8-11-2017

# Insurance and self-protection for increased risk aversion

Jian ZHANG

# Terms of Use

# INSURANCE AND SELF-PROTECTION FOR INCREASED RISK AVERSION

ZHANG JIAN

MPHIL

LINGNAN UNIVERSITY

2017

# INSURANCE AND SELF-PROTECTION FOR INCREASED RISK AVERSION

by
ZHANG Jian
張健

A thesis
submitted in partial fulfillment
of the requirements for the Degree of
Master of Philosophy in Business

Lingnan University

2017

**ABSTRACT**

Insurance and Self-protection for Increased Risk Aversion

by

ZHANG Jian

Master of Philosophy

We re-examine the classic problem of risk aversion and self-protection in this paper. In the beginning of this paper, we conduct comparative statics of risk aversion and prevention efforts based on the mono-periodic two states model of choice under risk. We show this new condition is effective with self-insurance-cum-protection model (Lee, 1998), in which the decision maker's activities to prevent the risk can sever both as self-insurance and self-protection. We suggest a new condition that increased risk aversion induces more prevention activities. This new condition requires only one assumption concerning fear of ruin coefficient, marginal effect of SICP activity on probability and marginal cost of SICP activity. By applying interval dominance order (Quah and Strulovici,2009), we find that a decision maker will exert higher level of SICP activity if he becomes more risk averse, under the condition that his hazard rate is higher than the 'boldness' coefficient (Aumann and Kurz,1977). This new condition is effective even when the optimal level for SICP activity is not interior solution. With our method, the assumption,that optimal solution is interior,is not necessary and marginal utility functions do not need to be monotonic on the interval $[0, w_0]$. Based on this, the optimal solution can be corner solution or inflection point solution. And the DM's attitude towards risk can be variable. Hence, the relation suggested by our findings is more consistent with real world situations.

**Key words**: Risk aversion, Self-protection, Insurance, Interval dominance order

## DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

Zhang Jian

ZHANG Jian

Date

Aug. 29. 2017

CERTIFICATE OF APPROVAL OF THESIS

INSURANCE AND SELF-PROTECTION FOR INCREASED RISK AVERSION

by

ZHANG Jian

Master of Philosophy

Panel of Examiners :

尚蔚鑫

(Prof. SHANG Weixin) (Chairman)

(Prof. Keith WONG K.P.) (External Member)

Li Jingyuan

(Prof. LI Jingyuan) (Internal Member)

Wei Lai

(Prof. WEI Lai) (Internal Member)

Supervisor :

Prof. LI Jingyuan

Approved for the Senate :

(Prof. MOK Ka Ho Joshua)

Chairman, Postgraduate Studies Committee

11 AUG 2017

Date

# Contents

**1 Introduction** **1**
1.1 Research Background . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
1.2 Literature Review . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
1.2.1 Self-insurance and Self-protection . . . . . . . . . . . . . . . . 3
1.2.2 Monotonic Comparative Statics . . . . . . . . . . . . . . . . . . 8
1.3 Contributions of This Thesis . . . . . . . . . . . . . . . . . . . . . . . 9

**2 Methodology and model setting** **10**
2.1 Review of Concepts . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
2.2 The models . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 11
2.2.1 Self-Insurance . . . . . . . . . . . . . . . . . . . . . . . . . . . 12
2.2.2 Self-insurance-cum-protection model . . . . . . . . . . . . . . 16
2.2.3 A Brief Discussion for Self-Protection . . . . . . . . . . . . . . 23

**3 Conclusion** **27**

## ACKNOWLEDGEMENT

Firstly, I would like to express my sincere gratitude to my advisor Prof. Li Jingyuan for the continuous support of my Mphil study and related research, for his patience, motivation, and immense knowledge. Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Weixin Shang, Prof. Kit-pong Wong, and Prof.Lai Wei, for their insightful comments and encouragement, which help me to improve my thesis and widen my research from various perspectives.

Furthermore, I wish to extend my appreciation to Prof.Gao Jin, Prof.Man-lai Wong, Prof.Winnie Poon, Prof.Zhao Xiaofeng, Dr. Ho-Yin Yick, Dr. Jianli Wang, Mr. Dongri Liu and all seminar participants at the Business School in Lingnan University for helpful comments and meaningful suggestions. I also want to thank Ms. Clara Hui and Ms.Agnes Tsui, for their kind help during my study.

Last but not the least, I would like to thank my fellow students, friends and my family for supporting me throughout writing this thesis and my life in general.

# Chapter 1 Introduction

## 1.1 Research Background

In the past 40 years, economics of risk and insurance begun to flourish in three main areas: optimal insurance and protection, market equilibrium under asymmetric information and insurance market structure. In the theory of risk management, insurance and prevention are among the tools available to manage risk. The goal for risk management is to find optimal effort for risk prevention or insurance activities. In order to mitigate risks, individuals may take actions either reducing the severity of potential loss(self-insurance or loss reduction)or reducing the probability of occurrence of a risk(self-protection or loss prevention). It is intuitive that people would take more efforts to reduce risk when they become more averse to take risk.

Ehrlich and Becker (1972) are the first to study the demand of self-insurance and self-protection [1]and their study focuses on the interaction between market insurance, self-insurance and self-protection. This paper is regarded as first article on theory of risk management. Another classic paper on self-insurance and self-protection is by Dionne and Eeckhoudt (1985). They show that a more risk averse decision maker would take higher self-insurance activities, while this more risk averse decision maker's choice on self-protection is ambiguous. After Dionne and Eeckhoudt's paper, a number of later research try to clarify the link between risk aversion and self-protection. Boyer and Dionne (1989) study the relation between increased exogenous risk and self-protection actions, but they

[1] They are the first to use the terms *self-insurance* and *self-protection*.

find that impact on the self-protection activities by increased risk is ambiguous. Briys and Schlesinger(1990) prove that the relation between risk aversion and self-insurance are still robust in several distinct settings, while the relation for self-protection and risk aversion cannot hold in these settings. Jullien and et al. (1999) suggest a utility-dependent threshold of probability, beneath which more self-protection is the result of increased risk aversion. Chiu(2000) analyzes the effect of prudence on this threshold. Eeckhoudt and Gollier(2005) propose some assumptions under which a risk-neutral agent invests less in self-protection than a prudent agent.

