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Transfer Pricing in a Multidivisional Firm: A Cooperative Game Analysis[1]

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Abstract

We consider the transfer pricing decision for a multidivisional firm with an upstream division and multiple downstream divisions. The downstream divisions can independently determine their retail prices, and decide on whether or not they will purchase from the upstream division at negotiated transfer prices. To allocate the firm-wide profit between upstream and downstream divisions, we construct a cooperative game, show the convexity of the game, and then compute the Shapley value-based transfer prices for the firm.

Key words: Transfer price, cooperative game theory, the core, Shapley value.

1 Introduction

The transfer pricing problem is of significant importance to multidivisional firms which need to consider the allocation of firm-wide profit between an upstream division and multiple downstream divisions. In such transfer pricing problems, all divisions of a firm can independently make their decisions as if they were operating in a decentralized setting. This means that all downstream divisions can determine their own retail prices, and decide on whether or not they will buy from the upstream division at negotiated transfer prices. For details regarding the transfer pricing decision in multidivisional firms, see online Appendix A.

In this paper, we consider the transfer pricing decisions of a multidivisional firm where the upstream and downstream divisions negotiate the transfer price that results in a fair allocation of the maximum system-wide profit surplus among three or more divisions. The downstream divisions then determine their retail prices. In this paper, to reflect the fact that customers are sensitive to retail prices, we assume that the demand in the market a downstream division serves is dependent on the division's retail pricing decision. We learn from Göx and Schiller [7] that most transfer pricing publications assumed the demand to be independent of the retail price, i.e., a constant ; and we find that only a few recent publications (e.g., Arya and Mittendorf [1], Baldenius and Reichelstein [2]) used a linear, deterministic, price-dependent demand function for transfer pricing problems.

We assume that the upstream division's unit production cost is not a constant but a decreasing, convex function of the production quantity—i.e., the downstream division's order quantity, or the demand faced by the downstream division. Such a modeling approach renders our model and analysis more realistic, because it is consistent with the wide existence of "economies of scale," see, e.g., Lingnau [12]. The quantity-dependent cost function also distinguishes our paper from other transfer pricing papers.

In reality, the upstream division of a multidivisional firm usually sells its intermediate products to multiple downstream divisions that are located in different marketing areas. In Section 2, we analyze the transfer pricing decisions for such a system by using cooperative game theory. We believe that this is an appropriate methodology for our transfer-pricing analysis because all downstream divisions of a multidivisional firm are "free" to determine whether or not they will buy from the upstream division at negotiated transfer prices. Specifically, each division is able to decide on whether it will trade with the upstream division in a non-cooperative setting or in a cooperative setting. In the non-cooperative setting, the upstream and downstream members make their transfer prices and retail prices in Stackelberg equilibrium, respectively. In the cooperative setting, the downstream members choose the globally-optimal retail prices that maximize the system-wide profit, and negotiate transfer prices with the upstream member. In order to guarantee that the downstream members are willing to adopt the globally-optimal retail prices, we will use cooperative game theory to find the negotiated transfer prices assuring that the downstream members are better (by achieving more profits) in the cooperative setting than in the non-cooperative setting.

Therefore, for the multidivisional firm with a single upstream division and n downstream divisions with $n \geq 2$, we construct an $(n + 1)$-division cooperative game in characteristic function form, and prove that the characteristic value function is supermodular, that is, the game is convex and thus superadditive. We also show that our game has a non-empty core, and use the concept of Shapley value (Shapley [15]) to find a unique allocation scheme. Note that Shapley value is a proper concept for our analysis because it is in the core due to the convexity of our game. We then calculate n transfer prices for the n downstream divisions, using the Shapley value-based allocation scheme. Our proofs for all theorems and corollaries are delegated to online Appendices C and D, respectively.

An important contribution of our paper to the literature is the application of n-player cooperative game theory (with $n \geq 3$) to transfer pricing problems. To the best of our knowledge, very few transfer pricing-related publications (for example, Rosenthal [14]) applied this important methodology. Our model differs from [14] because of the following three facts: (i) As mentioned above, we consider the quantity-dependent production cost and the price-sensitive demand functions; Rosenthal [14] assumed a constant production cost and a deterministic demand. (ii) We investigate the two-echelon system involving a single upstream division and multiple downstream divisions, whereas Rosenthal [14] considered an n-echelon ($n \geq 3$) system in which there is a single division at each level. (iii) We compute the firm-wide profit surplus as the difference between the profit in the cooperative setting and that in the non-cooperative setting. In [14], each division's profit in the non-cooperative setting was assumed to be zero; thus, each player's profit surplus was equal to the player's profit in the cooperative setting.

2 Transfer Pricing Decisions

In this section, we investigate the transfer pricing problem for a two-echelon system (multidivisional firm) involving an upstream division U and $n \geq 2$ downstream divisions (i.e., D_j, $j = 1, 2, \ldots, n$). Such a system is very common in practice. For example, Ford Motor Company's Struandale engine plant in South Africa (an upstream division) supplies the Duratorq TDCi diesel engines to the firm's global assembly plants (multiple downstream divisions) which make the Ford Ranger pick-up trucks [5]. Those assembly plants are actually "free" to decide on whether they can buy from the plant in South Africa at negotiated transfer prices in the cooperative setting or at non-cooperative (Stackelberg equilibrium) transfer prices. Walmart's office "GP USA Export" at its Arkansas headquarters (an upstream division), as a branch of the Walmart Global Procurement, is mainly responsible to purchase and sell quality U.S. products to its global stores, which are allowed to negotiate transfer prices with headquarters in the cooperative setting or buy at the transfer prices in the non-cooperative (Stackelberg game) setting [20].

In the multidivisional firm under study, the upstream division U sells its intermediate products—which are identical to each other—to $n \geq 2$ downstream divisions. We assume that, to improve the firm-wide performance, the n divisions D_j ($j = 1, 2, \ldots, n$) are located in, and serve, n different markets to reduce the possibility of competition between two, or among three or more, downstream divisions. Each division is "free" to decide on whether it trades with other divisions in a non-cooperative setting or in a cooperative setting. In the non-cooperative setting, the upstream division U first announces its transfer price to the downstream divisions D_j ($j = 1, 2, \ldots, n$), who then respond by determining their retail prices. For this "sequential-move" scenario, divisions U and D_j act as the "leader" and the "follower," respectively; accordingly, we need to find Stackelberg equilibrium to characterize the two echelons' decisions. In the cooperative setting, the $n + 1$ divisions—including the upstream division U and $n \geq 2$ downstream divisions—jointly make their decisions to maximize the system-wide profit that is the sum of all divisions' profits.

In order to entice $n+1$ divisions to cooperate, the multidivisional firm should divide the profit surplus—which is the difference between the system-wide profit in the cooperative setting and that in the non-cooperative setting—under a *fair* allocation scheme that is *acceptable* to all divisions. To do so, we develop a cooperative game model in characteristic-function form, and use the solution concepts of core and Shapley value to determine a fair allocation scheme and calculate the transfer price between the upstream division U and each downstream division. Note that, since there are $n \geq 2$ downstream divisions, we need to determine n transfer prices.

Next, we develop the profit functions for divisions U and D_j ($j = 1, 2, \ldots, n$). Division D_j ($j = 1, 2, \ldots, n$) makes its retail pricing decision p_j and sells $q_j(p_j)$ units of its final products to serve market j. Similar to [1] and [2], division D_j's sales quantity is determined by the deterministic, linear demand function $q_j(p_j) = a_j - b_j p_j$ with $a_j, b_j > 0$ and $p_j \leq a_j/b_j$. Note that all downstream members face independent demands [i.e., $q_j(p_j)$ (for $j = 1, 2, \ldots, n$) are independent of each other], because they are located in different marketing areas, as assumed in Section 1. Thus, this division's profit $\pi_j(p_j)$ is calculated as,

$$\pi_j(p_j) = (p_j - T_j)q_j(p_j) = (p_j - T_j)(a_j - b_j p_j), \tag{1}$$

where T_j denotes the transfer price that division D_j pays to division U.

Since division U sells its intermediate products to serve all of n downstream divisions, division U's total sale quantity is $Q(\mathbf{q}) \equiv \sum_{j=1}^{n} q_j(p_j)$, where $\mathbf{q} \equiv (q_1(p_1), \ldots, q_n(p_n))$. As discussed in Section 1, this division's unit production cost is dependent on the production quantity, because of the existence of economies of scale, for details, see, e.g., Lingnau [12]. Hence, the unit production cost—which is incurred by division U when this division makes $Q(\mathbf{q})$ units of intermediate products—can be written as $c(Q(\mathbf{q}))$. We assume that $c(\cdot)$ is a decreasing, convex function of the production quantity; that is, $c'(\cdot) \leq 0$ and $c''(\cdot) \geq 0$. Moreover, similar to Lingnau [12], we assume that $1 + b_j c'(\cdot) \geq 0$, for $j = 1, \ldots, n$. This means that the system-wide unit profit—i.e., the sum of the upstream division's and n downstream divisions' unit profits—is increasing in each downstream division's retail price. For details, see online Appendix B. For other publications involving a linear, decreasing cost function of the quantity, see, for example, Gray *et al.* [8], Jaag [10], and Moorthy [13]. Differing from the above publications, we do not consider any specific function but use the general form $c(Q(\mathbf{q}))$ for our analysis.

