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Precautionary Saving with Selden/Kreps-Porteus Preferences Revisited

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Abstract

This paper re-examines precautionary saving with general Selden/Kreps-Porteus preferences. The conditions existing in the literature are much more complex than in the Expected Utility framework. We obtain a simple and intuitive result on precautionary savings via disentangling time preference and risk preference effects.

Key words: Precautionary saving; Prudence; Selden/Kreps-Porteus Preferences

JEL classification: D81, D91, E21

1 Introduction

It is well-known that current savings may be raised by the uncertainty of future incomes. This motive for saving is called “precautionary saving”. The precautionary saving problem is first studied by Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972). Under the framework of Expected-Utility (EU), Kimball (1990) introduces the terminology of prudence: an agent is prudent if the uncertainty affects future incomes raises a motive for current saving. He also shows that an agent is prudent if and only if she has a convex marginal utility of future consumption. In the framework of EU, there is now a wide and lively literature on this line of study.

Kimball and Weil (1992, 2009) point out that the traditional EU theory of precautionary saving dose not disentangle aversion to risk and resistance to intertemporal substitution, because it is built on intertemporal EU maximization. Therefore, it cannot address some fundamental problems of consumption under labor income risk. For instance, given the same intertemporal substitution, how do risk aversion changes affect the intensity of the precautionary saving motive? They use a two-period model with Selden/Kreps-Porteus preferences to investigate whether the results from an intertemporal EU framework continue to hold when we disentangle aversion to risk and aversion to intertemporal substitution. Their answer is positive but under a restriction: a decreasing marginal utility of saving (DMUS). Gollier (2001, p300-303) provides a similar study on the motive for precautionary saving. K-W-G[1] provide two sufficient conditions for DMUS: (i) the resistance to intertemporal substitution is greater than risk aversion, or (ii) the certainty equivalent function of future consumption is concave. They show that a risk preference with concave absolute risk tolerance guarantees condition (ii). These two conditions limit the conditions leading to precautionary saving. Because there are many preferences that violate condition (i), and “there is no obvious argument favoring the concavity or the convexity of absolute risk tolerance (Gollier 2001, p165-166),” therefore it is better to dispense the constraint on DMUS.

K-W-G model focuses on additive Selden/Kreps-Porteus preferences. Such preferences assume that the subject discount factor (or marginal impatience) is independent of current wealth. However this assumption is inconsistent with our intuition. Therefore, it is better to study precautionary saving in a general Selden/Kreps-Porteus preferences framework.

[1]Kimball and Weil (1992, 2009) and Gollier (2001) is abbreviated as “K-W-G”.

We notice that K-W-G's studies are based on Implicit Function Theorem. So they have to impose the assumption of DMUS to assure concavity of the value function. In this paper, we re-investigate K-W-G's precautionary saving model without the assumption of DMUS. We show that supermodularity of value function provides a more general analysis for precautionary saving.

We first disentangle time preference and risk preference effects on the optimal level of saving with general Selden/Kreps-Porteus preferences and show that the assumption of DMUS in Kimball and Weil (1992, 2009)'s study is redundant. The result extends Gollier (2001, Proposition 78 part 2)'s conclusion. Then we study, given the same time aggregator, how risk aversion changes affect the intensity of the precautionary saving motive. It can be shown that some results of Kimball and Weil (2009)'s comparative statics are independent of DMUS. Therefore, our results increase our knowledge of precautionary saving.

2 Prudence

Following the idea of K-W-G, we consider the precautionary saving with general Selden/Kreps-Porteus preferences, which can be described as

$$\max_s V(s,\tilde{x}) = W(w_0 - s, M(s,\tilde{x})), \tag{1}$$

where w_0 is the initial wealth in the first period, s is the saving and W is the time aggregator. $M(s,\tilde{x})$ is implicitly defined by $v(M(s,\tilde{x})) = Ev(w_1+\tilde{x}+\rho s)$ where $\tilde{x}$ is a risk such that $E\tilde{x}=0$, w_1 is the wealth the agent receives in the second period and ρ is the risk-free interest rate. We assume v is an increasing and concave function. The concavity of v measures the degree of risk aversion, while the properties of W are related to intertemporal substitution. The axiomatic foundations of model (1) were examined by Kreps and Porteus (1978) and Selden (1978).