Interestingly, scholars used first order condition to conduct comparative statics of risk aversion and prevention efforts in these studies. A major limitation is these results require some technical assumptions of second order conditions for utility function[2]. Methods of monotonic comparative statics can help us gain some results with less assumptions. For example, single crossing condition(Milgrom and Shannon,1994) and interval dominance order (Quah and Strulovici,2009) enables one to better analyze this comparative statics problem of risk aversion and self protection with less assumptions regarding the second order conditions for utility function. By applying single crossing condition and interval dominance order, we show the positive relationship between increased risk aversion and self-insurance is still effective without second order assumptions. And we suggest under a new condition increased risk aversion increases SICP efforts.

The structure of this paper is as follows. In next section, we introduce

[2]for example, concave of utility function

a brief review of literature, which concerns self-insurance, self-protection,single crossing condition and interval dominance order. Chapter 2 provides settings of the model and derives our condition for higher protection under increased risk aversion. Chapter 3 concludes the paper.

## 1.2 Literature Review

Prior studies have found that increased risk aversion induces more self-insurance activities when DMs are risk averse, while this relation is ambiguous for increased risk-aversion and self-protection activities. Under some assumptions about utility function and initial probability of loss, increased self-protection activity is the reaction to increased risk aversion.

In this section, I first briefly review the literature on self-insurance and self-protection. I will then review the existing studies on monotonic comparative statics, which is our main method to reach conclusions in this paper.

### 1.2.1 Self-insurance and Self-protection

In the first place, we will summarize the literature on risk aversion and prevention efforts. Prevention can be understood as the efforts to avoid the risk or, at least, to reduce the losses caused by this risk. Compared with precaution, prevention is a static concept while precaution is a dynamic concept with varying information available over time. Two major types of prevention effort are self-insurance and self-protection.

Self-insurance refers to the activity that aims to reduce the severity of loss when risk occurs at the cost of lower wealth in all states. While self-protection is the activity to lower the probability of loss occurs. Self-insurance offers a channel to transfer the wealth from states where marginal utility of wealth is lower to where it is higher, while self-protection only increases the probability of higher utility state at the expense of reducing wealth in all states. Risk preferences is the decision maker's attitude towards uncertainty about his wealth. There are three types of risk preference:risk-loving, risk-neutral and risk averse. If an agent dislikes every lottery with an expected payoff of zero, even at any level of wealth, then this agent is risk-averse. The absolute value of risk-aversion is indicated by the inverse for quotient of second order derivative of utility function to first order derivative of utility function $(-\frac{u''}{u'})$. This has been developed by Arrow(1963) and Pratt(1964) independently in 1960's.

Ehrlich and Becker (1972) are the first to study the demand of self-insurance and self protection. Their study examines the interaction between market insurance and prevention efforts. Their findings are mainly three-fold:

First, when no market insurance is available, the risk-averse decision maker will engage in self-insurance and self-protection activities. The optimal level of these prevention actions depends on the cost and complete elimination of the risk is generally not optimal.

Second, market insurance and self-insurance are substitute. The increased amount of protection from the insurer induces a rational decision maker to reduces his investment into self-insurance. However it is widely believed that this effect of market insurance may lead to 'moral hazard'. They show that, under some conditions, the market insurance may lead to a decrease in the probability of hazardous events.

Third, self-protection and market insurance may be complement or substitute depending on the sensitivity of the insurance premium to the effect of self-protection.

After this paper, increasing number of important contributions concerning prevention efforts and risk aversion are made. A decade later, Dionne and Eeckhoudt (1985) show that increased risk aversion for a decision maker results in higher self-insurance activities, while this relation is not valid for self-protection. They are the first to conduct comparative statics of risk aversion and prevention efforts and first to suggest the ambiguous effect of increased risk aversion on self-protection. They think the key for a clear relation between self-protection and increased risk aversion is the value of transformation for more risk averse agent's utility function.

In Boyers and Dionne s' (1989) paper, they study the relation between increased exogenous risk and self-protection activities and suggest that higher risk has an ambiguous impact on the self-protection activities. If decision maker's utility functions are non-DARA, they will spend more in self-protection as the reaction to increased risk. They also note that self-insurance leads to larger changes in distribution of risk than self-protection does. This idea links self-insurance and self-protections activities to the first order stochastic improvement of utility.

Briys and Schlesinger(1990) prove that the relation between risk aversion and self-insurance are still robust in several distinct settings, such as state-dependent utility, the presence of background and random initial wealth, while the relation for self-protection still cannot hold under these settings.

Briys, Schlesinger and Schuenburs(1991) study the relation between market insurance, self-insurance and self-protection if the reliability is not guaranteed. As a result, some original relations do not hold, such as the inductive relation between higher risk and higher self-insurance and the substitute relation of market insurance and self-insurance. Additionally, no definite result for self-protection is reached in this study.

Sweeney and Beard(1992) focus on the comparative statics of self-protection solely and investigate the effects of altered probability of loss and loss sizes on optimal self-protection. They find the effect of loss size and loss probability have mixed effect on self-protection, and thresholds for these two cases both relates to absolute risk aversion function.

Jullien, Salanie and Salanie (1999) analyze the effect of prevention efforts on distribution of loss. They examine the effect of self-insurance and self-protection from the perspective of distribution of loss. They conclude that a more risk averse agent always chooses a higher-level of self-insurance and efforts in self-protection would be higher for more risk-averse under the condition that the initial loss probability is low enough. This condition is really helpful in understanding the relation between risk-aversion and self-protection since it links the probability of loss and protection efforts. Moreover they extend this result to a general type of probability and find it effective in general type model.