Division U's profit generated by trading with division D_j ($j = 1, 2, \ldots, n$), denoted by Π_i, is calculated as its sale revenue minus its production cost, i.e., $\Pi_j = T_j q_j(p_j) - c(Q(\mathbf{q}))q_j(p_j)$. Total profit Π that division U can realize by trading with n downstream divisions are thus found as,

$$\Pi = \sum\nolimits_{j=1}^{n} \Pi_j = \sum\nolimits_{j=1}^{n} [T_j - c(Q(\mathbf{q}))]q_j(p_j). \tag{2}$$

2.1 Cooperative Game Model

We now use multi-person cooperative game theory to solve the transfer pricing problem for $n+1$ divisions with $n \geq 2$. Following von Neumann and Morgenstern [19, Ch. VI], we construct a cooperative game in characteristic-function form by computing the characteristic value of each possible coalition, which is defined as the *minimum* profit surplus that all divisions in the coalition can guarantee to achieve jointly when all the other divisions form an opposing coalition and try to minimize the profit surplus to the coalition. Next, we compute the characteristic values of all possible coalitions. In the empty coalition $\varnothing$, there is no division and the profit surplus is certainly zero. Thus, the characteristic value of the empty coalition is $v(\varnothing) = 0$.

2.1.1 The Characteristic Values of One-Division Coalitions

If division $i = U, D_1, D_2, \ldots, D_n$ does not cooperate with any other division(s), then it forms the one-division coalition (i), where it is the only member. For our problem, $v(i)$ is the "security-level" profit surplus that division i can achieve by itself when other divisions form an opposing coalition.

Theorem 1 For the cooperative game under consideration, we always have $v(U) = 0$. If $2c'(Q) + c''(Q)Q \leq 0$ for any production quantity Q, then the characteristic value of each single-division coalition must be zero, i.e., $v(i) = 0$, for $i = D_j$ $(j = 1, 2, \ldots, n)$. ■

The above theorem indicates that whether $v(D_i) = 0$ (for $i = 1, \ldots, n$) depends on the condition $2c'(Q) + c''(Q)Q \leq 0$ for any production quantity Q. We note that, if the upstream division U's unit production cost $c(\cdot)$ is specified as a linear, decreasing function as in Gray *et al.* [8], Jaag [10], and Moorthy [13], then $c'(\cdot) < 0$ and $c''(\cdot) = 0$, and thus, the condition that $2c'(Q) + c''(Q)Q \leq 0$ must be satisfied. To facilitate our analysis, we hereafter assume that $2c'(Q) + c''(Q)Q \leq 0$ for any production quantity Q.

2.1.2 The Characteristic Values of k-Division Coalitions with $k \geq 2$

We calculate the characteristic value of the k-division coalition $\mathbb{C}_k$ $(2 \leq k \leq n+1)$, in which k divisions trade in the cooperative setting. When $k = n + 1$, $\mathbb{C}_k = \mathbb{C}_{n+1}$, which is called the grand coalition in which all of $n + 1$ divisions cooperate. We note that the coalition $\mathbb{C}_k$ may or may not include the upstream division U. If the coalition $\mathbb{C}_k$ involves U, then there are $k - 1$ downstream divisions in $\mathbb{C}_k$; otherwise, all of k members in $\mathbb{C}_k$ are downstream divisions.

To find the characteristic value $v(\mathbb{C}_k)$, we need to calculate the *minimum* profit surplus that the k members in $\mathbb{C}_k$ jointly achieve by their *own* efforts. Hence, we should consider the impact of the decisions of the other $(n - k + 1)$ divisions—who are not in the coalition $\mathbb{C}_k$—on total profit of the k members in $\mathbb{C}_k$. As discussed previously, we calculate the value of $v(\mathbb{C}_k)$, assuming that the $(n - k + 1)$ divisions (that are not in $\mathbb{C}_k$) do not cooperate but behave in the non-cooperative setting—where the upstream division U acts as the leader and the downstream divisions that are not in $\mathbb{C}_k$ act as the followers—and choose the transfer prices in Stackelberg equilibrium. Similar to the proof of Theorem 1, we find that, if division U is not in the k-division coalition $\mathbb{C}_k$, then the profit surplus of each member in $\mathbb{C}_k$ is zero. That is, if $U \notin \mathbb{C}_k$, then $v(\mathbb{C}_k) = 0$.

Next, we calculate $v(\mathbb{C}_k)$ when $U \in \mathbb{C}_k$. Note that, if the upstream division joins $\mathbb{C}_k$, then $(k - 1)$ downstream divisions are in $\mathbb{C}_k$ and thus, there are $C_{k-1}^n = n!/[(k-1)!(n-k+1)!]$ possible k-division coalitions, which are denoted by $\mathbb{C}_k^r$, $r = 1, 2, \ldots, C_{k-1}^n$. Note that the upstream division U and the $(n - k + 1)$ divisions $D_j \notin \mathbb{C}_k^r$ adopt their transfer prices $\hat{T}_j^{(k;r)}$ and retail prices $\hat{p}_j^{(k;r)}$ in Stackelberg equilibrium, respectively. [In this paper, the symbol (ˆ) indicates the Stackelberg equilibrium.] To find Stackelberg equilibria for the divisions U and $D_j \notin \mathbb{C}_k^r$, we use the following three steps. In the first step, given the retail price $p_j^{(k;r)}$, we maximize the upstream division U's profit Π in (2) to find its best-response transfer price. In the second step, we substitute the best-response transfer price into the downstream division D_j's profit $\pi_j(p_j)$ in (1), and maximize it to obtain D_j's Stackelberg equilibrium-characterized retail price as $\hat{p}_j^{(k;r)} = [a_j + 2q_j(\hat{p}_j^{(k;r)})]/(2b_j) + [c(Q(q;k,r)) + c'(Q(q;k,r))Q(q;k,r)]/2$, where $Q(q;k,r) \equiv \sum_{\{i|D_i\in\mathbb{C}_k^r\}} q_i(p_i^{(k;r)}) + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} q_j(\hat{p}_j^{(k;r)})$ with $p_i^{(k;r)}$ denoting the retail price of division D_i in the coalition $\mathbb{C}_k^r$. In the third step, we substitute $\hat{p}_j^{(k;r)}$ into the upstream division U's best response function, and obtain the transfer price in Stackelberg equilibrium $\hat{T}_j^{(k;r)}$ as,

$$\hat{T}_j^{(k;r)} = \frac{2}{b_j} q_j(\hat{p}_j^{(k;r)}) + c(Q(\mathbf{q};k,r)) + c'(Q(\mathbf{q};k,r))Q(\mathbf{q};k,r). \tag{3}$$

Next, we calculate the characteristic value $v(\mathbb{C}_k^r)$. In the coalition $\mathbb{C}_k^r$, division U and the $(k-1)$ downstream divisions cooperate to jointly determine the optimal retail pricing decisions (i.e., $p_i^{(k;r)*}$, for $i \in \{i \mid D_i \in \mathbb{C}_k^r\}$) maximizing their total profit $\Phi^{(k;r)}$, which is given as,

$$\Phi^{(k;r)} = \sum_{\{i|D_i\in\mathbb{C}_k^r\}} p_i^{(k;r)} q_i(p_i^{(k;r)}) - c(Q(\mathbf{q};k,r))Q(\mathbf{q};k,r) + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} \hat{T}_j^{(k;r)} q_j(\hat{p}_j^{(k;r)}). \tag{4}$$

Theorem 2 Consider the k-division $(3 \le k \le n+1)$ coalition $\mathbb{C}_k^r$ $(r = 1, 2, \ldots, C_{k-1}^n)$ including the upstream division U and the $(k-1)$ downstream divisions. The downstream divisions' optimal retail prices $p_i^{(k;r)*}$ $(i \in \{i \mid D_i \in \mathbb{C}_k^r\})$—that maximize $\Phi^{(k;r)}$ in (4)—satisfy the following equation:

$$2p_i^{(k;r)*} = \frac{a_i}{b_i} + c(Q^*(\mathbf{q};k,r)) + c'(Q^*(\mathbf{q};k,r))Q^*(\mathbf{q};k,r), \text{ for } D_i \in \mathbb{C}_k^r, \tag{5}$$

where $Q^*(\mathbf{q};k,r) \equiv \sum_{\{i|D_i\in\mathbb{C}_k^r\}} q_i(p_i^{(k;r)*}) + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} q_j(\hat{p}_j^{(k;r)})$. We also find that the upstream division U's transfer prices $\hat{T}_j^{(k;r)}$ that is paid by the downstream divisions $D_j \notin \mathbb{C}_k^r$ can be obtained by substituting $Q^*(\mathbf{q};k,r)$ into (3). Moreover, the characteristic value $v(\mathbb{C}_k^r)$ is calculated as,

$$\begin{aligned} v(\mathbb{C}_k^r) \;\; = & \sum_{\{i|D_i\in\mathbb{C}_k^r\}} \{[p_i^{(k;r)*} - c(Q^*(\mathbf{q};k,r))]q_i(p_i^{(k;r)*}) - [\hat{p}_i^{(1)} - c(\hat{Q}(\mathbf{q}))]q_i(\hat{p}_i^{(1)})\} \\ & + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} \{[\hat{T}_j^{(k;r)} - c(Q^*(\mathbf{q};k,r))]q_j(\hat{p}_j^{(k;r)}) - [\hat{T}_j^{(1)} - c(\hat{Q}(\mathbf{q}))]q_j(\hat{p}_j^{(1)})\}. \quad \blacksquare \end{aligned} \tag{6}$$