When $W(x,y) = u(x) + U(y)$,

$$W(w_0 - s, M(s,\tilde{x})) = u(w_0 - s) + U(M(s,\tilde{x})). \tag{2}$$

We are back to K-W-G model.

Let $\max_s V(s,0) = W(w_0 - s, M(s,0))$ denote the optimal saving problem without the risk $\tilde{x}$ (i.e., $\tilde{x} \equiv 0$) and $\max_s V(s,\tilde{x}) = W(w_0 - s, M(s,\tilde{x}))$ denote the optimal problem with the risk $\tilde{x}$. Define $s_1 \in \arg\max V(s,0)$ and $s_2 \in \arg\max V(s,\tilde{x})$.

Following K-W-G, we link prudence with precautionary saving in the following definition.

Definition 2.1 *Suppose that the agents have general Selden/Kreps-Porteus preferences described by (1). An agent is prudent if* $s_1 \leq s_2$*.*

The following proposition provides a set of sufficient conditions for precautionary saving.

Proposition 2.2 *If* $W_{12} \geq 0$*,* $W_{22} \leq 0$ *and* v *is strictly Decreasing Absolute Risk Aversion (DARA), then the agent is prudent, that is,* $s_1 \leq s_2$*.*

Proof See Appendix. Q.E.D.

Proposition 2.2 disentangles the set of sufficient conditions for precautionary saving into the following two classes:

(i) time preference conditions: $W_{12} \geq 0$ and $W_{22} \leq 0$.

(ii) risk preference condition: strictly DARA.

W_2 is defined as the subject discount factor (see, Koopmans, 1960). Hence $W_{12} \geq 0$ means decreasing marginal impatience (DMI). The validity of DMI is supported by Lawrance (1991), Samwick (1998), Harrison et al. (2002) and Ikeda (2006). $W_{22} \leq 0$ means the DM has preferences for a late resolution of uncertainty (PLRU) (Kreps and Porteus 1978). A DM is PLRU if she prefers to observe $\tilde{x}$ at the second period than at the first period. The PLRU is supported by many empirical studies (e.g., Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011).

Proposition 2.2 concludes that DMI, PLRU and strictly DARA are the traits for prudence in the framework of general Selden/Kreps-Porteus preferences. This disentangling of risk and time preferences provides an important intuition about the property of precautionary saving.

When $W(x, y) = u(x) + U(y)$, $W_{12} = 0$ and $W_{22} = U''$. Proposition 2.2 implies the following result:

Proposition 2.3 *If* $U'' \leq 0$ *and* v *is strictly DARA, then the agent is prudent, that is,* $s_1 \leq s_2$*.*

For $W(x, y) = u(x) + U(y)$, Gollier (2001, Proposition 78) proposes the following result: assume that M is concave in s, then $s_1 \leq s_2$ if (i) $v''' \geq 0$ and U is more concave than v; or (ii) v is DARA. Kimball and Weil (1992, 2009) use the assumption of decreasing marginal utility of saving (e.g., $U'(M(s, \tilde{x}))M_s(s, \tilde{x})$ is decreasing in s) to guarantee that $s_1 \leq s_2$. They provide two sufficient conditions for the assumption: (i) U is more concave than v, or (ii) M is concave. They show that a risk preference with concave absolute risk tolerance guarantees concavity of

M. They also point out that, only for v with concave absolute risk tolerance, M is always concave. Proposition 2.3 indicates that this constraint on decreasing marginal utility of saving is not necessary. Therefore, our results enlarge the precautionary saving behavior.

In the proof of Proposition 2.2, we use the supermodularity property of the value function, while K-W-G use the Implicit Function Theorem to obtain their results. The assumption of concavity of the value function is a key to Implicit Function Theorem. For the value function defined by (2), decreasing marginal utility of saving is equivalent to the concavity of the value function. However, there are no concavity assumptions needed for supermodularity property (see e.g., Sundaram 1996, p253, or Amir 2005, p639-640). This is the reason why Proposition 2.2 provides a more useful result.

We also notice that, in K-W-G, the utility function u of the first period must be concave, so as to assure that the the solution of optimal saving is uniquely determined[2]. But our work doesn't need this strict assumption, that is, u can be non-concave. Especially, Crainich, Eeckhoudt and Trannoy (2013) show that one risk seeking agent can own positive precautionary saving. Thus, we cannot ignore the saving behavior of risk seeking agents. From this point, our approach has a wider application.