Chiu(2000) studies individual's propensity to self-protect and concludes that both marginal and average propensity to self-protect is in relation to the initial loss probability and degree of risk aversion. He also finds the average propensity to self-protect is decreasing with the initial loss probability, and the same is true for the marginal propensity under a mild restriction. In short, when initial probability is small enough, higher risk-aversion leads to higher self-protection efforts.

Eeckhoudt and Gollier (2005) examine the relation between high degree of risk averse-prudence, and optimal level of effort. They show a prudent agent, whether risk-loving or risk-averse, will spend less in protection efforts than a risk-neutral one does.

Meyer and Meyer (2011) use the procedure by Dimanod and Stiglitz to study the impact on self-protection by changes in risk. This method not only allows the findings to be generalized, and also study the self-protection activity in a larger sense.

Hofmann and Peter (2015) extend the study on effect of risk preference on self-insurance and self-protection to a two-period model. Their conclusions is consistent with previous mono-periodic models, that increased risk aversion induces higher self-insurance activities and higher self-protection activities when initial probability for loss is small enough. When the model includes endogenous savings, the agent with more concave utility function will always select more self-insurance, will take more self-protection only when the probability of loss is small enough. When there is no endogenous savings, the agent will take higher effort in either self-insurance or self-protection only if current consumption is sufficiently large. In fact the two-period model separates the cost and benefits of prevention efforts, which is relatively clear to understand the relation between risk preference and prevention efforts.

### 1.2.2 Monotonic Comparative Statics

Comparative statics is a commonly used method in economics, which compares different outcomes when there are changes in the parameters. In this paper, we mainly use the monotonic comparative statics to solve optimization problems.The major methods in this are the 'single crossing condition' by Milgrom and Shannon (1994) and 'interval dominance order' by Quah and Strulovici(2009).

Some early contributions in literature of monotone comparative statics are by Milgrom and Roberts(1990), Vives(1990)and Topkis(1998).

Milgrom and Shannon (1994) characterize the single crossing condition and demonstrate its application to several settings, such as competitive firm, the Bertrand oligopolistist and so on.

Quah and Strulovici(2009) identify the interval dominance, which complement the application of comparative statics when single crossing property is violated.

## 1.3 Contributions of This Thesis

My study offers a new condition for positive relation between risk-aversion and self-insurance-cum-protection activities and we try to extend this condition to self-insurance and self-protection model.

Firstly, we resort to a new method-interval dominance order, to conduct comparative statics of risk aversion and optimal prevention activities. This method has been applied to study background risk, high order risk preference and precautionary paying in previous literatures(Wang and Li 2015, Wang et al 2016). But there is no study using this method to focus on the risk preference and prevention efforts.

Secondly, our study extend the effectiveness of previous results(Dionne and Eeckhoudt 1985, Lee 1998, Jullien et al 1999). Our finding is valid even when optimal solution for SICP or self-insurance activity is corner solution or inflection point solution.

Unlike the previous literature on risk aversion and self-protection using the first order condition, whose conclusion requires a threshold for initial probability of loss and second order condition for utility function, results for our study requires one assumption about the relation between hazard rate and boldness coefficient.In this way, our study releases assumptions in previous literature.

# Chapter 2 Methodology and model setting

## 2.1 Review of Concepts

To compare two sets,Topkis(1998)defines an order of sets.

**Definition 2.1.1** *Let $S^{'}$ and $S^{''}$ be two subsets of R. $S^{''}$ dominates $S^{'}$ in the strong set order ($S^{''} \geq_{SSO} S^{'}$)if for any $x^{''} \in S^{''}$ and $x^{'} \in S^{'}$,we have $max\{x^{''}, x^{'}\} \in S^{''}$ and $min\{x^{''}, x^{'}\} \in S^{'}$.*

Milgrom and Shannon(1994)propose the single crossing property.

**Definition 2.1.2** *Let $S^{'}$ and $S^{''}$ be two subsets of R. $S^{''}$ dominates $S^{'}$ in the strong order set ($S^{''} \geq_{SSO}$)$S^{'}$ if for any $x^{''} \in S^{''}$ and $x^{'} \in S^{'}$,we have* $\max\{x'', x'\} \in S''$ *and* $\min\{x'', x'\} \in S'$.

**Definition 2.1.3** *The family $\{f(x,s)\}_{s \in S}$ obeys single crossing property if for all $x^{''} > x^{'}$ and $s^{''} > s^{'}$*

$$f(x^{''}, s^{'}) - f(x^{'}, s^{'}) \Rightarrow f(x^{''}, s^{''}) - f(x^{'}, s^{''}).$$

Define $\Delta(s) = f(x^{''}, s) - f(x^{'}, s)$, then $\Delta(s)$ is an increasing function and crosses x-axis only once under the condition of single crossing.

Single crossing property is useful in comparing the optimal solutions in some situations.

**Theorem 2.1.4** *The family $\{f(x,s)\}_{s \in S}$ obeys single crossing difference if and only if $argmax_{x \in Y} f(x,s)$ is increasing in s in the sense of $\geq_{SSO}$ for all $Y \subseteq R$.*

In some cases, single crossing property is not effective. Quah and Strulovici(2009) propose the interval dominance order.