Note that in (6), $\hat{Q}(\mathbf{q}) \equiv \sum_{i=1}^n q_i(\hat{p}_i^{(1)})$; $\hat{T}_i^{(1)}$ and $\hat{p}_i^{(1)} = (a_i + b_i\hat{T}_i^{(1)})/(2b_i)$ $(i = 1, 2, \ldots, n)$ respectively denote the upper division U's and downstream division D_i's Stackelberg equilibria when U trades with each downstream division in the non-cooperative setting. We can also find that, under the assumption that $2c'(Q) + c''(Q)Q \le 0$ for any production quantity Q, both $p_i^{(k;r)*}$ and $\hat{T}_j^{(k;r)}$ are decreasing in k. This means that, if more downstream divisions join a coalition, then the optimal retail prices of all downstream divisions in the coalition and the upstream division U's Stackelberg transfer prices to the downstream divisions $D_j \notin \mathbb{C}_k^r$ should be reduced.

We also learn from (5) that, if division D_i joins the coalition $\mathbb{C}_k^r$, then division D_i's and division U's total profit margin $[p_i^{(k;r)*} - c(Q^*(\mathbf{q}; k, r))]$ is computed as $q_i(p_i^{(k;r)*})/b_i + c'(Q^*(\mathbf{q}; k, r))Q^*(\mathbf{q}; k, r)$; and division U's profit margin $[\hat{T}_j^{(k;r)} - c(Q^*(\mathbf{q}; k, r))]$ for its trade with division $D_j \notin \mathbb{C}_k^r$ is calculated as $2q_j(\hat{p}_j^{(k;r)})/b_j + c'(Q^*(\mathbf{q}; k, r))Q^*(\mathbf{q}; k, r)$.

Corollary 1 If division D_i is in the coalition $\mathbb{C}_k^r$, then total profit margin [i.e., $p_i^{(k;r)*} - c(Q^*(\mathbf{q}; k, r))$] of divisions U and D_i is increasing in k; but, the profit margin is a convex function of $q_i(p_i^{(k,r)*})$ if $c'''(\cdot) \geq 0$. Similarly, we find that, if division $D_j \notin \mathbb{C}_k$, then division U's profit margin [i.e., $\hat{T}_j^{(k;r)} - c(Q^*(\mathbf{q}; k, r))$] is increasing in k, but it is a convex function of $q_j(\hat{p}_j^{(k;r)})$ if $c'''(\cdot) \geq 0$. ■

We find that, in most relevant publications such as [8], [10], and [13], the authors assumed the linearity of the function $c(\cdot)$, which implies $c'''(\cdot) = 0$. In our paper, we do not impose a specific form on the function $c(\cdot)$ for our game analysis but only assume that it is decreasing and convex with the property $c'''(\cdot) \geq 0$.

We note that, when $k = n+1$, division U and all downstream divisions form the grand coalition $\mathbb{C}_{n+1}$ to jointly make their globally-optimal decisions. Following Theorem 2, we find that the characteristic value $v(\mathbb{C}_{n+1})$—which denotes the profit surplus generated when all divisions of the multidivisional firm cooperate—can be written as $v(\mathbb{C}_{n+1}) = \sum_{i=1}^{n}\{[p_i^{(n+1)*} - c(Q^*(\mathbf{q}; n+1))]q_i(p_i^{(n+1)*}) - [\hat{p}_i^{(1)} - c(\hat{Q}(\mathbf{q}))]q_i(\hat{p}_i^{(1)})\}$, which is the sum of all divisions' profit surpluses. We also find that $v(\mathbb{C}_{n+1}) \geq 0$, because, according to Corollary 1, total profit surplus of division U and the downstream division D_i—i.e., $[p_i^{(n+1)*} - c(Q^*(\mathbf{q}; n+1))]q_i(p_i^{(n+1)*}) - [\hat{p}_i^{(1)} - c(\hat{Q}(\mathbf{q}))]q_i(\hat{p}_i^{(1)})$—is non-negative.

Corollary 2 The characteristic value $v(\mathbb{C}_k^r)$ is an increasing function of k; i.e., as more divisions join a coalition to cooperate, they can jointly realize a higher profit surplus. ■

2.2 Profit Allocation and Transfer Prices

The characteristic values $[v(\varnothing); v(i), i = U, D_1, D_2, \ldots, D_n; v(\mathbb{C}_k), 2 \leq k \leq n+1]$—that we computed in Section 2.1—constitute the $(n+1)$-division cooperative game model $\mathcal{G}$ for our transfer pricing problem involving division U and n divisions D_j $(j = 1, 2, \ldots, n)$. It is interesting—and, we think, important—to determine whether or not our cooperative game in characteristic-function form is *superadditive* and *convex*; for more information about superadditive and convex games, see, for example, Straffin [17]. Since any convex cooperative game must be superadditive, we subsequently show the convexity of our game. To do so, we need to examine the supermodularity of the characteristic function, because a cooperative game is *convex* and also superadditive if its characteristic function is supermodular (Shapley [16]).

Theorem 3 For our $(n+1)$-division cooperative game, the characteristic function is supermodular; thus, the game $\mathcal{G}$ is convex and also superadditive. ■

As Theorem 3 implies, when more divisions form a coalition, the characteristic value of the coalition is higher. It thus follows that all divisions in our $(n+1)$-player cooperative game in characteristic-function form should have the incentive to join the grand coalition $\mathbb{C}_{n+1}$. This means that the grand coalition $\mathbb{C}_{n+1}$ is stable if $v(\mathbb{C}_{n+1})$ is allocated to all divisions in a *fair* manner. Next, we consider the fair allocation of the characteristic value (total profit surplus) $v(\mathbb{C}_{n+1})$. We let y_U denote the profit surplus allocated to division U, and y_{D_j} denote the surplus allocated to division D_j $(j = 1, 2, \ldots, n)$. Using y_i, for $i = U, D_1, D_2, \ldots, D_n$, we can characterize a proper allocation scheme by using an $(n+1)$-tuple of numbers $y \equiv (y_U, y_{D_1}, y_{D_2}, \ldots, y_{D_n})$ with the following two properties: (i) *individual rationality*, i.e., $y_i \geq v(i) = 0$, for all $i \in \mathbb{C}_{n+1} = (U, D_1, D_2, \ldots, D_n)$; (ii) *collective rationality*, i.e., $\sum_{i \in \mathbb{C}_{n+1}} y_i = v(\mathbb{C}_{n+1})$; see Straffin [17]. In cooperative game theory there are a number of concepts that could be used for our analysis of the $(n+1)$-player cooperative game in characteristic form. As Leng and Parlar [11] described, one of the most important concepts is the *core* (Gillies [6]).

Theorem 4 For our $(n+1)$-division cooperative game, the core is non-empty. ■

As the above theorem indicates, any point in the non-empty core represents a fair allocation scheme. To find a unique allocation solution, we next focus on the fair allocation in terms of the Shapley value [15], which is the unique imputation $(y_U, y_{D_1}, y_{D_2}, \ldots, y_{D_n})$ where the payoffs y_i $(i = U, D_1, D_2, \ldots, D_n)$ are distributed "fairly" by an "arbitrator." Note that, in this paper, the headquarters of the multidivisional firm act as the arbitrator; in fact, as Göx and Schiller [7] reviewed, in many transfer pricing-related publications, the headquarters were assumed to be responsible for the coordination of all divisions.

Theorem 5 For the downstream division D_j $(j = 1, 2, \ldots, n)$, the Shapley value $y^*_{D_j}$ is calculated as

$$y^*_{D_j} = \frac{\sum_{k=2}^{n+1} \sum_{r=1}^{C_{k-2}^{n-1}} (k-1)!(n+1-k)![v(\mathbb{C}_k^r(U, D_j)) - v(\mathbb{C}_k^r(U, D_j) - \{D_j\})]}{(n+1)!}, \tag{7}$$

where $v(\mathbb{C}_k^r(U, D_j))$ and $v(\mathbb{C}_k^r(U, D_j) - \{D_j\})$ can be calculated by using (6). For the upstream division U, the Shapley value y^*_U is calculated as $y^*_U = v(\mathbb{C}_{n+1}) - \sum_{j=1}^n y^*_{D_j}$. ■

Next, we use the Shapley value given in the above theorem to compute the transfer prices for n downstream divisions D_j $(j = 1, 2, \ldots, n)$, which is shown in this following theorem.

