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Managing sales and product returns under the word-of-mouth effect : pricing, quality, and restocking fee decisions

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by
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ABSTRACT

Managing Sales and Product Returns under the Word-of-Mouth Effect: Pricing, Quality, and Restocking Fee Decisions

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This paper considers a two-echelon supply chain involving a manufacturer and a retailer who make their pricing, quality, and restocking fee decisions under the word-of-mouth (WOM) effect. To investigate the decision-making problem for the sales and product returns, we construct a leader-follower game model in which the manufacturer first determines his quality effort and the wholesale price and the retailer then decides on her retail price and the restocking fee. Our results show that the wholesale and retail prices under no WOM effect are smaller than those when the WOM effect exists; and, as the WOM has a higher impact on consumer purchases, the retailer should increase her retail price, and the manufacturer should also spend more quality control effort and raise his wholesale price. In addition, when the WOM effect exists, both the restocking fee and the retail price are decreasing in the mismatch probability. It is interesting to learn that a positive WOM may not help achieve more total sales and realized sales than a negative WOM, and the realized sales under the full refund policy may be lower than those under a partial refund policy. We also find that the manufacturer and the retailer may not benefit under the WOM effect, which mainly depends on how consumers are sensitive to the restocking fee and the WOM. The retailer can benefit from the full refund policy when the mismatch chance is not high, whereas the manufacturer can always benefit from the full refund policy.

Key words: word-of-mouth, returns, pricing, restocking fee, game analysis.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

(Hui Sun Yuen)
Date:

CERTIFICATE OF APPROVAL OF THESIS

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1 Introduction

A couple of decades ago consumers could not access the Internet and had to visit physical stores for shopping. Since the early 1990s information technology has been advancing rapidly and allowing consumers to shop online with increasing convenience and efficiency. In today's electronic market, a consumer can place an online order with only a few clicks anytime at any place, and can then receive the goods at a designated address in a short waiting time. According to the data collected by Loechner (2008), around 85% of consumers in the world have online shopping experience, which is mainly ascribed to the fact that consumers usually expect to save their times and easily search for what they desire to purchase. Retailers can also benefit from online shopping by managing their retailing businesses more efficiently with much less cost compared with running a brick-and-mortar store. That is, eBusiness reconstructs the information flow, money flow, and supply chain, and also helps improve the benefits of both companies and consumers.

As Fortune magazine (Hao 2013) reported, eBusiness giants in some countries have gained a significant share of the Internet economy by developing their online services ahead of most of other companies. For example, in the United States, there are Google ($293 billion market capitalization), Amazon ($125 billion), eBay ($66 billion), and Facebook ($58 billion). In China, Baidu ($33 billion) is a leading Chinese search engine, Alibaba is viewed as a dominant B2B company, and Tencent ($69 billion) appears to be the principal player in social networking and gaming. The number of products sold online is around 5% of total sales in the United States and the number is 6% in China. As electronic market has been a "battlefield" for many companies to reveal their values worldwide, the competition for the market share becomes more and more intense (Hao 2013), which is demonstrated by the evidence that a number of companies (e.g., Walmart and Jet.com) have made a large investment to improve their online businesses. The booming of eBusiness also induces supply chain partners (i.e., manufacturers and retailers) to develop more effective strategies to reduce their costs as well as increase their market shares and profits.

In order to achieve high sales and increase profits, sellers need to meet the expectations of customers who are likely to learn other consumers' reviews on products or services prior to making a decision. As the Nielsen (2013)—a global performance management company in the United States—reported, 84% of customers trust their friends' recommendations and 68% trust online reviews. Such recommendations and online reviews are generally called "word of mouth" (WOM). Gwinner et al. (2004) formally defined the online WOM as "any positive or negative statement made by potential, actual, or former customers about a product or company, which is available to a multitude of people and institutions via the Internet." The WOM can greatly influence customers' purchase decisions and help companies effectively enlarge their consumer bases and enjoy higher profits (American Marketing Association 2016). Marketing staffs may adopt the WOM as a way of promoting their products or services. For example, Amazon is the first online company to publicly release consumer reviews with a motivation to reduce the large expenses mainly caused by product returns. Although some online consumer reviews might be negative, in today's business world, the majority of companies have allowed consumers to post their reviews online. As a leading source of product reviews in the world, Amazon has created wish lists for consumers to share favorite products in 1999 and launched discussion hubs in 2006, which helps Amazon obtain valuable opinions as well as a huge success in marketing. Expedia, a website famous for collecting consumer reviews about hotels, car-rentings, and others, even developed a policy to encourage experienced customers (who have made at least one transaction with Expedia online) to leave their thoughts.

Motived by the successful experiences of online giants such as Amazon and Expedia, we investigate whether the influence of the WOM is essential to online firms that intentionally adopts the WOM as a tool to expand their sales. In practice, under the WOM effect, an online firm's success is largely dependent on its pricing, quality, and restocking fee decisions, which play significant roles in the management of sales and product returns. In the past years, the increasingly important value of WOM has induced many researchers to examine the pricing, quality, or restocking fee decisions. Although no publication has considered the three decisions jointly, Kwark et al. (2014) and Li et al. (2014) concluded that the reveal of customer reviews may entice manufacturers and retailers to reduce their prices and attract customers. To increase sales, manufacturers and retailers may intend to improve their qualities, which can generate a higher cost for quality control and may induce those firms to increase their prices. As a result, the demand may be reduced and those firms may lose their profits. But, the quality improvement can influence the WOM and then the demand, viz., a high quality can result in a positive WOM value, which may be helpful to increase the demand. Although a low quality usually generates a negative WOM value and then reduces the demand, it can also save a firm's quality control cost. In addition, even when a firm reduces the quality but increases the price, the firm may still enjoy a higher profit particularly when the cost is very low and the marginal profit is large enough to cover the demand loss. Therefore, prior to performing our analysis, we cannot conclude that a negative WOM value certainly hurts the firms, or a positive WOM can always benefit them. It thus behooves us to examine the pricing and quality decisions for a manufacturer and the return policy for a retailer in a supply chain.

When a customer is dissatisfied with a product due to the mismatch between the actual product and her expectation, the customer is likely to return the product to the retailer, who has to respond by implementing an effective strategy to avoid product returns. As in practice, the firm can charge a restocking fee for any product return, which can be viewed as a cost to be absorbed by customer returning their mismatched products. The restocking fee has been regarded as an effective tool in controlling consumers' returns and affecting a firm's profit. As Shulman et al. (2011) observed, a firm tends to set a high restocking fee that could be above the retail price to avoid product returns. Similarly, Koessler and Renault (2012) revealed that a firm is inclined to increase the restocking fee to discourage customers from returning high-quality products. A high restocking fee can reduce returns, whereas a low one may maintain loyal consumers. We also note that some online retailers such as Amazon, Zara, and H&M adopt a zero restocking fee policy, which raises a research question: could it be optimal to determine a zero restocking fee? It would be thus interesting and important to find an optimal restocking fee for online retailers.

According to our above discussion, we jointly consider the quality, pricing, and restocking fee decisions for a two-echelon supply chain in a leader-follower setting, where the manufacturer first determines his quality level and the wholesale price, and the retailer then decides on her retail price and the restocking fee. We use the backward induction to solve the decision problem. In Section 2 we provide a comprehensive review of extant publications related to the pricing, quality, and restocking fee decisions under the WOM effect. The review indicates the originality of our research problem in this paper.

In Section 3, we provide our preliminary discussions regarding the WOM, the return of mismatched products, and sales function. In section 4, we analyze the decision problems for both the manufacturer and the retailer. Solving the game, we obtain two firms' decisions in Stackelberg equilibrium. Then, we perform sensitivity analysis to draw managerial implications concerning the impact of important parameters on the pricing, quality, and restocking fee decisions as well as the total sales, the realized sales (i.e., the number of sold products that are not returned), and two firms' profits.

We have drawn a number of interesting managerial insights. For example, when the WOM influences consumers' purchases, the equilibrium restocking fee is decreasing in consumers' sensitivity to the restocking fee, whereas the equilibrium retail price is increasing in this sensitivity. Both the manufacturer and the retailer are worse off when consumers are more sensitive to the restocking fee.

Under the WOM effect, when consumers are more sensitive to the retail price, the manufacturer and the retailer are more likely to reduce their wholesale and retail prices, respectively. In order to mitigate the impact of the consumers' sensitivity to the retail price on sales, the retailer should charge a small restocking fee. Moreover, the manufacturer should spend less effort for quality control in order to assure his profit margin. We also find that as consumers' purchases are more dependent on the average WOM value (i.e., the overall social evaluation of the product), the retailer should increase her retail price, and the manufacturer should also spend more effort on his quality control and raise his wholesale price. In addition, when the WOM effect exists, both the restocking fee and the retail price are decreasing in the mismatch probability. To improve consumers' purchase incentives, the retailer needs to decrease her retail price. This improves the manufacturer's profit, which implies that the manufacturer is better off from the mismatch.

In Section 5, we conduct extensive discussions of the WOM effect and the full-refund return policy. Specifically, we first analyze our decision problem with no WOM effect, and compare our results with those when the WOM effect exists as in section 4. Such a comparison indicates how the WOM influences the firms' decisions and profits. We find that the WOM may not always help the retailer achieve more total sales and realized sales. Moreover, if each consumer returns his or her mismatched product with a sufficiently large chance, then the realized sales under the full refund policy are higher than those under a partial refund policy.

The wholesale and retail prices with no WOM effect are smaller than those when the WOM effect exists, whereas the restocking fee remains unchanged regardless of whether the WOM effect exists or not. We also find that the manufacturer and the retailer may not always benefit from the WOM effect. For example, the WOM effect makes the manufacturer worse (better) off if the consumers' sensitivity to the restocking fee is sufficiently small (large), whereas the retailer always benefits under the WOM effect. When the consumers' sensitivity to the WOM is sufficiently small, the WOM effect makes both firms worse off compared to the case with no WOM effect. In addition, the manufacturer always profits from the WOM effect, whereas the retailer benefits from the WOM effect only when the mismatch probability is sufficiently small. The retailer can be more beneficial under the full refund policy than under a partial refund policy when the mismatch chance is not high, whereas the manufacturer can always benefit from the full refund policy. This paper ends with a summary of managerial implications in Section 6.

2 Literature Review

We review major publications which are concerned with online sales and product returns under the WOM effect. Recent empirical studies (e.g., Brynjolfsson, Hu, and Smith 2006, and Elberse and Oberholzer-Gee 2007) have revealed that the WOM effect is of great significance to influence customers' purchase decisions and the sales at online retail stores (e.g., Amazon) that sell video games and books online. If there are a number of long and positive online reviews for a product, then the sales of the product are very likely to be higher than others even when the product is a niche one. Judith and Dina (2006) analyzed the user review data collected from public web sites, and discovered that negative reviews are more effective in decreasing sales comparing to positive reviews in increasing sales.

