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SELLING TO REFERENCE-DEPENDENT CONSUMERS

LEUNG HIN SHING

MPHIL

LINGNAN UNIVERSITY

2020

SELLING TO REFERENCE-DEPENDENT CONSUMERS

by
LEUNG Hin Shing
梁軒誠

A thesis
submitted in partial fulfillment
of the requirements for the Degree of
Master of Philosophy in Economics

Lingnan University

2020

ABSTRACT

Selling to Reference-dependent Consumers

by

LEUNG Hin Shing

Master of Philosophy

This research studies the optimal price charged by a monopoly firm when it is facing reference-dependent consumers (Kőszegi and Rabin, 2006). In the model, a consumer's total utility from purchasing a good consists of intrinsic utility and gain-loss utility, the latter of which is determined by an exogenously determined reference point. The firm's optimal pricing depends on the level of the reference point. In particular, the firm optimally adopts a gain-type pricing strategy when the reference point is low. In this equilibrium, all consumers who purchase enjoy gain utility (gain-type consumers). When the reference point is high the firm optimally chooses a loss-type pricing strategy. In this equilibrium, some consumers who purchase incur loss utility (loss-type consumers). Moreover, the firm's equilibrium profit decreases when the reference point increases. Finally, I discuss various extensions such as selling cost, participation cost and competition.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

SIGNED

LEUNG Hin Shing
Date: 3rd Mar 2021

CERTIFICATE OF APPROVAL OF THESIS

SELLING TO REFERENCE-DEPENDENT CONSUMERS

by

LEUNG Hin Shing

Master of Philosophy

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CONTENTS

LIST OF TABLES

Table 1. Effect of the Change in α, λ, θ to the Optimal Price, Demand, and Profit for Different θ

		α	λ	θ
$\theta \leq \frac{\alpha+1}{\alpha+2}$	${P_G}^*$	+		-
	${Dd_G}^*$	-		-
	${\pi_G}^*$	+		-
$\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$	${P_\theta}^* = \theta$			+
	${Dd_\theta}^* = 1-\theta$			-
	${\pi_\theta}^* = \theta(1-\theta)$			-
$\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$	${P_L}^*$	+	+	-
	${Dd_L}^*$	-	-	-
	${\pi_L}^*$	+	+	-

I. INTRODUCTION

When a consumer is making a decision on purchasing a good, he would consider the utility received from the good versus the cost of obtaining the good. The utility is generated from the intrinsic value of the good. However, in reality, we sometimes cannot know the true value of the good before we really come to the shop and examine the good. For example, we can only know the functions and specifications of a new iPhone model from the Apple Conference and the website provided by Apple Inc. The iPhone is not be released to the market so we can only have certain beliefs on the functions of the product. This would constitute a reference point, which is a level of "expectation" on the true intrinsic value of that iPhone formed before examining it. When a consumer go to the store, he could examine the iPhone model and know the true intrinsic value of it. Sometimes we would compare the true intrinsic value of the iPhone with our belief or expectation (the reference point) for making a decision of a purchase. When we found that the true intrinsic value of the iPhone is greater than our belief, we would consider it as a "gain". We would consider such gain as a "surprise" and it would generate extra utility to us. In order words, such comparison between the intrinsic value and the expectation of the intrinsic value of the iPhone would add extra utility we received from purchasing it. On the other hand, if the true intrinsic value of the iPhone is lower than the belief, we might not purchase the iPhone as we would consider such transaction is "lower than our expectation" on the product. However, sometimes in reality even though the true value of the product is lower than our belief, we would still decide to buy it as long as the total utility received is greater than the cost of obtaining the good. In this case, although we may regard this purchase as a "loss", such loss would not prevent us from purchasing the product. It is different from the traditional economics theory which states that consumers would only compare the intrinsic value (which generates utility) with the price (which is the cost of the good), and then make a decision of purchase. In this case we would consider the situation that there is a belief (or expectation) of the intrinsic value of the product before knowing the true intrinsic value of the good, and consumers would compare the true intrinsic value with their beliefs. Such belief is regarded as the reference point, and the reference point represents consumers are reference-dependent in reality. Also, the assumption of the comparison between the true intrinsic value and the reference point by a consumer implies that the total utility received from purchasing a good by a reference-dependent consumer consists of two parts: (i) utility generated from the true intrinsic value of good, and (ii) utility generated from comparing the intrinsic value with the reference point. It serves an important basis for studying the optimal price set by a monopoly when it faces reference-dependent consumers. The reference point can be formed from various sources. The case mentioned previously states that the reference point is constituted by those information provided by a firm. As the new iPhone model is not launched to the market yet, those information such as functions and specifications of the iPhone, are all provided by the firm. Consumers could only access to those information provided by the firm and form the reference point. In reality, in addition to the information provided by the firm, there are some other sources which would constitute a formation of a reference point, such as the consumers' expectations towards the products (which might not necessarily involve the information provided from firm), status quo (for example, the value received from the product they are currently using), past events (previous purchase of the good from the same firm), or product from another firm (similar good which is offered by another firm) etc. Although there are various sources of formation of a reference point, from consumers' perspective, the decision of the purchase made by a consumer involves merely comparing the true intrinsic value with the reference point no matter how the source of the reference point was formed.

A monopoly firm, who would like to maximize the profit when facing reference-dependent consumers, now has to consider extra variables such as the reference point and the factors that affect the magnitude of reference point such that they would influence the utility received by the consumers. It is because those factors would affect the demand faced by the firm. Then firm has to set the optimal price, based on the reference point observed by the firm. As it is mentioned that consumers would consider their purchase as a gain or a loss, the demand faced by the firm might include these two types of consumers. Different prices set by the firm under a same level of reference point could lead to two scenarios. Firstly, the demand, according to the price set by the firm, consists of both two types of consumers. Another scenario would be, the price set by firm might drive out those consumers who would consider the transaction as loss from the market, and thus the demand consists of those consumers who would consider the transaction as gain. Therefore, to maximize the profit, the firm has to consider the demand as a result of the price it set for given a level of reference-dependence. If the firm found that it is more profitable to set a price such that all consumers are holding a belief on the good that is greater than the true intrinsic value of good, then it would be the optimal price for the firm. Therefore, the optimal pricing strategies of the firm depends on the reference point formed by consumers. In fact, consumers would prefer to have a low reference point. The intuition is that, a low reference point could bring a high surprise (for a case of gain) or low disappointment (for a case of loss) as a result. However, the relationship between the reference point and the profit received by firm might not be so obvious. Is a low reference point beneficial to the firm? In reality, we often observe that a firm would make effort on keeping consumers' high "expectation" towards its product. For example, through spending a huge amount on advertisement to promote the product or spending some expenditures to maintain a good reputation in order to rise the reference point of consumers. It seems that there is an incentive for a firm to keep a high reference point, while consumers would prefer a low reference point. Therefore, the questions would be, firm would prefer a higher or a lower reference point in order to maximize the profit? Also, accordingly, how would the firm set the price under different levels of reference point?

They provide the motivation to address these questions. This research proposes a model of pricing strategies for a firm when it faces reference-dependent consumers. This research focuses on the following questions including (i) What would be the pricing strategies for a corresponding reference point? (ii) Firm would prefer a higher or lower reference point to maximize the profit? (iii) What would be the optimal level of the reference point and the corresponding pricing strategy when the reference point can be chosen by the firm? (iv) How to connect the model to real-life situation? This thesis is organized as follow, section II is the literature review. It includes the researches which are related to the topic of reference-dependent consumers and pricing strategies that provide inspirational insights for this thesis. In section III I will present the proposed model for maximizing the profit received by the firm and the corresponding price (the optimal pricing strategies) when consumers are reference-dependent. The details of the model would be explained in this section. Section VI is analysis, which includes the contents of deriving the optimal pricing strategies, the optimal demand, and thus the maximized profit using the proposed model, with certain assumptions be mentioned in this section. Section V is discussion. In this section I will examine the results with the results shown in the previous section when some variations are added into the model, such as assuming the reference point is now endogenously determined. In addition, I will connect the model to the real-life situation and try to explain the behaviours of firm which are related to the research questions by considering several scenarios. Lastly, section VI is the conclusion. I would summarize the results of my research about those findings and the research questions.

II. LITERATURE REVIEW

The model proposed by this study is mainly based on the concept of a reference point, which was mentioned in the research done by Kahneman and Tversky (1979). In their work, they proposed the prospect theory and the model of reference dependence, which stated that people usually would consider an outcome as a gain or a loss, rather than their ultimate personal wealth or welfare. Also, they proposed a concept of loss-aversion, which is meant by the utility received by a person would be reduced with a greater magnitude when facing a loss than the increased utility received when facing a gain, for the same amount of changes in actual wealth. They illustrated this by considering a value function, which is concave for the domain of gain while convex for the domain of loss. The function is steeper in loss domain than gain domain. Later, there was another research done by Kahneman and Tversky (1991). They presented a model of reference-dependent theory in their work, which is a theory of consumer choice and they held the assumption of loss-aversion in their model. For this research, the concept of loss-aversion would be captured in the model, associated with the reference point. However, the model presented in this study focuses on firm's side, which is related to the optimal price set by a monopoly firm when facing reference-dependent consumers.

In addition, Kőszegi and Rabin (2006) proposed a model of reference-dependent preference, which extended and modified the model proposed by Kahneman and Tversky. They suggested that the total utility received by a person from a transaction depends on both "consumption bundle" and "reference bundle", which refer to an intrinsic "consumption utility" of the good and "gain-loss utility" that is derived by considering the consumption utility with the reference point. Their research provides an insight for developing the framework of my research. The model in this study follows their approach and divides the utility into two parts, such that the total utility received by a consumer consists of the intrinsic value and the gain-loss utility. However, instead of focusing on consumer choices, this research would like to study the firm's side decision making. This research tries to figure out the optimal prices set by a firm under different reference point, and the total utility received by a reference-dependent consumer would be related to the optimal price. There are several studies done by some researchers which focus on monopoly and loss-averse consumers. For example, the work done by Spiegler (2012) assumed that all consumers faced by a monopoly are loss-averse. It was found that a monopoly firm's optimal pricing that causes an unexpected rise in price would reduce consumers' willingness to pay. This might suggest that a rise in reference point would reduce the profit received by a model firm, the intuition is similar to a higher reference point would reduce the "gain-utility" (or rise "loss-disutility") to a gain-consumer (or loss-consumer). However, the dimension of the reference point in my model is different to that of in Spiegler's (2012) model. The reference point in my model refers an expectation on the intrinsic value of good offered by a firm, while Spiegler's (2012) focused on the expectation on price (reference price). There was another research done by Heidhues and Kőszegi (2014), which focused on pricing strategy by a monopolist and loss-averse consumers. It suggested that consumers' loss-aversion might lead to an introduction of risk into the pricing strategies by the firm. In their work, they proposed two sets of pricing strategies for any degree of loss-aversion of consumers. In my research, there are two sets of pricing strategies would be proposed. However, those two pricing strategies are based on the types of consumers faced by a firm. Also, the focus of my research is different from the work done by Heidhues and Kőszegi (2014). My model focuses on the optimal pricing strategies for a given level of reference-dependence. I would examine the resulting profits due to different prices set by the firm, without introducing a risk into my model. Moreover, Rosato (2016) studied the optimal pricing strategies and product-availability strategies for a monopoly firm who is facing loss-averse consumers and selling two goods which are substitutes to each other. In the research, it was found that the firm could maximize the profit by raising the reference point of consumers. Rosato's (2016) research is related to my study in the sense that it was studying the how the reference point could affect the profit received by firm. However, we have different settings in the model, as Rosato (2016) assumed that the firm is selling two products which are substitutes to each other, while there is only one product sold by a firm in my model. However, as Rosato (2016) concluded that raising the reference point by a firm could maximized the profit received, it is still worth to examine whether a same conclusion could be drawn in my model, which is now involves one product sold by firm, rather than two products. This would answer the research question on firm’s preference of the (exogenously determined) reference point, based on the assumptions and settings in my research. Also, it seems that those works were mainly focusing on loss-averse consumers (or some would assume consumers are loss-averse), instead of considering a monopoly firm which faces both gain-type and loss-type consumers. This research aims to study the firm’s optimal price for a given reference point. It is assumed that consumers are reference-dependent, instead of merely assuming all are loss-averse. Therefore, this research could add some contributions to the existing literatures which would assume all consumers are loss-averse.