3 Changes in risk aversion

In this section, given the same time aggregator, we link the aversion to risk to the intensity of the precautionary saving motive. Consider

$$\max_s V_i(s,\tilde{x}) = W(w_0 - s, M_i(s,\tilde{x})), \qquad for\ i = 1,2, \tag{3}$$

where $M_i(s,\tilde{x})$ is defined by $v_i(M_i(s,\tilde{x})) = Ev_i(w_1 + \tilde{x} + \rho s)$.

Define $s_i \in \arg\max V_i(s,\tilde{x}) = W(w_0 - s, M_i(s,\tilde{x}))$ for $i = 1,2$. We propose the following two propositions.

Proposition 3.1 *Suppose two Selden/Kreps-Porteus utility functions share the same time aggregator W but have different inner interpossibility functions v_1 and v_2, if*

[2]Noting equation (8) in Kimball and weil (2009), if the first-period utility function (i.e. u) is not global concave, then decreasing marginal utility of saving still can't assure that the the solution of (8) is uniquely determined. But the work of Kimball and Weil (2009) is conducted under the assumption of the unique solution, thus, their work must assume both $u'' < 0$ and decreasing marginal utility of saving.

(i) $W_{12} \geq 0$,

(ii) $-\frac{W_{22}}{W_2} \geq -\frac{v_1''}{v_1'}$,

(iii) v_2 *is more risk averse than* v_1,

(iv) v_2' *is more risk averse than* v_1',

(v) v_2 *has decreasing absolute risk aversion, and*

(vi) at least one condition in (ii) to (v) holds strictly,

then $s_1 \leq s_2$.

Proof See Appendix. Q.E.D.

Proposition 3.2 *Suppose two Selden/Kreps-Porteus utility functions share the same time aggregator* W *but have different inner interpossibility functions* v_1 *and* v_2*, if*

(i) $W_{12} \geq 0$,

(ii) the resistance to intertemporal substitution is greater than 1 $(-\frac{W_{22}}{W_2} \geq 1)$,

(iii) v_2 *is more risk averse than* v_1,

(iv) v_1 *has constant or increasing relative risk aversion,*

(v) v_2 *has constant or decreasing relative risk aversion, and*

(vi) at least one condition in (ii) to (v) holds strictly,

then $s_1 \leq s_2$.

Proof See Appendix. Q.E.D.

When $W(x, y) = u(x) + U(y)$, $W_{12} = 0$ and $-\frac{W_{22}}{W_2} = -\frac{U''}{U'}$. From Propositions 3.1 and 3.2, we obtain

Proposition 3.3 *Suppose two Selden/Kreps-Porteus utility functions share the same time aggregator* $W(x, y) = u(x) + U(y)$ *but have different inner interpossibility functions* v_1 *and* v_2*, if*

(i) $-\frac{U''}{U'} \geq -\frac{v_1''}{v_1'}$,

(ii) v_2 *is more risk averse than* v_1,

(iii) v_2' *is more risk averse than* v_1',

(iv) v_2 *has decreasing absolute risk aversion, and*

(v) at least one condition in (i) to (iv) holds strictly,

then $s_1 \leq s_2$.

Proposition 3.4 *Suppose two Selden/Kreps-Porteus utility functions share the same time aggregator $W(x,y) = u(x) + U(y)$ but have different inner interpossibility functions v_1 and v_2, if*

(i) the resistance to intertemporal substitution is greater than 1 ($-\frac{U''}{U'} \geq 1$),

(ii) v_2 is more risk averse than v_1,

(iii) v_1 has constant or increasing relative risk aversion,

(iv) v_2 has constant or decreasing relative risk aversion, and

(v) at least one condition in (i) to (iv) holds strictly,

then $s_1 \leq s_2$.

For $W(x,y) = u(x) + U(y)$, Kimball and Weil (2009, Propositions 5 and 6) examine how aversion to risk affects the strength of the precautionary saving motive under the same aversion to intertemporal substitution. Their study relies on the assumptions of DMUS and concavity of u. Propositions 3.3 and 3.4 show that these two assumptions are redundant. Therefore Kimball and Weil (2009, Propositions 5 and 6)'s conclusions can take care much more cases than we thought.