The WOM-related academic study is very limited in the past years, although the WOM plays an important role in online operations, and a number of scholars have published their findings related to the impact of pricing, quality, and restocking fee decisions on the sales and product returns. Most of relevant publications only focus on the impact of one of the three decision variables, and others jointly consider any two of these three decisions. No paper has jointly considered these three decisions especially under the WOM effect. Next, we briefly summarize our reviews of major publications that concern the pricing, quality, or restocking fee decisions.

The Pricing Decision under the WOM Effect Price is the most important and effective factor in sales and product returns. Kwark et al. (2014) investigated how the WOM influences price competitions among manufacturers as well as retailers. They found that the information of quality difference indicated by customer reviews increases the competition among manufacturers, thus reducing their wholesale prices and the retail prices. If customers' reviews indicate that manufacturers' products possess a quality as those customers expect, then the competition decreases and the wholesale and retail prices increase. In order to make the WOM valuable to customers in their decision-makings, firms need to enlarge its user base and embrace more customer reviews. In a more recent study, Li (2014) treated the WOM as a signal of quality, examined the impact of pricing strategies on the WOM, and found that the sales reflects the information accuracy and a fit to customer needs. The author also disclosed that an early-period price cut can attract more customers and generate more online reviews, thus improving the accuracy of quality information.

The Restocking Fee Decision under the WOM Effect A number of publications are focused on the role of restocking fee in operations management; see, e.g., Kroll et al. (1999) and Swinney (2011), who showed that, for a firm, the restocking fee is an effective method in preventing product returns and is also a source of the firm's profit. They also found that less-informed consumers are more likely to return their purchased products. As a response, the firm may transfer returns-caused costs to consumers by charging a restocking fee, which can reduce product returns. Therefore, the restocking fee plays both a cost-defrayment role and a role in altering consumer behavior. This is in agreement with the findings in some earlier publications by, e.g., Davis et al. (1998) and Peterson and Kumar (2010) who had mainly discussed return policies that a higher production cost causes a more restrict return policy. Shulman et al. (2009) also demonstrated that a retailer may raise the hassle cost to discourage excessive returns; and when consumers strongly expect to obtain the right product, a higher restocking fee can effectively prevent them from exchanging and thus improve the retailer's profit.

The Quality Decision under the WOM Effect Quality is a major issue for consumers to generate the WOM values regarding their use experiences. Firms may intentionally improve product quality to reduce product returns (Kuzma and Shanklin 1992, Powell 1995, and Hendricks and Singhal 1996). As Wright and Heiens (1999) and Espejel-Blanco and Fandos-Herrera (2008) revealed, the chance for product returns is low if consumers are confident with the product quality, which implies that consumers possess loyalties toward high quality products. As a response, in practice many firms have invested enormously to improve the quality of their products even with a target of a very high level. However, as recent studies (Manchanda and Chintagunta 2006 and Manduchi 2010) explored, the WOM has an indirect impact on the quality decision and thus, a number of firms have overstated the value of their products, which do not match customer needs. In fact, when a product matches customer needs, the WOM is useful to delivering values, effectively improving the sales, and reducing product returns.

Joint Pricing and Quality Decisions under the WOM Effect In the last decade, very few publications were concerned with the pricing and quality decisions under the WOM effect. As a seminal paper, Mathios (2000) performed an empirical study for the salad industry, using the data from upscale supermarkets in the state of New York—i.e., duopoly markets where salad dressings are either high or low in fat. The results revealed that firms are willing to disclose product information when the quality of their products is able to meet customers' expectations on, e.g., the low-fat salad dressings. Koessler and Renault (2012) found that quality changes with the price swing. Full disclosure of the product information helps achieve the price equilibrium in a market and maximize the profits of firms in the market. Under such a price equilibrium top quality firms intentionally utilize the WOM and sell their products at high prices; see, e.g., Koessler and Renault (2012). In addition, Board (2009) showed that a low-quality firm may set a high price and then enjoy free ride from high-quality firms, which would decide to share the product information even though a higher competition leads the price to fall.

Joint Pricing and Restocking Fee Decisions under the WOM Effect In practice, consumers usually prefer to learn about product features prior to their purchases, which is more likely to occur when the refund is smaller. Recent studies (e.g., Hess et al. 1996) have shown that when the value of a product is higher, the opportunistic returns of the product rise. As a response, firms increase their prices and also nonrefundable charges (i.e., restocking fees), as such charges make firms more profitable. Ofek et al. (2011) have found that a consumer has a higher risk of buying a defective product at an online store than at an brick-and-mortar store. To solve this problem, online retailers may invest to reduce the rate of defects; as a result, a product that is available for online sale could have a higher price than that sold at a physical store. Otherwise, the pricing and restocking fee decisions play important roles in reducing product returns. A firm may set a restocking fee above its cost to prevent consumers from buying from its competitors, as reported by Shulman et al. (2011).

After we review relevant publications and reports, we found that there is no publication specifically regarding joint quality and restocking fee decisions under the word-of-mouth effect. And we can conclude that our paper significant differs from any extant publication. As briefly described in Section 1, in this paper we jointly analyze the impact of pricing, quality, and restocking fee decisions under the WOM effect. We construct a leader-follower game model in which a manufacturer first determines his wholesale price and the quality level and a retailer then determines her retail price and the restocking fee. We solve the game to attain the two firms' pricing and quality decisions in Stackelberg equilibrium.

Our decision-making models have not been considered in the literature. That is, a major contribution of our analysis is to explore the complex impact of the major three decisions for online operations (i.e., price, quality, and restocking fee) under the WOM effect. Such an investigation is important because consumer reviews has significantly influenced the management of online sales and product returns in today's electronic markets, as the rapid development of information technology enables customers to acquire more information about products easily and quickly. Our results are expected to expose the WOM effect on the pricing, quality, and restocking fee decisions in online operations.

3 Preliminaries

We consider a supply chain involving a manufacturer and a retailer who serve consumers in a market. The manufacturer produces a specific product and sells it to the retailer, who then satisfies consumer demand. As each consumer may return his or her mismatched product to the retailer, we consider the sales and product return decision-making stages in the supply chain. Next, we discuss some issues in the two stages.

3.1 Quality and Word-of-Mouth

In reality, the WOM usually plays a vital role in each consumer's purchasing decision-making process. Specifically, prior to a consumer's purchase, she cannot exactly ensure the product's quality, attributes, and performance, and thus usually has to search these information by reading previous consumers' (online) product reviews and/or consulting her friends (a.k.a. WOM). The WOM can be viewed as a reference value for the consumer on which she can decide whether or not to buy. Since previous consumers often follow their experiences to assess the product with, for example, a grade between 0 (worst) and 5 (best), we can estimate the average WOM value for the product according to previous consumers' grades. Note that the WOM can affect the consumer's purchase intention *psychologically*; and as usual, the higher the average WOM value is, the more the consumer is willing to buy. According to the average WOM value, the consumer can have a general image about the product. Naturally, a greater quality level of the product can induce consumers to make a higher grade. That is, the average WOM value is increasing in the product performance mainly in terms of quality level, as discussed by Wright and Heiens (1999), Manchanda and Chintagunta (2010), Espejel-Blanco and Fandos-Herrera (2008), Manduchi (2010), and others. The manufacturer can spend more effort on the quality improvement and raise the average WOM value, which then increases the demand. But, this entails the manufacturer incurring a higher cost. Therefore, it is important for the manufacturer to make a proper decision on his effort on quality improvement. Hereafter, the effort is simply represented by the manufacturer's quality level q. In this paper, we denote the average WOM value by $r(q)$. As discussed above, $r(q)$ is an increasing, concave function of q, i.e., $r'(q) \geq 0$ and $r''(q) \leq 0$.

As in practice, the average WOM value $r(q)$ may be positive, zero, or negative. Specifically, a positive WOM value (i.e., $r(q) > 0$) means that past consumers, in general, accept the product quality. A zero WOM value (i.e., $r(q) = 0$) implies that overall, consumers evaluate the product neutrally, which may not significantly influence new consumers' purchasing decisions. A negative WOM value (i.e., $r(q) < 0$) represents previous consumers' negative views on the product quality; this discourages new consumers to buy from the retailer. As a higher quality level results in a greater WOM value, there should exist a reference point $q_0 > 0$ on the quality level such that $r(q) > 0$ if $q > q_0$, $r(q_0) = 0$, and $r(q) < 0$ if $q < q_0$. In this paper, we specify the average WOM value function in a logarithmic form, i.e., $r(q) = k \ln(q/q_0)$, where $k > 0$ means the increase in the average

WOM value resulting from an one percent increase in the quality cost, measuring the impact of the manufacturer's quality effort on the WOM.

3.2 Restocking Fee, Realized Sales and Returns

A consumer may purchase a product that mismatches her expectation on the product performance. When a consumer observes a mismatch, she may decide to return this product to the retailer. As a result, we should consider the "realized sales" (i.e., the number of the sold products that are not returned, denoted by D_1) and "unrealized sales" (i.e., the number of the sold products that are returned, denoted by D_2).

The retailer pays the wholesale price w to procure the product from the manufacturer, and sells it to the consumer at the retail price p. Therefore, the retailer's profit generated from the realized sales is $(p-w)D_1$. If a product does not match a consumer's preference and is thus returned to the retailer, then the retailer should refund the consumer's purchase payment. As in many real operations, the retailer may or may not charge a restocking fee on each consumer's return, and she can also obtain a salvage value s from dealing with each unit return. Suppose that $s < c$ (c is the manufacturer's unit production cost), which means that the consumer return is socially inefficient and thus generates a negative social welfare. In this paper, we consider a fixed restocking fee $\$f$ per unit, which is consistent with a number of firms' operational decisions in practice; see examples in Table 1. As a result, the retailer's profit from the unrealized sales is $(f + s - w)D_2$. Note that, if the retailer guarantees a full refund policy for consumers, then the retailer can obtain a profit from such returns as $(s - w)D_2$, where $f = 0$. Accordingly, the retailer's total expected profit is given as $(p - w)D_1 + (f + s - w)D_2$.

Next, we discuss the realized and unrealized sales D_1 and D_2. In this paper, the mismatch occurs with likelihood $\tau \in (0, 1)$ and thus, the product satisfies the consumer's need with probability $1 - \tau$. Moreover, as the restocking fee is increased, each consumer has less intention to return his or her mismatched

Firm	Restocking Fee ($/unit)
Premier Lacewigs (http://www.premierlacewigs.com/exchange-return-policy)	$25/unit
US Air Purifiers LLC. (http://www.usairpurifiers.com/returns-exchanges.html)	$25/ unit
Evannex (http://evannex.com/pages/returns)	$75/unit
Road Armor (http://www.roadarmor.com/warranty-and-terms-of-service)	$350/unit
Sea Catch (http://www.seacatch.com/prices.htm)	$25/unit
Oar Board (http://www.oarboard.com/policies/)	$40/unit

Table 1: Practical examples of the restocking fee under partial refund policies.

product. Accordingly, we assume that a mismatched product is returned with a probability $\beta(f) \in [0, 1]$ that is a decreasing, convex function of f. We specify $\beta(f)$ in exponential function form: $\beta(f) = \exp(-\gamma f)$, where $\gamma > 0$ represents the instantaneous reduction rate of the return probability with per incremental restocking fee, measuring the impact of restocking fee on the consumer returns. Hence, a sold product will be returned with probability $\tau\beta(f)$.