Although this research does not cover the formation of a reference point, it is worth to take those researches which are related to the formation of the reference point as reference. It is because they might provide some useful insight for my research to connect my model to reality. According to literatures, reference point formation is ambiguous. Also, there were different sources of reference point formation suggested by different researches. Pesendorfer (2006) even argued that researchers might treat the reference point as a free variable such that it is chosen to match phenomenon or behaviors observed. Although Kahneman and Tversky (1979) mentioned the idea of the reference point, the formation of reference point was not be discussed. Even in the later study done by Kahneman and Tversky (1991), in which they proposed a theory of reference-dependent for analyzing consumer’s choice, the formation of reference point was still not be mentioned. However, Samuelson and Zeckhauser (1988) referred status quo as a reference point. They stated that a decision maker would consider either to do nothing or to stick with previous or current decision. In their work, they conducted a series of experiments to support their view of using status quo as a reference point. In the experimental research done by Knetsch (1989) which focused on endowment effect, there were some results with experimental evidences which showed that subjects were loss-averse associated with status quo. Knetsch (1989) interpreted those results which supported the view of equating the reference point as status quo. However, in the influential model presented by Kőszegi and Rabin (2006), the reference point is assumed to be formed by recent expectations. They proposed that the reference point was a probabilistic belief that a person held recently regarding the outcomes. They argued that referring status quo as a reference point is based on an assumption such that people would expect to stay at status quo. However, when there is a difference between an expectation and the status quo, Kőszegi and Rabin (2006) would prefer to equate the reference point as an expectation. Ericson and Fuster (2009) conducted an experimental research, in which they induced expectations exogenously. Although the focus of their research is associated with endowment effect experiment, their results supported that the reference point was determined by expectations. In addition, Baucells et al. (2008) proposed a model with several experiments in a financial context to study the formation of a reference point and its updating. In their work, they assumed that the reference point is a function which includes all available information. This available information is weighted by a function called information weighting function. Their experimental results showed that the purchased stock price and the current stock price would receive the highest weighting that affect the reference price the most. However, there were other researches associated with the financial context which regarded reference point as a past event. In the research done by Chen and Rao (2002), they regarded reference point as a stimulus and studied the change in reference point to explain the phenomenon of surprising reversal. Besides, Compete and Jehiel (2003) applied an idea of the reference point in their bargaining model and assumed the reference point is affected by prior offers. However, these researches which incorporate reference points in their models are context specific. In spite of context specific, the formation of reference points under different situations are still ambiguous. Lastly, there was another source of the reference point formation, which regards the reference point as the first product a consumer had considered. The research done by Zhou (2012) focused on the implications of consumers' reference-dependence and the impacts on market competition. It was found that when consumers regard a good, such as the first good they had considered, as a reference point and use it to evaluate other goods, consumers' loss-aversion could be observed in such case.

This result shows that a reference point can be regarded as a product that a consumer had considered, and that product might not necessarily be purchased before (i.e. a good purchased in the past that is not exactly the same). Although in this research I would not focus on the discussion of the reference point formation, it is worth to understand various sources of reference point formation such that it might help connecting the model into reality. In this study, the reference point is regarded as the expectation on the intrinsic value of good offered by a monopoly firm. Later in the session of Hotelling competition, the reference point refers to the expectation on the preference of purchasing the good from a particular firm. It would be explained in details in that session.

Lastly, the reference point management by the firm is another focus in this research. The reference point is one of the variables that affects the maximized profit. It would also affect the optimal price set by firm. Therefore, it is worth to examine how the firm influences the reference point such that the profit could be maximized. In Karle's (2013) research, it was suggested that a firm can influence the willingness to pay of loss-averse consumers through informative advertising rather than price. Karle (2013) found that a partial information disclosure by a monopolistic firm would lead to the greatest consumers' willingness to pay, which is optimal to the firm. Inspired by the research of Karle (2013), this study would examine the factors that affect the reference point, and how those factors would affect the maximized profit received by a firm. It might answer why sometimes a firm would prefer a certain level of consumers' expectation towards its product in reality.

III. THE MODEL

Consider a good generates a value $v \sim U[0,1]$, where the value v represents the intrinsic value of the good received by a consumer. Consumer receive utility $u = v$ and they would only purchase the good as long as $u \geq P$, where P is the price set by a monopoly firm. The demand for the good is $Dd = 1 - v$ and thus the profit received by the monopoly firm $\pi = P \times Dd$. As the firm sets an optimal price $P = P^*$to maximize profit π^*. In this case, solving for the optimal price and the corresponding profit, we have

$$P^* = \frac{1}{2} \tag{1}$$

$$Dd^* = \frac{1}{2} \tag{2}$$

$$\pi^* = \frac{1}{2} \tag{3}$$

This is called a "standard model" and (1) is the optimal pricing strategy when the firm does not consider consumers are reference-dependent. However, when the reference point $\theta \in [0,1]$ is introduced into the model for analysis, the monopoly firm has to consider its consumers are reference-dependent. The utility function faced by a single consumer would become

$$u = \begin{cases} v + \alpha(v - \theta) & if \quad v > \theta \\ \theta & if \quad v = \theta \\ v - \alpha\lambda(\theta - v) & if \quad v < \theta \end{cases} \tag{4}$$

The total utility received by a consumer consists of two parts, the *intrinsic value* and the *gain-loss utility*. For the gain-loss utility, there is a reference-dependent multiplier $0 < \alpha < 1$ for both of the situations of gain ($v > \theta$) and loss ($v < \theta$) faced by a consumer. Additionally, there is a loss-averse multiplier $\lambda > 1$ for the situation of loss, due to the assumption that consumers are loss-averse. Under this model, it is assumed that there are two types of consumers, namely gain-type and loss-type, and a consumer is either gain-type or loss-type. A gain-type consumer receives the intrinsic value of the good that is greater than the reference point formed before the purchase is made, which is represented by $v > \theta$, while a loss-type consumer receives the intrinsic value that is less than the reference point, which is represented by $v < \theta$. It is worth to notice that loss-type consumers would still purchase the good from firm as long as they receive total utility $u \geq P$, even if they regard the purchase as a loss, that is when the intrinsic value of good is less than the reference point formed prior to the purchase ($v < \theta$).

The utility function shows that the reference point θ would only affect the gain-loss utility, and both of the gain-type consumers and the loss-type consumers would prefer a low θ as possible because it adds extra gain-loss utility and thus it brings a higher total utility received by a gain-type consumer as a result. This would be the situation that is regarded as a "surprise" faced by a gain-type consumer which adds extra utility received by him, and such "surprise" would be higher when the original expectation on a product is low, which is θ is low. It is worth to notice that the term "expectation" used in this research does not refer to expected value. Instead, it is interchangeable with "reference point". However, for loss-type consumer, as the gain-loss utility would reduce the total utility received, a higher θ would lead to a higher reduction in total utility received by a loss-type consumer as a result. This would be the situation such as a high "disappointment" faced by a loss-type consumer which reduces utility received by him. The "disappointment" would be higher when that consumer initially held expectation on the product, which means that the reference point θ is high. In other words, the "disappointment" for loss-type would be reduced when θ is low. Therefore, both gain-type consumers and loss-type consumers would prefer the reference point θ as low as possible in order to receive more total utility. This is the intuition of low reference point is always preferred by consumers.

The firm sets $P = P(\alpha, \theta, \lambda)$ to maximize the profit $\pi = \pi(P)$. The firm can freely set any price P but there might be an optimal pricing strategy which depends on α, θ and λ, which would be different from the standard model as (1). The demand faced by the firm can still be derived by $Dd = 1 - v$, but now Dd would also depend on α, θ and λ, because the derivation of demand involves P. Therefore, the maximized profit received by the firm might depend on the price set by monopoly firm which is required to consider the variables α, θ and λ.

The sequence of the purchase:

1. Consumers' form a reference point θ towards a good
2. Firm sets a price P
3. Consumers examine the intrinsic value v of the good
4. Consumers refer to the utility function they are facing and make the decision of purchase if total utility received $u \geq P$

IV. ANALYSIS

The model defines the two types of consumers faced by the monopoly firm, namely the gain-type consumer and loss-type consumer. It is assumed that a consumer is either gain-type consumer or loss-type consumer, depending on the intrinsic value v and the level of reference-dependence θ.

For the monopoly firm who faces the reference-dependent consumers to maximize the profit, the firm has to consider the demand for the good after the price P is set. It is assumed that the reference point θ is exogenously determined and the product cost is zero cost faced by the firm. To derive the demand Dd corresponding to the price P set by the firm, it is required to consider the critical consumer. The critical consumer is indifferent from deciding to purchase or not to purchase when the firm sets the price P, as this consumer receives zero utility when $u = P$. The critical consumer gain-type receives the value, which is

$$\widehat{v_G} = \frac{P + \alpha\theta}{1 + \alpha} \tag{5}$$

Similarly, the critical loss-type consumer will receive

$$\widehat{v_L} = \frac{P + \alpha\theta\lambda}{1 + \alpha\lambda} \tag{6}$$

Then the demand for good can be derived by $Dd = 1 - \widehat{v_L}$ and thus the profit π can be calculated. There are two sets of pricing strategies proposed in this study which are derived from the utility function in (4), namely gain-type pricing strategy and loss-type pricing strategy, based on gain-type consumers and loss-type consumers respectively. It is worth to notice that when the price is set by the firm, there are only two cases: (i) the demand consists of gain-type consumers only (all consumers are gain-type consumers), or (ii) the demand consists of both gain-type consumers and loss-type consumers. Therefore, the fundamental difference between the gain-type pricing strategy and loss-type pricing strategy is that, the gain-type pricing strategy involves serving all gain-type consumers only, while the loss-type pricing strategy involves serving both gain-type consumers and (some of) loss-type consumers.