Theorem 6 The transfer price T^*_j $(i = 1, 2, \ldots, n)$—that division D_j pays to division U—is calculated as,

$$T^*_j = p_j^{(n+1)*} - \frac{4b_j y^*_{D_j} + (a_j - b_j \hat{T}_j^{(1)})^2}{4b_j q_j(p_j^{(n+1)*})}, \tag{8}$$

where $\hat{T}_j^{(1)}$, $p_j^{(n+1)*}$ and $y^*_{D_j}$ can be computed by using Theorems 2 and 5. ■

To illustrate our above analysis, we provide the following numerical example.

Example 1 We consider a multidivisional firm involving an upstream division (U) and 3 downstream divisions D_j $(j = 1, 2, 3)$. Division D_j purchases the intermediate products from U at the transfer price $T_j^{(k;r)}$, and then determines their retail prices $p_j^{(k;r)}$ and satisfies the demand $q_j(p_j^{(k;r)}) = a_j - b_j p_j^{(k;r)}$ with the following parameter values: $a_1 = 100$, $b_1 = 0.5$; $a_2 = 90$, $b_2 = 0.2$; $a_3 = 110$, $b_3 = 1$. The upstream division U's unit production cost function is given as $c(Q(\mathbf{q})) = 5 - 0.01Q(\mathbf{q})$, where $\mathbf{q} = (q_1(p_1), q_2(p_2), q_3(p_3))$ and $Q(\mathbf{q}) = \sum_{j=1}^3 q_j(p_j)$.

We can calculate the characteristic values for all possible coalitions, as specified in online Appendix E, and thus develop our cooperative game as follows:

$$v(\varnothing) = v(U) = v(D_1) = v(D_2) = v(D_3) = v(D_1D_2) = v(D_1D_3) = v(D_2D_3) = 0,$$
$$v(UD_1) = 1,218.45,\ v(UD_2) = 2,501.28, \quad v(UD_3) = 722; \quad v(D_1D_2D_3) = 0,$$
$$v(UD_1D_2) = 3,732.02, \quad v(UD_1D_3) = 1,953.72, \quad v(UD_2D_3) = 3,236.54; \quad v(UD_1D_2D_3) = 4,479.49.$$

We can easily find that the above game is a convex game with an non-empty core, as shown in Theorems 3 and 4. Using Theorem 5, we calculate the Shapley value—which is used to allocate the system-wide profit surplus $v(UD_1D_2D_3) = 4,479.49$ among the four divisions—as follows: $y_U^* = \$2,418.61$, $y_{D_1}^* = \$566.71$, $y_{D_2}^* = \$1,154.67$, and $y_{D_3}^* = \$339.5$. Then, we use Theorem 6 to find transfer prices as: $T_1^* = \$65.31$, $T_2^* = \$144.75$, $T_3^* = \$36.72$. $\triangleleft$

We learn from the above example that the profit surpluses ($y_{D_j}^*$, for $j = 1, 2, 3$) allocated to three downstream divisions depend on different values of three divisions' demand parameters (i.e., a_j and b_j). Specifically, we note that, if the downstream division j has a larger value of the ratio a_j/b_j, then the division should gain more allocations, which is possibly attributed to the following reason: Equation (5) shows that, in the grand coalition $\mathbb{C}_{n+1}$, a larger value of a_j/b_j will result in a higher retail price $p_j^{(n+1)*}$ for the downstream division j (compared with other divisions), because the value of $c(Q^*(q; n+1)) + c'(Q^*(q; n+1))Q^*(q; n+1)$ is the same to all downstream divisions. As discussed in Section 2.1.2, the system-wide unit profit is $\sum_{j=1}^{n}[p_j^{(n+1)*} - c(Q^*(q; n+1))]$, where the division j with a higher retail price $p_j^{(n+1)*}$ makes a larger contribution. This may lead the Shapley value to suggest a higher profit surplus allocated to the division j. Noting that the inverse form of our linear demand function is $p_j = a_j/b_j - q_j(p_j)/b_j$, we find that a_j/b_j can be interpreted as the upper limit for division j's retail price. Thus, we conclude from the above that a downstream division with a higher upper limit for its retail price should obtain a larger allocation of the system-wide profit surplus.

3 Conclusions

In this paper, we consider the transfer pricing decisions for a multidivisional firm with a single upstream division and multiple downstream divisions. The upstream division manufactures its intermediate products and incurs a quantity-dependent production cost, and each downstream division uses the intermediate products to make final products and satisfy the retail price-sensitive demand. We compute the firm-wide profit surplus as the profit in the cooperative setting minus that in the non-cooperative setting, construct an $(n + 1)$-division cooperative game model, and use cooperative game theory to fairly allocate the profit surplus among three or more divisions. We show that, as more divisions join a coalition to cooperate, each downstream division's retail price decreases. Moreover, we prove that the characteristic function of the cooperative game is supermodular, which means that our game is convex and superadditive. We also show that the core of this game is non-empty. Then, we compute Shapley value for our game to find a unique, fair allocation scheme, which is in the core because of the convexity of our game. Thus, the allocation scheme suggested by the Shapley value can assure the stability of the grand coalition. Using the Shapley value, we find analytical transfer-pricing decisions that the n downstream divisions pay to the upstream division. In this paper, the application of cooperative game theory with three or more players to transfer pricing analysis is our most important contribution; this modeling approach is expected to help other cooperation-related research projects.

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Online Supplements
"Transfer Pricing in a Multidivisional Firm: A Cooperative Game Analysis"
M. Leng and M. Parlar

Appendix A A Brief Description of the Transfer Pricing Decision in Multidivisional Firms

The transfer pricing problem is of significant importance to multidivisional firms which need to consider the allocation of firm-wide profit between an upstream division and multiple downstream divisions. More precisely, in a multidivisional firm, an upstream division sells its intermediate products to a downstream division which then makes the final products to serve a market. The side-payment from the downstream division to the upstream division is calculated as the transfer price times the downstream division's purchase quantity. Since the upstream division's sales revenue is equal to the downstream division's total purchase cost, the side-payment does not impact the firm-*wide* profit (i.e., the sum of the two divisions' profits), which only depends on the retail price at which the downstream division serves its market. However, in order to fairly allocate the firm-wide profit between the two divisions, the firm must make the transfer pricing decision judiciously. Since for each unit of the intermediate product, the downstream division pays the transfer price to the upstream division, a higher transfer price brings more profit to the upstream division but a lower transfer price benefits the downstream division. When the firm's upstream and downstream divisions jointly determine the retail price to maximize the firm-wide profit, the firm must address the critical question of setting the transfer price to allocate the maximum firm-wide profit between the two divisions.[1]

It is natural to expect that, in practice, a multidivisional firm may desire to coordinate its divisions for the maximization of firm-wide after-tax profit through the transfer pricing decision. Specifically, if the tax rate for the firm's downstream division is lower than that for its upstream division, then the firm may reduce the transfer price to increase the downstream division's taxable profit and decrease the upstream division's taxable profit. As a result, the firm's total after-tax profit—i.e., the sum of the downstream division's and the upstream division's after-tax profits—rises. (If the upstream division is located in a lower-tax jurisdiction, then increasing the transfer price results in an increase in the firm's after-tax profit.) For example, as upheld by Canada's Federal Court of Appeal, the General Electric Capital Canada Inc. developed a transfer pricing rule to deduct from its income a C$136.4 million fee paid to its U.S. parent. The Walmart store in Bentonville in Arkansas recently reported that the retailer had achieved significant tax savings in 2010, which were largely related to changes in transfer pricing policies in a foreign jurisdiction. For more examples, see, e.g., a recent report [18] by Tax Management Inc., a subsidiary of The Bureau of National Affairs, Inc.

As observed in the General Electric and Walmart examples, such a transfer pricing decision could result in a loss of tax revenue in a high-tax jurisdiction (province or state), which may thus desire to impose a "fair" transfer pricing rule on multidivisional firms. To prevent multidivisional firms from intentionally transferring their profits from high-tax jurisdictions to low-tax jurisdictions, all divisions of a firm can independently make their decisions as if they were operated in a decentralized setting. This means that all downstream divisions are able to determine their own retail prices and decide on whether or not they will buy from the upstream division at negotiated transfer prices. The transfer pricing decision plays a very important role in the operation of a multidivisional firm, and it has been of interest to accountants, economists, and managers. As Borstell and Hobster (on behalf of Ernst & Young) reported in [3], 75% of parent and 81% of subsidiary respondents believe that transfer pricing is "absolutely critical" or "very important" to their organizations.

[1] We note that the transfer price paid by the downstream division to the upstream division is similar to the wholesale price paid by a retailer to the manufacturer. But, the difference in this case is that both divisions belong to a single firm, whereas the retailer and the manufacturer are usually assumed to be in a decentralized system.