**Definition 2.1.5** *let $X \subseteq R$. The set $Y \subseteq X$ is an interval of $X$ if, whenever $x'$ and $x''$ are in $X$, then any $x \in X$ such that $x' \leq x \leq x''$ is also in $Y$.*
*The family $\{f(x,s)\}_{s \in S}$ obeys the interval dominance order if for any $x^{''} > x^{'}$ and $s^{''} > s^{'}$ such that $f(x^{''},s^{'}) - f(x,s^{'}) \geq 0$ for all $x \in [x', x'']$, we have*

$$f(x^{''},s^{'}) - f(x^{'},s^{'}) \Rightarrow f(x^{''},s^{''}) - f(x^{'},s^{''}).$$

**Theorem 2.1.6** *The family $\{f(x,s)\}_{s \in S}$ obeys obeys the interval dominance order if and only if $argmax_{x \in Y} f(x,s)$ is increasing in $s$ all intervals $Y \subseteq X$.*

Quah and Strulovici(2009)also provide a simple way of checking that a family obeys the interval dominance order.

**Theorem 2.1.7** *Suppose $X$ is a interval of $R$, the functions $f$, $g$: $X \to R$ are absolutely continuous on compact intervals in $X$. If there is an increasing and strictly positive function $\alpha : X \to R$ such that $g'(x) \geq \alpha(x) f'(x)$ for all $x$, then $g$ dominates $f$ by the interval dominance order.*

## 2.2 The models

Consider an individual with an initial wealth $w_0$ is facing an event with potential risk. Let $p \in [0,1]$ defines the probability of risk happens and $(1-p)$ is the state when risk does not happen.Assume this individual has von Neumann-Morgenstern utility function $u$, and utility function is differentiable and strictly increasing ($u^{'} > 0$). The individual can engage in self-insurance activities to reduce the potential losses of the accident, self-protection activities to guarantee a better chance of accidents do not happen and self-insurance-cum-protection activities to reduce the size and probability of losses at the same time.The decision maker's effort in self-insurance or self-protection is equal to or smaller than his initial wealth.

We will first consider self-insurance activities and self-protection activities separately, then we study SICP model.

### 2.2.1 Self-Insurance

Assume an individual's level of self-insurance is $x$ and his self-insurance is within the interval $[0, w_0]$ .Monetary cost for self-insurance activity is $c(x)$ and increasing marginal cost indicates $c^{'}(x) > 0$.Potential loss is reduced by self-insurance activities, thus losses is a function of level of self-insurance $l(x)$. Because marginal effect of self-insurance is decreasing,$l^{'}(\cdot) < 0$ holds.

The expected utility function for an individual $u$ can be put:

$$U(x) = pu(w_0 - c(x) - l(x)) + (1 - p)u(w_0 - c(x))$$

Assume another individual $v$ is more risk-averse than $u$ and his expected utility can be represented by a concave transformation $k(u)(k^{'}(\cdot) > 0, k^{''}(\cdot) < 0)$ of $u$'s expected utility.

$$
\begin{aligned}
& pv(w_0 - c(x) - l(x)) + (1-p)v(w_0 - c(x)) \\
& = pk(u((w_0 - c(x) - l(x))) + (1-p)k(u(w_0 - c(x)))
\end{aligned}
$$

Let $x_u$ and $x_v$ represent the optimal self-insurance level for $u$ and $v$.

In Dionne and Eeckhoudt(1985)'s paper, they show that to guarantee the optimal self-insurance level for $u$ is positive ($x_u > 0$), we need

$$
-l^{'}(x) - c^{'}(x) > 0
$$

which suggests the marginal effect on risk of self-insurance must be larger than its marginal cost.

Dionne and Eeckhoudt(1985) resort to first order condition to show $v$ takes higher self-insurance activities than $u$ does and finds $v$'s marginal utility is larger than zero at the optimal self-insurance level($x_u$) for $u$. The FOC of $v$ is:

$$
-pv^{'}(w_0 - c(x_u) - l(x_u))(c^{'}(x_u) + l^{'}(x_u)) - (1-p)v(w_0 - c(x_u))c^{'}(x_u) > 0
$$

This can be put:

$$
\begin{aligned}
& - pk^{'}(u(w_0 - c(x) - l(x)))u^{'}(w_0 - c(x_u) - l(x_u))(c^{'}(x_u) + l^{'}(x_u)) \\
& - (1-p)k^{'}(u(w_0 - c(x)))u^{'}(w_0 - c(x_u))c^{'}(x_u) > 0 \qquad (2.1)
\end{aligned}
$$

Compare with the FOC for $u$, (2.1) is positive when the coefficient for $u$'s marginal utility when loss happens is larger than that for $u$'s marginal utility when no loss happens. Hence,

$$k^{'}(u(w_0 - c(x) - l(x))) > k^{'}(u(w_0 - c(x)))$$

This holds because $k^{''}(\cdot) < 0$ and $u^{'}(\cdot) > 0$.

A major limitation for Dionne and Eeckhoudt(1985)'s conclusion is they require the first order derivatives of $U(x)$ and $V(x)$ are monotonic ($U^{''}(x) < 0, V^{''}(x) < 0$).

However, if the first order derivative is not monotonic, $-pv^{'}(w_0 - c(x_u) - l(x_u))(c^{'}(x_u) + l^{'}(x_u)) - (1-p)v(w_0 - c(x_u))c^{'}(x_u) > 0$ does not guarantee higher optimal care for $v$ ($x_v > x_u$).

Here, we resort to single crossing condition, which is still effective when monotonicity of first order condition is released. Our proposition is

**Proposition 2.2.1** *If $v$ is more risk averse than $u$ in Arrow-Pratt sense,regardless of $u$'s preference towards risk ($u^{''}(\cdot) = 0, > 0, < 0$),$v$ will exert more self-insurance than $u$ do.*

**Proof** For the sake of convenience, define $A = w_0 - c(x) - l(x)$ and $B = w_0 - c(x)$. $U(x), V(x)$ are both in the family $\{W(x)\}$. Because $v(x)$ can be gained by an increasing concave transformation $k(x)(k^{'}(x) > 0)$ of $u(x)$ ($v(x) = k(u(x))$).