4 Conclusion

This paper revisits the work of Gollier, Kimball and Weil on precautionary savings with general Selden/Kreps-Porteus preferences. The main weakness of the existing results was that they required a decreasing marginal utility of saving (DMUS)-an assumption for which various unappealing sufficient conditions were derived, among which the unintuitive concavity of absolute risk tolerance. We do away with DMUS and instead assume that the value function is supermodular. Under that assumption, we show that the main comparative results of Kimball and Weil (2009) on the effect of prudence and risk aversion on precautionary saving go through, with supermodularity, without requiring DMUS. The study extends K-W-G's studies on precautionary motives and improves our understanding of the determinants of precautionary savings. The study also provides a new application for supermodularity in economics.

5 Appendix

5.1 Proof of Proposition 2.2

First we prove a lemma.

Suppose $s_1 > s_2$. From

$$
\begin{aligned}
& V(s_1,\tilde{x}) + V(s_2,0) > V(s_1,0) + V(s_2,\tilde{x}) && (4)\\
\Leftrightarrow\ & W(w_0 - s_1, M(s_1,\tilde{x})) + W(w_0 - s_2, M(s_2,0)) \\
& > W(w_0 - s_1, M(s_1,0)) + W(w_0 - s_2, M(s_2,\tilde{x})) \\
\Leftrightarrow\ & W(w_0 - s_1, M(s_1,\tilde{x})) - W(w_0 - s_1, M(s_1,0)) \\
& > W(w_0 - s_2, M(s_2,\tilde{x})) - W(w_0 - s_2, M(s_2,0)) \\
\Leftrightarrow\ & W(w_0 - s, M(s,\tilde{x})) - W(w_0 - s, M(s,0)) \textit{ is increasing in } s \\
\Leftrightarrow\ & W_1(w_0 - s, M(s,0)) - W_1(w_0 - s, M(s,\tilde{x})) \\
& +W_2(w_0 - s, M(s,\tilde{x}))M_s(s,\tilde{x}) - \rho W_2(w_0 - s, M(s,0)) > 0 \quad (By\ M(s,0) = w_1 + \rho s),
\end{aligned}
$$

we know that

$$W_1(w_0 - s, M(s,0)) \geq W_1(w_0 - s, M(s,\tilde{x})) \tag{5}$$

and

$$W_2(w_0 - s, M(s,\tilde{x}))M_s(s,\tilde{x}) > \rho W_2(w_0 - s, M(s,0)) \tag{6}$$

imply (4).

From

$$v(M(s,\tilde{x})) = Ev(w_1 + \tilde{x} + \rho s) \leq v(w_1 + E\tilde{x} + \rho s) = v(M(s,0)), \tag{7}$$

we obtain

$$M(s,\tilde{x}) \leq M(s,0). \tag{8}$$

Therefore $W_{12} \geq 0$ implies (5).

From the assumption "v is strictly DARA", we obtain[3]

$$Ev(w_1 + \tilde{x} + \rho s) = v(M(s,\tilde{x})) \Rightarrow Ev'(w_1 + \tilde{x} + \rho s) > v'(M(s,\tilde{x})). \tag{9}$$

From $v(M(s,\tilde{x})) = Ev(w_1 + \tilde{x} + \rho s)$, we have

$$M_s(s,\tilde{x}) = \rho \frac{Ev'(w_1 + \tilde{x} + \rho s)}{v'(M(s,\tilde{x}))}. \tag{10}$$

[3] See the inequality (20.14) in Gollier (2001).

Therefore, both (9) and (10) imply

$$M_s(s,\tilde{x}) > \rho. \tag{11}$$

Finally, (8), (11) and $W_{22} \leq 0$ imply (6).

We summarize the above result in the following lemma[4]:

Lemma 5.1 *When $s_1 > s_2$, $W_{12} \geq 0$, $W_{22} \leq 0$ and v is strictly DARA,*

$$V(s_1,\tilde{x}) + V(s_2,0) > V(s_1,0) + V(s_2,\tilde{x}). \tag{13}$$

Then we prove the main proposition via a contradiction:

Suppose $s_1 > s_2$. Since

$$s_1 \in \arg\max V(s,0) \Rightarrow V(s_1,0) - V(s_2,0) \geq 0, \tag{14}$$

then $W_{12} \geq 0$, $W_{22} \leq 0$ and v is strictly DARA imply

$$\begin{aligned} V(s_1,\tilde{x}) - V(s_2,\tilde{x}) &> V(s_1,0) - V(s_2,0) \; (by \; Lemma \; 5.1) \\ &\geq 0 \quad (by (14)), \end{aligned} \tag{15}$$

and hence $V(s_1,\tilde{x}) > V(s_2,\tilde{x})$. This is a contradiction to $s_2 \in \arg max V(s,\tilde{x})$, that is, $s_1 \leq s_2$.