Since a multidivisional firm should *not* use transfer pricing as a tool for reducing the firm's total tax payment, in our models we assume that tax rates in all jurisdictions are approximately equal to each other so that we can exclude the impact of tax rates on a multidivisional firm's transfer pricing decision. This is in line with most publications on transfer pricing problems which have not considered the tax-related matters. For details, see a recent and fairly complete review by Göx and Schiller [7] who surveyed a large number of transfer-pricing literature starting from Hirshleifer's seminal, standard model [9].

Appendix B Explanation on the Assumption $1 + b_j c'(\cdot) \geq 0$

To explain the assumption that $1 + b_j c'(\cdot) \geq 0$, we show that the sum of the upstream member's and each downstream member's unit profits is increasing in the downstream member's retail price, because the downstream divisions are independent of each other. We consider the downstream division i, which determines its retail price p_i. The upstream division's unit profit from trading with the division i is calculated as the transfer price T_i—paid by the downstream division i to the upstream division—minus the upstream division's unit production cost $c(Q(\mathbf{q}))$, i.e., $T_i - c(Q(\mathbf{q}))$.

Since the downstream division i pays the transfer price T_i to the upstream division and achieves the unit sales revenue p_i, the division's unit profit is $p_i - T_i$. Therefore, the total profit of the upstream division and the downstream division i is computed as $\Lambda_i \equiv p_i - c(Q(\mathbf{q}))$.

Note that $Q(\mathbf{q})$ is the upstream division's total production quantity, i.e., $Q(\mathbf{q}) = \sum_{j=1}^{n} q_j(p_j)$, where $q_j(p_j) = a_j - b_j p_j$ (for $j = 1, \ldots, n$) is the downstream division j's order quantity. For our proof, we re-write $Q(q)$ as the sum of the downstream division i's order quantity and other divisions' quantities, i.e., $Q(\mathbf{q}) = q_i(p_i) + \sum_{j \neq i} q_j(p_j)$.

The first-order derivative of Λ_i w.r.t. p_i can be computed as,

$$\frac{\partial \Lambda_i}{\partial p_i} = \frac{\partial [p_i - c(Q(\mathbf{q}))]}{\partial p_i} = 1 - b_j c'(Q(\mathbf{q})),$$

which is greater than or equal to zero, i.e., $1 - b_j c'(Q(\mathbf{q})) \geq 0$ (for $j = 1, \ldots, n$), if and only if Λ_i is increasing in p_i. Actually, this assumption is reasonable, because a firm's unit profit should be usually increasing in its retail price. But, the firm's total profit is not always increasing in the price, because the sales quantity is usually decreasing in the price.

Appendix C Proofs of Theorems

Proof of Theorem 1. Since division i may be division U or may be division D_j ($j = 1, 2, \ldots, n$), we need to analyze the impact of cooperation (between two, or among three or more, of the other divisions) on division U and that on division D_j. If division i is U who does not cooperate with any other division D_j, then whether or not n downstream divisions cooperate does not affect division U's profit, because of the following fact: When k ($1 \leq k \leq n$) downstream divisions D_j ($j = 1, 2, \ldots, k$) cooperate, we can easily find from (1) that these divisions' optimal retail prices maximizing $\sum_{i=1}^{k} \pi_i(p_i)$ is the same as those maximizing $\pi_i(p_i)$ (for $i = 1, 2, \ldots, k$). Therefore, if $i = U$, then $v(i) = 0$.

We next consider the case that division i is a downstream division, e.g., division D_n. Then, division U may cooperate with one or more of the other $(n-1)$ downstream divisions (i.e., D_j, $j = 1, 2, \ldots, n-1$). Without loss of generality, we assume that division U cooperates with k ($1 \leq k \leq n-1$) downstream divisions D_z, $z = 1, 2, \ldots, k$; and as a result, the $(k+1)$-division coalition $\mathbb{C}_{k+1} \equiv (U, D_z, z = 1, 2, \ldots, k)$ forms. For this coalition, divisions U and D_z ($z = 1, 2, \ldots, k$) need to determine the k downstream divisions' globally-optimal retail prices $p_z^{(k+1)*}$ ($z = 1, 2, \ldots, k$), in which the superscript $(k+1)$ means that the optimal prices are made when divisions U and D_z ($z = 1, 2, \ldots, k$) form the $(k+1)$-division coalition $\mathbb{C}_{k+1}$. Moreover, since division U also sells its intermediate products to the other $(n-k-1)$ downstream divisions (i.e., D_j, $j=k+1,\ldots,n-1$) in the non-cooperative setting, the upstream division should determine its Stackelberg equilibrium transfer prices to division D_j $(j=k+1,\ldots,n-1)$. Thus, we should also determine transfer prices $\hat{T}_j^{(k+1)}$ $(j=k+1,\ldots,n-1)$, which denotes the Stackelberg equilibria when divisions U and D_z $(z=1,2,\ldots,k)$ form the $(k+1)$-division coalition $\mathbb{C}_{k+1}$. The upstream division U's and the downstream division D_j's Stackelberg equilibria (for $j=k+1,\ldots,n-1$) can be respectively calculated as,

$$\begin{aligned}\hat{T}_j^{(k+1)} &= \frac{2}{b_j} q_j(\hat{p}_j^{(k+1)}) + c(Q^*(\mathbf{q};k+1)) + c'(Q^*(\mathbf{q};k+1))Q^*(\mathbf{q};k+1),\\ \hat{p}_j^{(k;r)} &= (a_j + b_j\hat{T}_j^{(k;r)})/(2b_j),\end{aligned}$$

where

$$Q^*(\mathbf{q};k+1) \equiv \sum_{z=1}^{k} q_z(p_z^{(k+1)*}) + \sum_{j=k+1}^{n-1} q_j(\hat{p}_j^{(k+1)}), \tag{9}$$

and $p_z^{(k+1)*}$ $(z=1,2,\ldots,k)$ denotes division D_z's globally-optimal retail price maximizing the total profit of divisions U and D_z in the coalition $\mathbb{C}_{k+1}$.

To find $p_z^{(k+1)*}$, for $z=1,2,\ldots,k$, we maximize the sum of the upstream division's profit and the downstream divisions' profits, which is written as,

$$\max_{p_z^{(k+1)}} \Phi = \sum_{z=1}^{k} p_z^{(k+1)} q_z(p_z^{(k+1)}) - c(Q(\mathbf{q}))Q(\mathbf{q}) + \sum_{j=k+1}^{n-1} \hat{T}_j^{(k+1)} q_j(\hat{p}_j^{(k+1)}), \tag{10}$$

where $Q(\mathbf{q}) = \sum_{z=1}^{k} q_z(p_z^{(k+1)}) + \sum_{j=k+1}^{n-1} q_j(\hat{p}_j^{(k+1)})$. The first- and second-order derivatives of Φ w.r.t. $p_z^{(k+1)}$ are computed as,

$$\begin{aligned}\frac{\partial \Phi}{\partial p_z^{(k+1)}} &= q_z(p_z^{(k+1)}) - b_z p_z^{(k+1)} + b_z c(Q(\mathbf{q})) + b_z c'(Q(\mathbf{q}))Q(\mathbf{q}), \qquad (11)\\ \frac{\partial^2 \Phi}{\partial (p_z^{(k+1)})^2} &= -2b_z[1 + b_z c'(Q(\mathbf{q}))] - b_z^2 c''(Q(\mathbf{q}))Q(\mathbf{q}) < 0,\end{aligned}$$

which implies that Φ is a concave function of $p_z^{(k+1)}$ given the values of other decision variables. Setting $\partial\Phi/\partial p_z^{(k+1)}$ to zero and solving them for $p_z^{(k+1)}$, we have,

$$p_z^{(k+1)*} = \frac{1}{b_z} q_z(p_z^{(k+1)*}) + c(Q^*(\mathbf{q};k+1)) + c'(Q^*(\mathbf{q};k+1))Q^*(\mathbf{q};k+1).$$

Next, we determine if the retail price is higher when more downstream divisions cooperate with the upstream division. If division D_z and division U cooperate and form a two-division coalition, division D_z's globally-optimal retail price can be determined by the following first-order condition,

$$p_z^{(2)*} - \frac{q_z(p_z^{(2)*})}{b_z} = c(q_z(p_z^{(2)*})) + c'(q_z(p_z^{(2)*}))q_z(p_z^{(2)*}). \tag{12}$$

We use $p_z^{(2)*}$ to replace p_z in the first-order derivative $\partial\Phi/\partial p_z$ in (11), and find that

$$\left.\frac{\partial\Phi}{\partial p_z}\right|_{p_z=p_z^{(2)*}} = b_z\left\{[c(\tilde{Q}(\mathbf{q}))+c'(\tilde{Q}(\mathbf{q}))\tilde{Q}(\mathbf{q})] - \left[p_z^{(2)*} - \frac{q_z(p_z^{(2)*})}{b_z}\right]\right\},$$

where $\tilde{Q}(\mathbf{q}) \equiv q_z(p_z^{(2)*}) + \sum_{i=1,\ldots,k,\ i\neq z} q_i(p_i^{(k+1)*}) + \sum_{j=k+1}^{n-1} q_j(\hat{p}_j^{(k+1)})$. Using (12), we rewrite the above to

$$\left.\frac{\partial\Phi}{\partial p_z}\right|_{p_z=p_z^{(2)*}} = b_z\{[c(\tilde{Q}(\mathbf{q}))+c'(\tilde{Q}(\mathbf{q}))\tilde{Q}(\mathbf{q})] - [c(q_z(p_z^{(2)*}))+c'(q_z(p_z^{(2)*}))q_z(p_z^{(2)*})]\},$$

which cannot be immediately determined as a positive or a negative value. Note that $q_z(p_z^{(2)*}) \leq \tilde{Q}(\mathbf{q})$. Therefore, if, for the production quantity Q, $2c'(Q)+c''(Q)Q \leq 0$, then $[c(Q)+c'(Q)Q]$ is decreasing in Q and thus $\partial\Phi/\partial p_z|_{p_z=p_z^{(2)*}} \leq 0$, which means $p_z^{(k+1)*} \leq p_z^{(2)*}$, because $\partial\Phi/\partial p_z$ is zero at the point $p_z = p_z^{(k+1)*}$. Noting that $p_z^{(2)*} \leq \hat{p}$, we find that the quantity $Q^*(\mathbf{q})$ in (9)—when division U cooperates with one or more downstream divisions—should be greater than or equal to that when there is no cooperation between U and any downstream division.