According to Theorem 2.1.4, if $\{W(x)\}$ obeys single crossing property, then

$$argmax_{x \in R^+} V(x) \geq_{SSO} argmax_{x \in R^+} U(x)$$

holds.Based on Definition 2.1.3

$\{W(x)\}$ obeys single crossing property

$\Leftrightarrow$

$$U(x'') - U(x') \geq (>) 0 \Rightarrow V(x'') - V(x') \geq (>) 0. \tag{2.2}$$

A sufficient condition for (2.2) is

$$\textit{U(x) is increasing with } x \Rightarrow \textit{ V(x) is increasing with } x. \tag{2.3}$$

Note that

$$\begin{aligned} & \textit{U(x) is increasing with } x \\ \Leftrightarrow \quad & -pu^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)u^{'}(B)c^{'}(x) \geq (>) 0 \end{aligned} \tag{2.4}$$

and

$$\begin{aligned} & \textit{V(x) is increasing with } x \\ \Leftrightarrow \quad & -pv^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)v^{'}(B)c^{'}(x) \geq (>) 0 \end{aligned} \tag{2.5}$$

From $v(x) = k(u(x))$, we have

$$
\begin{aligned}
& -pv^{'}(A)(c^{'}(x)+l^{'}(x)) - (1-p)v^{'}(B)c^{'}(x) \geq (>)0 \\
\Leftrightarrow\ & -pk^{'}(u(A))u^{'}(A)(c^{'}(x)+l^{'}(x)) - (1-p)k^{'}(u(B))u^{'}(B)c^{'}(x) \geq (>)0 \\
\Leftrightarrow\ & k^{'}(u(A))[-pu^{'}(A)(c^{'}(x)+l^{'}(x)) \\
& -(1-p)\frac{k^{'}(u(B))}{k^{'}(u(A))}u^{'}(B)c^{'}(x)] \geq (>)0 \qquad (2.6)
\end{aligned}
$$

From $k^{'}(\cdot) > 0, k^{''}(\cdot) < 0$ and $u(A) < u(B)$, we obtain $\frac{k^{'}(u(B))}{k^{'}(u(A))} < 1$, which implies

$$-pu^{'}(A)(c^{'}(x)+l^{'}(x))-(1-p)\frac{k^{'}(u(B))}{k^{'}(u(A))}u^{'}(B)c^{'}(x) \geq -pu^{'}(A)(c^{'}(x)+l^{'}(x))-(1-p)u^{'}(B)c^{'}(x)$$

Therefore (2.4) implies (2.5).

**Q.E.D**

An advantage of our result is:this conclusion is independent of the restrictions on second order derivatives of utility functions. Thus, this result extends the effectiveness of conclusions by Dionne and Eeckhoudt(1985)to the situations where the optimal solutions are not inertial solutions.

### 2.2.2 Self-insurance-cum-protection model

The Self-insurance-cum-protection model (SICP) is formally studied by Lee (1998). This model examines the case when one's effort simultaneously influences both probability of risk and the size of loss. In practice, SICP model is more consistent with some real world examples. For example, those who wear helmet while cycling are tend to be more cautious about their behavior than those do not wear any protection. Thus they are less likely to suffer from the accidents and exposure to lower probability of accidents. The helmet can reduce the seriousness of potential injures. Given this, these cyclists exposure to both lower probability of accidents and less potential injure. Another example can be found with high quality brakes on viechles reduce the probability of an automobile accident(such as ABS system can guarantee the viechle tractable in harsh situations) and lower the magnitude of a loss if the viechle clashes another one.

The model is as following. The expected utility for two agents are:

$$U(x) = p(x)u(w_0 - c(x) - l(x)) + (1 - p(x))u(w_0 - c(x))$$

$$V(x) = p(x)v(w_0 - c(x) - l(x)) + (1 - p(x))v(w_0 - c(x))$$

Assume Agent $v$ is more risk averse than $U$,and the utility function for $v$ can be represented by a concave transformation of utility function for $u$,($v(x) = k(u(x))$. In our model, there is no assumptions on the second order derivative of $u$, which implies the DM can be risk-averse, risk-neutral or even risk-loving. $x$ is the level of self-insurance-cum-protection (SICP) activities taken by the decision maker. Let $x_u$ and $x_v$ represent the optimal SICP level for $u$ and $v$ respectively. The probability and size of losses decrease by higher SICP activities. Thus, $p(x) \in [0, 1], p^{'}(x) < 0$ and $l(x) > 0, l^{'}(x) < 0$. Monetary cost for SICP activites is represented by $c(x)$ and its marginal cost is increasing ($c^{'}(x) > 0$).

Lee's(1998) paper has studied the effect of an increase in risk aversion on SICP activities and shows that the effect depends in part on the shape of the loss function, relating the size of a potential loss to SICP expenditures. Particularly, if the marginal reduction for a loss is larger than the marginal increase in the cost of SICP expenditures, more risk-averse individuals invest more in SICP.

His proposition goes as follows.

**Theorem 2.2.2 (Lee,1998)**

1. *Assume that, for a given risk-averse individual with the utility function $U$, $\frac{\partial EU}{\partial x}|_{x=0} > 0$ and $\frac{\partial EU}{\partial x}|_{x=w_0} < 0$ hold and hence an interior solution, $x_{u'}$ exists. Then, a sufficient condition for more risk-averse individual to invest more in SICP is $c^{'}(x_u) + l^{'}(x_u) \leq 0$.*

2. *Assume that, for a given risk-averse individual with the utility function $U$, $\frac{\partial EU}{\partial x}|_{x=0} > 0$ and $\frac{\partial EU}{\partial x}|_{x=w_0} < 0$ hold and hence an interior solution, $x_{u'}$ exists, and that $c^{'}(x_u) + l^{'}(x_u) > 0$.Then, there is some number $p_u^* \in (0,1)$ such that more risk-averse individuals invest more(less) in SICP if $p(x_u) < (>) p_u^*$.*

In Lee(1998)'s proposition, he considers when mainly two different situations, based on the sign of $c^{'}(x_u) + l^{'}(x_u)$. When $c^{'}(x_u) + l^{'}(x_u) \leq 0$, the results is much like that for self-insurance case. While $c^{'}(x_u) + l^{'}(x_u) > 0$, some additional assumptions are needed to guarantee relation between SICP and increased risk-aversion, since the increase in probability of loss $p$ will decrease DM's benefit from SICP activities.

