_{cd}(k) - \pi_c(k) = |O(k)|$*;*

(ii) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{I}_N$ *and* $(-1)^m u_{12(m-1)} \geq 0$ *for* $m = 1, 2, ..., N+1,$ *then the agent's attitude towards risk is first-order conditional dependent risk aversion and* $\pi_{cd}(k) - \pi_c(k) = |O(k)|$*;*

(iii) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{H}_N$ *and* $(-1)^m u_{12(m-1)} \geq 0$ *for* $m = 1, 2, ..., N+1,$ *then the agent's attitude towards risk is first-order conditional dependent risk aversion and* $\pi_{cd}(k) - \pi_c(k) = -|O(k)|$*;*

(iv) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{I}_N$ *and* $(-1)^m u_{12(m-1)} \leq 0$ *for* $m = 1, 2, ..., N+1,$ *then the agent's attitude towards risk is first-order conditional dependent risk aversion and* $\pi_{cd}(k) - \pi_c(k) = -|O(k)|$*.*

Eeckhoudt et al. (2007, p120) also provide an intuitive interpretation for the meaning of the sign of the higher order cross-derivatives of utility function, $u_{12(k)}$. For example, $u_{122} > 0$ is a necessary and sufficient condition for "cross-prudence in wealth," meaning that higher wealth reduces the detrimental effect of the background risk. We consider two examples to illustrate Proposition 5.3.

Example 3. Consider the additive background risk case $u(x, y) = U(x + y)$. Since $(-1)^m u_{12(m-1)} \leq 0 \Leftrightarrow (-1)^m U^{(m)} \leq 0$, parts (i) and (iv) of Proposition 5.3 imply that if the agent is kth degree risk averse (See Ekern, 1980 and Eeckhoudt and Schlesinger, 2006 for more discussions of kth-degree risk aversion.) for $m = 1, 2, ..., N+1$ and $\tilde{\varepsilon}$ is positive (negative) Nth expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) > (<) \pi_c(k)$.

Example 4. Consider the multiplicative background risk case $u(x, y) = U(xy)$. Since

$$(-1)^m u_{12(m-1)} \leq 0 \Leftrightarrow (-1)^m xy \frac{U^{(m+1)}(xy)}{U^{(m)}(xy)} \geq m, \ for \ m = 1, 2, ..., N+1 \tag{12}$$
$$(multiplicative \ risk \ apportionment \ of \ order \ m \ for \ m = 1, 2, ..., N+1)$$

(See Eeckhoudt et al., 2009, Wang and Li, 2010, and Chiu et al., 2010 for more discussions of multiplicative risk apportionment of order m.) Proposition 5.3 implies that (i) if $(-1)^m xy \frac{U^{(m+1)}(xy)}{U^{(m)}(xy)} \geq m$ for $m = 1, 2, ..., N+1$ and $\tilde{\varepsilon}$ is positive (negative) expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) > (<) \pi_c(k)$; (ii) if $(-1)^m xy \frac{U^{(m+1)}(xy)}{U^{(m)}(xy)} \leq m$ for $m = 1, 2, ..., N+1$ and $\tilde{\varepsilon}$ is positive (negative) expectation dependent on the background risk $\tilde{y}$, then the agent's attitude towards risk is first-order conditional dependent risk aversion and $\pi_{cd}(k) < (>) \pi_c(k)$.

6 Applications: the importance of background risk in risk diversification and portfolio choice

In this section, we illustrate the applicability of our results, specifically, how they can be used to gain additional insight into risk diversification in the presence of a dependent background risk. We also show how our framework extends the understanding of insurance supply, public investment decisions, naive diversified portfolio model, stock market participation puzzle, bank lending, and the lottery business in the presence of a dependent background risk.