We then compute division D_n's profit when k downstream divisions D_z $(z=1,2,\ldots,k)$ cooperate with the upstream division U. We find that the optimal transfer price from D_n to U is

$$\hat{T}_n^{(k+1)} = \frac{a_n}{2b_n} + \frac{c(Q^*(\mathbf{q}))+c'(Q^*(\mathbf{q}))Q^*(\mathbf{q})}{2}.$$

If $2c'(Q)+c''(Q)Q \leq 0$ for any production quantity Q, we find that, when the $(k+1)$-division coalition $\mathbb{C}_{k+1}$ forms, $\hat{T}_n^{(k+1)}$ is lower than $\hat{T}_n$—Stackelberg equilibrium when division U does not cooperate with any downstream division—and the D_n's profit $\pi(\hat{p}_n^{(k+1)}) = (a_n - b_n\hat{T}_n^{(k+1)})^2/(4b_n)$ is thus higher than that when there is no cooperation between division U and any downstream division. This means that division D_n benefits from the cooperation between divisions U and D_z $(z=1,2,\ldots,k)$; hence, its characteristic value—division D_n's *minimum* profit surplus—should be zero when divisions U and D_z $(z=1,2,\ldots,k)$ does not cooperate, i.e., $v(D_n)=0$. Similarly, we can find that $v(D_i)=0$, for $i=1,\ldots,n-1$. This theorem is thus proved. ■

Proof of Theorem 2. In order to compute $v(\mathbb{C}_k^r)$, we need to find $(k-1)$ optimal retail prices $p_i^{(k;r)*}$ $(i\in\{i \mid D_i\in\mathbb{C}_k^r\})$ that maximize total profit of all members in $\mathbb{C}_k^r$. Similar to our discussion in Section 2.1.1, we can calculated $p_i^{(k;r)*}$ as shown in (5), and also found $\hat{T}_j^{(k;r)}$ as in this theorem. Moreover, we find that, under the assumption that $2c'(Q)+c''(Q)Q \leq 0$ for any production quantity Q, then $p_i^{(k;r)*}$ and $\hat{T}_j^{(k;r)}$ decreases as k rises, which means that if more divisions cooperate, then their retail prices and division U's transfer prices should be reduced.

Substituting $p_i^{(k;r)*}$ $(i\in\{i\mid D_i\in\mathbb{C}_k^r\})$ and $\hat{T}_j^{(k;r)}$ $(j\in\{j\mid D_j\notin\mathbb{C}_k^r\})$ into (4) gives the maximum profit as,

$$\Phi^{(k;r)*} = \sum_{\{i|D_i\in\mathbb{C}_k^r\}} [p_i^{(k;r)*} - c(Q^*(\mathbf{q};k,r))]q_i(p_i^{(k;r)*}) + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} [\hat{T}_j^{(k;r)} - c(Q^*(\mathbf{q};k,r))]q_j(\hat{p}_j^{(k;r)}),$$

where $\hat{p}_j^{(k;r)} = (a_j + b_j\hat{T}_j^{(k;r)})/(2b_j)$. Note that the profit surplus $v(\mathbb{C}_k^r)$ is equal to total profit $\Phi^{(k;r)*}$—when divisions U and D_i $(i\in\{i\mid D_i\in\mathbb{C}_k^r\})$ join $\mathbb{C}_k^r$ to cooperate for maximizing their total profit—minus that when these divisions do not cooperate but make their decisions in the non-cooperative setting. In the non-cooperative setting, all divisions that are not in $\mathbb{C}_k^r$ choose their Stackelberg equilibria, and thus, the sum of division U's and division D_i's ($i \in \{i \mid D_i \in \mathbb{C}_k^r\}$) profits is calculated as,

$$\hat{\Phi}^{(k;r)} = \sum_{\{i|D_i \in \mathbb{C}_k^r\}} [\hat{p}_i^{(1)} - c(\hat{Q}(\mathbf{q}))] q_i(\hat{p}_i^{(1)}) + \sum_{\{j|D_j \notin \mathbb{C}_k^r\}} [\hat{T}_j^{(1)} - c(\hat{Q}(\mathbf{q}))] q_j(\hat{p}_j^{(1)}),$$

where $\hat{p}_i^{(1)} = (a_i + b_i \hat{T}_i^{(1)})/(2b_i)$ ($i = 1, 2, \ldots, n$) denotes division D_i's Stackelberg equilibrium when division U does not cooperate with any downstream division; and $\hat{Q}(\mathbf{q}) \equiv \sum_{i=1}^n q_i(\hat{p}_i^{(1)})$. We then compute the characteristic value (profit surplus achieved by $\mathbb{C}_k^r$) as $v(\mathbb{C}_k^r) = \Phi^{(k;r)*} - \hat{\Phi}^{(k;r)}$, which can specified as in (6). Since $p_i^{(k;r)*}$ ($i \in \{i \mid D_i \in \mathbb{C}_k^r\}$) and $\hat{T}_j^{(k;r)}$ ($j \in \{j \mid D_j \notin \mathbb{C}_k^r\}$) are the global solution maximizing $\Phi^{(k;r)}$, we can conclude that $\Phi^{(k;r)*} \geq \hat{\Phi}^{(k;r)}$ and $v(\mathbb{C}_k^r) \geq 0$. ■

Proof of Theorem 3. Since, if a cooperative game's characteristic function is supermodular, then the game must be convex and superadditive, we next need to show the supermodularity. Using Driessen's approach [4], we should prove that $v(\mathbb{S}_1 \cup \{i\}) - v(\mathbb{S}_1) \leq v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2)$, for all $\mathbb{S}_1 \subseteq \mathbb{S}_2 \subseteq \mathbb{C}_{n+1} \backslash \{i\}$. We can easily find that, if division U does not belong to $\mathbb{C}_2$, then this division is not in the coalition $\mathbb{S}_1$, and thus $v(\mathbb{S}_1 \cup \{i\}) - v(\mathbb{S}_1) = v(\mathbb{S}_1 \cup \{i\})$ and $v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2) = v(\mathbb{S}_2 \cup \{i\})$. Since $v(\mathbb{S}_1 \cup \{i\}) \leq v(\mathbb{S}_2 \cup \{i\})$ as indicated by Corollary 2, we have $v(\mathbb{S}_1 \cup \{i\}) - v(\mathbb{S}_1) \leq v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2)$. If division U is in $\mathbb{S}_2$ but is not in the coalition $\mathbb{S}_1$, then $v(\mathbb{S}_1 \cup \{i\}) = v(\mathbb{S}_1) = 0$ and $v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2) \geq 0$, and it thus follows that $v(\mathbb{S}_1 \cup \{i\}) - v(\mathbb{S}_1) \leq v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2)$.