Gain-type Pricing Strategy

This is one of the pricing strategies for a monopoly firm who faces reference-dependent consumers. As a consumer is assumed to be either a gain-type consumer or a loss-type consumer, this pricing strategy refers to the optimal price set by the firm such that all consumers are gain-type consumers only, for a given level of reference-dependence θ. In other words, it is the price when all loss-type consumers are driven out from the market. Denote $P_G \in [0,1]$ as the price set by firm when the gain-type pricing strategy is adopted. To find the optimal $P_G = {P_G}^*$, we have to consider the critical gain-type consumer who receives the total utility $\hat{u} = {P_G}^*$ would receive the value $\widehat{v_G}$, that is

$$\widehat{v_G} = \frac{{P_G}^* + \alpha\theta}{1 + \alpha} \quad (7)$$

Based on (7), we could derive the maximized profit ${\pi_G}^*$, based on the optimal price ${P_G}^*$ and the demand the firm faced ${Dd_G}^* = 1 - \widehat{v_G}$, so we have

$${P_G}^* = \frac{\alpha - \alpha\theta + 1}{2} \quad (8)$$

Substituting (8) into (7), we can rewrite the critical consumer $\widehat{v_G}$ from (7) and derived demand ${Dd_G}^*$as

$$\widehat{v_G} = \frac{\alpha + \alpha\theta + 1}{2(1 + \alpha)} \quad (9)$$

$$Dd^* = \frac{\alpha - \alpha\theta + 1}{2(1 + \alpha)} \quad (10)$$

Then, using (8) and (10) to calculate the optimal profit ${\pi_G}^*$, we have

$${\pi_G}^* = \frac{(\alpha - \alpha\theta + 1)^2}{4(1 + \alpha)} \quad (11)$$

From (8), the derivatives of $P_G{}^*$ with respect to α and θ are

$$\frac{dP_G{}^*}{d\alpha} = \frac{1-\theta}{2} \tag{12}$$

$$\frac{dP_G{}^*}{d\theta} = -\frac{\alpha}{2} \tag{13}$$

The derivatives show that the optimal gain-type price $P_G{}^*$ increases as the gain-loss multiplier α increases ($\frac{dP_G{}^*}{d\alpha} > 0$), while $P_G{}^*$ decreases as the level of reference-dependence of consumers θ increases($\frac{dP_G{}^*}{d\theta} < 0$). The result suggests that the firm adopts gain-type pricing strategy would set a high price when consumers care more about the gain-loss utility (α increases). However, the firm would set a lower price when the consumers' reference point towards the product is high (θ increases).The intuition is that, a higher α would bring more gain-utility to a gain-type consumer, which enables the firm to set a higher price $P_G{}^*$, while a higher θ bring would less gain-utility to a gain-type consumer, and thus the price has to set a lower price $P_G{}^*$.

To take the derivatives of demand $Dd_G{}^*$ from (10) with respect to α and θ respectively, we have

$$\frac{dDd_G{}^*}{d\alpha} = -\frac{\theta}{2(\alpha+1)^2} \tag{14}$$

$$\frac{dDd_G{}^*}{d\theta} = -\frac{\alpha}{2(\alpha+1)^2} \tag{15}$$

The derivatives of the demand $Dd_G{}^*$under the optimal gain-type price $P_G{}^*$ shows that the demand $Dd_G{}^*$ decreases whenever either α or θ increases ($\frac{dDd_G{}^*}{d\alpha} < 0$ and $\frac{dDd_G{}^*}{d\alpha} > 0$). These result suggest that the demand for the good decreases when consumers would care more about the gain-loss utility received for a purchase, or when consumers would hold a higher expectation (i.e. reference point) towards the good. It makes sense because a higher reference point would reduce the number of gain-type consumers in the market (i.e. less consumers face $v > \theta$ when θ increases), so the monopoly firm who adopts the gain-type pricing strategy (to sever gain-type consumers only) would serve fewer consumers. As a result, the demand faced by the firm would be lower.

Lastly, for the profit ${\pi_G}^*$ from (11), the derivatives of the maximized profit under gain-type pricing strategy with respect to α and θ, would be

$$\frac{d{\pi_G}^*}{d\alpha} = \frac{[(\theta - 1)\alpha - 1][(\theta - 1)\alpha + 2\theta - 1]}{4(\alpha + 1)^2} \tag{16}$$

$$\frac{d{\pi_G}^*}{d\theta} = \frac{\alpha(\alpha\theta - \alpha - 1)}{2(\alpha + 1)} \tag{17}$$

The derivatives of the profit shows that ${\pi_G}^*$ increases as α increases ($\frac{d{\pi_G}^*}{d\alpha} > 0$), while ${\pi_G}^*$ decreases as θ increases ($\frac{d{\pi_G}^*}{d\theta} < 0$). From (12) and (14), it can be concluded that the combining effect on profit ${\pi_G}^*$ from the increase in price is greater than that of the decrease in demand, as a result of an increase in gain-loss multiplier, and thus it increases in profit ${\pi_G}^*$. However, as an increase in reference point would cause both price and demand to fall, the profit ${\pi_G}^*$ would decrease when θ increases. The result suggests that when the firm adopts gain-type pricing strategy to set the price, it would prefer a low reference point θ formed by consumers.

Loss-type Pricing Strategy

Different from the gain-type pricing strategy set by a monopoly firm such that it serves gain-type consumers only, the loss-type pricing strategy involves setting the optimal price by a monopoly that serve both gain-type and some of the loss-type consumers, when consumers are considered as reference-dependent, for a given level of reference-dependence θ. To explain why it is impossible to set a price such that there are loss-type consumers only, consider a given θ, the gain-type consumers are those who face $v > \theta$, and the demand from the gain-type consumers would be $1 - \theta$. The loss-type consumers face $v < \theta$, and the demand from this group of consumers is $\theta - \widehat{v_L}$. Combining them and we would have the total demand $Dd_L = 1 - \widehat{v_L}$. Therefore, when loss-type pricing strategy is adopted by a monopoly firm to set price ${P_L}^*$ for a given θ, the corresponding demand must include both gain-type consumers and loss-type consumers.

Considering the firm sets $P = P_L$, and the critical loss-type consumer who receives the utility $\hat{u} = P_L{}^*$ would receive the value $\widehat{v_L}$, that is

$$\widehat{v_L} = \frac{P_L{}^* + \alpha\lambda\theta}{1 + \alpha\lambda} \tag{18}$$

As the total demand, which consists of both gain-type consumers and loss-type consumers, is just $Dd_L = 1 - \widehat{v_L}$. According to (18), the maximized profit $\pi_L{}^*$ and the corresponding price $P_L{}^*$ and demand $Dd_L{}^*$could be derived from the corresponding critical loss-type consumer $\widehat{v_L}$, as shown below:

$$P_L{}^* = \frac{\alpha\lambda - \alpha\lambda\theta + 1}{2} \tag{19}$$

$$\widehat{v_L} = \frac{\alpha\lambda + \alpha\lambda\theta + 1}{2(\alpha\lambda + 1)} \tag{20}$$

$$Dd_L{}^* = \frac{\alpha\lambda - \alpha\lambda\theta + 1}{2(\alpha\lambda + 1)} \tag{21}$$

$$\pi_L{}^* = \frac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(\alpha\lambda + 1)} \tag{22}$$

To examine how would the optimal loss-type price $P_L{}^*$, the demand $Dd_L{}^*$ and maximized profit $\pi_L{}^*$ change with the gain-loss multiplier α, the loss-averse multiplier λ, and the reference point θ, we can take the first-order derivatives for $P_L{}^*$, $Dd_L{}^*$ and $\pi_L{}^*$ with respect to α, λ and θ.

The derivatives of the optimal loss-type price $P_L{}^*$ from (19) with respect to α, λ and θ, would be

$$\frac{dP_L{}^*}{d\alpha} = \frac{\lambda(1 - \theta)}{2} \tag{23}$$

$$\frac{dP_L{}^*}{d\lambda} = \frac{\alpha(1-\theta)}{2} \tag{24}$$

$$\frac{dP_L{}^*}{d\theta} = -\frac{\alpha\lambda}{2} \tag{25}$$

The results of the derivatives of $P_L{}^*$ with respect to α, λ and θ show that the optimal price $P_L{}^*$ increases when either α or λ increases ($\frac{dP_L{}^*}{d\alpha} > 0$ and $\frac{dP_L{}^*}{d\lambda} > 0$), while $P_L{}^*$ decreases when θ increases ($\frac{dP_L{}^*}{d\theta} < 0$). The optimal price PL* will increase when gain-loss multiplier increases, or when loss-averse multiplier increases. They could be the cases that the optimal price set by the monopoly firm increases when consumers care more about their gain-loss utility, which is similar to the result of gain-type pricing strategy. However, it is worth to mention that different from the gain-type consumer, the loss-type consumer would receive less utility when the purchase is regarded as loss. Recall the utility function mentioned in (4), the reduction in the total utility received by a loss-type consumer (i.e. the term of gain-loss utility $-\alpha\lambda(\theta - v)$) would be enlarged when α or λ increases. For the reference point θ, the result is similar to the gain-type pricing strategy, the optimal price $P_L{}^*$ decreases reduced when θ increases. A higher the reference point θ would enables the monopoly firm to set a higher price, as loss-type consumers would receive less disutility for a higher θ.

For the demand $Dd_L{}^*$ in (21) faced by the firm for adopting loss-type pricing strategy, the derivatives would be

$$\frac{dDd_L{}^*}{d\alpha} = -\frac{\lambda\theta}{2(\alpha\lambda+1)^2} \tag{26}$$

$$\frac{dDd_L{}^*}{d\lambda} = -\frac{\alpha\theta}{2(\alpha\lambda+1)^2} \tag{27}$$

$$\frac{dDd_L{}^*}{d\theta} = -\frac{\alpha\lambda}{2(\alpha\lambda+1)} \tag{28}$$

The derivatives of the demand $Dd_L{}^*$ with respect to α, λ and θ are all negative ($\frac{dDd_L{}^*}{d\alpha} < 0$, $\frac{dDd_L{}^*}{d\lambda} < 0$ and $\frac{dDd_L{}^*}{d\theta} < 0$). The results suggest that the demand decreases when there is an increase in the gain-loss multiplier α, the loss-averse multiplier λ, or the reference point θ. These results are consistent with the results of the gain-type pricing strategy that taking the derivatives of demand with respect to α and θ). As mentioned previously that both α and λ represent the size of the reduction in the utility received by the loss-type consumers, so it follows that an increase in α and λ will enlarge the effect of reducing utility and thus the demand for good decreases.

Lastly, for the derivatives of the maximized profit ${\pi_L}^*$ from (22) with respect to α, λ and θ, we have

$$\frac{d{\pi_L}^*}{d\alpha} = \frac{\lambda[(\theta-1)\alpha\lambda-1][(\theta-1)\alpha\lambda+2\theta-1]}{4(\alpha\lambda+1)^2} \tag{29}$$

$$\frac{d{\pi_L}^*}{d\lambda} = \frac{\alpha[(\theta-1)\alpha\lambda-1][(\theta-1)\alpha\lambda+2\theta-1]}{4(\alpha\lambda+1)^2} \tag{30}$$

$$\frac{d{\pi_L}^*}{d\theta} = -\frac{\alpha\lambda(\alpha\lambda\theta-\alpha\lambda-1)}{2(\alpha\lambda+1)} \tag{31}$$

The derivatives of the maximized profit ${\pi_L}^*$with respect to the gain-loss multiplier α and the loss-averse multiplier λ are positive ($\frac{d{\pi_L}^*}{d\alpha} > 0$ and $\frac{d{\pi_L}^*}{d\lambda} > 0$), while the derivatives of the maximized profit ${\pi_L}^*$ with respect to the reference point θ is negative ($\frac{d{\pi_L}^*}{d\theta} < 0$) The result is similar to the gain-type pricing strategy which suggests that maximized profit increases as α increases, while the profit decreases as θ increases. It could by explain by due to the total demand consists of not only loss-type consumers but also gain-type consumers, and both loss-type consumers and gain-type consumers prefer θ as low as possible.