5.2 Proof of Proposition 3.1

First we prove a lemma.

Suppose $s_1 > s_2$. From

$$\begin{aligned} & V_2(s_1,\tilde{x}) + V_1(s_2,\tilde{x}) > V_1(s_1,\tilde{x}) + V_2(s_2,\tilde{x}) \qquad (16) \\ \Leftrightarrow\; & W(w_0 - s_1, M_2(s_1,\tilde{x})) + W(w_0 - s_2, M_1(s_2,\tilde{x})) > W(w_0 - s_2, M_2(s_2,\tilde{x})) + W(w_0 - s_1, M_1(s_1,\tilde{x})) \\ \Leftrightarrow\; & W(w_0 - s_1, M_2(s_1,\tilde{x})) - W(w_0 - s_1, M_1(s_1,\tilde{x})) > W(w_0 - s_2, M_2(s_2,\tilde{x})) - W(w_0 - s_2, M_1(s_2,\tilde{x})) \\ \Leftrightarrow\; & W(w_0 - s, M_2(s,\tilde{x})) - W(w_0 - s, M_1(s,\tilde{x})) \; is \; increasing \; in \; s, \end{aligned}$$

[4](13) can be written as

$$V(a_1,t_2) + V(a_2,t_1) > V(a_1,t_1) + V(a_2,t_2), \tag{12}$$

where $a_2 \geq a_1$ and t_1 dominates t_2 in the sense of an increase in risk (Rothschild and Stiglitz, 1970). Therefore, V is strictly decreasing differences and hence is strictly submodular (see e.g., Topkis 1998, p42-43). We can also obtain Lemma 5.1 from a result in Topkis's Monotonicity Theorem (see e.g., Sundaram 1996, Theorem 10.6; or Topkis 1998, Theorem 2.8.5; or Vives 2000, Theorem 2.3). Wang and Li (2014) provide an intuitive illustration for this lemma via Eeckhoudt and Schlesingers (2006) harms disaggregation idea.

we know

$$W_1(w_0 - s, M_1(s, \tilde{x})) \geq W_1(w_0 - s, M_2(s, \tilde{x})) \tag{17}$$

and

$$W_2(w_0 - s, M_2(s, \tilde{x}))M_{2s}(s, \tilde{x}) > W_2(w_0 - s, M_1(s, \tilde{x}))M_{1s}(s, \tilde{x}) \tag{18}$$

imply (16)

Since "v_2 is more risk averse than v_1" implies $M_1(s, \tilde{x}) \geq M_2(s, \tilde{x})$, "$v_2$ is more risk averse than v_1" together with $W_{12} \geq 0$ imply (17).

From a proof of Kimball and Weil (2009, p273-275), when we define $U(.) = W(w_0 - s, .)$, we know that, if (i) $-\frac{W_{22}}{W_2} \geq -\frac{v_1''}{v_1'}$, (ii) v_2 is more risk averse than v_1, (iii) v_2' is more risk averse than v_1', and (iv) v_2 has decreasing absolute risk aversion, then

$$W_2(w_0 - s, M_2(s, \tilde{x}))M_{2s}(s, \tilde{x}) \geq W_2(w_0 - s, M_1(s, \tilde{x}))M_{1s}(s, \tilde{x}). \tag{19}$$

If at least one condition in (i) to (iv) holds strictly, then

$$W_2(w_0 - s, M_2(s, \tilde{x}))M_{2s}(s, \tilde{x}) > W_2(w_0 - s, M_1(s, \tilde{x}))M_{1s}(s, \tilde{x}). \tag{20}$$

Therefore, we can propose the following lemma.