6.1 Background risk and risk diversification

Common wisdom suggests that diversification is a good way to reduce risk. Consider a set of n lotteries whose net gains are characterized by $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$,...,$\tilde{\varepsilon}_n$ which are assumed to be independent and identically distributed. Define the sample mean $\tilde{\varepsilon} = \frac{1}{n}\sum_{i=1}^{n}\tilde{\varepsilon}_i$ then, when w is not random,

$$Eu(w + E\tilde{\varepsilon} - \pi_c(\frac{1}{n}), \tilde{y}) = Eu(w + \tilde{\varepsilon}, \tilde{y}), \quad \textit{where } \tilde{\varepsilon} \textit{ and } \tilde{y} \textit{ are independent}, \tag{13}$$

and

$$Eu(w + E\tilde{\varepsilon} - \pi_{cd}(\frac{1}{n}), \tilde{y}) = Eu(w + \tilde{\varepsilon}, \tilde{y}), \quad \textit{where } \tilde{\varepsilon} \textit{ and } \tilde{y} \textit{ are not necessary independent}. \tag{14}$$

From (5), we know that $\pi_c(\frac{1}{n}) = O(\frac{1}{n^2})$. When $n \to \infty$, $\pi_c(\frac{1}{n}) \to 0$ because diversification is an efficient way to reduce risk. With an independent background risk, diversification can eliminate idiosyncratic risk at the rate of $\frac{1}{n^2}$ and the agent is second-order risk averse. This is the well known benefit of diversification. However, with a dependent background risk, it is not clear that the benefit of diversification holds for a correlation-averse agent.

From Proposition 5.3 and equation (5), we obtain:

Proposition 6.1 *(i) If* $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{H}_N$ *and* $(-1)^m u_{12^{(m-1)}} \leq 0$ *for* $m = 1, 2, ..., N+1$, *then* $\pi_{cd}(\frac{1}{n}) = |O(\frac{1}{n})|$;

(ii) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{I}_N$ *and* $(-1)^m u_{12^{(m-1)}} \geq 0$ *for* $m = 1, 2, ..., N+1$, *then* $\pi_{cd}(\frac{1}{n}) = |O(\frac{1}{n})|$;

(iii) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{H}_N$ *and* $(-1)^m u_{12^{(m-1)}} \geq 0$ *for* $m = 1, 2, ..., N+1$, *then* $\pi_{cd}(\frac{1}{n}) = -|O(\frac{1}{n})|$;

(iv) If $(\tilde{\varepsilon}, \tilde{y}) \in \mathcal{I}_N$ *and* $(-1)^m u_{12^{(m-1)}} \leq 0$ *for* $m = 1, 2, ..., N+1$, *then* $\pi_{cd}(\frac{1}{n}) = -|O(\frac{1}{n})|$.

Proposition 6.1 signs the effect of dependent background risk on the benefits of diversification: if $\tilde{\varepsilon}$ and $\tilde{y}$ are positive (negative) expectation dependent and the agent is correlation-averse, then $\pi_{cd}(\frac{1}{n})$ will be greater (less) than zero. Proposition 6.1 also shows that in the presence of an expectation dependent background risk, diversification can eliminate idiosyncratic risk ($\pi_{cd}(\frac{1}{n}) \to 0$, as $n \to \infty$). Therefore, for correlation-averse agents, the benefit of diversification still holds. However, the convergence rate is $\frac{1}{n}$ rather than $\frac{1}{n^2}$ which implies that if we use the wrong convergence rate to approximate $\pi_{cd}(\frac{1}{n})$, then the error will be large in the presence of an expectation dependent background risk.

6.2 Public investment decisions

Arrow and Lind (1970) investigate the implications of uncertainty for public investment decisions. They consider the case where all individuals have the same preferences U, and their disposable incomes are identically distributed random variables represented by $\tilde{A}$. Suppose that the government undertakes an investment with returns represented by $\tilde{B}$, which are independent of $\tilde{A}$. Let $\bar{B} = E\tilde{B}$ and $\tilde{X} = \tilde{B} - \bar{B}$. Consider a specific taxpayer and denote his fraction of this investment by s with $0 \leq s \leq 1$. Suppose that each taxpayer has the same tax rate and that there are n taxpayers, then $s = \frac{1}{n}$. Arrow and Lind (1970) show that