It is important to highlight that the monopoly firm could set the price P freely, for a given level of reference-dependence θ, which cannot be chosen by the monopoly firm. The two pricing strategies mentioned above suggest the optimal price set by firm to maximize for a given θ. For a given θ, the monopoly firm can set any price and decide to serve only gain-type consumers, or both gain-type and loss-type consumers. The following tries to find out the optimal price set by firm in three intervals of the level of reference-dependence θ.

When $\boldsymbol{\theta} \leq \frac{\boldsymbol{\alpha}+\mathbf{1}}{\boldsymbol{\alpha}+\mathbf{2}}$

Firstly, the maximum $\theta = \frac{2}{3}$ as $0 < \alpha < 1$. The monopoly firm would maximize the profit by setting the price $P = P_G{}^*$ using the gain-type pricing strategy and receives the corresponding profit $\pi = \pi_G{}^*$ within this interval. As mentioned in (11), the gain-type pricing strategy, the profit received by the firm is

$$\pi_G{}^* = \frac{(\alpha - \alpha\theta + 1)^2}{4(\alpha + 1)}$$

To prove that $P_G{}^*$ is optimal, firstly, consider any price $P \neq P_G{}^*$. Assuming $P = \varepsilon\theta$, where ε represents the factor that measures the different between the price set by the monopoly without using the gain-type pricing strategy and the reference point such that $\varepsilon = \frac{P}{\theta}$, and we firstly assume that $P \neq \theta$ so $\varepsilon \neq 1$. For the price P we have the corresponding critical value for the gain-type consumer $v = \frac{\varepsilon\theta+\alpha\theta}{1+\alpha}$. As there is a critical value for the reference point $\theta = \frac{\alpha+1}{\alpha+2}$ within this interval, we would have the corresponding $v = \frac{\alpha+\varepsilon}{\alpha+2}$ and thus the profit

$$\pi = \frac{\theta}{\alpha + 2}\varepsilon(2 - \varepsilon) \tag{32}$$

and we have $0 < \varepsilon < 2$. The maximum value for the term $\varepsilon(2 - \varepsilon) = 1$ when $\varepsilon = 1$ (i.e. $P = \theta$). As long as $\varepsilon \neq 1$, the profit would be less than the case when $P = \theta$. It also shows that, if the price set by the monopoly is different from the reference point (i.e. $\varepsilon \neq 1$) without using the gain-type pricing strategy, the resulting profit received by the monopoly would be lower than the profit received by using gain-type pricing strategy when $\theta \leq \frac{\alpha+1}{\alpha+2}$ for $0 < \varepsilon < 2$, i.e. $\pi < \pi_G{}^*$ for $P \neq P_G{}^*$. If we just consider $0 < \varepsilon < 1$, it at least ensures that any price $P \neq P_G{}^*$ and $P < \theta$ will not maximize profit.

So, what if $P > \theta$ (and $1 < \varepsilon < 2$)? Firstly, it could be true that $P_G{}^* \geq \theta$ such that $\frac{P_G{}^*}{\theta} < 2$ for $\theta \leq \frac{\alpha+1}{\alpha+2}$. However, excluding the cases for $P_G{}^*$ and for $\varepsilon < 2$, any price $P > \theta$ but $P \neq P_G{}^*$ would generate the corresponding profit $\pi < \pi_G{}^*$. Also, all of the loss-type consumers will be driven out from the market when $P > \theta$ for this interval. That means, there are only gain-type consumers. When the firm is facing gain-type consumers, for a given reference point θ, setting $P = P_G{}^*$ would generate the maximized profit $\pi_G{}^*$. It is worth to notice that (12) does not include the price as a variable, which implies that the maximized profit does not depend on the price set by firm, but α and θ instead. It means that, for $P > \theta$, as there are only gain-type consumers served by the monopoly firm, the maximized profit will be just equal to $\pi_G{}^*$.

For $P = \theta$, considering $\pi = \frac{\theta}{\alpha+2}\varepsilon(2-\varepsilon)$. As $\varepsilon = 1$, the maximum profit would be $\pi = \frac{\theta}{\alpha+2}$. It shows that the profit π increases as $\pi\theta$ increases. However, as the reference point is at most $\pi = \frac{\alpha+1}{\alpha+2}$ for this interval, the maximum $\pi = \frac{\theta}{\alpha+2}\varepsilon(2-\varepsilon)$ can be written as

$$\pi = \frac{\alpha+1}{\alpha+2} \tag{33}$$

assuming $\alpha = 0$, the maximum value would be $\pi = \frac{\alpha+1}{(\alpha+2)^2} = \frac{1}{4}$. As $0 < \alpha < 1$, the range for profit $\pi \subset (\frac{2}{9}, \frac{1}{4})$. In fact, the profit can also be written as $\pi = P \times (1 - \frac{P+\alpha\theta}{1+\alpha})$. As $P = \theta$, we would have $v = \theta$ and $\pi = \theta(1-\theta) = P(1-P) = v(1-v)$. It becomes the standard model mentioned previously and the maximum profit received would be just $\frac{1}{4}$. However, when we compare this profit π with the maximized profit by using gain-type pricing strategy $\pi_G{}^*$, as long as $0 \leq \theta \leq 1$, we would have $\frac{1}{4} \leq \pi_G{}^* \leq \frac{1}{2}$. Therefore, the above analysis shows that any price $P \neq P_G{}^*$ would generate the corresponding profit $\pi < \pi_G{}^*$. In other words, the gain-type pricing strategy would maximize the profit received by the monopoly firm for $\theta \leq \frac{\alpha+1}{\alpha+2}$.

For the loss-type pricing strategy, using the approach mentioned above directly cannot derive the profit received by the firm correctly because we would have the intrinsic value received all of the consumers $v \geq \theta$ (i.e. no loss-type consumers) under this interval $\theta \leq \frac{\alpha+1}{\alpha+2}$. The loss-type price ${P_L}^*$ would give the value received by the critical loss-type consumer $\widehat{v_L} > \theta$ if we follow the analytical framework under loss-type pricing strategy to find ${P_L}^*$ and the corresponding ${\pi_L}^*$ directly using (19) and (22). The result would be inaccurate as it violates the assumption of the loss-type pricing strategy such that there would are no consumers face the intrinsic value $v < \theta$. In fact the monopoly firm could set the price such that there are loss-type consumers for the price when $\theta \leq \frac{\alpha+1}{\alpha+2}$, but the profit of serving the loss-type consumers (together with gain-type consumers) would be strictly lower than ${\pi_G}^*$, as proved above any price P $\neq$ ${P_G}^*$ would generate the corresponding $\pi < {\pi_G}^*$.

In short, the monopoly firm can maximize the profit by setting the using the gain-type pricing strategy when facing reference-dependent consumers when the level of reference-dependence falls within $\theta \leq \frac{\alpha+1}{\alpha+2}$.

When $\boldsymbol{\theta} \geq \frac{\boldsymbol{\alpha\lambda+1}}{\boldsymbol{\alpha\lambda+2}}$

The monopoly firm would maximize the profit by setting the price $P = {P_L}^*$ using the loss-type pricing strategy and receives the corresponding profit $\pi = {\pi_L}^*$ within this interval. As mentioned in (18), we have the value received by the critical loss-type consumer such that

$$\widehat{v_L} = \frac{{P_L}^* + \alpha\lambda\theta}{1 + \alpha\lambda}$$

It assumes that the critical consumer is the loss-type consumer who faces $\widehat{v_L} \leq \theta$. If ${P_L}^* = \theta$, then the critical loss-type consumer faces $\widehat{v_L} = {P_L}^* = \theta$. To prove that ${P_L}^*$is optimal, consider any price $P \neq {P_L}^*$.When $P > \theta$ all loss-type consumers are driven out from the market, as none of them face the value $v < \theta$ but at the same would purchase from the shop because of the total utility received $u \geq P$. As there is no loss-type consumer in the market, all consumers are gain-type consumers. In this case, the corresponding profit π is derived by consider the price $P > \theta$, and the demand $(1 - v_G)$. However, the profit π is strictly less than ${\pi_L}^*$. That is, $\frac{(\alpha\lambda-\alpha\lambda\theta+1)^2}{4(\alpha\lambda+1)} > P(1 -$

$\frac{P+\alpha\theta}{\alpha+1}$), where $P > \theta$. As the higher price $P > \theta$ would even reduce more demand of gain-type consumers.

When $P = \theta$, the profit $\pi = \theta(1-\theta) = \frac{\alpha\lambda+1}{\alpha\lambda+2}\left(1-\frac{\alpha\lambda+1}{\alpha\lambda+2}\right) = \frac{\alpha\lambda+1}{(\alpha\lambda+2)^2}$. Also, the derivative of $\pi = \theta(1-\theta)$ with respect to θ is $\frac{d\pi}{d\theta} = 1-2\theta$. As the highest θ for this interval is $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$, the reference point $\theta > \frac{1}{2}$ as long as $0 < \alpha < 1$ and $\lambda > 1$. So, we would have $\frac{d\pi}{d\theta} < 0$. For the maximized loss-type profit $\pi_L{}^* = \frac{(\alpha\lambda-\alpha\lambda\theta+1)^2}{4(\alpha\lambda+1)} = \frac{\alpha\lambda+1}{(\alpha\lambda+2)^2}$ when $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$. Therefore, when $P = \theta$, the profit $\pi = \pi_L{}^*$ as $P_L{}^* = \theta$ at $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$. The derivative of maximized loss-type profits $\pi_L{}^*$ with respect to the reference point θ is shown in (31) is negative ($\frac{d\pi_L{}^*}{d\theta} < 0$). Although we have $\frac{d\pi}{d\theta} < 0$ and $\frac{d\pi_L{}^*}{d\theta} < 0$, and it is found that $\frac{d\pi}{d\theta} > \frac{d\pi_L{}^*}{d\theta}$. Additionally, as we have $\pi = \pi_L{}^*$, we could claim that setting price $P = \theta$ and loss type pricing strategy would generate the same profit for $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ when $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$, but as $\theta > \frac{\alpha\lambda+1}{\alpha\lambda+2}$, the maximized profit by adopting loss-type pricing strategy is higher than setting $P = \theta$ (i.e. $\pi_L{}^* > \pi$ for $\theta > \frac{\alpha\lambda+1}{\alpha\lambda+2}$).

When $P \neq P_L{}^*$ and $P < \theta$, there are both gain-type and loss-type consumers in the market. To derive the optimal pricing strategy, it would returns to the loss-type pricing strategy as mentioned previously, at a given reference point θ. Therefore, for any price $P < \theta$ and $P \neq P_L{}^*$, the profit $\pi < \pi_L{}^*$.