In our proposition, we suggest a new condition, which expand the effectiveness of the relation in first situations ($c^{'}(x_u) + l^{'}(x_u) \leq 0$) in Lee(1998)'s paper.

**Proposition 2.2.3** *Assume agent $v$ is more risk averse than $u$. The utility is always positive for both agents. $c^{'}(x_u) + l^{'}(x_u) \leq 0$. Under the condition that hazard rate of the loss is higher than 'boldness' coefficient of $u$ in no risk state ($\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u'(w_0-c(x))}{u(w_0-c(x))}$), $v$ will take higher SICP efforts than $u$ do.*

**Proof** For convenience, let $A = w_0 - c(x) - l(x), B = w_0 - c(x)$.

Define $g(x) = k(x) - k'(x)x$

$$g'(x) = k'(x) - k''(x)x - k'(x) = -k''(x)x \geq 0 \text{ for all } x \geq 0$$

Thus

$$g(u(A)) = k(u(A)) - k'(u(A))u(A) < g(u(B)) = k(u(B)) - k'(u(B))u(B)$$

The first order derivatives for two agents' utility function are

$$\begin{aligned}
U^{'}(x) &= p'(x)u(A) - p(x)(c'(x) + l^{'}(x))u'(A) - p'(x)u(B) - (1-p(x))c'(x)u'(B) \\
V'(x) &= p'(x)v(A) - p(x)(c'(x) + l^{'}(x))v'(A) - p'(x)v(B) - (1-p(x))c'(x)v'(B) \\
&= p'(x)k(u(A)) - p(x)(c'(x) + l^{'}(x))k'(u(A))u'(A) \\
&\quad -p'(x)k(u(B)) - (1-p(x))c'(x)k'(u(B))u'(B)
\end{aligned}$$

$$= p'(x)\{[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)]\}$$

$$
\begin{aligned}
&+k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l'(x))u'(A)] \\
&-k'(u(B))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
\geq\ & k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l'(x))u'(A)] \\
&-k'(u(B))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
\geq\ & k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l'(x))u'(A)] \\
&-k'(u(A))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
=\ & k'(u(A))U'(x), \qquad (2.7)
\end{aligned}
$$

The first inequality holds because $p'(x) < 0$ and $[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)] < 0$.The secondly inequality holds if $(k'(u(A)) - k'(u(B))) * [p'(x)u(B) + (1 - p(x))c'(x)u'(B)] > 0$. Since $k'' < 0$,$k'(u(A)) - k'(u(B)) > 0$. However, conditions do not guarantee $p'(x)u(B)+(1-p(x))c'(x)u'(B)$ is positive. So we assume $p'(x)u(B) + (1 - p(x))c'(x)u'(B) > 0$.

Given the above (2.7) holds,

$$V(x) \succeq_I U(x)$$

$$argmax_{x \in R^+} V(x) \geq argmax_{x \in R^+} U(x)$$

More risk-averse decision maker will take higher SICP effort.

**Q.E.D**

The condition in this proposition can be put:

$$\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u^{'}(w_0-c(x))}{u(w_0-c(x))} \tag{2.8}$$

$$HR(x) \geq boldness(x)$$

$$1 \geq HR(p(x))FR(u(x)) > 0$$

The term of left side of first inequality is *hazard rate*, measures change in probability for risk when no risk happens at the current activity level. The term of right side is the boldness rateand its inverse as *fear of ruin*( Aumann and Kurz,1977)[1],which indicates an agents willingness to risk all his fortune against a small potential gain. This condition can be put in another way that the less risk averse decision maker's marginal decrease in probability to survival probability is higher than the marginal cost of utility to total utility.

With interval dominance order, our conclusion extends to the situation, when second order derivative is not given or optimal solution is not interior solution. Compare with Lee(1998)'s result, when $c^{'}(x) + l^{'}(x) \leq 0$, our method extend the effectiveness even when the optimal solution is not interior. In Lee(1998)'s proposition,the condition $\frac{\partial EU}{\partial x}|_{x=0} > 0$ and $\frac{\partial EU}{\partial x}|_{x=w_0} < 0$ is required. With our method, this assumption is not necessary. The marginal utility functions do not need to be monotonic on the interval $[0, w_0]$. Given this, the optimal solution maybe corner solution or inflection point solution. This is more consistent with real world situations. For example, decision makers may have psychological thresholds for protection activities. If they take higher pro-

[1]also see Foncel and Treich(2005)

tection activities than this threshold in mind, their attitude towards risk may change, such as turn risk neutral or even risk loving from risk averse.

In fact, the model of self-insurance and self-protection can be derived from SICP model. If the probability of loss is independent of the activity level($p^{'} = 0$), this SICP model is reduced to self-insurance model. Our findings are still effective under this circumstance. The marginal utility for $u$ can be positive even when $-p'(x)u(B) - (1 - p(x))c'(x)u'(B)$ is negative. Because the absolute value of marginal effect on loss is larger then that of marginal cost $(c^{'}(x_u) + l^{'}(x_u) \leq 0)$ is the implied condition in this model and, hence, $u$'s optimal self-insurance level maybe positive. Single crossing condition suggests $v$ takes higher self-insurance even second order derivatives for $u$ or $v$ is not defined.