$$EU(\tilde{A} + \frac{\bar{B}}{n} + r(n)) = EU(\tilde{A} + \frac{\bar{B} + \tilde{X}}{n}), \tag{15}$$

where $r(n)$ is the risk premium of the representative individual. They demonstrate that not only does $r(n)$ vanish when $n \to \infty$, but so does the total of the risk premiums for all individuals: $nr(n)$ approaches zero as n rises. This result implies that the total cost of risk-bearing ($nr(n)$) goes to zero as the population of taxpayers increases and the expected value of net benefit ($\bar{B}$) closely approximates the correct measure of net benefits in terms of willingness to pay.

Proposition 6.1 allows us to investigate the cases where $\tilde{A}$ and $\tilde{B}$ are dependent. Since (15) can be rewritten as

$$EU(\tilde{A} + \frac{\bar{B}}{n} + r(n)) = EU(\tilde{A} + \frac{\tilde{B}}{n}), \tag{16}$$

from Proposition 6.1, we obtain:

Proposition 6.2 *(i) If $(\tilde{B}, \tilde{A}) \in \mathcal{H}_N$ and $(-1)^m U^{(m)} \leq 0$ for $m = 1, 2, ..., N+1$, then $r(n) = -|O(\frac{1}{n})|$;*

(ii) If $(\tilde{B}, \tilde{A}) \in \mathcal{I}_N$ and $(-1)^m U^{(m)} \leq 0$ for $m = 1, 2, ..., N+1$, then $r(n) = |O(\frac{1}{n})|$.

Therefore, when $\tilde{A}$ and $\tilde{B}$ are expectation dependent, $nr(n)$ cannot vanish as n increases. Proposition 6.2 shows that if the return on the investment and the disposable incomes are positive (negative) expectation dependent and the society is risk-averse, then, as the population of taxpayers increases, the total cost of risk-bearing will remain less (greater) than zero and the expected value of net benefit ($\bar{B}$) overestimates (underestimates) the correct measure of net benefits in terms of willingness to pay.

6.3 Stock market participation puzzle

The stock market participation puzzle states that even though the stock market has a positive mean return, a large proportion of the population does not hold any stock (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995). This puzzle has various explanations. For example, Barberis et al. (2006) show that utility functions that combine first-order risk aversion with narrow framing can easily address the puzzle. Another approach examines whether nonstockholders have background risks correlated with the stock market risks (Heaton and Lucas, 1997, 2000; Vissing-Jorgensen, 2002). However, Heaton and Lucas (2000) find that stock market risks have a correlation close to zero with other important risks, such as labor income risk, proprietary income risk, and house price risk. Curcuru et al. (2005) question whether the correlation of the stock market return with the background risk of nonstockholders is high enough to explain the participation puzzle.

Our results offer a new explanation for the stock market participation puzzle by adding first-order risk aversion to the standard expected utility framework and by proposing expectation dependence, which is a stronger definition of dependence than covariance. We showed that for expected utility preferences, it is the expectation dependence between the stock market risks and background risks, rather than the covariance, that determines households' risk premiums. From the definition of expectation dependence, we know that positive (negative) correlated random variables are not necessary positive (negative) expectation dependent. Here, we provide a simple example of two random variables that are positive correlated but not positive expectation dependent.

Example 5. Let $\tilde{x}$ be normally distributed with $E\tilde{x} = \mu > 0$ and $var(\tilde{x}) = \sigma^2$. Let $\tilde{y} = \tilde{x}^2$. Since $E\tilde{x}^2 = \mu^2 + \sigma^2$ and $E\tilde{x}^3 = \mu^3 + 3\mu\sigma^2$, then

$$\begin{aligned} cov(\tilde{x}, \tilde{y}) &= E\tilde{x}\tilde{y} - E\tilde{x}E\tilde{y} \\ &= E\tilde{x}^3 - E\tilde{x}E\tilde{x}^2 \\ &= \mu^3 + 3\mu\sigma^2 - \mu(\mu^2 + \sigma^2) = 2\mu\sigma^2 > 0. \end{aligned} \tag{17}$$