For gain-type pricing strategy, as the level of reference-dependence is high $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, the value faced by the critical gain-type consumer is $\widehat{v_G} < \theta$, which is less the reference point. However, it violates the assumption of the gain-type pricing strategy. Therefore, using the gain-type strategy to derive the profit directly would receive an incorrect result. As mentioned that when $P > \theta$, there are only gain-type consumers, however, considering (9), when θ is high ($\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$), the gain-type pricing strategy is no applicable. It is true that the profit can be derived for the firm serves only gain-type consumer $P > \theta$ in this interval. The resulting profit would be strictly less then profit generated by loss-type pricing strategy ($\pi < {\pi_L}^*$) as mentioned above.

In short, the monopoly firm can maximize the profit by setting the using the loss-type pricing strategy when facing reference-dependent consumers when the level of reference-dependence equals to $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$.

When $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$

As long as the loss-averse multiplier $\lambda > 1$, there is an interval for the reference point $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. For this interval, as mentioned previously that the gain-type pricing strategy and the loss-type pricing strategy will give an inaccurate result for deriving the maximized profits ${\pi_G}^*$ and ${\pi_L}^*$ for $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ and $\theta \leq \frac{\alpha+1}{\alpha+2}$ respectively. The optimal pricing strategy for a monopoly firm is to set the price equals to the reference point θ and thus the profit ${\pi_\theta}^*$ would be maximized when $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$

To prove that profit ${\pi_\theta}^*$is maximized when monopoly firm sets $P_\theta = \theta$ for $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. Considering the value received by the critical gain-type consumer $\widehat{v_G} = \frac{P+\alpha\theta}{1+\alpha}=$, when $P = \theta$, this critical gain-type consumer receives $\widehat{v_G} = P = \theta$. For a critical loss-type consumer $\widehat{v_L} = \frac{P+\alpha\lambda\theta}{1+\alpha\lambda}$, this consumer would receive $\widehat{v_L} = P = \theta$ when $P = \theta$. When the firm sets a price $P \neq P$ and $P < \theta$, as the threshold for a critical consumer to purchase $\hat{v}$ is reduced, there would be more loss-type consumer (i.e. demand increases), and the profit for the price $P < \theta$ would generate the profit $\pi = P(1 - \frac{P+\alpha\lambda\theta}{1+\alpha\lambda})$ where $P < \theta$, but this profit $\pi < {\pi_\theta}^*$. It is worth to notice that the value received by the critical gain-type consumer would be $\widehat{v_G} < \theta$ so it would be inaccurate to derive the profit π by considering the firm is facing the demand equals to $1 - \widehat{v_G}$ instead of $1 - \widehat{v_L}$ for $P < \theta$. Similarly, if the firm sets a price $P \neq P$ and $P > \theta$, as the threshold for a critical consumer to purchase $\hat{v}$ is increased, there will be no loss-type consumers (as $\widehat{v_L} > \theta$). Therefore, the profit $\pi = P(1 - \frac{P+\alpha\theta}{1+\alpha})$ where $P > \theta$, but this profit $\pi < {\pi_\theta}^*$.

Therefore, the optimal price, and the corresponding demand and thus the profit, when $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, would be

$$ {P_\theta}^* = \theta \quad (34) $$

$$ {Dd_\theta}^* = 1 - \theta \quad (35) $$

$$ {\pi_\theta}^* = \theta(1 - \theta) \quad (36) $$

The derivatives for the price ${P_\theta}^*$ and profit ${\pi_\theta}^*$under this pricing strategy are simply 1 and $1 - 2\theta$. For the derivative of ${\pi_\theta}^*$ with respect to θ ($\frac{d{\pi_\theta}^*}{d\theta} = 1 - 2\theta$) is negative because of the condition $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ and $0 < \alpha < 1$. That is, for any $0 < \alpha < 1$ and the condition for this pricing strategy $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ such that profit ${\pi_\theta}^* = \theta(1 - \theta)$, it can be found that $\theta \geq \frac{1}{2}$. It follows that $\frac{d{\pi_\theta}^*}{d\theta} < 0$. Different from the pricing strategies mentioned previously, this pricing strategy involves setting a higher price as there is an increase in reference point θ. However, it is same as those pricing strategies that the profit would be reduced as θ increases.

In short, when $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ where using both gain-type pricing and loss-type pricing strategies to derive the corresponding profit will be inaccurate, the maximized profit received by the monopoly firm is ${\pi_\theta}^*$ as long as the firm sets the price as ${P_\theta}^* = \theta$.

To sum up, for a given θ, considering the optimal price for all of the three intervals of reference point $\theta \leq \frac{\alpha+1}{\alpha+2}$, $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ and $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. When $\theta \leq \frac{\alpha+1}{\alpha+2}$, the monopoly firm could maximize the profit by setting price $P = {P_G}^*$. When $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, the monopoly firm could maximize the profit by setting price $P = {P_\theta}^* = \theta$. When $\theta \leq \frac{\alpha+1}{\alpha+2}$, the monopoly firm could maximize the profit by setting price $P = {P_L}^*$.

Any price other than $P = P_G{}^*$ for $\theta \leq \frac{\alpha+1}{\alpha+2}$, $P = P_\theta{}^*$ for $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, and $P = P_L{}^*$ for $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ would generate less profit to the monopoly firm. Therefore, there is a theorem for a monopoly firm to maximize profit by setting the optimal price.

Theorem 1: a monopoly firm who is facing reference-dependent consumers maximizes its profit by setting the optimal price $P = P_G{}^*$ for $\theta \leq \frac{\alpha+1}{\alpha+2}$, $P = P_\theta{}^* = \theta$ for $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, and $P = P_L{}^*$ for $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$.

Combining those previous results which maximize the profit received by the monopoly firm when an exogenously determined reference point θ falls into different intervals, given that consumers are reference-dependent, we can conclude that

$$P^* = \begin{cases} P_G^* & for \quad \theta \leq \dfrac{\alpha+1}{\alpha+2} \\ P_\theta^* = \theta & for \quad \dfrac{\alpha+1}{\alpha+2} \leq \theta \leq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \\ P_L^* & for \quad \theta \geq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \end{cases} \tag{37}$$

For a given θ, any price set by the monopoly firm other than P^*as in (37) would generate less than the profit. To summarize the profit corresponding to the optimal price set by the monopoly firm under those three intervals of θ as mentioned in (37), we have

$$\pi^* = \begin{cases} \pi_G^* = \dfrac{(\alpha - \alpha\theta + 1)^2}{4(\alpha+1)} & for \quad \theta \leq \dfrac{\alpha+1}{\alpha+2} \\ \pi_\theta^* = \theta(1-\theta) & for \quad \dfrac{\alpha+1}{\alpha+2} \leq \theta \leq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \\ \pi_L^* = \dfrac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(\alpha\lambda+1)} & for \quad \theta \geq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \end{cases} \tag{38}$$

As we are interested in studying how would the profit change as the reference point θ increases, taking the derivatives of profits from (38) with respect to θ, we have

$$\frac{d\pi}{d\theta} = \begin{cases} \dfrac{d\pi_G^*}{d\theta} = \dfrac{\alpha(\alpha\theta - \alpha + 1)}{2(\alpha+1)} & for \quad \theta \leq \dfrac{\alpha+1}{\alpha+2} \\ \dfrac{d\pi_\theta^*}{d\theta} = 1 - 2\theta & for \quad \dfrac{\alpha+1}{\alpha+2} \leq \theta \leq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \\ \dfrac{d\pi_L^*}{d\theta} = \dfrac{\alpha\lambda(\alpha\lambda\theta - \alpha\lambda + 1)}{2(\alpha\lambda+1)} & for \quad \theta \geq \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \end{cases} \tag{39}$$

It shows that those three derivatives are all negative with respect to θ ($\frac{d\pi_G^*}{d\theta} < 0$, $\frac{d\pi_\theta^*}{d\theta} < 0$ and $\frac{d\pi_L^*}{d\theta} < 0$). It is worth to mention that $\pi_G^{*''} < \pi_L^{*''}$ so π^* decreases at a faster rate when $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ than when $\theta \leq \frac{\alpha+1}{\alpha+2}$. It implies consumers' loss-aversion and it is captured by the loss-averse multiplier λ. In addition, at the critical point $\theta = \frac{\alpha+1}{\alpha+2}$, it is found that $\pi_G^* = \theta(1-\theta) = \pi_\theta^*$. Similarly, at another critical point $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$ it is found that $\pi_L^* = \theta(1-\theta) = \pi_\theta^*$. Therefore, according to these results, the optimal profit mentioned (38) is a continuous and monotonic decreasing function with respect to θ. Additionally, we could conclude that (i) $\pi_G{}^*(\theta_1) \geq \pi_\theta{}^*(\theta_2) \geq \pi_L{}^*(\theta_3)$ for $\theta_1 \leq \theta_2 \leq \theta_3$ but$\theta_1 < 3$, where $\theta_1 \leq \frac{\alpha+1}{\alpha+2}$, $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ and $\theta_3 \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$; and (ii) $\pi_G{}^*(\theta_1) = \pi_\theta{}^*(\theta_2)$ only when $\theta_1 = \theta_2$, and $\pi_\theta{}^*(\theta_2) \geq \pi_L{}^*(\theta_3)$ only when $\theta_2 = \theta_3$.

V. DISCUSSION

When Reference Point is Endogenously Determined

When the reference point θ is not exogenous determined, which means that the monopoly firm can now influence and control the reference point θ to maximize the profit. It is found that when the reference point θ is exogenously determined, the maximized profit would be shown in (38) and the derivatives with respect to the reference point θ as shown in (39) are all negative. These results show that firm would prefer to have the reference point θ as low as possible when θ is exogenously determined. When θ is endogenously determined by the monopoly firm, it would be a corollary that the firm would choose the lowest θ. For that θ chosen by firm, the optimal price, corresponding demand and the maximize profit would be same as covered in the previous content where θ is exogenously determined, as they are derived based on a given θ.

Selling Cost

The result suggests that the firm would receive higher maximized profit for a lower the reference point /theta. However, there are situations in which a firm may choose a higher reference point. To illustrate this point, consider a selling cost faced by the monopoly firm. For example, it is a cost spent by firm to maintain its reputation so that those retailers would be willing to sell the products offered by a monopoly, or it could be a cost for promoting the products such that more consumers would know, or even refer the product to others. The selling cost and the reference point θ are positively related. The model presented previously assumes there is a zero production cost faced by the firm. This assumption is still held for the following discussion. However, a selling cost would be introduced to explain the firm's behavior of keeping a high reference point by consumers, instead of a low reference point.

Considering a selling cost $c = c(\theta)$, which is a function of reference point such that $c'(\theta) < 0$ and $c''(\theta) < 0$. The selling cost is a decreasing function with respect to reference point θ. The selling cost is assumed to be a lump sum cost instead of proportionate to the size of demand. It is assumed that this amount could not be avoided by firm. For example, if we consider this selling cost as a cost faced by a monopoly to maintain a good reputation otherwise those retailers would not sell those products in their stores. If the reference point is low, a monopoly firm has to spend more such that to have a good reputation by consumers. However, the effect of such promotion activities on maintaining reputation would be diminished as θ increases. Therefore, the selling cost is a decreasing function with respect to reference point such that $c'(\theta) < 0$ and $c''(\theta) < 0$. In addition, consumers could help promoting the product through various platforms when the reference point is high. Consumers would be more willing to refer a product to others, if the product comes from the firm which has a good reputation in market. It might be related to network externality but it would not be covered in the model in this research. Despite of this, holding this assumption, the network externality could cause the firm to spend even less as a selling when the reference point is high such that $c''(\theta) < 0$.