On the other hand, the consumer's purchase decision actually depends on the retail price p as well as the average WOM value $r(q)$. Accordingly, we construct a p- and $r(q)$-dependent total demand function as follows:

$$D(r(q), p) = a \exp(-b_1 p + b_2 r(q)), \tag{1}$$

where $a > 0$ denotes the market size; b_1, $b_2 > 0$ represent the instantaneous reduction rate of the total sales with an increment in retail price p and a decrement in the average WOM value $r(q)$, respectively. Hereafter, we simply call b_1 and b_2 the "pricing effect on total sales" and "WOM effect on total sales", respectively. As indicated by a recent comprehensive review (Huang, Leng, Parlar 2013), the exponential model structure in (1) has been used in a number of relevant publications (e.g., Jeuland and Shugan 1988, Hanssens and Parsons 1993, Song et al. 2008, and Xu 2009). We also learn from KelloggInsight (2012) and `Chargebacks911.com` (2015) that the impact of the restocking fee on product returns is very likely to be smaller than the impact of the retail price on total sales. This implies that $\gamma < b_1$, which means that consumers are more sensitive to the retail price in their purchases than to the restocking fee in their product returns. According to these discussions, we can write the realized sales and the returns as $D_1(r(q), p, f) = (1 - \tau\beta(f)) \times D(r(q), p)$ and $D_2(r(q), p, f) = \tau\beta(f) \times D(r(q), p)$, respectively. Therefore, the total sales is $D_1(r(q), p, f) + D_2(r(q), p, f) = D(r(q), p, f)$.

4 Model Analysis and Managerial Implications

We consider a leader-follower game in which the manufacturer first determines his quality level and wholesale price, and the retailer then decides on her retail price and restocking fee. Solving the game we can find the two firms' decisions in Stackelberg equilibrium. Next, we start with the best response analysis for the retailer.

4.1 Retailer's Pricing and Restocking Fee Decisions

As discussed in Section 3.2, the retailer's expected profit including those generated from both the realized sales $D_1(p, r(q))$ and the returns $D_2(p, r(q))$ is written as

$$\begin{aligned}
\pi_R(p, f) &= (p - w)(1 - \tau\beta(f))D(p, r(q)) + (f + s - w)\tau\beta(f)D(p, r(q)) \\
&= [(p - w)(1 - \tau\beta(f)) + (f + s - w)\tau\beta(f)]D(p, r(q)).
\end{aligned}$$

Given the manufacturer's wholesale price w and quality level q, we can find the retailer's best response on the retail price and the restocking fee.

Proposition 1 The retailer's optimal retail price p^* and restocking fee f^* are determined as

$$p^* = w + \frac{1}{b_1} + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right) \text{ and } f^* = z - s + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right), \tag{2}$$

where $z \equiv 1/b_1 + 1/\gamma$ and $W(\cdot)$ is the Lambert W (omega) function. ■

We learn from the above proposition that the optimal restocking fee is decreasing in the salvage value, which means that the retailer prefers to charge a lower restocking fee if returns have higher salvage values. The reason is that a major aim of charging the restocking fee is to compensate the cost resulting from managing, restoring, repacking, and reshipping the returned products. Moreover, we also find that the restocking fee f^* is independent of w, because the restocking fee mainly functions to control returns.

The term $W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)$ in (2) must be a negative real number, because, for any values of parameters γ, b_1, and $\tau \in (0, 1)$, we find that $\gamma z = 1 + \gamma/b_1 > 1$ and $\tau\gamma z \exp\left(-\gamma z\right) \leq \exp(-1)$. Thus, $-\exp(-1) \leq -\tau\gamma z \exp\left(-\gamma z\right) < 0$, and $W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)$ is negative real number in the range $[-1, 0]$.

4.2 Manufacturer's Wholesale Pricing and Quality Decisions

Using the retailer's best response on the retail price and restocking fee, the manufacturer determines his wholesale price and quality level. In the supply chain, the manufacturer incurs the production cost c as well as the quality control cost q_c, and enjoys the unit revenue w. Thus, the manufacturer's profit is given as

$$\pi_M(w, q_c) = (w - c - q_c) D(p^*, r(q)). \tag{3}$$

Proposition 2 In Stackelberg equilibrium, the manufacturer's quality level q^*

and wholesale price w^* are derived as

$$q^* = \frac{b_2}{b_1} \quad \text{and} \quad w^* = c + \frac{1+b_2}{b_1}. \quad \blacksquare \tag{4}$$

The above proposition indicates that the manufacturer's wholesale pricing and quality decisions are associated with the relative significance between the impact of retail price on sales (i.e., b_1) and the impact of WOM on sales (i.e., b_2). That is, if consumers are more sensitive to the WOM compared with the retail price in their purchases, viz., the value of b_2/b_1 increases, then the manufacturer should raise his quality control effort, which increases the manufacturer's unit quality cost. In order to assure his profit, the manufacturer also needs to increase the wholesale price. The retailer then responds by increasing her retail price, as indicated by the the retail price in Stackelberg equilibrium given below.

Substituting the manufacturer's equilibrium decisions into p^* and f^* in (2), we can obtain the retailer's retail price and restocking fee in Stackelberg equilibrium as

$$p^* = c + \frac{2+b_2}{b_1} + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right) \text{ and } f^* = z - s + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right). \tag{5}$$

4.3 Sensitivity Analysis

We investigate the impacts of major parameters (including γ, b_1, b_2 and τ) on firms' pricing, quality, and restocking decisions in Stackelberg equilibrium, and also on the sales and two firms' profits. To do so, we perform sensitivity analysis for each parameter and draw managerial implications.

4.3.1 The Impact of γ

Parameter γ measures the impact of the retailer's restocking fee on the consumer's return decision. We next examine the effect of γ on the retailer's decisions, the expected sales, and two firms' profits.

Corollary 1 As the impact of the restocking fee on returns increases, the restocking fee f^* decreases (i.e., $\partial f^*/\partial\gamma < 0$) whereas the retail price p^* increases ($\partial p^*/\partial\gamma > 0$). ■

The results in Corollary 1 are justified as follows. If consumers are more sensitive to the restocking fee, then the retailer has to reduce the restocking fee, which induces more consumers to return their mismatched products. In order to compensate the loss from the increasing returns, the retailer needs to raise her retail price. The expected sales in Stackelberg equilibrium are $D(p^*, r(q^*)) = a\exp(-b_1p^*+b_2r(q^*))$. Therefore, $\partial D(p^*, r(q^*))/\partial\gamma = -b_1D(p^*, r(q^*))\times\partial p^*/\partial\gamma < 0$. This suggests that a decrease in the value of γ can help increase the expected sales.

We next discuss the impact of γ on two firms' profits. The manufacturer's profit under the Stackelberg equilibrium is $\pi_M(w^*, q^*) = D(p^*, r(q))/b_1$. We thus have $\partial\pi_M(w^*, q^*)/\partial\gamma = (\partial D(p^*, r(q^*))/\partial\gamma)/b_1 < 0$, which indicates that an increase in consumers' sensitivity to the restocking fee can reduce the manufacturer's expected profit. Moreover, differentiating $\pi_R(p^*, f^*)$ once w.r.t. γ gives

$$\frac{\partial\pi_R(p^*, f^*)}{\partial\gamma} = \frac{1}{b_1}\left[\gamma\tau\beta(f^*)D(p^*, r(q^*))\frac{\partial f^*}{\partial\gamma} - b_1(1-\tau\beta(f^*))D(p^*, r(q^*))\frac{\partial p^*}{\partial\gamma}\right],$$

which is also negative because $\partial f^*/\partial\gamma < 0$ and $\partial p^*/\partial\gamma > 0$. This implies that an increase in the value of γ makes the retailer worse off.

4.3.2 The Impact of b_1

We perform sensitivity analysis to investigate the impact of parameter b_1 on two firms' decisions in Stackelberg equilibrium, the expected sales, and two firms' profits.

Corollary 2 Under a partial refund policy, the impacts of b_1 on two firms' decisions are characterized as follows: the retailer's restocking fee and retail price are both decreasing in b_1 (i.e., $\partial f^*/\partial b_1 < 0$ and $\partial p^*/\partial b_1 < 0$, and the manufacturer's wholesale price and the quality level are also both decreasing in b_1 (i.e., $\partial w^*/\partial b_1 < 0$ and $\partial q^*/\partial b_1 < 0$). ■

As the above corollary indicates, if consumers have a greater concern about the retail price when they make their purchase decisions, then the retailer should charge a smaller restocking fee. This happens mainly because a reduction in the restocking fee can help mitigate the impact of consumers' greater price sensitivity on sales. Moreover, a higher consumer sensitivity on the retail price in sales induces the retailer to reduce her retail price, simply because the retailer intends to prevent the total sales from dropping due to the higher consumer sensitivity. Corollary 2 also indicates that the manufacturer responds to a higher consumer sensitivity to the retail price by decreasing his wholesale price and unit quality cost. This is mainly attributed to the fact that a reduction in the wholesale price follows a decrease in the retail price, and also induces the manufacturer to spend less effort on his quality control.

4.3.3 The Impact of b_2

We now examine the impact of b_2—the parameter measuring a consumer's sensitivity to the WOM in his or her purchase—on the manufacturer's and the retailer's decisions in Stackelberg equilibrium. The restocking fee f^* is independent of b_2, as the WOM influences sales rather than product returns which are solely related to the restocking fee. From (5), we can observe that the retail price rises as the consumer sensitivity to the WOM increases. This means that as consumers pay more attention to the average WOM value (i.e., the overall social evaluation of the product), the retailer would respond by increasing her retail price. In addition, we learn from (4) that the manufacturer's quality and wholesale price are both increasing in the consumer sensitivity to the WOM. This implies that the consumer's higher concern about the WOM leads the manufacturer to improve his quality (by increasing the unit quality cost), which in return increases the average WOM value. However, a greater quality level reduces the manufacturer's profit margin; as a response to this reduction, she decides to raise the wholesale price.