By definition,

$$ED(-\sqrt{\mu^2+\sigma^2}) = E\tilde{y} - E(\tilde{y}|\tilde{x} \leq -\sqrt{\mu^2+\sigma^2}) \quad (18)$$
$$= E\tilde{x}^2 - E(\tilde{x}^2|\tilde{x} \leq -\sqrt{\mu^2+\sigma^2})$$
$$= \mu^2+\sigma^2 - E(\tilde{x}^2|\tilde{x} \leq -\sqrt{\mu^2+\sigma^2}) < 0,$$

and we obtain $(\tilde{y}, \tilde{x}) \notin \mathcal{F}_1$.

Our results indicate that although positive covariance between $\tilde{x}$ and $\tilde{y}$ cannot guarantee first-order risk aversion, expectation dependence can. Therefore, we may rephrase the question about the participation puzzle and background risk as whether the expectation dependence of the stock market with the background risk of nonstockholders is high enough to explain the low participation in the stock market. Our contribution suggests that new empirical tests based on expectation dependence between stock market risk and background risk exposure should be developed.

6.4 Naive diversified portfolio model

The naive portfolio diversification rule is defined as one in which a fraction $\frac{1}{n}$ of wealth is allocated to each of the n assets available for investment at each rebalancing date. This rule is easy to implement because it relies neither on estimation nor on optimization. Many investors continue to use this simple rule to allocate their wealth across assets (see Benartzi and Thaler 2001; Huberman and Jiang 2006). DeMiguel et al. (2009) find that there is no single model that consistently delivers a Sharpe ratio or a certainty-equivalent return that is higher than that of the $\frac{1}{n}$ portfolio rule.

Suppose that $\tilde{\varepsilon}_i$ is the return of stock i, $\tilde{\varepsilon}$ is the return of a portfolio consisting of $\frac{1}{n}$ shares of each stock, and $\pi_{cd}(\frac{1}{n})$ and $\pi_c(\frac{1}{n})$ are minimum risk premiums the investor will demand for this portfolio. Proposition 6.1 shows that in the presence of a dependent background risk, the investor can not always take advantage of the benefit of diversification, and the portfolio risk will be eliminated only at the rate of $\frac{1}{n}$. If $\tilde{\varepsilon}$ and $\tilde{y}$ are positive (negative) expectation dependent and the investor is correlation-averse, then the return of the naive diversified portfolio will be higher (lower) than that corresponding to the portfolio's expected return.

6.5 Insurance supply

It is well known that the Law of Large Numbers is the actuarial basis of insurance pricing. By pooling the risks of many policyholders, the insurer can take advantage of this Law. While Li (2011) and Soon et al. (2011) investigate how dependent background risk affects demand for insurance, Proposition 6.1 shows how dependent background risk affects insurance supply. If $\frac{1}{n}\sum_{i=1}^{n}\tilde{\varepsilon}_i$ and $\tilde{y}$ are positive (negative) expectation dependent and the insurer is correlation-averse, then the insurance premium will be higher (lower) than the actuarially fair premium. Suppose that $\tilde{\varepsilon}_i$ is the loss for the insured i, and $\pi_{cd}(\frac{1}{n})$ and $\pi_c(\frac{1}{n})$ are the risk premiums of the insurance company for the individual loss $\tilde{\varepsilon}_i$. Proposition 6.1 implies that in the presence of a dependent background risk, the insurer cannot always take advantage of the benefit of diversification because the insurance risk will be eliminated only at the rate of $\frac{1}{n}$.

6.6 Other examples

We now discuss our result in relation with two other examples. Suppose that $\tilde{\varepsilon}_i$ is the default risk of borrower i, and $\pi_{cd}(\frac{1}{n})$ and $\pi_c(\frac{1}{n})$ are the yield spreads charged by the banker. Proposition 6.1 shows that if $\tilde{\varepsilon}$ and $\tilde{y}$ are positive (negative) expectation dependent and the banker is correlation-averse, then the yield spreads will be higher (lower) than that corresponding to the expected loss of default risk.