Therefore, with an introduction of the selling cost, the "profit" in (38) now becomes the "revenue" received by the firm. Considering (38) and $c = c(\theta)$, the new profit with selling cost π_C received by the firm becomes

$$\pi_C{}^* = \begin{cases} \pi_{cG}{}^* = \dfrac{(\alpha - \alpha\theta + 1)^2}{4(\alpha + 1)} - c(\theta) & for \quad \theta \leq \dfrac{\alpha + 1}{\alpha + 2} \\ \pi_{c\theta}{}^* = \theta(1 - \theta) - c(\theta) & for \quad \dfrac{\alpha + 1}{\alpha + 2} \leq \theta \leq \dfrac{\alpha\lambda + 1}{\alpha\lambda + 2} \\ \pi_{cL}{}^* = \dfrac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(\alpha\lambda + 1)} - c(\theta) & for \quad \theta \geq \dfrac{\alpha\lambda + 1}{\alpha\lambda + 2} \end{cases} \quad (40)$$

It is mentioned in (39) that the function π is steeper when $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ than when $\theta \leq \frac{\alpha+1}{\alpha+2}$. It implies that $\pi_G{}^*(\theta_1) \geq \pi_L{}^*(\theta_2)$, where $\theta_1 < \theta_2$ and $\theta_1 \leq \frac{\alpha+1}{\alpha+2}$, $\theta_2 \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. However, with an introduction of the selling cost c such that such that $c'(\theta) < 0$ and $c''(\theta) < 0$, it is possible that $\pi_{cG}{}^* < \pi_{cL}{}^*$ when θ is sufficiently high. That is, $\pi_{cG}{}^*(\theta_1, c(\theta_1)) <$

${\pi_{cL}}^*(\theta_2, c(\theta_2))$, where $\theta_1 < \theta_2$ and $\theta_1 \leq \frac{\alpha+1}{\alpha+2}$, $\theta_2 \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. Depending on the selling cost function $c = c(\theta)$, if $c(\theta)$ is sufficiently high at $\theta = 0$ and $c'' > \pi''$, then it is possible to have $\frac{d\pi_c}{d\theta} > 0$ for certain level of reference-dependence θ. As a result, it will give ${\pi_{cG}}^* < {\pi_{cL}}^*$ and thus this could explain in reality a monopoly firm could receive more profit when the level of reference-dependence θ is not the lowest.

Therefore, if the reference point θ is endogenously determined by the monopoly firm, it would not always prefer the lowest reference point θ. It is because setting the price according to the lowest possible reference point θ would not always guarantee generating highest maximized profit. The optimal reference point θ would depend on the properties of the selling cost function $c(\theta)$, that is $c'(\theta) < 0$ and $c''(\theta) < 0$. In addition, if there is an inflection point such that $c''(\theta) < 0$ becomes $c''(\theta) > 0$. The inflection point might be resulting from an ineffective advertising strategy. As a result, it might create an optimal point for π_c (a local maximum). In this case, there would be an optimal reference point θ, which is a target θ for firm. For example, if the monopoly firm knows π_c for each θ and θ can be determined by firm. Firm could receive the optimal maximized profit by choosing θ and spend corresponding the selling cost for this θ. Therefore, the selling cost might be one of the possibilities that firm could receive higher profit under a certain level of consumers' reference point, instead of a lowest possible reference point (or even $\theta = 0$).

Participation Cost

I next consider a situation in which a consumer incurs a participation cost when inspecting the good. Assuming a consumer would participate only if the consumer has a certain extend of expectation of purchasing a high-value product, which could be regarded as a high reference point θ towards the product. In other words, when the reference point θ is high, the participation rate of consumers would be high, so the demand would be high. When θ is low, those with high participation cost would drop out, so demand would decrease. Therefore, the total demand for the product would decrease when θ decreases. It is found that demand is a decreasing function with respect to θ and ${Dd^*}'(\theta_L) < {Dd^*}'(\theta_H)$, where $\theta_L < \theta_H$, $\theta_L \leq \frac{\alpha+1}{\alpha+2}$ and $\theta_H \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$.

Considering there are n potential consumers and each of them has to face their inspect cost. However, the inspect costs are different for each consumer, assuming the inspect cost follows the uniform distribution, for example, the nth person faces n units of cost, the n-1th one faces $n-1$ units of cost, and so on. As mentioned that consumers would only purchase the product after inspecting it, and they would inspect the product only when it is expected to be valuable (i.e. when θ is sufficiently high). Consumer would then compare the total utility received with the inspect cost and price they faced. When θ decreases, consumers with high inspect cost will drop out because the inspect cost is too high such that adding the inspect cost to price of good would exceed the total utility they received from purchasing it.

Denotes $\mu(n)$ as the participation cost function, which is the function of n as it represents the units of the participation cost for each consumer and each consumer has different n. Consumers' purchase decision involves considering $u(\theta) \geq P(\theta) + \mu(n)$. For certain level of reference point θ that consumers are facing their corresponding participation cost with function $\mu(n)$, consumers would still purchase the product as long as $u \geq P + \mu$. It is mentioned that consumers would prefer low θ as possible such that $\frac{du}{d\theta} < 0$ and it is shown that $\frac{dP}{d\theta} < 0$. With the new participation cost function $\mu(n)$, the critical consumer would receive total utility $u = P + \mu$. However, I would firstly not to derive the result by considering the critical consumer at $u = P + \mu$. Instead, I would consider the participation cost which would directly affect the total demand only.

Considering a new total demand function which includes another component $r(\theta)$, where $r(\theta)$ represents the reduction in demand as a result of a change in reference point θ. It is because as θ decreases, and some consumers with high participation cost would drop out. Therefore, $r(\theta)$ is a function of the reference point θ such that the new total demand now becomes $Dd - r(\theta)$. In addition, the demand would be decreased at a decreasing rate with respect to θ. Given that the price of good set by firm remains unchanged, as assumed that the optimal price of good is not set based on $u = P + \mu$, and the participation cost would only affect the total demand. That is, the participation cost is not part of a consideration for a monopoly firm to set its price, but it would in fact affect the demand for good. It can be considered as a situation that consumers has certain threshold for the reference point such that they would put an effort in inspecting a good if the reference point is at least higher than the threshold, but such threshold would not be considered by a monopoly firm in setting the price of good. There would be two opposite forces, (i) firm prefers a low reference point θ because the profit would decrease as θ increases, but at the same time (ii) a low θ would reduce the total demand for a good when consumers are facing the participation cost. Therefore, there would be an optimal θ such that the firm would no longer prefer the lowest possible θ, or even $\theta = 0$, as a low reference point leads to a low participation rate.

Now considering the situation if a monopoly firm would consider consumers' participation cost when setting the price of a good. Denote this version of participation cost as i. Therefore, this participation cost would not only affect the total demand, but also the price and the profit. The firm has to consider consumers' total utility received versus the cost they faced. So, consumers would purchase as long as $u \geq P + i$. As the level of reference point θ is assumed to be exogenously determined, the following analysis includes the effect of participation cost i on the optimal price, the total demand and the corresponding profit for all three intervals as mentioned previously. The aim is to show the effect of the introduction of the participation cost on demand and profit, when the monopoly firm consider consumers' participation cost when set a price, for a given level of reference-dependence. Consumers would now purchase the product only if $u - i \geq P$. Using the approach of finding the critical gain-type and loss-type consumers and derive the corresponding optimal price, the demand, as well as the profit as mentioned previously, we would have

$$P = \begin{cases} P_G{}^* = \dfrac{\alpha - \alpha\theta + 1}{2} - \dfrac{i}{2} & for \quad \theta \leq \dfrac{\alpha + 1}{\alpha + 2} \\ P_\theta{}^* = \theta - i & for \quad \dfrac{\alpha + 1}{\alpha + 2} \leq \theta \leq \dfrac{\alpha\lambda + 1}{\alpha\lambda + 2} \\ P_L{}^* = \dfrac{\alpha\lambda - \alpha\lambda\theta + 1}{2} - \dfrac{i}{2} & for \quad \theta \geq \dfrac{\alpha\lambda + 1}{\alpha\lambda + 2} \end{cases} \tag{41}$$

$$
Dd = \begin{cases} Dd_G^* = \dfrac{\alpha - \alpha\theta + 1}{2(\alpha+1)} - \dfrac{i}{2(\alpha+1)} & for \quad \theta \le \dfrac{\alpha+1}{\alpha+2} \\ Dd_\theta^* = 1 - \theta + i & for \quad \dfrac{\alpha+1}{\alpha+2} \le \theta \le \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \\ Dd_L^* = \dfrac{\alpha\lambda - \alpha\lambda\theta + 1}{2(\alpha\lambda+1)} - \dfrac{i}{2(\alpha\lambda+1)} & for \quad \theta \ge \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \end{cases} \tag{42}
$$

$$
\pi = \begin{cases} \pi_G^* = \dfrac{(\alpha - \alpha\theta + 1)^2}{4(\alpha+1)} + \dfrac{i^2 + 2\alpha\theta i - 2\alpha i - 2i}{4(\alpha+1)} & for \quad \theta \le \dfrac{\alpha+1}{\alpha+2} \\ \pi_\theta^* = (\theta - i) - (\theta - i)^2 & for \quad \dfrac{\alpha+1}{\alpha+2} \le \theta \le \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \\ \pi_L^* = \dfrac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(\alpha\lambda+1)} + \dfrac{i^2 + 2\alpha\lambda\theta i - 2\alpha\lambda i - 2i}{4(\alpha\lambda+1)} & for \quad \theta \ge \dfrac{\alpha\lambda+1}{\alpha\lambda+2} \end{cases} \tag{43}
$$

There is an interesting result, the effect of the participation cost on reducing the demand when the reference point is low $(\theta \le \frac{\alpha+1}{\alpha+2})$ is greater than that of the situation when the reference point is high $(\theta \ge \frac{\alpha\lambda+1}{\alpha\lambda+2})$, as shown in (42) and we have $\frac{i}{2(\alpha+1)} > \frac{i}{2(\alpha\lambda+1)}$. In addition, both of the $\frac{i^2+2\alpha\theta i-2\alpha i-2i}{4(\alpha+1)}$ for π_G^* and $\frac{i^2+2\alpha\lambda\theta i-2\alpha\lambda i-2i}{4(\alpha\lambda+1)}$ for $\pi_L^*$are increasing functions, although they are both negative for $\theta \in [0,1]$. It is also worth to mention that the profit $\pi = (\theta - i) - (\theta - i)^2$ is increasing for $\frac{\alpha+1}{\alpha+2} \le \theta \le \frac{\alpha\lambda+1}{\alpha\lambda+2}$. As π_L^* is a decreasing function, there is a new local maximum for the profit function (43) at $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$ for $\theta \in [0,1]$. The result suggests that when there is an existence of the participation cost, a monopoly firm would not prefer a lowest possible level of reference dependence. If the reference point θ can be chosen by firm, then setting the price by adopting gain-type pricing strategy (with $\theta = 0$) would not be optimal, as there is a new local maximum at $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$. It is found that $\frac{d\pi_G^*}{d\theta} < 0$ for $\theta \le \frac{\alpha+1}{\alpha+2}$, which suggests that π_G^* is the highest at $\theta = 0$. However, it is also found that $\frac{(\alpha\lambda-\alpha\lambda\theta_L+1)^2}{4(\alpha\lambda+1)} + \frac{i^2+2\alpha\lambda\theta_L i-2\alpha\lambda i-2i}{4(\alpha\lambda+1)} < (\theta_H - i) - (\theta_H - i)^2$, where $\theta_L < \theta_H$. Therefore, the introduction of the participation cost could provide an explanation for a monopoly firm to keep a certain level of reference point θ in reality, instead of preferring a lowest possible reference point θ as mentioned in (38). Also, it is mentioned that consumers would participate only if they have certain level of reference point towards a high-value product. In other words, instead of the demand is started at

$\theta = 0$, the demand function may start at certain level of reference-dependence where consumers would be willing to bear the participation cost for inspecting the product. This could also explain that the local maximum is no longer necessarily at $\theta = 0$ in reality.