Interestingly, our finding fails to extend its effectiveness when loss is independent from protection activity and only probability is affected by self-protection. Since size of loss is unrelated to the activity level($l^{'}(\cdot) = 0$), the model is reduced to self-protection model. However, our condition does not indicate a clear result for self-protection. The problem lies in the marginal utility of $u$ is always negative. Because $c^{'}(x_u) + l^{'}(x_u) = c^{'}(x_u) > 0$ and sum of the first two items in marginal utility for $u$ is samller than zero. Additionally,our condition assumes the sum of last two items in marginal utility of $u$ is negative. Therefore,$u$'s utility decreases as he takes increasing level of self-protection and his optimal protection level is zero, which is inconsistent with most cases in real world. Even $v$'s marginal utility can be represented by a increasing transformation of $u$'s, it can not guarantee $v$ has positive marginal utility from self-protection. Some additional assumptions are needed to conclude this relation in this case.

### 2.2.3 A Brief Discussion for Self-Protection

In this part we consider self-protection case. Self-protection only increases the probability of good outcome at the expense of decreasing the utility of all states. Assume the level of self-protection is $x$,and $x \in [0, w_0]$. The probability of loss can be expressed as a function of self-protection level $p(x)$ and $p^{'}(\cdot) < 0$ because the marginal effect of self-protection is decreasing. The monetary cost $c(x)$ and $c^{'}(\cdot) > 0$ for increasing marginal cost. Here, we assume self-protection can only influence the probability of risk occurrence,while it does not affect the severity of loss. The size of loss is independent of self-protection and represented by a constant $l, (l > 0)$.

The expected utility function for two decision makers are:

$$U(x) = p(x)u(w_0 - c(x) - l) + (1 - p(x))u(w_0 - c(x))$$

$$V(x) = p(x)v(w_0 - c(x) - l) + (1 - p(x))v(w_0 - c(x))$$

Let $x_u$ and $x_v$ represent the optimal self-protection level for $u$ and $v$.

In Dionne and Eeckhoudt(1985)'s paper, the conventional first order condition does not suggest a clear comparison for the optimal self-protection level.

Because the sign of

$$V'(x_u) = p'(x_u)v(w_0 - c(x_u) - l) - p(x_u)c'(x_u)v'(w_0 - c(x_u) - l)$$
$$-p'(x_u)v(w_0 - c(x_u)) - (1 - p(x_u))c'(x_u)v'(w_0 - c(x_u))$$

is not decided by $k^{'}(\cdot)$ and $k^{''}(\cdot)$. This formula does not indicate the relation between increased risk-aversion and self-protection. To find the relation between $x_u$ and $x_v$ they discuss the value of $k^{'}(\cdot)$.

**Theorem 2.2.4 (Dionne and Eeckhoudt,1985)**

1. $k^{'} > 1(< 1)$ *everywhere in the interval* $[A, B]$ *(they assume* $A = w_0 - c(x) - l(x), B = w_0 - c(x)$*) implies that both the marginal benefit and the marginal cost of* $x$ *increases (decreases) for* $v$ *such that the net effect on* $x$ *is ambiguous.*

2. $k^{'} > 1$ *at* $A$*,*$k^{'} < 1$ *at* $B$ *and* $k^{'} = 1$ *at* $C$ *with* $A < C < B$*. It is impossible without more restrictive assumptions to predict the sign of the variation both for the marginal benefit and for the marginal cost.*

Dionne and Eeckhoudt(1985) also consider several concrete examples including quadratic utility function, logarithmic utility function and exponential utility function. They find that the the optiaml self-protection level for more risk averse agent is higher ($x_v > x_u$) if $p > \frac{1}{2}$ for quadratic utility functions, while the relation is ambiguous with logarithmic type and exponential type.

Jullien and et al. (1999) suggested a more risk-averse agent will take higher self-protection activities if and only if the probability of loss is lower than a threshold. The idea for Jullien et al(1999)'s proving is to find $v$'s first order condition is positive with $u$'s optimal effort.

**Theorem 2.2.5 (Jullien,Salanie and Salanie,1999)**

*Assume condition:*

1. *$U$ is concave;$c(x)$,$c(x)+l$ are increasing convex;$p(x)$ is decreasing convex and $p^{''}(x)p(x) \geq 2p^{'}(x)$;*

2. *The level of effort of $u$ is stricly between $0$ and $w_0$.*

*hold for u and v, and v is more risk-averse than u. Then self-protection is higher for v than for u if and only if the probability is smaller than a threshold ($p(e_u) < p^*$).*

$$\frac{p^*}{1-p^*} = \left(\frac{U^{'}(B)\Delta V - V^{'}(B)\Delta U}{V^{'}(A)\Delta U - U^{'}(A)\Delta V}\right)\frac{c^{'}(e_u)}{d^{'}(e_u)},$$

$$\Delta V = V(B) - V(A), \Delta U = U(B) - U(A)$$

$$A = W - d(e_u), B = W - c(e_u),$$

*$e_u$ is the optimal self-protection level for agent u.*

This results implies if the initial probability for loss is low enough, more risk-averse decision maker will take higher self-protection activities.

We resort to interval dominance order and try to suggest a new condition, which requires one assumption concerning fear of ruin coefficient and marginal effect of self-protection on probability and marginal cost of self-protection. However, our result is not desirable. Our condition is theoretically practical. While its economic intuition is unreasonable. Because based on our assumption, the less risk averse DM always takes no self-protection activity since his marginal utility is always negative and taking self-protection activity decreases his utility. For the more risk averse agent, his optimal level of prevention cannot be decided since of the sign of his marginal utility from self-protection is unclear. Marginal utility for $v$ can be put as:

$$\begin{aligned} V'(x) &= p'(x)k(u(A)) - p(x)c'(x)k'(u(A))u'(A) - p'(x)k(u(B)) \\ &\quad -(1-p(x))c'(x)k'(u(B))u'(B) \\ &= p'(x)\{[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)]\} \\ &\quad +k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\ &\quad -k'(u(B))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\ &> k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\ &\quad -k'(u(B))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\ &> k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\ &\quad -k'(u(A))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\ &= k'(u(A))U'(x) \end{aligned} \tag{2.9}$$

However, the sign of $V^{'}(x)$ is not defined in this inequality. Because the marginal utility for $u$ is always negative, while the sign for $v$'s marginal utility is not clear. Hence the more risk averse agent's choice for self-protection can not be determined by this method. We need more conditions to suggest a clear relation between risk aversion and self-protection.