It is believed that the lottery business is rather safe, because the Law of Large Numbers entails that the average of the results from a large number of independent bets is quasi-constant (with a very small variance). Suppose that $\tilde{\varepsilon}_i$ is the payment to a winner i, $\pi_{cd}(\frac{1}{n})$ and $\pi_c(\frac{1}{n})$ are the average risk premiums for a lottery ticket. Proposition 6.1 shows that if $\tilde{\varepsilon}$ and $\tilde{y}$ are positive (negative) expectation dependent and the lottery business is correlation-averse, then the price for a lottery ticket must be higher (lower) than the expected payment of the lottery game.

7 Conclusion

In this study, we have extended the concept of first-order conditional risk aversion to first-order conditional dependent risk aversion. We have shown that first-order conditional dependent risk aversion can appear in the framework of the expected utility function hypothesis. Our contribution provides insight into the difficulty of obtaining risk diversification in the presence of a dependent background risk.

Recent studies show that background risk is more significant to explain portfolio choices. The decision of a household to participate in the stock market is a function of many random factors such as labor income, housing risk, private business income, and health. The availability of health insurance is observed to increase the financial behavior of households (Goldman and Maestas, 2007). More surprisingly, background risk becomes more significant than previously documented to affect stock risk premiums and improve the performance of asset pricing models (Palia et al., 2009): "When all background risk variables shift one standard deviation from their sample means, a household will decrease its likelihood to participate in the stock market by twelve percent and reduce the proportion of stock holdings by four percent." It seems that low market participation is significantly related to the heterogeneity in household background risk exposures.

These new results increase the previously documented magnitude of background risk effect on stock market participation. Palia et al. (2009) estimate the standard deviations of the growth rates of labor income, home equity, and private business income, as well as their correlations, to measure the background risk effect. Using more general measures of risk dependence, such as first-order expectation dependence, and even second-order expectation dependence, should increase the effect of background risk on stock market participation for expected utility households by allowing consideration of the first-order risk aversion effect.

8 Appendix: Proof of Lemma 4.4

From the definition of $\pi_{cd}(k)$, we know that

$$Eu(w + Ek\tilde{\varepsilon} - \pi_{cd}(k), \tilde{y}) = Eu(w + k\tilde{\varepsilon}, \tilde{y}). \tag{19}$$

Differentiating with respect to k yields

$$\pi'_{cd}(k) = \frac{E\tilde{\varepsilon}Eu(w + Ek\tilde{\varepsilon} - \pi_{cd}(k), \tilde{y}) - E[\tilde{\varepsilon}u_1(w + k\tilde{\varepsilon}, \tilde{y})]}{Eu_1(w - \pi_{cd}(k), \tilde{y})}. \tag{20}$$

Since $\pi_{cd}(0) = 0$, we have

$$\pi'_{cd}(0) = \frac{E\tilde{\varepsilon}Eu_1(w, \tilde{y}) - E[\tilde{\varepsilon}u_1(w, \tilde{y})]}{Eu_1(w, \tilde{y})}. \tag{21}$$

Note that

$$E[\tilde{\varepsilon}u_1(w, \tilde{y})] = E\tilde{\varepsilon}Eu_1(w, \tilde{y}) + Cov(\tilde{\varepsilon}, u_1(w, \tilde{y})) \tag{22}$$

and the covariance can always be written as (see Cuadras (2002), Theorem 1)

$$Cov(\tilde{\varepsilon}, u_1(w,\tilde{y})) \quad = \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[F(\varepsilon,y) - F_\varepsilon(\varepsilon)F_y(y)]d\varepsilon du_1(w,y). \tag{23}$$