Selling to reference-dependent consumers under competition

This subsection considers competition. Considering a Hotelling setting where there are two identical firms located at the two extreme points. Firstly, we have $x \sim U[0,1]$ where x is the preference of purchasing from firm A, so $1 - x$ represents the preference of purchasing the good from firm B. As two firms are locating at two extreme point respectively, firm A locates at $x = 0$ while firm B locates at $x = 1$. These two firms are selling identical product with same production cost (assumed to be zero) to reference-dependent consumers. Assuming there is a reference point θ which is exogenously determined and it is regarded as the expectation on the preference x by reference-dependent consumers. It is worth to notice that the reference point θ for this context, unlike the previous model, does not refer to the expectation on intrinsic value v. When θ is high numerically (i.e. θ is closer to 1), it is interpreted as a low expectation on the preference of purchasing the good from firm A (i.e. x), as it is far away from firm A (located at $x = 0$). In other words, it would be same to consider a high θ numerically means a high expectation on the preference of purchasing the good from B. In a standard Hotelling setting, consumers have to bear a transportation cost t. Under this setting, the transportation cost t is regarded as the cost for purchasing the good from either firm. So the total cost faced by a consumer located at x who purchases the good from firm A would be $P_A + tx$, while the total cost for purchasing the good from firm B would be $P_B + t(1 - x)$. As two firms offer identical products, consumers would receive same intrinsic value v as a result.

When $x < \theta$, the realized preference of purchasing from firm A is less than the reference point. It could be interpreted as there is a consumer who expects to pay $P_A + tx$ for purchasing the good from firm A, but he realized that his preference is x and $x < \theta$. So the total cost that the consumer has to pay is $P_A + tx$ only. It is less than what he expects, that is, $P_A + tx < P_A + t\theta$ when purchasing the good from firm A.

In order words, that consumer is in fact facing a lower disutility than he had expected. Therefore, $x < \theta$ represents a gain under this setting, and the gain is $t(\theta - x)$. However, when $x > \theta$, the realized preference of purchasing from firm A is greater than the reference point, which means that the consumer has to pay more if he purchases from firm A, that is, $P_A + tx > P_A + t\theta$ In this case there would be a loss faced by the consumer for purchasing the good from firm A, and the loss is $t\lambda(x - \theta)$, where λ is a variable that measures the magnitude of loss-aversion and $\lambda > 1$, similar to the model mentioned in the previous sections of this research. In addition, consumers who face a loss may still purchase the good from the corresponding firm as long as $[v - t\lambda(\theta - x)] > (P + tx)$. Therefore, there are two types of consumers, gain-type consumers and loss-type consumers. The total demand for goods from firm A consists of x_1 and x_2, where $x_1 < \theta$ represents the gain-type consumers and $x_2 > \theta$ represents the loss-type consumers. The total demand for goods offered by firm A would be

$$Dd_A = x_1 + x_2 - \theta \tag{44}$$

As it is under a Hotelling setting, the total demand for goods offered by firm B would be

$$Dd_B = 1 - Dd_A \tag{45}$$

To derive the price P_A and P_B set by firm A and firm B respectively, we have to consider the critical gain-type consumer and the critical loss-type consumer who purchase the good from either firm A or firm B is indifferent for both $x < \theta$ and $x > \theta$. This is the case that the consumer is a critical gain-type consumer for firm A, but a critical loss-type consumer for firm B, so it is indifferent for that consumer to purchase goods from between A and firm B. For the critical gain-type consumer who faces$x_1 < \theta$, when the utility of purchasing from firm A received equals to the utility of purchasing from firm B, we have

$$[v + t(\theta - x_1)] - (P_A + \mathrm{t}x_1) = [v - t\lambda(\theta - x_1)] - [P_B + t(1 - x_1)]$$

$$x_1 = \frac{P_B - P_A + t + t\theta + t\lambda\theta}{t(\lambda + 3)} \tag{46}$$

where the critical gain-type consumer receives the intrinsic value v and gain-utility $t(\theta - x_1)$, but at the same time he has to pay the cost for purchasing the good from firm A $P_A + tx_1$, which is equal to when he is receiving the intrinsic value v but a loss-utility $-t\lambda(\theta - x_1)$, with the cost $P_B + t(1 - x_1)$ when he decides to purchase the good from firm B. As $x_1 < \theta$ only represents part of the gain-type demand for good for firm A (and loss-type demand for firm B), to derive the total demand for both firms, we have to consider the case when $x_2 > \theta$ as well, which represents the loss-type demand for good from firm A (and gain-type demand for firm B). Similarly, we have

$$[v + t\lambda(x_2 - \theta)] - (P_A + \mathrm{t}x_2) = [v - t(x_2 - \theta)] - [P_B + t(1 - x_2)]$$

$$x_2 = \frac{P_B - P_A + t + t\theta + t\lambda\theta}{t(\lambda + 3)} \tag{47}$$

The total demand faced by firm A and B derived from (47) and (48) respectively would be

$$Dd_A{}^* = \frac{2P_B - 2P_A + 2t - t\theta + t\lambda\theta}{t(\lambda + 3)} \tag{48}$$

$$Dd_B{}^* = \frac{2P_A - 2P_B + t + t\lambda + t\theta - t\lambda\theta}{t(\lambda + 3)} \tag{49}$$

Then, to derive the optimal price by considering the maximized profit and taking the first order condition, we have

$$P_A{}^* = \frac{t(5 + \lambda\theta + \lambda - \theta)}{6} \tag{50}$$

$$P_B{}^* = \frac{t(4 - \lambda\theta + 2\lambda + \theta)}{6} \tag{51}$$

The derivatives of the prices set by firm A $P_A{}^*$ and firm B $P_B{}^*$ with respect to θ are $\frac{dP_A{}^*}{d\theta} = \frac{t(\lambda-1)}{6} > 0$ and $\frac{dP_B{}^*}{d\theta} = -\frac{t(\lambda-1)}{6} < 0$. It shows that the price $P_A{}^*$ increases when the reference point θ numerically increases for firm A (i.e. a low expectation on the preference of purchasing the good from A), while the opposite is true for firm B. Under the Hotelling setting, it means that firm A would set a higher price when consumers have a lower expectation towards the preference of purchasing the good from firm A, such that the θ is closer to another firm. The intuition is that, a lower expectation towards preference of purchasing the good from a firm gives more room for that firm to set a higher price as consumer prefers a lower expectation θ that generates higher resulting total utility (it is true for both gain-type or loss-type consumers). For the total demand for good faced by firm A and firm B, using the derived optimal prices set by firm A (53) and firm B (54), the total demand faced by firm A (51) and firm B (52) can be written as

$$Dd_A{}^* = \frac{5 + \lambda - \theta + \lambda\theta}{3(\lambda + 3)} \tag{52}$$

$$Dd_B{}^* = \frac{4 - \lambda\theta + 2\lambda + \theta}{3(\lambda + 3)} \tag{53}$$

We could derive the derivatives with respect to θ such that we have $\frac{dDd_A{}^*}{d\theta} = \frac{\lambda-1}{3(\lambda+3)} > 0$, while $\frac{dDd_B{}^*}{d\theta} = -\frac{\lambda-1}{3(\lambda+3)} > 0$. It means that when the expectation of the preference of purchasing the good from A decreases (and the reference point decreases is represented by θ numerically increases as it is closer to 1), the total demand for A would increases. It is because when consumers would expect they would be less preferred to purchase the good from firm A as consumers prefer to have a lower reference point, the realized preference x would lead to greater "surprise" to consumers who purchase from firm A such that the total utility received from purchasing from firm A increases. Therefore, the reference point θ that is closer to 1 would increase the demand for good from A. In addition, as both of the price and the total demand set by firm A increase when the reference point θ is closer to 1, we could predict that the maximized profit received by firm A would also be increased when the reference point θ is closer to 1. Combining (50) with (52) for firm A and (51) with (53) for firm B, the maximized profit for two firms would be

$$\pi_A{}^* = \frac{t(5 + \lambda - \theta + \lambda\theta)^2}{18(\lambda + 3)} \quad (54)$$

$$\pi_B{}^* = \frac{t(4 - \lambda\theta + 2\lambda + \theta)^2}{18(\lambda + 3)} \quad (55)$$

The derivative of the maximized profit with respect to the reference point θ for firm A is $\frac{d\pi_A{}^*}{d\theta} = \frac{t(\lambda-1)(5+\lambda-\theta+\lambda\theta)}{9(\lambda+3)} > 0$, while the derivative for firm B is $\frac{d\pi_B{}^*}{d\theta} = -\frac{t(\lambda-1)(4-\lambda\theta+2\lambda+\theta)}{9(\lambda+3)} < 0$. The result confirms that the maximized profit received by firm A would increase as the reference point decreases (represented by θ increases numerically). That is, when there is a higher expectation towards the preference of purchasing the good from firm A, firm A would receive a lower maximized profit. The intuition is that, for a lower reference point, consumers would receive a higher gain-utility from purchasing at a firm, which leads to a higher demand and at the same time it enables firm to set a higher price. The result from this example which considers a competition under Hotelling setting is consistent with the model proposed by this study which assumes a monopoly firm facing reference dependent consumers. This could be an example for showing the model that is also applicable to the situation of competition under Hotelling setting.