# Chapter 3 Conclusion

In this dissertation, I re-examine Dionne and Eechkdout(1985)'s topic on risk aversion and self-insurance and self-protection activities and Lee(1998)'s SICP model. With single crossing condition and interval dominace order,I conclude a new condition for the positive relation between risk aversion and SICP activities. Additionally, we extend our results to self-protection (SICP) model. Our method in this study has not been conducted in previous studies.

In this simple case of choice under low order risk, we show that increased risk aversion will induce both self-insurance and SICP activities.The condition for SICP requires property of hazard rate and fear of ruin coefficient. This study still opens to further examination. For example, this model can be expanded to a two-period model setting (Menegatti 2009,2011) and more general cases with possibility of various states in form of density function. Moreover, comparative statics for high order risk preference and prevention efforts is another interesting topic. Furthermore, with single crossing condition and interval dominance order, the comparative statics of relation between ambiguity aversion and precaution actions is another potential research topic.

Now the model is restricted to the two-state model, namely only loss and safe states. It is of great interest to investigate continuous states of nature and the general type of distribution function. Furthermore, utility is bi-variate or even multivariate function is not included, which can more effectively suggest injurers and victims' behavior and care level. The bivariate model can be applied to diverse settings, such as risk aversion and contest. These extensions are my future research topics.

# References

Aumann, R. J., Kurz, M. (1977), Power and taxes. Econometrica: Journal of the Econometric Society, 1137-1161.

Boyer, M., Dionne, G.(1989), More on insurance, protection, and risk,The Canadian Journal of Economics/Revue canadienne d'Economique, 22,202-204.

Briys, E., Schlesinger, H.(1990),Risk aversion and the propensities for self-insurance and self-protection,Southern Economic Journal,458-467.

Brown, J. P. (1973), Toward an economic theory of liability. The Journal of Legal Studies, 2(2), 323-349.

Chiu, W. H. (2000), On the propensity to self-protect,Journal of Risk and Insurance,555-577.

Cornes, R., & Hartley, R. (2012), Risk aversion in symmetric and asymmetric contests. Economic Theory, 51(2), 247-275.

Diamond, P. A. (1974), Accident law and resource allocation. The Bell Journal of Economics and Management Science, 366-405.

Diamond, P. A. (1974), Single activity accidents. The Journal of Legal Studies, 3(1), 107-164.

Diamond, P. A., & Stiglitz, J. E.(1974),Increases in risk and in risk aversion. Journal of Economic Theory, 8(3), 337-360.

Dionne, G., Eeckhoudt, L.(1985),Self-insurance, self-protection and increased risk aversion, Economics Letters, 17,39-42.

Eeckhoudt, L. R., Hammitt, J. K. (2001), Background risks and the value of a statistical life. Journal of risk and uncertainty, 23(3), 261-279.

Eeckhoudt, L.,& Gollier, C.(2005), The impact of prudence on optimal prevention. Economic Theory, 26(4), 989-994.

Ehrlich, I., Becker, G. S.(1972),Market insurance, self-insurance, and self-protection, Journal of political Economy, 80, 623-648.

Fagart, M. C.,& Fluet, C.(2009), Liability insurance under the negligence rule. The RAND Journal of Economics, 40(3), 486-508.

Foncel, J., & Treich, N. (2005), Fear of ruin. Journal of Risk and Uncertainty, 31(3), 289-300.

Jullien, B., Salanie, B., Salanie, F.(1999),Should more risk-averse agents exert more effort?, The Geneva Papers on Risk and Insurance Theory 24,19-28.

Konrad, K. A., & Schlesinger, H. (1997), Risk Aversion in Rent - Seeking and Rent - Augmenting Games. The Economic Journal, 107(445), 1671-1683.

Lee, K. (1998). Risk aversion and self-insurance-cum-protection. Journal of Risk and Uncertainty, 17(2), 139-151.

Meyer, D.J.&Meyer, J.(2011), A Diamond-Stiglitz approach to the demand for self-protection.Journal of Risk and Uncertainty, 42, 45-69

Milgrom, P., Roberts, J.(1990), Rationalizability, learning, and equilibrium in games with strategic complementarities,Econometrica: Journal of the Econometric Society, 1255-1277.

Milgrom, P., Shannon, C. (1994), Monotone comparative statics. Econometrica: Journal of the Econometric Society, 157-180.

Quah, J. K. H., Strulovici, B. (2009), Comparative statics, informativeness, and the interval dominance order. Econometrica, 77(6), 1949-1992.

Shavell, S. (1980), Strict liability versus negligence. The Journal of Legal Studies, 9(1), 1-25.

Shavell, S. (1987,2009), Economic analysis of accident law. Harvard University Press.

Skaperdas, S., & Gan, L. (1995), Risk aversion in contests. The Economic Journal, 951-962.

Tibiletti, L. (2006), A Shortcut Way of Pricing Default Risk Through Zero-Utility Principle. Journal of Risk and Insurance, 73(2), 303-308.

Treich, N. (2010), Risk-aversion and prudence in rent-seeking games. Public Choice, 145(3-4), 339-349.

Topkis, D.M., Supermodularity and Complementarity, 1998.

Viscusi, W. K. (1993), The value of risks to life and health. Journal of economic literature, 31(4), 1912-1946.

Vives, X.(1990), Nash equilibrium with strategic complementarities, Journal of Mathematical Economics, 19(3), 305-321.

Zivin, J. G., Just, R. E., & Zilberman, D. (2005), Risk Aversion, Liability Rules, and Safety. International Review of Law and Economics, 25(4), 604-623.