Since we can always write (see e.g. Tesfatsion (1976), Lemma 1)

$$\int_{-\infty}^{\infty}[F_\varepsilon(\varepsilon|\tilde{y} \le y) - F_\varepsilon(\varepsilon)]d\varepsilon = E\tilde{\varepsilon} - E(\tilde{\varepsilon}|\tilde{y} \le y), \tag{24}$$

hence, by straightforward manipulations, we find

$$\begin{aligned} Cov(\tilde{\varepsilon}, u_1(w,\tilde{y})) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[F(\varepsilon,y) - F_\varepsilon(\varepsilon)F_y(y)]u_{12}(w_0,y)d\varepsilon dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[F_\varepsilon(\varepsilon|\tilde{y} \le y) - F_\varepsilon(\varepsilon)]d\varepsilon F_y(y)u_{12}(w,y)dy \\ &= \int_{-\infty}^{\infty}[E\tilde{\varepsilon} - E(\tilde{\varepsilon}|\tilde{y} \le y)]F_y(y)u_{12}(w,y)dy \qquad (by\ (24)) \\ &= \int_{-\infty}^{\infty} ED(y)u_{12}(w,y)F_y(y)dy. \end{aligned} \tag{25}$$

Finally, we get

$$\pi'_{cd}(0) = -\frac{\int_{-\infty}^{\infty} ED(y)u_{12}(w,y)F_y(y)dy}{Eu_1(w,\tilde{y})}. \tag{26}$$

Using a Taylor expansion of π around $k = 0$, we obtain that

$$\pi_{cd}(k) = \pi_{cd}(0) + \pi'_{cd}(0)k + O(k^2) = -k\frac{\int_{-\infty}^{\infty} ED(y)u_{12}(w,y)F_y(y)dy}{Eu_1(w,\tilde{y})} + O(k^2). \tag{27}$$

Q.E.D.

9 Appendix: Proof of Lemma 5.2

From (22) and (24), we know that

$$E[\tilde{\varepsilon}u_1(w,\tilde{y})] = E\tilde{\varepsilon}Eu_1(w,\tilde{y}) + Cov(\tilde{\varepsilon}, u_1(w,\tilde{y})) = E\tilde{\varepsilon}Eu_1(w,\tilde{y}) + \int_{-\infty}^{\infty} ED(y)u_{12}(w,y)F_y(y)dy. \tag{28}$$

We now integrate the last term of (28) by parts repeatedly until we obtain:

$$\begin{aligned} Cov(\tilde{\varepsilon}, u_1(w,\tilde{y})) &= \sum_{m=2}^{N}(-1)^m u_{12^{(m-1)}}(w,d)m^{th}ED(\tilde{x}|d) \\ &\quad + \int_c^d(-1)^{N+1}u_{12^{(N)}}(w,y)N^{th}ED(\tilde{x}|y)dy,\ for N \ge 2. \end{aligned} \tag{29}$$

From (21), we have

$$\begin{aligned} &\pi'_{cd}(0) \\ = \quad &\frac{E\tilde{\varepsilon}Eu_1(w,\tilde{y}) - E[\tilde{\varepsilon}u_1(w,\tilde{y})]}{Eu_1(w,\tilde{y})} \\ &-k\frac{\sum_{m=2}^{N}(-1)^m u_{12^{(m-1)}}(w,d)m^{th}ED(\tilde{x}|d) + \int_c^d(-1)^{N+1}u_{12^{(N)}}(w,y)N^{th}ED(\tilde{x}|y)dy}{Eu_1(w,\tilde{y})}. \end{aligned} \tag{30}$$

Using a Taylor expansion of π around $k=0$, we obtain that

$$\begin{aligned}
& \pi_{cd}(k) && (31)\\
= \; & \pi_{cd}(0) + \pi'_{cd}(0)k + O(k^2) \\
= \; & -k\frac{\sum_{m=2}^{N}(-1)^m u_{12^{(m-1)}}(w,d)m^{th}ED(\tilde{x}|d) + \int_c^d (-1)^{N+1}u_{12^{(N)}}(w,y)N^{th}ED(\tilde{x}|y)dy}{Eu_1(w,\tilde{y})} \\
& +O(k^2).
\end{aligned}$$

Q.E.D.

10 References

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