The first-order derivative of the total sales $D(p^*, r(q^*))$ w.r.t. b_2 is given as

$$\frac{\partial D(p^*, r(q^*))}{\partial b_2} = D(p^*, r(q^*)) \left[-b_1 \frac{\partial p^*}{\partial b_2} + r(q^*) + b_2 r'(q^*) \frac{\partial q^*}{\partial b_2} \right] = D(p^*, r(q^*)) r(q^*),$$

which means that the impact of the consumer sensitivity to the WOM on the total sales depends on whether there is a positive or negative overall WOM valuation. If the average WOM value is negative, then an increase in the consumer sensitivity to the WOM decreases the sales; otherwise, if consumers positively evaluate the product, then a higher consumer sensitivity to the WOM can encourage more consumers to buy. Noting that the realized sales are $(1 - \tau\beta(f^*))D(p^*, r(q^*))$, we can conclude that the consumer sensitivity to the WOM influences the realized sales in a same mode as the WOM affects the total sales. Moreover, the impact of b_2 on the manufacturer's and the retailer's profits similarly depends on the 1
average WOM value $r(q^*)$.

4.3.4 The Impact of τ

The parameter τ characterizes the mismatch chance, which does not influence the manufacturer's wholesale pricing and quality decisions in Stackelberg equilibrium, because such decisions solely affect the consumer purchase and the WOM value, thus influencing the sales. We observe from (2) and (5) that both the restocking fee f^* and the retail price p^* are decreasing in τ, which results from the following fact: for a higher mismatch chance, a larger number of consumers experience the mismatch between their expectations and the product performance. To improve consumers' purchase motivations, the retailer needs to decrease her retail price (as the average WOM value is unchanged), which then raises the total sales $D(p^*, r(q^*))$. Therefore, the manufacturer's profit is also increasing in the mismatch chance, which implies that the manufacturer is better off from the mismatch. This occurs mainly because the manufacturer is not involved into the management of product returns, as discussed by KelloggInsight (2012).

The retailer reduces her restocking fee to allow more consumers to return their mismatched products; this may not decrease the realized sales (i.e., $(1 - \tau\beta(f^*))D(p^*, r(q^*))$). The first-order derivative of the realized sales w.r.t. γ is

$$-\frac{D(p^*, r(q^*))}{[1 + W(-\tau\gamma z \exp(-\gamma z))]} \left\{ \beta(f^*) + \frac{b_1}{\gamma\tau}(1 - \tau\beta(f^*))W(-\tau\gamma z \exp(-\gamma z)) \right\},$$

where the first and second terms in the curly brackets "{ }" are positive and negative, respectively. Moreover, the first term is independent of τ and the second term is decreasing in τ. Accordingly, there exists a threshold $\hat{\tau}$ such that if $\tau \leq \hat{\tau}$, then the realized sales increase when the mismatch chance is higher; otherwise, the realized sales is decreasing in the mismatch chance. That is, when a sufficiently small number of consumers experience the mismatch, an increase in the mismatch probability does not significantly increase the number of returned products but it reduces the retail price and raises the sales. As a consequence, the realized sales increase when $\tau \leq \hat{\tau}$. When the mismatch happens with a larger chance, there are a greater number of returned products and thus, the realized sales are smaller. The retailer's profit exhibits a similar changing pattern as the realized sales.

5 Effects of WOM and Restocking Fee

In the preceding section, we have analyzed the pricing, quality, and restocking fee decisions made by two firms to manage sales and product returns under the WOM effect. To further examine the impact of the WOM and the refund amount, we study the supply chain with no WOM effect or with full refund for all product returns, and compare our results with those in Section 4 to draw managerial implications regarding the effects of the WOM and refund policy on the supply chain.

5.1 Implications for the WOM Effect

We analyze the supply chain under no WOM effect. In the absence of a WOM effect, the sales function in (1) is reduced to $D(p) = a\exp(-b_1 p)$, which is only dependent on the retail price p. In such a setting, the manufacturer does not make the quality decision but only decides on the wholesale price. The manufacturer's and the retailer's profits are given as

$$\begin{cases} \bar{\pi}_M(w) = (w - c)D(p), \\ \bar{\pi}_R(p, f) = [(p - w)(1 - \tau\beta(f)) + (f + s - w)\tau\beta(f)]D(p), \end{cases}$$

where the symbol "–" refers to the case without a WOM effect in the market.

5.1.1 The Equilibrium Decisions under No WOM Effect

We compute the manufacturer's and the retailer's decisions in Stackelberg equilibrium.

Proposition 3 When there is no WOM effect in the market, the retailer's equilibrium restocking fee, the equilibrium wholesale and retail prices are determined as

$$\bar{w}^* = c + \frac{1}{b_1}, \quad \bar{f}^* = z - s + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right), \text{ and } \quad \bar{p}^* = c + \frac{2}{b_1} + \frac{W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)}{\gamma}. \quad (6)$$ ■

The above indicates that both the wholesale price $\bar{w}^*$ and the retail price $\bar{p}^*$ are decreasing in the consumer's sensitivity to the retail price in their purchases, which is similar to the impact of b_1 on the retail price p^* as shown in Section 4.3.2. Moreover, we also observe that the parameter γ affects the restocking fee $\bar{f}^*$ and the retail price $\bar{p}^*$ in a way similar to its effect on f^* and p^*. We compare the pricing and restocking fee decisions $(\bar{w}^*, \bar{p}^*, \bar{f}^*)$ with those when the WOM takes effect, and draw relevant results below.

Theorem 1 The equilibrium wholesale and retail prices under no WOM effect are smaller than those when the WOM effect exists. But, the equilibrium restocking fee is independent of whether there is a WOM effect or not. ■

The above theorem shows that, in a market with a WOM effect on consumer purchase decisions, both the manufacturer and the retailer have incentives to raise their wholesale and retail prices, respectively. When consumers do not consider the WOM in their purchase decisions, we find that the total sales and the realized sales are $D(\bar{p}^*)$ and $(1 - \tau\beta(\bar{f}^*))D(\bar{p}^*)$, respectively. Comparing them to those under the WOM effect—i.e., $D(p^*, r(q^*))$ and $(1 - \tau\beta(f^*))D(p^*, r(q^*))$ yields the following important results.

Theorem 2 The retailer may not achieve higher total sales and realized sales under the WOM effect, which depends on the ratio of the equilibrium quality control level q^* to the initial quality level q_0. Specifically, if

$$\frac{q^*}{q_0} \leq \exp(1), \tag{7}$$

then the WOM effect reduces the total sales and realized sales; otherwise, both of these two sales rise. ■

The above theorem indicates that if the equilibrium quality q^* is smaller than the initial quality level q_0, then the WOM would have a negative effect on consumers' purchase decisions, which reduces consumers' incentives to buy. Otherwise, there is a positive WOM effect. But, it may not be helpful to increase the expected sales. That is, when the equilibrium quality level q^* is sufficiently high such that the ratio q^*/q_0 is larger than the cutoff level $\exp(1)$, the positive WOM effect can result in more expected sales. However, if the manufacturer chooses a quality level such that $q_0 < q^* \leq \exp(1)q_0$, then although the WOM has a positive effect on consumer purchases, the sales are still smaller under the WOM effect. This reflects the fact that prior to buying a product, each consumer needs to have an expectation on the product quality that is significantly higher than his or her reference point level q_0. Accordingly, in order to increase the sales, the manufacturer needs to exert an effort to guarantee a sufficiently high WOM so that such a WOM can encourage more consumers to purchase. According to (4) (i.e., $q^* = b_2/b_1$), we can rewrite the condition (7) as

$$b_2 \leq \exp(1) q_0 b_1, \tag{8}$$

which means that if consumers' sensitivity to the retail price (i.e., b_1) and/or the threshold for the quality cost (i.e., q_0) are significantly low, then the WOM effect is very likely to enlarge the sales.

5.1.2 The Impacts of Major Parameters under No WOM Effect

We next investigate the impacts of major parameters γ, b_1, b_2 and τ on the manufacturer's and the retailer's expected profits. In what follows, we conduct several numerical analyses to examine these effects. Figure 1 illustrates the impact of consumers' sensitivity to the restocking fee (i.e., γ) on two firms' profits. We increase the value of γ from 0.05 to 0.5 in steps of 0.045. We observe that as γ is very low, the manufacturer will become worse off if he exerts the effort on the quality improvement to raise the WOM effect; but, when γ is sufficiently large, he is better off from his effort. However, the retailer can always benefit from the WOM effect. This is justified as follows: we learn from Theorem 1 that, regardless of how consumers are sensitive to the restocking fee in their returns, the retail price in Stackelberg equilibrium under the WOM effect is always higher than that when there is no WOM effect and thus, the retailer's pricing decision discourages consumers to buy and reduces the sales compared with the scenario with no WOM effect. For the case of a smaller value of γ, more consumers return their mismatched products, and the retailer can still benefit mainly because she enjoys a high marginal profit. But, this enforces the manufacturer to spend more effort on quality control. As a result, although the manufacturer also responds to the WOM effect by raising his wholesale price, he still experiences a lose resulting from the WOM effect because he needs to afford the cost for quality improvement. However, when consumers become more sensitive to the restocking fee in their returns, consumers' returns are significantly reduced, and the retailer is thus willing to set a lower price than that when consumers are less sensitive, which yields more sales. As a consequence, both the manufacturer and the retailer benefit from the WOM effect.

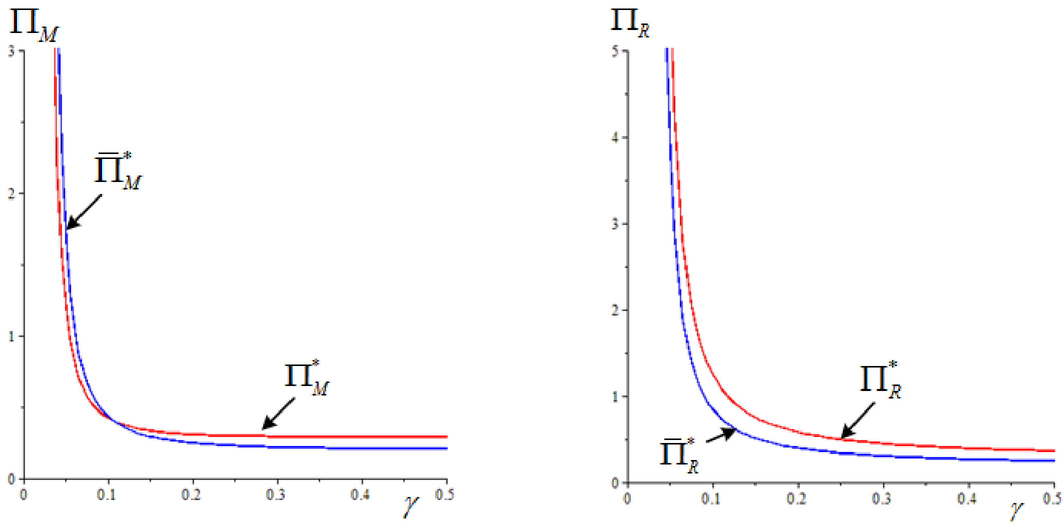

Figure 1: The impact of γ on two firms' profits in terms of their equilibrium decisions when the WOM effect exists and those when there is no WOM effect.