VI. Conclusion

In this research I proposes a model to study the optimal price set by a monopoly firm who faces reference-dependent consumers, under a given level of reference-dependence. It is intuitive that consumers would prefer a low reference point as they can receive highest total utility, because a low reference point could bring higher surprise or lower disappointment to consumers. However, the preferred reference point level may not be so obvious to firm. So, I proposed two pricing strategies, namely the gain-type pricing strategy and the loss-type pricing strategy. Then I try to compare the profits as a result of different prices set by a monopoly firm, for a given level of reference-dependence. It is found that, firm could receive a highest maximized profit to set the price when(i) using the gain-type pricing strategy for $\theta \leq \frac{\alpha+1}{\alpha+2}$, (ii) setting price equals to the level of reference-dependence for $\frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$, and (iii) using the loss-type pricing strategy for $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$. Any other prices could not generate a higher maximized profit to firm. In addition, the results show that firm would prefer (an exogenously determined) reference point that is as low as possible, as a lower reference point would generate higher profit to firm. It is shown by the function of maximized profit decreases as the reference point increase. Additionally, it is expected that the result would be the same when the reference point is endogenously determined, as it is just a corollary of the case of an exogenously determined reference point. Therefore, we can conclude that when the monopoly firm is facing reference dependent consumers, the firm would generally prefer the lowest possible reference point, unless there are some variations added to the model which would fit some situations in reality, such as the selling cost and participation cost. Then I consider a situation for a firm which has to compete with another identical firm in the same market with reference-dependent consumers under the Hotelling setting, for a given reference point which is exogenously determined. Even though the reference point refers to an expectation on the preference of purchasing the product from that firm by a reference-dependent consumer, instead of an expectation on intrinsic value of the product offered by that firm, it is found that a firm would receive higher maximized profit for a lower reference point, and the price set by firm would be higher for a lower reference point (but it is represented by a higher θ numerically in the model of Hotelling setting). It could be concluded that firms who are facing competition under Hotelling setting would prefer the lowest reference point. That means, a firm would prefer consumers have a lower expectation on the preference of purchasing a product from that firm. This result is consistent with the situation of a monopoly firm who is facing reference-dependent consumers and there is no competition in the market. The intuitions for those two cases are the same, consumers would receive higher total utility for a lower reference point.

The proposed model could be improved in several aspects. Firstly, the intrinsic value v is assumed to follow a uniform distribution. It is worth to examine the results if the model is generalized, in which would be assumed to follow a more general distribution, rather than the uniform distribution. Therefore, it could show that those results found by this study are only applicable for assuming v follows uniform distribution, or those results still hold under a more general distribution. Secondly, this research does not examine the formation of the reference point. The reference point is assumed to be exogenously determined in the model. However, it is still worth to study the formation of the reference point as it could help us to have a better understanding about the observed phenomenon in reality which are different from the results derived from the model, for example, a firm would wish to keep certain level of reference point in reality, instead of a lowest possible reference point. Lastly, further researches in future on this topic could be done by releasing some of the assumptions for the model. For example, some varieties could be added to the model, such as allowing the firm to sell more than one product, or allowing the competition that involve more than two firms, instead of applying the Hotelling model to study the situation of the competition of two firms. I believe that future researches on these aspects would help improving and enriching the model such that it would be more comprehensive and complete.

VII. APPENDIX

Maximizing the Profit $\pi_G{}^*$ when $P = P_G{}^*$

From (1), a gain-type consumer faces $v > \theta$ and that consumer would only purchase the good as long as $u \geq P$. From (7), the critical gain-type consumer would have $\widehat{v_G} = \frac{P_G{}^* + \alpha\theta}{1+\alpha}$, and the demand faced by the firm would then be $Dd_G{}^* = 1 - \widehat{v_G}$, so the profit $\pi_G{}^*$ would be

$$\pi_G{}^* = P_G{}^*(1 - \frac{P_G{}^* + \alpha\theta}{1 + \alpha})$$

Considering the first-order-condition of $\pi_G{}^*$ with respect to $P_G{}^*$, we have

$$\frac{d\pi_G{}^*}{dP_G{}^*} = 0$$

$$1 - \frac{2}{1 + \alpha}(P_G{}^*) - \frac{\alpha\theta}{1 + \alpha} = 0$$

$$P_G{}^* = \frac{\alpha - \alpha\theta + 1}{2}$$

It is (8). Then substituting (8) back into (7), we have

$$\widehat{v_G} = \frac{\frac{\alpha - \alpha\theta + 1}{2} + \alpha\theta}{1 + \alpha}$$

$$\widehat{v_G} = \frac{\alpha + \alpha\theta + 1}{2(1 + \alpha)}$$

which is (9). Then substitutes (9) into the demand $Dd_G{}^* = 1 - \widehat{v_G}$, we have

$$Dd_G{}^* = 1 - \widehat{v_G}$$

$$Dd_G{}^* = 1 - \frac{\alpha + \alpha\theta + 1}{2(1 + \alpha)}$$

$$Dd_G{}^* = \frac{\alpha - \alpha\theta + 1}{2(1 + \alpha)}$$

It is (10). Then combining (8) and (10) to derive the maximized profit $\pi_G{}^*$, we have

$$\pi_G{}^* = P_G{}^*(Dd_G{}^*)$$

$$\pi_G{}^* = \frac{\alpha - \alpha\theta + 1}{2}\left(\frac{\alpha - \alpha\theta + 1}{2(1 + \alpha)}\right)$$

$$\pi_G{}^* = \frac{(\alpha - \alpha\theta + 1)^2}{4(1 + \alpha)}$$

Maximizing the Profit $\pi_L{}^*$ when $P = P_L{}^*$

From (1), a loss-type consumer faces $v < \theta$ and that consumer would only purchase the good as long as $u \geq P$. From (18), the critical loss-type consumer would have $\widehat{v_L} = \frac{P_L{}^* + \alpha\lambda\theta}{1+\alpha\lambda}$, and the demand faced by the firm would then be $Dd_L{}^* = 1 - \widehat{v_L}$, so the profit $\pi_L{}^*$ would be

$$\pi_L{}^* = P_L{}^*\left(1 - \frac{P_L{}^* + \alpha\lambda\theta}{1 + \alpha\lambda}\right)$$

Considering the first-order-condition of ${\pi_L}^*$ with respect to ${P_L}^*$, we have

$$\frac{d{\pi_L}^*}{d{P_L}^*} = 0$$

$$1 - \frac{2}{1+\alpha\lambda}({P_L}^*) - \frac{\alpha\lambda\theta}{1+\alpha\lambda} = 0$$

$${P_L}^* = \frac{\alpha\lambda - \alpha\lambda\theta + 1}{2}$$

It is (19). Then substituting (19) back into (18), we have

$$\widehat{v_L} = \frac{\frac{\alpha\lambda - \alpha\lambda\theta + 1}{2} + \alpha\lambda\theta}{1+\alpha\lambda}$$

$$\widehat{v_L} = \frac{\alpha\lambda + \alpha\lambda\theta + 1}{2(1+\alpha\lambda)}$$

which is (20). Then substitutes (20) into the demand ${Dd_L}^* = 1 - \widehat{v_L}$, we have

$${Dd_L}^* = 1 - \widehat{v_L}$$

$${Dd_L}^* = 1 - \frac{\alpha\lambda + \alpha\lambda\theta + 1}{2(1+\alpha\lambda)}$$

$${Dd_L}^* = \frac{\alpha\lambda - \alpha\lambda\theta + 1}{2(1+\alpha\lambda)}$$

It is (21). Then combining (19) and (21) to derive the maximized profit ${\pi_L}^*$, we have

$${\pi_L}^* = {P_L}^*({Dd_L}^*)$$

$${\pi_L}^* = \frac{\alpha\lambda - \alpha\lambda\theta + 1}{2}\left(\frac{\alpha\lambda - \alpha\lambda\theta + 1}{2(1+\alpha\lambda)}\right)$$

$${\pi_L}^* = \frac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(1+\alpha\lambda)}$$

Profit Curve is Continuous over the Three Intervals $\boldsymbol{\theta \leq \frac{\alpha+1}{\alpha+2}, \frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}}$ and $\boldsymbol{\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}}$

The following shows a way to show that the profit curve as (38) is continuous. Considering the values of the critical points at $\theta = \frac{\alpha+1}{\alpha+2}$ and $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$ such that the profit received by the firm when setting $P_G{}^*$ or $P = \theta$ are equal at $\theta = \frac{\alpha+1}{\alpha+2}$. Similarly, profit received by the firm when setting $P_L{}^*$or $P = \theta$ are equal at $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$. So, considering $\theta = \frac{\alpha+1}{\alpha+2}$,

$$\pi_G{}^* = \frac{(\alpha - \alpha\theta + 1)^2}{4(\alpha+1)} = \frac{[\alpha - \alpha\left(\frac{\alpha+1}{\alpha+2}\right) + 1]^2}{4(\alpha+1)}$$

which has to be equal to $\pi = \theta(1-\theta) = (\frac{\alpha+1}{\alpha+2})(1 - \frac{\alpha+1}{\alpha+2})$. Considering the critical value $\alpha = 0, \pi_G{}^* = \frac{(\alpha-\alpha\theta+1)^2}{4(\alpha+1)} = \frac{[\alpha-\alpha\left(\frac{\alpha+1}{\alpha+2}\right)+1]^2}{4(\alpha+1)} = \left(\frac{\alpha+1}{\alpha+2}\right)\left(1 - \frac{\alpha+1}{\alpha+2}\right) = \frac{1}{2}$. For another critical value $\alpha = 1, \pi_G{}^* = \frac{[\alpha-\alpha\left(\frac{\alpha+1}{\alpha+2}\right)+1]^2}{4(\alpha+1)} = \left(\frac{\alpha+1}{\alpha+2}\right)\left(1 - \frac{\alpha+1}{\alpha+2}\right) = \frac{2}{9}$. Therefore, the profit received by the firm are equal when the firm is setting either $P_G{}^*$ or $P = \theta(1-\theta)$ at $\theta = \frac{\alpha+1}{\alpha+2}$ for $1 < \alpha < 0$. Similarly, for $P_L{}^*$ and $P = \theta(1-\theta)$, considering $\theta = \frac{\alpha\lambda+1}{\alpha\lambda+2}$

$$\pi_L{}^* = \frac{(\alpha\lambda - \alpha\lambda\theta + 1)^2}{4(\alpha\lambda+1)}) = \frac{[\alpha\lambda - \alpha\lambda\left(\frac{\alpha\lambda+1}{\alpha\lambda+2}\right) + 1]^2}{4(\alpha\lambda+1)}$$

The critical values are $\alpha = 0$, $\alpha = 1$ and $\lambda = 1$. When $\alpha = 0$ and $\lambda = 1$, $\pi_L{}^* = \frac{[\alpha\lambda-\alpha\lambda\left(\frac{\alpha\lambda+1}{\alpha\lambda+2}\right)+1]^2}{4(\alpha\lambda+1)} = \left(\frac{\alpha\lambda+1}{\alpha\lambda+2}\right)\left(1 - \frac{\alpha\lambda+1}{\alpha\lambda+2}\right) = \frac{1}{4}$, while when $\alpha = 1$ and $\lambda = 1$, $\pi_L{}^* = \frac{[\alpha\lambda-\alpha\lambda\left(\frac{\alpha\lambda+1}{\alpha\lambda+2}\right)+1]^2}{4(\alpha\lambda+1)} = \left(\frac{\alpha\lambda+1}{\alpha\lambda+2}\right)\left(1 - \frac{\alpha\lambda+1}{\alpha\lambda+2}\right) = \frac{2}{9}$. It also shows that the profits are equal when the firm is setting either $P_L{}^*$ or $P = \theta(1-\theta)$ at $\frac{\alpha\lambda+1}{\alpha\lambda+2}$ for $1 < \alpha < 0$ and $\lambda > 1$. Given that $\pi_G{}^*$, $\pi = \theta(1-\theta)$ and $\pi_L{}^*$ are continuous at $\theta \leq \frac{\alpha+1}{\alpha+2}, \frac{\alpha+1}{\alpha+2} \leq \theta \leq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ and $\theta \geq \frac{\alpha\lambda+1}{\alpha\lambda+2}$ respectively. According to the results above, we can conclude that profit curve as (38) is continuous.

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