Next, we consider the case in which division U is in the coalition $\mathbb{S}_1$. This means that division U is also in $\mathbb{S}_2$. W.l.o.g., we assume that $\mathbb{S}_1$ and $\mathbb{S}_2$ are k_1- and k_2-division coalitions with $k_2 \geq k_1$. Using Theorem 2, we find that

$$[v(\mathbb{S}_2 \cup \{i\}) - v(\mathbb{S}_2)] - [v(\mathbb{S}_1 \cup \{i\}) - v(\mathbb{S}_1)] = \kappa_1 + \kappa_2 + \kappa_3 + \kappa_4, \tag{13}$$

where

$$\begin{aligned} \kappa_1 \;\; \equiv \;\; & \sum_{j \in \mathbb{S}_1} \{[p_j^{(k_2+1)*} - c(Q^*(\mathbf{q}; k_2+1))] q_j(p_j^{(k_2+1)*}) - [p_j^{(k_2)*} - c(Q^*(\mathbf{q}; k_2))] q_j(p_j^{(k_2)*})\} \\ & - \sum_{j \in \mathbb{S}_1} \{[p_j^{(k_1+1)*} - c(Q^*(\mathbf{q}; k_1+1))] q_j(p_j^{(k_1+1)*}) - [p_j^{(k_1)*} - c(Q^*(\mathbf{q}; k_1))] q_j(p_j^{(k_1)*})\}, \end{aligned}$$

which is non-negative, because, as Corollary 1 indicates, $[p_j^{(k)*} - c(Q^*(\mathbf{q}; k))] q_j(p_j^{(k)*})$ is an increasing, convex function. In (13), κ_2 is defined as,

$$\begin{aligned} \kappa_2 \;\; \equiv \;\; & \sum_{j \notin \mathbb{S}_1} \{[\hat{T}_j^{(k_2+1)} - c(Q^*(\mathbf{q}; k_2+1))] q_j(\hat{p}_j^{(k_2+1)}) - [\hat{T}_j^{(k_2)} - c(Q^*(\mathbf{q}; k_2))] q_j(\hat{p}_j^{(k_2)})\} \\ & - \sum_{j \notin \mathbb{S}_1} \{[\hat{T}_j^{(k_1+1)} - c(Q^*(\mathbf{q}; k_1+1))] q_j(\hat{p}_j^{(k_1+1)}) - [\hat{T}_j^{(k_1)} - c(Q^*(\mathbf{q}; k_1))] q_j(\hat{p}_j^{(k_1)})\}, \end{aligned}$$

which is also non-negative because $[\hat{T}_j^{(k)} - c(Q^*(\mathbf{q}; k))] q_j(\hat{p}_j^{(k)})$ is an increasing and convex function, as indicated in Corollary 1. The term κ_3 in (13) is defined as,

$$\begin{aligned} \kappa_3 \;\; \equiv \;\; & [p_i^{(k_2+1)*} - c(Q^*(\mathbf{q}; k_2+1))] q_i(p_i^{(k_2+1)*}) - [p_i^{(k_1+1)*} - c(Q^*(\mathbf{q}; k_1+1))] q_i(p_i^{(k_1+1)*}) \\ & -[\hat{T}_i^{(k_2)} - c(Q^*(\mathbf{q}; k))] q_i(\hat{p}_i^{(k_2)}) + [\hat{T}_i^{(k_1)} - c(Q^*(\mathbf{q}; k_1))] q_i(\hat{p}_i^{(k_1)}), \end{aligned}$$

which is non-negative, as we show in the proof of Corollary 2. The term κ_4 in (13) is as,

$$\begin{aligned}\kappa_4 \equiv & \sum_{j\in\mathbb{S}_2-\mathbb{S}_1}\{(p_j^{(k_2+1)*}-T_j^{(k_2+1)*})q_j(p_j^{(k_2+1)*})-(p_j^{(k_2)*}-T_j^{(k_2)*})q_j(p_j^{(k_2)*})\} \\ & + \sum_{j\in\mathbb{S}_2-\mathbb{S}_1}\{[T_j^{(k_2+1)*}-c(Q^*(\mathbf{q};k_2+1))]q_j(p_j^{(k_2+1)*})-[T_j^{(k_2)*}-c(Q^*(\mathbf{q};k_2))]q_j(p_j^{(k_2)*})\} \\ & - \sum_{j\in\mathbb{S}_2-\mathbb{S}_1}\{[\hat{T}_j^{(k_2+1)}-c(Q^*(\mathbf{q};k_2+1))]q_j(\hat{p}_j^{(k_2+1)})-[\hat{T}_j^{(k_2)}-c(Q^*(\mathbf{q};k_2))]q_j(\hat{p}_j^{(k_2)})\},\end{aligned}$$

where is non-negative according to Corollary 1.

In conclusion, the characteristic function of this game is supermodular, and thus, the game is convex and superadditive. ■

Proof of Theorem 4. This theorem follows from the superadditivity of our $(n+1)$-division cooperative game (which is shown in Theorem 3). More specifically, we assume that all divisions form z ($z \geq 2$) disjoint, less-than-$(n+1)$-division but non-empty coalitions $\mathbb{C}'_1, \mathbb{C}'_2, \ldots, \mathbb{C}'_z$; that is, $\mathbb{C}'_i \neq \varnothing$ and $\mathbb{C}'_i \subset \mathbb{C}_{n+1}$, for $i = 1, 2, \ldots, z$; $\mathbb{C}'_i \cap \mathbb{C}'_j = \varnothing$, for $i, j = 1, 2, \ldots, z$, $i \neq j$; and $\cup_{i=1}^{z}\mathbb{C}'_i = \mathbb{C}_{n+1}$. Thus, the total profit surpluses achieved by all divisions in the coalitions $\mathbb{C}'_i$ ($i = 1, 2, \ldots, z$) is $\sum_{i=1}^{z} v(\mathbb{C}'_i)$, which is no more than $v(\mathbb{C}_{n+1})$, because the cooperative game is superadditive according to Theorem 3. This proves the theorem. ■

Proof of Theorem 5. In order to calculate Shapley value, we need to identify all possible coalitions that each division joins. Next, we consider division D_j, $j = 1, 2, \ldots, n$, and calculate Shapley value y_{D_j} for this division. When division D_j does not cooperate with any other divisions but joins the one-division coalition (D_j), we find from Section 2.1.1 that $v(D_j) = 0$. It thus follows that $v(D_j) - v(\varnothing) = 0$.

If division D_j cooperates with one or more of the other downstream divisions to form a coalition, we then need to consider whether or not division U is also in the coalition since, as discussed previously, the characteristic value of a coalition that does not include the U is zero. Therefore, if division D_j joins the coalition $\mathbb{S}$ (i.e., $D_j \in \mathbb{S}$) but division U does not join $\mathbb{S}$ (i.e., $U \notin \mathbb{S}$), then we have $v(\mathbb{S}) - v(\mathbb{S} - \{D_j\}) = 0$.

We next compute the value of $v(\mathbb{S}) - v(\mathbb{S} - \{D_j\})$, where $\mathbb{S}$ includes division U (i.e., $U \in \mathbb{S}$) and it is a possible coalition that division D_j joins. Assume that $\mathbb{S}$ is a k-division coalition $\mathbb{C}_k(U, D_j)$ ($2 \leq k \leq n+1$) that includes U, D_j and, if $k \geq 3$, one or more of the other downstream divisions. Note that there are C_{k-2}^{n-1} $k-$division coalitions, which are denoted by $\mathbb{C}_k^r(U, D_j)$, $r = 1, 2, \ldots, C_{k-2}^{n-1}$. Hence, for a given value of k, we can find that

$$\begin{aligned}& \frac{\sum_{j\in\mathbb{S}}(|\mathbb{S}|-1)!(n+1-|\mathbb{S}|)![v(\mathbb{S})-v(\mathbb{S}-\{j\})]}{(n+1)!} \\ = \;& \frac{\sum_{r=1}^{C_{k-2}^{n-1}}(k-1)!(n+1-k)![v(\mathbb{C}_k^r(U,D_j))-v(\mathbb{C}_k^r(U,D_j)-\{D_j\})]}{(n+1)!},\end{aligned}$$

where $v(\mathbb{C}_k^r(U, D_j))$ and $v(\mathbb{C}_k^r(U, D_j) - \{D_j\})$ can be calculated by using (6).

The Shapley value y_i^* ($i = D_1, D_2, \ldots, D_n$) is thus calculated as in (7). Since $\sum_{i\in\mathbb{C}_{n+1}} y_i^* = v(\mathbb{C}_{n+1})$, we easily find that the Shapley value for division U is $y_U^* = v(\mathbb{C}_{n+1}) - \sum_{j=1}^{n} y_{D_j}^*$. Since our $(n+1)$-division cooperative game is convex as shown in Theorem 3, the Shapley value must be in the core. ■

Proof of Theorem 6. Since the allocation to division i is y_i^*, for $i = U, D_1, D_2, \ldots, D_n$, we can find that $\pi(p_j^{(n+1)*}) - \pi(\hat{p}_j^{(1)}) = y_{D_j}^*$, for division D_j ($j = 1, 2, \ldots, n$). Because $\pi(p_j^{(n+1)*}) = (p_j^{(n+1)*} - T_j^*)q_j(p_j^{(n+1)*})$ and $\pi(\hat{p}_j^{(1)}) = (a_j - b_j\hat{T}_j^{(1)})^2/4b_j$, we can easily calculate T_j^* as shown in (8). ■