Figure 2 indicates the impact of consumers' sensitivity to the retail price (i.e., b_1) on two firms' profits when the firms make their decisions in Stackelberg equilibrium. We increase the value of γ from 0.05 to 0.5 in steps of 0.045. We find that if this sensitivity is very low, then both the manufacturer and the retailer can benefit from the WOM effect; otherwise, the WOM makes both firms worse off. This is ascribed to the fact that, as consumers are less sensitive to the retail price, in order to expand the market sales, the manufacturer may exert a larger effort on quality control (which results in a higher WOM value), and both firms thus obtain higher profits. However, when consumers are more sensitive to the retail price, the manufacturer invests less on quality control, which reduces the sales. Actually, we can learn from our numerical experiments that under the WOM effect, both the total sales and the realized sales are decreasing in the value of b_1. As a result, both firms become worse off from the WOM effect.

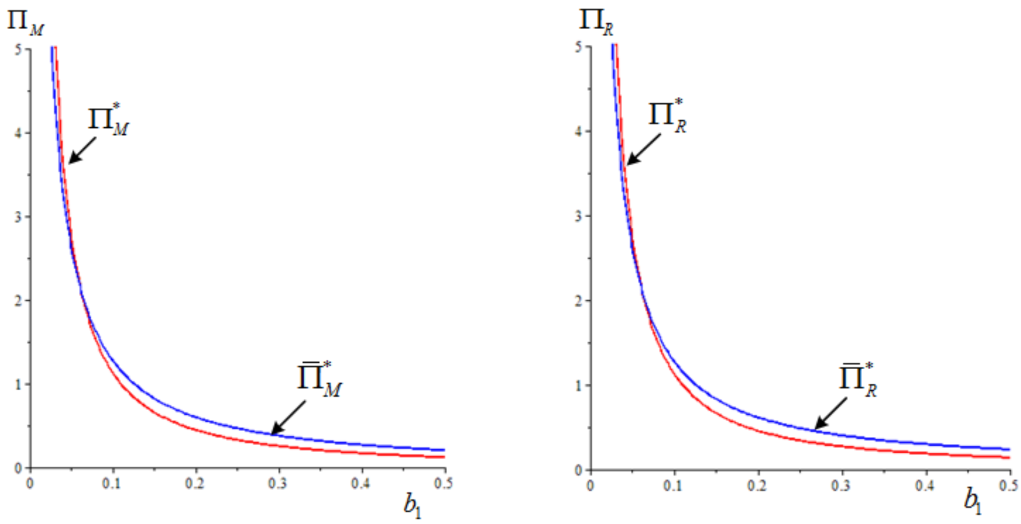

Figure 2: The impact of b_1 on two firms' profits in terms of their equilibrium decisions when the WOM effect exists and those when there is no WOM effect.

We depict Figure 3 to investigate the effect of consumers' sensitivity to the WOM (i.e., b_2) on two firms' profits. We increase the value of γ from 0.05 to 0.5 in steps of 0.045. If the value of this sensitivity is low, then both firms are more beneficial in the absence of WOM in the market; otherwise, the WOM can help improve both firms' profits. This occurs because when consumers are less sensitive to the WOM, the sales are lower than those without a WOM effect, even if the WOM has a positive effect on consumers' purchase decisions. But, when the WOM value is sufficiently high, the sales are larger than those in the absence of a WOM, which thus makes both firms better off. We also find that both the manufacturer and the retailer are more beneficial when consumers' sensitivity to the WOM is higher.

Figure 4 illustrates the impact of the mismatch probability τ on two firms' profits. We increase the value of γ from 0.05 to 0.5 in steps of 0.045. We find that the WOM hurts the retailer when τ is very high, because, otherwise, the expected returns are low and thus, the retailer can benefit from the WOM by achieving more sales. But, when the value of τ is very large, each consumer has a high chance to experience a mismatch and the expected returns are thus high, which makes the retailer worse off. We find that the manufacturer can always benefit from the WOM for any value of τ, because in the supply chain, the retailer is

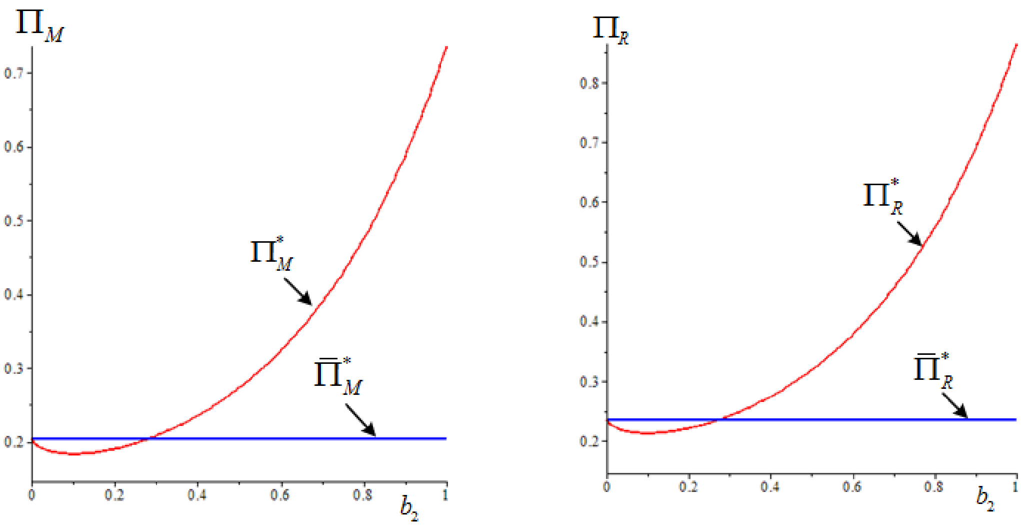

Figure 3: The impact of b_2 on two firms' profits in terms of their equilibrium decisions when the WOM effect exists and those when there is no WOM effect.

solely responsible for all the cost associated with product returns; moreover, an increase in the mismatch chance induces the retailer to reduce her retail price, which encourages more consumers to buy. Therefore, the manufacturer can profit from the WOM effect. We also observe an interesting (and surprising) result as follows: the manufacturer's profit is increasing in the value of τ. This is mainly attributed to the following reason: an increase in the mismatch probability leads the retailer to reduce her retail price, thus attracting more consumers to buy. This makes the manufacturer better off. However, the retailer's profit is increasing in the value of τ only when τ is sufficiently small, because as the mismatch likelihood is small, in order to attract consumers to buy, she has to charge a low retail price and the sales are thus high, which makes the retailer better off. But, the retailer's profit is decreasing in the value of τ when the mismatch probability is sufficiently large, because product returns significantly increase and thus the retailer experiences a profit loss.

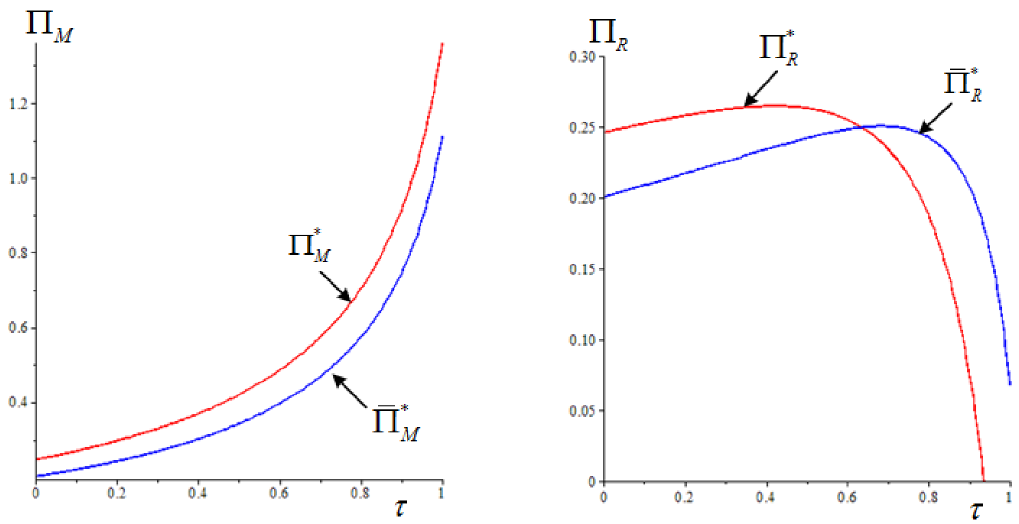

Figure 4: The impact of τ on two firms' profits in terms of their equilibrium decisions when the WOM effect exists and those when there is no WOM effect.

5.2 Implications for the Restocking Fee in the Returns of Mismatched Products

We consider a scenario in which the retailer provides a full refund to each consumer who returns his or her mismatched product, i.e., $f = 0$. In this scenario, the mismatch chance is still τ. The manufacturer's and the retailer's profits are written as

$$\begin{aligned} \hat{\pi}_M(w) &= (w - c - q)D(p, r(q)), \\ \hat{\pi}_R(p) &= (p - w)(1 - \tau)D(p, r(q)) + (s - w)\tau D(p, r(q)) \\ &= [(p - w)(1 - \tau) + (s - w)\tau]D(p, r(q)). \end{aligned}$$

5.2.1 The Equilibrium Decisions under the Full Refund Policy for Product Returns

We solve the leader-follower game in which the manufacturer first makes his wholesale price and quality decisions, and the retailer then decides on her retail price and restocking fee.

Proposition 4 Under the full refund policy, the manufacturer's and the retailer's equilibrium decisions can be determined as

$$\hat{w}^* = c + \frac{(1-\tau)(1+b_2)}{b_1},\ \hat{q}^* = \frac{(1-\tau)b_2}{b_1},\ \text{and}\ \hat{p}^* = \frac{c-\tau s}{1-\tau} + \frac{2+b_2}{b_1}. \quad \blacksquare$$

According to the above proposition and our analytical results in Section 4.2, we find that $w^* > \hat{w}^*$ and $q^* > \hat{q}^*$. However, the retail price $\hat{p}^*$ may be higher or may be lower than p^*, which depends on the difference between the cost c and the salvage value s. Specifically, if this difference is sufficiently large such that $c - s > (1-\tau)W(-\tau\gamma z \exp(-\gamma z))/(\tau\gamma)$, then $\hat{p}^* > p^*$; otherwise, $\hat{p}^* \le p^*$. This means that only when the salvage value is so large that the difference between c and s is significantly low, the retailer would charge a retail price lower than that under a partial refund policy; for other cases, the retailer would set a higher retail price under the full refund policy. This result is justified as follows: under the full refund policy, customers are more likely to return their mismatched products and thus, in order to reduce the loss generated by the increasing returns, the retailer would set a higher retail price when the the salvage value is significantly small. To compensate the retailer, the manufacturer would also charge a low wholesale price under the full refund policy. In order to obtain an acceptable profit margin, the manufacturer also decides to reduce her investment on quality control.