Lemma 5.2 *When $s_1 > s_2$ if*

(i) $W_{12} \geq 0$,

(ii) $-\frac{W_{22}}{W_2} \geq -\frac{v_1''}{v_1'}$,

(iii) v_2 is more risk averse than v_1,

(iv) v_2' is more risk averse than v_1',

(v) v_2 has decreasing absolute risk aversion, and

(vi) at least one condition in (ii) to (v) holds strictly,

then

$$V_2(s_1, \tilde{x}) + V_1(s_2, \tilde{x}) > V_1(s_1, \tilde{x}) + V_2(s_2, \tilde{x}). \tag{21}$$

Then we prove the main proposition via a contradiction:

Suppose $s_1 > s_2$. Since

$$s_1 \in \arg\max V_1(s, \tilde{x}) \Rightarrow V_1(s_1, \tilde{x}) - V_1(s_2, \tilde{x}) \geq 0, \tag{22}$$

then conditions (i), (ii), (iii), (iv), (v) and (vi) in Lemma 5.2 imply

$$\begin{aligned} V_2(s_1, \tilde{x}) - V_2(s_2, \tilde{x}) &> V_1(s_1, \tilde{x}) - V_1(s_2, \tilde{x}) \ (\textit{by Lemma } 5.2) \\ &\geq 0 \quad (\textit{by}(22)), \end{aligned} \tag{23}$$

and hence $V_2(s_1,\tilde{x}) > V_2(s_2,\tilde{x})$. This is a contradiction to $s_2 \in \arg max V_2(s,\tilde{x})$, that is, $s_1 \leq s_2$.

5.3 Proof of Proposition 3.2

From the proof of Proposition 3.1, we know that, when $s_1 > s_2$,

$$W_1(w_0 - s, M_1(s,\tilde{x})) \geq W_1(w_0 - s, M_2(s,\tilde{x})) \tag{24}$$

and

$$W_2(w_0 - s, M_2(s,\tilde{x}))M_{2s}(s,\tilde{x}) > W_2(w_0 - s, M_1(s,\tilde{x}))M_{1s}(s,\tilde{x}) \tag{25}$$

imply

$$V_2(s_1,\tilde{x}) + V_1(s_2,\tilde{x}) > V_1(s_1,\tilde{x}) + V_2(s_2,\tilde{x}). \tag{26}$$

Again, "v_2 is more risk averse than v_1" together with $W_{12} \geq 0$ imply (24).

From a proof of Kimball and Weil (2009, p275-276), we know that, if (i) the resistance to intertemporal substitution is greater than 1 ($-\frac{W_{22}}{W_2} \geq 1$), (ii) v_2 is more risk averse than v_1, (iii) v_1 has constant or increasing relative risk aversion, and (iv) v_2 has constant or decreasing relative risk aversion, then

$$W_2(w_0 - s, M_2(s,\tilde{x}))M_{2s}(s,\tilde{x}) \geq W_2(w_0 - s, M_1(s,\tilde{x}))M_{1s}(s,\tilde{x}). \tag{27}$$

If at least one condition in (i) to (iv) holds strictly, then

$$W_2(w_0 - s, M_2(s,\tilde{x}))M_{2s}(s,\tilde{x}) > W_2(w_0 - s, M_1(s,\tilde{x}))M_{1s}(s,\tilde{x}). \tag{28}$$

Therefore, we can propose the following lemma.

Lemma 5.3 *When $s_1 > s_2$, if*

(i) $W_{12} \geq 0$,

(ii) the resistance to intertemporal substitution is greater than 1 ,

(iii) v_2 is more risk averse than v_1,

(iv) v_1 has constant or increasing relative risk aversion,

(v) v_2 has constant or decreasing relative risk aversion, and

(vi) at least one condition in (ii) to (v) holds strictly,

then

$$V_2(s_1,\tilde{x}) + V_1(s_2,\tilde{x}) > V_1(s_1,\tilde{x}) + V_2(s_2,\tilde{x}). \tag{29}$$

Then we prove the main proposition via a contradiction:

Suppose $s_1 > s_2$. Since

$$s_1 \in \arg\max V_1(s, \tilde{x}) \Rightarrow V_1(s_1, \tilde{x}) - V_1(s_2, \tilde{x}) \geq 0, \tag{30}$$

then conditions (i), (ii), (iii), (iv), (v) and (vi) in Lemma 5.3 imply

$$\begin{aligned} V_2(s_1, \tilde{x}) - V_2(s_2, \tilde{x}) &> V_1(s_1, \tilde{x}) - V_1(s_2, \tilde{x}) \ (by\ Lemma\ 5.3) \\ &\geq 0 \ \ (by (30)), \end{aligned} \tag{31}$$

and hence $V_2(s_1, \tilde{x}) > V_2(s_2, \tilde{x})$. This is a contradiction to $s_2 \in \arg max V_2(s, \tilde{x})$, that is, $s_1 \leq s_2$.

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