Appendix D Proofs of Corollaries

Proof of Corollary 1. Using Theorem 2, we can calculate the profit margin of divisions U and D_i as $m_i \equiv q_i(p_i^{(k;r)*})/b_i + c'(Q^*(\mathbf{q};k,r))Q^*(\mathbf{q};k,r)$, where $Q^*(\mathbf{q};k,r) \equiv \sum_{\{i|D_i\in\mathbb{C}_k^r\}} q_i(p_i^{(k;r)*}) + \sum_{\{j|D_j\notin\mathbb{C}_k^r\}} q_j(\hat{p}_j^{(k;r)})$. Differentiating m_i once and twice w.r.t. $q_i(p_i^{(k)*})$ gives

$$\begin{aligned}
\frac{\partial m_i}{\partial q_i(p_i^{(k;r)*})} &= 1/b_i + c'(Q^*(\mathbf{q};k,r)) + c''(Q^*(\mathbf{q};k,r))Q^*(\mathbf{q};k,r) \geq 0,\\
\frac{\partial^2 m_i}{\partial [q_i(p_i^{(k;r)*})]^2} &= 2c''(Q^*(\mathbf{q};k,r)) + c'''(Q^*(\mathbf{q};k,r))Q^*(\mathbf{q};k,r) \geq 0,
\end{aligned}$$

if $c'''(\cdot) \geq 0$. Note that, as k increases, then $p_i^{(k;r)*}$ decreases (as shown in Theorem 2) and $q_i(p_i^{(k;r)*})$ increases, and thus m_i increases. Similarly, we can show that division U's profit margin [i.e., $\hat{T}_j^{(k;r)} - c(Q^*(\mathbf{q};k,r))$] is increasing in $q_j(\hat{p}_j^{(k;r)})$, but it is a convex function of $q_j(\hat{p}_j^{(k;r)})$ if $c'''(\cdot) \geq 0$. ■

Proof of Corollary 2. We consider the case that the upstream division U and $(k-1)$ downstream divisions (e.g., D_j, $j = 1, 2, \ldots, k-1$)—who are now in the coalition $\mathbb{C}_k$—decide to cooperate the downstream division D_k, and thus form a $(k+1)$-division coalition $\mathbb{C}_{k+1}$. Using (6) we can write the characteristic value $v(\mathbb{C}_{k+1})$ as

$$\begin{aligned}
v(\mathbb{C}_{k+1}) &= \sum_{i=1}^{k}\{[p_i^{(k+1)*} - c(Q^*(\mathbf{q};k+1))]q_i(p_i^{(k+1)*}) - [\hat{p}_i^{(1)} - c(\hat{Q}(\mathbf{q}))]q_i(\hat{p}_i^{(1)})\}\\
&\quad + \sum_{j=k+1}^{n}\{[\hat{T}_j^{(k+1)} - c(Q^*(\mathbf{q};k+1))]q_j(\hat{p}_j^{(k+1)}) - [\hat{T}_j^{(1)} - c(\hat{Q}(\mathbf{q}))]q_j(\hat{p}_j^{(1)})\}
\end{aligned}$$

We then calculate $v(\mathbb{C}_{k+1}) - v(\mathbb{C}_k)$ as,

$$\begin{aligned}
v(\mathbb{C}_{k+1}) - v(\mathbb{C}_k) &= [p_k^{(k+1)*} - c(Q^*(\mathbf{q};k+1))]q_k(p_k^{(k+1)*}) - [\hat{p}_k^{(1)} - c(\hat{Q}(\mathbf{q}))]q_k(\hat{p}_k^{(1)})\\
&\quad -[\hat{T}_k^{(k)} - c(Q^*(\mathbf{q};k))]q_k(\hat{p}_k^{(k)}) + [\hat{T}_k^{(1)} - c(\hat{Q}(\mathbf{q}))]q_k(\hat{p}_k^{(1)})\\
&\quad +\sum_{i=1}^{k-1}\{[p_i^{(k+1)*} - c(Q^*(\mathbf{q};k+1))]q_i(p_i^{(k+1)*}) - [p_i^{(k)*} - c(Q^*(\mathbf{q};k))]q_i(p_i^{(k)*})\}\\
&\quad + \sum_{j=k+1}^{n}\{[\hat{T}_j^{(k+1)} - c(Q^*(\mathbf{q};k+1))]q_j(\hat{p}_j^{(k+1)})\\
&\quad -[\hat{T}_j^{(k)} - c(Q^*(\mathbf{q};k))]q_j(\hat{p}_j^{(k)})\}, \qquad (14)
\end{aligned}$$

which is non-negative according to Corollary 1. More precisely, from Corollary 1, we find that $[p_k^{(k+1)*} - c(Q^*(\mathbf{q};k+1))]q_k(p_k^{(k+1)*}) \geq [p_k^{(k)*} - c(Q^*(\mathbf{q};k))]q_k(p_k^{(k)*})$. Thus, the first two terms in (14) can be written as

$$\begin{aligned}
&[p_k^{(k+1)*} - c(Q^*(\mathbf{q};k+1))]q_k(p_k^{(k+1)*}) - [\hat{p}_k^{(1)} - c(\hat{Q}(\mathbf{q}))]q_k(\hat{p}_k^{(1)})\\
&-[\hat{T}_k^{(k)} - c(Q^*(\mathbf{q};k))]q_k(\hat{p}_k^{(k)}) + [\hat{T}_k^{(1)} - c(\hat{Q}(\mathbf{q}))]q_k(\hat{p}_k^{(1)})\\
\geq\ & (p_k^{(k)*} - \hat{T}_k^{(k)})q_k(p_k^{(k)*}) - (\hat{p}_k^{(1)} - \hat{T}_k^{(1)})q_k(\hat{p}_k^{(1)}),
\end{aligned}$$

which is non-negative, according to Corollary 1. It also follows from Corollary 1 that both the third and the fourth terms are non-negative. This corollary is thus proved. ■

Appendix E Calculation of the Characteristic Values in Example 1

As discussed in Section 2.1.1, the characteristic values (profit surpluses) of an empty coalition and one-division coalitions are zero, i.e., $v(\varnothing) = v(U) = v(D_1) = v(D_2) = v(D_3) = 0$. Moreover, as discussed in Section 2.1.2, for the two- and three-division coalitions that do not include U, the characteristic values are also zero; that is, $v(D_1D_2) = v(D_1D_3) = v(D_2D_3) = v(D_1D_2D_3) = 0$.

To find the characteristic values of the other two- and three-division coalitions and that of the four-division (grand) coalition, we should use Theorem 2 to calculate all divisions' optimal decisions in all possible coalition structures, which are presented in Table 1, where $\{(U), (D_1), (D_2), (D_3)\}$ denotes the coalition structure in which there is no cooperation between any two, or among any three or four, divisions; $\{(U), (D_1D_2), (D_3)\}$ denotes the coalition structure in which only divisions D_1 and D_2 cooperate whereas divisions U and D_3 make their decisions in the non-cooperative setting; and so on.

For a coalition structure, division(s) who join a coalition choose the globally-optimal *retail pricing* decisions that maximize the total profit in the coalition; and division(s) who do not join any coalition choose the Stackelberg equilibrium as their *retail* and *transfer pricing* decisions. As Theorem 2 indicates, if a coalition involves the upstream division U and one or more downstream divisions, these divisions in the coalition only need to determine *retail pricing* decisions to maximize their total profit. Their transfer pricing decisions should be determined by using Theorem 6 so as to allocate their total profit fairly.

In addition, one may note that, for some coalition structures (e.g., $\{(U), (D_1), (D_2), (D_3)\}$, $\{(U), (D_1D_2), (D_3)\}$, etc.) in Table 1, optimal decisions are the same; this occurs because, in those coalition structures, the upstream division does not cooperate with any downstream divisions and all divisions' decisions are Stackelberg equilibria.

Coalition	**Optimal Decisions**					
Structure	T_1	T_2	T_3	p_1	p_2	p_3
$\{(U), (D_1), (D_2), (D_3)\}$	\$102.01	\$227.02	\$57.00	\$151.00	\$338.51	\$83.50
$\{(U), (D_1D_2), (D_3)\}$	\$102.01	\$227.02	\$57.00	\$151.00	\$338.51	\$83.50
$\{(U), (D_1), (D_2D_3)\}$	\$102.01	\$227.02	\$57.00	\$151.00	\$338.51	\$83.50
$\{(U), (D_1D_3), (D_2)\}$	\$102.01	\$227.02	\$57.00	\$151.00	\$338.51	\$83.50
$\{(UD_1), (D_2), (D_3)\}$	–	\$226.52	\$56.52	\$101.52	\$338.26	\$83.26
$\{(UD_2), (D_1), (D_3)\}$	\$101.54	–	\$56.56	\$150.76	\$226.54	\$83.28
$\{(UD_3), (D_1), (D_2)\}$	\$101.50	\$226.50	–	\$150.75	\$338.25	\$56.50
$\{(UD_1D_2), (D_3)\}$	–	–	\$56.29	\$101.29	\$226.29	\$83.14
$\{(UD_1D_3), (D_2)\}$	–	\$226.24	–	\$101.24	\$338.12	\$56.24
$\{(UD_2D_3), (D_1)\}$	\$101.29	–	–	\$150.63	\$226.27	\$56.27
$\{(U), (D_1D_2D_3)\}$	\$102.01	\$227.02	\$57.00	\$151.00	\$338.51	\$83.50
$\{(UD_1D_2D_3)\}$	–	–	–	\$101.02	\$226.02	\$56.02

Table 1: All divisions' optimal decisions in all possible coalition structures.

We notice from Table 1 that all divisions' retail prices and transfer prices decrease when more divisions cooperate, as indicated in Theorem 2.