We next compare the Stackelberg euqilibrium-dependent sales between the full and the partial refund policies.

Theorem 3 If the probability of returning a mismatched product is sufficiently large such that

$$\beta(f^*) \ge \frac{1}{\tau}\left[1 - (1-\tau)\exp\left(\frac{b_1\tau(s-c)}{1-\tau} - \frac{\tau b_2^2}{b_1} + \frac{b_1}{\gamma}W(-\tau\gamma z \exp(-\gamma z))\right)\right], \tag{9}$$

then the realized sales under the full refund policy (i.e., $f = 0$) are higher than those under a partial refund policy; otherwise, the realized sales are lower under the full refund policy. $\blacksquare$

The above theorem indicates that implementing the full refund policy may or may not generate more realized sales. That is, when the probability of returning a mismatched product is significantly small, the restocking fee can effectively prevent consumers from returning; thus, the realized sales under a partial refund policy are higher than those under the full refund policy.

5.2.2 The Impacts of Major Parameters under the Full Refund Policy

We investigate the effects of major parameters on the manufacturer's and retailer's profits. Figure 5 shows that as a result of an increase in consumers' sensitivity to the restocking fee, two firms can both benefit more from a partial refund policy than from the full refund policy. We increase the value of γ from 0.05 to 0.5 in steps of 0.045. This can be justified as follows: the full refund policy usually encourages more consumers to return. As a response to the increasing returns, the retailer may choose to raise her retail price, which then results in less sales. Thereby, the full refund policy makes the retailer worse off. In order to compensate the retailer for her responsibility on consumer returns, the manufacturer chooses to reduce his wholesale price, which decreases his marginal profit and thus the total profit.

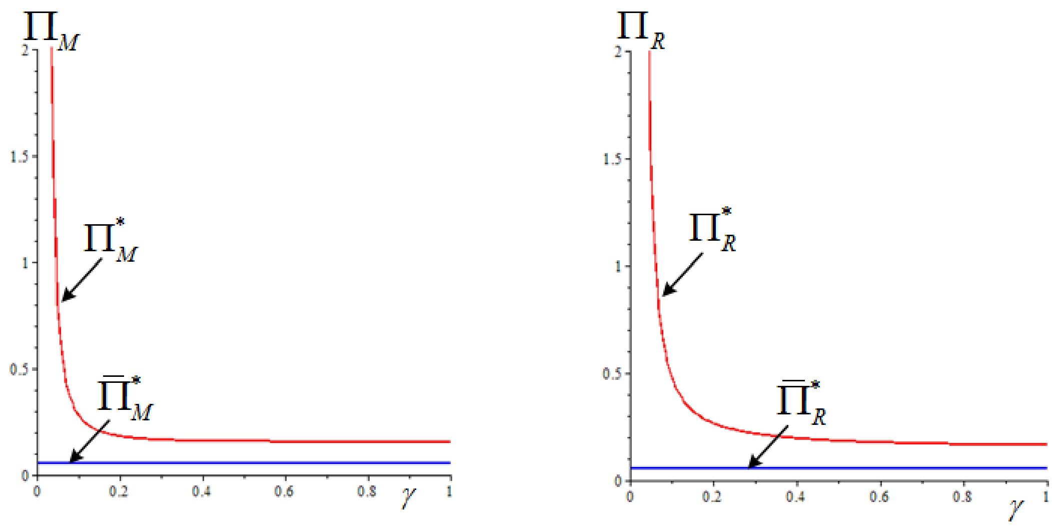

Figure 5: The impact of γ on two firms' profits in terms of their equilibrium decisions under the full refund policy and those under a partial refund policy.

Because of a similar reason, we can also justify the comparison between the two firms' profits when consumers' sensitivity to the retail price (or to the WOM level) increases, wherein both the manufacturer and the retailer can always benefit from the full refund policy; see Figures 6 and 7. We increase the value of γ from 0.05 to 0.5 in steps of 0.045.

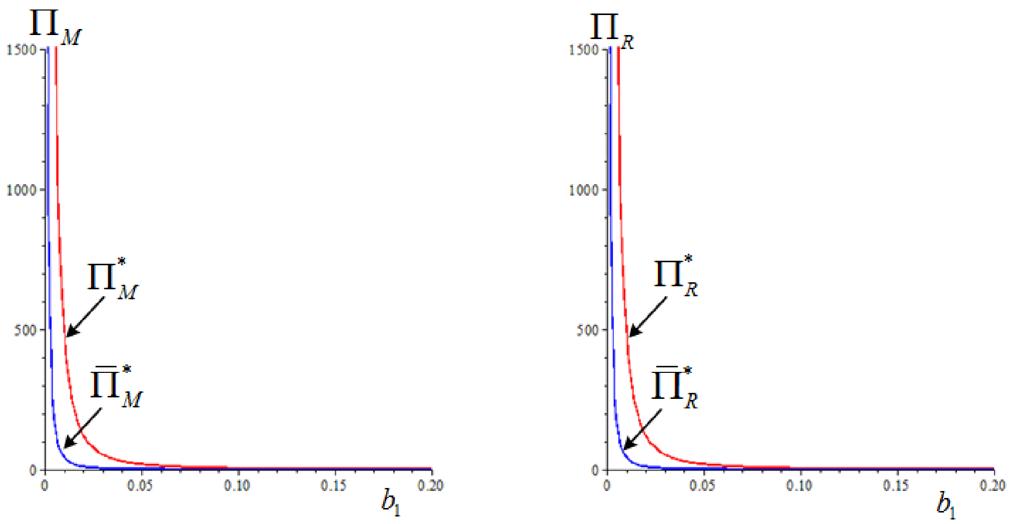

Figure 6: The impact of b_1 on two firms' profits in terms of their equilibrium decisions under the full refund policy and those under a partial refund policy.

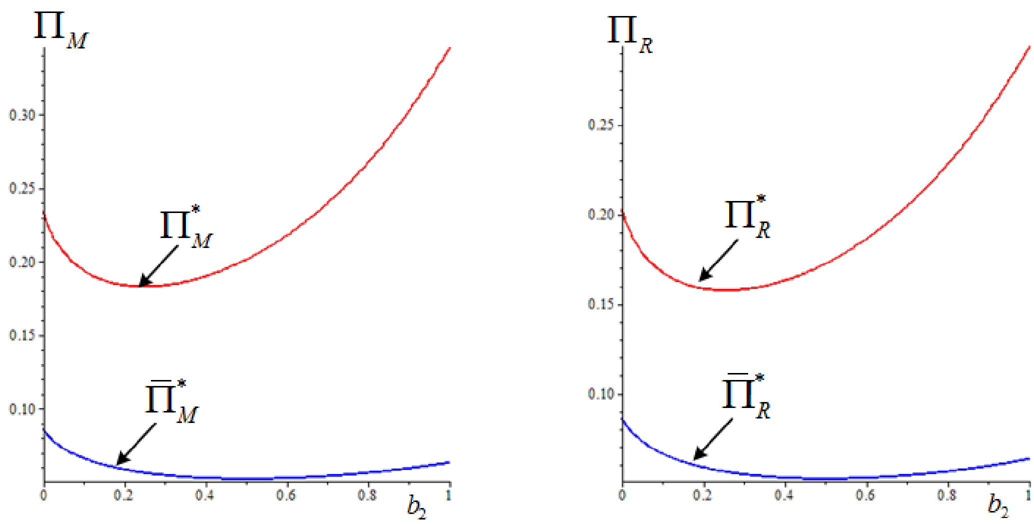

Figure 7: The impact of b_2 on two firms' profits in terms of their equilibrium decisions under the full refund policy and those under a partial refund policy.

As Figure 8 indicates, when the value of τ changes, we find that if the value of τ is very high, then the full refund policy makes the retailer worse off; otherwise, she can profit more under the full refund policy.We increase the value of γ from 0.05 to 0.5 in steps of 0.045. However, the manufacturer can always enjoy a higher profit under the full refund policy. This occurs because, for the case of a very high mismatch likelihood, the consumer returns are also very high; in order to offset the loss resulting from the increasing returns, the retailer would significantly reduce her retail price and thus obtain a smaller profit.

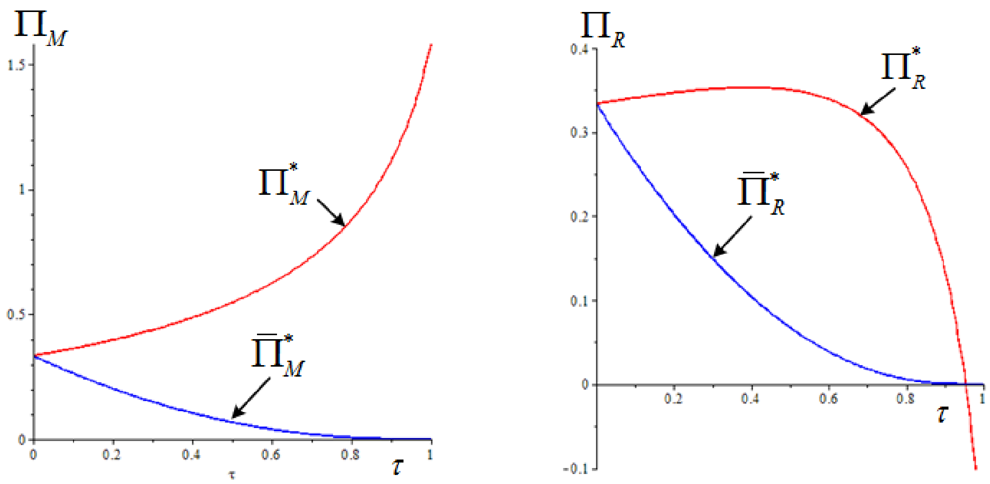

Figure 8: The impact of τ on two firms' profits in terms of their equilibrium decisions under the full refund policy and those under a partial refund policy.

6 Summary and Concluding Remarks

In this paper we consider a two-echelon supply chain in which the manufacturer determines his wholesale pricing and quality decisions and the retailer then decides on her retail price and restocking fee under the WOM effect. We construct a price- and WOM-dependent total sales model, and describe this problem as a leader-follower game where the manufacturer acts as the "leader" and the retailer as the "follower." Solving the game, we obtain the Stackelberg equilibrium. We conduct sensitivity analyses to investigate the impact of some major parameters (including γ, b_1, b_2 and τ) on the two firms' decisions, demand, and their profits; see Table 2. From Table 2, we learn that the retail price is the only decision variable that is affected by all the parameters. This may mean that the retail price plays a significant role in the management of sales and product returns under the WOM effect.

	γ	b_1	b_2	τ
f^*	$\downarrow$	$\downarrow$	—	$\downarrow$
p^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
w^*	—	$\downarrow$	$\uparrow$	—
q^*	—	$\downarrow$	$\uparrow$	—
$D(p^*, r(q^*))$	$\downarrow$	$\downarrow$	$\begin{cases} \downarrow \text{ if } b_2 < q_0 b_1, \\ \uparrow \text{ if } b_2 \geq q_0 b_1. \end{cases}$	$\uparrow$
$(1-\tau\beta(f^*))$ $\times D(p^*, r(q^*))$	$\uparrow$	$\downarrow$	$\begin{cases} \downarrow \text{ if } b_2 < q_0 b_1, \\ \uparrow \text{ if } b_2 \geq q_0 b_1. \end{cases}$	$\begin{cases} \uparrow \text{ if } \tau \leq \hat{\tau}, \\ \downarrow \text{ if } \tau > \hat{\tau}. \end{cases}$
π_M	$\downarrow$	$\downarrow$	$\begin{cases} \downarrow \text{ if } b_2 < q_0 b_1, \\ \uparrow \text{ if } b_2 \geq q_0 b_1. \end{cases}$	$\uparrow$
π_R	$\downarrow$	$\downarrow$	$\begin{cases} \downarrow \text{ if } b_2 < q_0 b_1, \\ \uparrow \text{ if } b_2 \geq q_0 b_1. \end{cases}$	$\begin{cases} \uparrow \text{ if } \tau \leq \hat{\tau}, \\ \downarrow \text{ if } \tau > \hat{\tau}. \end{cases}$

Table 2: The impact of major parameters on the two firms' decisions, sales, and profits.

In order to explore the WOM effect, we also analyze the setting in which there is no WOM effect, and compare our results with those in the presence of the WOM effect. Our results indicate how the WOM influences the firms' decisions and profits. In addition, to find the impact of restocking fee, we investigate the scenario in which all returns are fully refunded, and compare the results with those under a partial refund policy. Below is a summary of relevant managerial insights.

1. The wholesale and retail prices under no WOM effect are smaller than those when the WOM effect exists, whereas the restocking fee remains unchanged regardless of whether the WOM effect exists or not. Moreover, when the WOM influences the consumer purchase, the retailer may not achieve more total sales and realized sales, which depends on the ratio of the value of the equilibrium quality control level to the initial quality level. Specifically, if this ratio is sufficiently high, then the WOM effect reduces the total sales and realized sales; otherwise, both of these two sales are enlarged.

2. The WOM effect makes the manufacturer worse (better) off if the con-

sumers' sensitivity to the restocking fee is sufficiently small (large), whereas the retailer always benefits under the WOM effect. Moreover, when consumers are less sensitive to the retail price, both firms can benefit more in the presence of WOM. When the consumers' sensitivity to the WOM is sufficiently small, the WOM effect makes the two firms' profits lower than those under no WOM effect.

3. The manufacturer always profits in the presence of the WOM effect, whereas the retailer profits under the WOM effect only when the mismatch probability is sufficiently small. Moreover, under the WOM effect, the manufacturer's profit is increasing in the mismatch probability, whereas the retailer's profit is concave in the mismatch probability.

4. If each consumer returns his or her mismatched product with a sufficiently large chance, then the realized sales under the full refund policy are higher than those under a partial refund policy. The retailer can benefit from the full refund policy when the mismatch chance is not high, whereas the manufacturer can always benefit from the full refund policy.

In future, we consider more realistic problems, and we may extend this paper to an experimental study concerned with the WOM effect and the return policies including refund policy, exchange policy, and refund-exchange policy—viz., consumers can decide either to receive refunds or to exchange for other products. Furthermore, a retailer has an option of charging or not charging a restocking fee. When we combine the return policies and the restocking fee decision, there will be some different return policies available to online retailers. Each of these policies has advantages and disadvantages. For instance, charging a high restocking fee can help offset the losses resulting from product returns. However, it also discourages online purchases, which may lead to more losses. However, if a retailer charges a very low or no restocking fee, then unexpected consumer behaviors such as wardrobing will be likely to appear, making the retailer to possibly loss. The exchange policy also has some disadvantages. For example, under such a policy consumers may perceive that the stock could comprise the used items, which means that the exchanged products may be difficult to sell. Therefore, we cannot immediately find which policy is the best one. It behooves us to compare all the possible policies and find the optimal return policy that can generate the maximum profit for the retailer.

Bibliography

Bardón, J. (2014). Word of mouth marketing reaches eCommerce. `http://www.brainsins.com/en/blog/word-of-mouth-marketing-reaches-ecommerce/3505` (URL last accessed on July 30, 2017).

Board, O. (2009). Competition and disclosure, *The Journal of Industrial Economics* **57**(1): 197–213.

Brynjolfsson, E., Hu, Y. & Smith, M. D. (2006). From niches to riches: Anatomy of the long tail, *Sloan Management Review* **47**(4): 67–71.

Chargebacks911.com (2015). 9 reasons why consumers return merchandise. `https://chargebacks911.com/knowledge-base/9-reasons-why-consumers-return-products/` (URL last accessed on July 30, 2017).

Chevalier, J. A. & Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews, *Journal of Marketing Research* **XLIII**: 345–354.

Davis, S., Hagerty, M. & Gerstner, E. (1998). Return policies and the optimal level of "hassle", *Journal of Economics and Business* **50**(5): 445–460.

Elberse, A. & Oberholzer-Gee, F. (2007). Superstars and underdogs: An examination of the long tail phenomenon in video sales, *Marketing Science* **24**(4): 49–72.

Espejel-Blanco, J. & Fandos-Herrera, C. (2008). Perceived quality as a antecedent for buying intention of the olive oil fron bajo aragon with protected desigdesign of origin, *Esic-Market* **131**: 231–251.

Hanssens, D. M. & Parsons, L. (1993). Econometric and time-series: Market response models, *in* J. Eliashberg & G. L. Lilien (eds), *Handbooks in Operations*

Research and Management Science: Marketing, Elsevier Science Publishers, Amsterdam (The Netherlands), pp. 409–464.

Hao, K. (2013). E-commerce and the global battle for internet economics, Fortune Magazine. `http://fortune.com/2013/06/21/e-commerce-and-the-global-battle-for-internet-economics/` (URL last accessed on July 30, 2017).

URL: *http://fortune.com/2013/06/21/e-commerce-and-the-global-battle-for-internet-economics/*

Hendricks, K. B. & Singhal, V. R. (1996). Quality awards and the market value of the firm: An empirical investigation, *Management Science,* **42**(3): 415–436.

Hennig-Thurau, T., Gwinner, K., G.Walsh & D.Gremler (2004). Electronic word-of-mouth via consumer-opinion platforms: What motivates consumers to articulate themselves on the internet?, *Journal of Interactive Marketing* **18**(1): 38–52.

Hess, J. D., Chu, W. & Gerstner, E. (1996). Controlling product returns in direct marketing, *Marketing Letters* **7**(4): 307–317.

Huang, J., Leng, M. & Parlar, M. (2013). Demand functions in decision modeling: A comprehensive survey and research directions, *Decision Sciences* **44**(3): 557–609.

Jeuland, A. P. & Shugan, S. M. (1988). Channel of distribution profits when channel members form conjectures, *Marketing Science* **7(2)**: 202–210.

KelloggInsight (2012). Why you should accept returns. `http://insight.kellogg.northwestern.edu/article/why_you_should_accept_returns` (URL last accessed on July 30, 2017).

Koessler, F. & Renault, R. (2012). When does a firm disclose product information?, *The RAND Journal of Economics* **43**(4): 630–649.

Kroll, M., Wright, P. & Heiens, R. A. (1999). The contribution of product quality to competitive advantage: Impacts on systematic variance and unexplained variance in returns, *Strategic Management Journal* **20**(4): 375–384.

Kuzma, A. & W.L.Shanklin (1992). How medium-market-share companies achieve superior profitability, *Journal of Business & Industrial Marketing* **7**(3): 29–39.

Kwark, Y., Chen, J. & Raghunathan, S. (2014). Online product reviews: Implications for retailers and competing manufacturers, *Information Systems Research* **25**(1): 93–110.

Li, T. & Wang, C. (2014). Nurturing the buzz - pricing of experience product with consumer-generated product information. `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2480981` (URL last accessed on July 30, 2017).

Loechner, J. (2008). 85% of world's online population shopped on the web. `http://www.mediapost.com/publications/article/76195/85-of-worlds-online-population-shopped-on-the-we.html` (URL last accessed on July 30, 2017).

Manduchi, A. (2010). Non-neutral information costs with match-value uncertainty, *Journal of Economics* **109**: 1–25.

Mathios, A. D. (2000). The impact of mandatory disclosure laws on product choice: An analysis of the salad dressing market, *Journal of Law and Economics* **43**(2): 651–677.

Nam, S., Manchanda, P. & Chintagunta, P. K. (2010). The effects of service quality and word of mouth on customer acquisition, retention and usage, *Marketing Science* **29**: 690–700.

Nielsen.com (2013). Under the influence: Consumer trust in advertising. `http://www.nielsen.com/us/en/insights/news/2013/`

under-the-influence-consumer-trust-in-advertising.html (URL last accessed on July 30, 2017).

Ofek, E., Katona, Z. & Sarvary, M. (2011). "bricks and clicks": The impact of product returns on the strategies of multichannel retailers, *Marketing Science* **30**(1): 42–60.

Petersen, J. A. & Kumar, V. (2010). Can product returns make you money?, *Sloan Management Review* **51**(3): 85–89.

Powell, T. C. (1995). Total quality management as competitive advantage: A review and empirical study, *Strategic Management Journal* **16**(1): 15–37.

Shulman, J. D., Coughlan, A. T. & Savaskan, R. C. (2009). Optimal restocking fees and information provision in an integrated demand-supply model of product returns, *Manufacturing & Service Operations Management* **11**(4): 577–594.

Shulman, J. D., Coughlan, A. T. & Savaskan, R. C. (2011). Managing consumer returns in a competitive environment, *Management Science* **57**(2): 347–362.

Song, Y. Y., Ray, S. & Li, S. L. (2008). Structural properties of buyback contracts for price-setting newsvendors, *Manufacturing & Service Operations Management* **10(1)**: 1–18.

Swinney, R. (2011). Selling to strategic consumers when product value is uncertain: The value of matching supply and demand, *Management Science* **57**(10): 1737–1751.

Xu, X. (2009). Optimal price and product quality decisions in a distribution channel, *Management Science* **55**(8): 1347–1352.

Appendix A Proofs

Proof of Proposition 1. Given the value of p, we find the optimal restocking fee maximizing $\pi_R(p,f) = [(p-w)(1-\tau\beta(f)) + (f+s-w)\tau\beta(f)]D(p,r(q))$. The first- and second-order derivatives of $\pi_R(p,f)$ w.r.t. f are computed as

$$\begin{aligned}
\frac{\partial \pi_R(p,f)}{\partial f} &= \tau\beta(f)[\gamma(p-w) + 1 - \gamma(f+s-w)]D(p,r(q)) \\
\frac{\partial^2 \pi_R(p,f)}{\partial f^2} &= -\gamma\tau\beta(f)D(p,r(q)) < 0,
\end{aligned}$$

which implies that $\pi_R(p,f)$ is a concave function of f, and the optimal restocking fee can be computed as

$$f^*(p,w) = p - s + \frac{1}{\gamma}. \tag{10}$$

Using $f^*(p,w)$ to replace f in $\pi_R(p,f)$, we have

$$\pi_R(p, f^*(p,w)) = \left(p - w + \frac{1}{\gamma}\tau\beta(f^*(p,w))\right) D(p,r(q)). \tag{11}$$

We compute the first- and second-order derivatives of $\pi_R(p, f^*(p,w))$ w.r.t. p as

$$\frac{\partial \pi_R(p, f^*(p,w))}{\partial p} = \left[1 - \left(1 + \frac{b_1}{\gamma}\right)\tau\beta(f^*(p,w)) - b_1(p-w)\right] D(p,r(q)),$$

and

$$\begin{aligned}
\frac{\partial^2 \pi_R(p, f^*(p,w))}{\partial p^2} &= \left[\gamma\left(1 + \frac{b_1}{\gamma}\right)\tau\beta(f^*(p,w)) - b_1\right] D(p,r(q)) \\
&\quad -b_1\left[1 - \left(1 + \frac{b_1}{\gamma}\right)\tau\beta(f^*(p,w)) - b_1(p-w)\right] D(p,r(q)).
\end{aligned}$$

Using (10), we find that at the point satisfying the first-order condition,

$$\frac{\partial^2 \pi_R(p, f^*(p,w))}{\partial p^2} = -\gamma(b_1 f^*(p,w) - 1)D(p,r(q)),$$

which is negative, because $f^*(p,w) = p - w + 1/\gamma \geq 1/\gamma > 1/b_1$. Then, the best response price p^* can be uniquely obtained by solving the following equation for p:

$$p^* = w + \frac{1}{b_1} - \left(\frac{1}{b_1} + \frac{1}{\gamma}\right)\tau\beta(f^*(p,w)).$$

Recalling from (10) that $f^*(p,w) = p - w + 1/\gamma$, we have p^* as in (2). Therefore, the optimal restocking fee is obtained as f^* in (10). This proposition is proved.

■

Proof of Proposition 2. Given the value of q, we partially differentiate $\pi_M(w,q)$ in (3) once w.r.t. w as

$$\frac{\partial \pi_M(w,q)}{\partial w} = D(p^*, r(q)) + (w - c - q)\frac{\partial D(p^*, r(q))}{\partial w}.$$

Since $D(p^*, r(q)) = a\exp(-b_1 p^* + b_2 r(q))$, we compute the first-order derivative of $D(p^*, r(q))$ w.r.t. w is

$$\frac{\partial D(p^*, f^*, r(q))}{\partial w} = -b_1 \frac{\partial p^*}{\partial w} D(p^*, r(q)) = -b_1 D(p^*, r(q)).$$

It follows that

$$\begin{aligned}\frac{\partial \pi_M(w,q)}{\partial w} &= D(p^*, f^*, r(q)) - b_1(w - c - q)D(p^*, r(q)) \\ &= [1 - b_1(w - c - q)]D(p^*, r(q)).\end{aligned}$$

The second-order derivative of $\pi_M(w,q)$ in (3) w.r.t. w is then computed as

$$\frac{\partial^2 \pi_M(w,q)}{\partial w^2} = -b_1[2 - b_1(w - c - q)]D(p^*, f^*, r(q)),$$

which is negative at the point w such that $\partial \pi_M(w,q)/\partial w = 0$. Given q, the optimal wholesale price $w^*(q)$ is found as $w^*(q) = 1/b_1 + c + q$. As a result,

$$\pi_M(w^*(q), q) = \frac{1}{b_1} D(p^*(w,q), r(q)), \tag{12}$$

where

$$p^*(w^*(q), q) = c + q + \frac{2}{b_1} + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right).$$

We then compute the first-order derivative of $\pi_M(w^*(q), q)$ w.r.t. q as

$$\begin{aligned}
\frac{\partial \pi_M(w^*(q), q)}{\partial q} &= \frac{1}{b_1} D(p^*(w, q), r(q)) \left(-b_1 \frac{\partial p^*(w^*(q), q)}{\partial q} + b_2 \frac{\partial r(q)}{\partial q}\right) \\
&= \frac{1}{b_1} D(p^*(w, q), r(q)) \left(-b_1 + b_2 \frac{\partial r(q)}{\partial q}\right),
\end{aligned}$$

and calculate the second-order derivative of $\pi_M(w^*(q), q)$ w.r.t. q as

$$\begin{aligned}
\frac{\partial^2 \pi_M(w^*(q), q)}{\partial q^2} &= \frac{1}{b_1} D(p^*(w, q), r(q)) \left(-b_1 + b_2 \frac{\partial r(q)}{\partial q}\right)^2 \\
&\quad + \frac{b_2}{b_1} D(p^*(w, q), f^*(q), r(q)) \frac{\partial^2 r(q)}{\partial q^2},
\end{aligned}$$

which is negative at the point satisfying the first-order condition $\partial \pi_M(w^*(q), q)/\partial q = 0$. Thus, q^* can be attained by solving the following equation for q: $r'(q) = b_1/b_2$. Since $r(q) = \ln(q/k_0)$, $q^* = b_2/b_1$. This proposition is thus proved. ■

Proof of Corollary 1. Using (5), we find the equilibrium retail price as

$$p^* = c + \frac{2 + b_2}{b_1} + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right).$$

The first-order derivative of p^* w.r.t. γ is computed as

$$\begin{aligned}
\frac{\partial p^*}{\partial \gamma} &= -\frac{W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)}{z[1 + W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)]} \frac{\gamma}{b_1^2} \\
&= -\frac{W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)}{(b_1\gamma + b_1^2)[1 + W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)]} > 0.
\end{aligned}$$

Next, we examine the impact of γ on the equilibrium restocking fee, $f^* = z - s + \frac{1}{\gamma} W\left(-\tau\gamma z \exp\left(-\gamma z\right)\right)$, where $\gamma z = 1 + \gamma/b_1 > 1$. Note that $W(x) < 0$ if

x is a real number. Differentiating f^* once w.r.t. γ yields

$$\begin{aligned}
\frac{\partial f^*}{\partial \gamma} &= -\frac{1}{\gamma^2}[1+W(-\tau\gamma z\exp(-\gamma z))] - \frac{W(-\tau\gamma z\exp(-\gamma z))}{z[1+W(-\tau\gamma z\exp(-\gamma z))]}\frac{\gamma}{b_1^2} \\
&= \frac{-[1+W(-\tau\gamma z\exp(-\gamma z))]^2(b_1\gamma+\gamma^2) - \gamma^2 W(-\tau\gamma z\exp(-\gamma z))}{\gamma^2(b_1\gamma+b_1^2)[1+W(-\tau\gamma z\exp(-\gamma z))]} \\
&< \frac{-[1+W(-\tau\gamma z\exp(-\gamma z))]^2(\gamma+b_1)b_1}{\gamma^2(b_1\gamma+b_1^2)[1+W(-\tau\gamma z\exp(-\gamma z))]} \\
&< 0.
\end{aligned}$$

We thus complete the proof of this corollary. ■

Proof of Corollary 2. Using (5), we find that $f^* = z + W(-\tau\gamma z\exp(-\gamma z))/\gamma$, where $\gamma z = 1+\gamma/b_1 > 1$. Differentiating f^* once w.r.t. b_1 yields

$$\frac{\partial f^*}{\partial b_1} = \frac{\gamma W(-\tau\gamma z\exp(-\gamma z)) - zb_1(1+W(-\tau\gamma z\exp(-\gamma z)))}{zb_1^2(1+W(-\tau\gamma z\exp(-\gamma z)))} < 0.$$

We also learn that $p^* = c + f^* + 1/b_1 + b_2/b_1 + h\tau\beta(f^*) - 1/\gamma$. The first-order derivative of p^* w.r.t. b_1 is given as

$$\frac{\partial p^*}{\partial b_1} = -\frac{1}{b_1^2}\left[(2+b_2) - \frac{W(-\tau\gamma z\exp(-\gamma z))}{1+W(-\tau\gamma z\exp(-\gamma z))}\left(1+\frac{1}{z\gamma}\right)\right] < 0.$$

We find from (4) that $\partial w^*/\partial b_1 = -(1+kb_2)/b_1^2 < 0$ and $\partial q^*/\partial b_1 = -kb_2/b_1^2 < 0$. We thus prove the corollary. ■

Proof of Proposition 3. Given the wholesale price w, the retailer's best response price is $p^* = w + 1/b_1 + W(-\tau\gamma z\exp(-\gamma z))/\gamma$. Replacing p in $\bar{\pi}_M(w)$ with $p^*(w)$ and differentiating the resulting function once and twice w.r.t. w, we have $\partial\bar{\pi}_M(w)/\partial w = [1-b_1(w-c)]D(p^*)$ and $\partial^2\bar{\pi}_M(w)/\partial w^2 = -b_1[2-b_1(w-c)]D(p^*)$, which is negative at the point satisfying the first-order condition. Therefore, the wholesale price in Stackelberg equilibrium is $\bar{w}^* = c+1/b_1$. We can then compute $\bar{p}^*$, as given in this proposition. ■

Proof of Theorem 1. Comparing w^* in (4) and p^* in (5) with $\bar{w}^*$ and $\bar{p}^*$ in

(6), respectively, we find that $\bar{w}^* < w^*$ and $\bar{p}^* < p^*$. In addition, $\bar{f}^* = f^* = z + W(-\tau\gamma z \exp(-\gamma z))/\gamma$. ■

Proof of Theorem 2. Since $\bar{f}^* = f^*$, we only need to compare $D(\bar{p}^*) = a\exp(-b_1\bar{p}^*)$ and $D(p^*, r(q^*)) = a\exp(-b_1 p^* + b_2 r(q^*))$. Although $\bar{p}^* < p^*$, as shown in Theorem 1. Using (5) and (6), we have

$$p^* = \bar{p}^* + \frac{b_2}{b_1},$$

and we compute

$$\begin{aligned} D(\bar{p}^*) - D(p^*, r(q^*)) &= a\exp(-b_1\bar{p}^*) - a\exp\left(-b_1\left(\bar{p}^* + \frac{b_2}{b_1}\right) + b_2 r(q^*)\right) \\ &= a\exp(-b_1\bar{p}^*) - a\exp(-b_1\bar{p}^* - b_2 + b_2 r(q^*)) \\ &= a\exp(-b_1\bar{p}^*)(1 - \exp(-b_2 + b_2 r(q^*))). \end{aligned}$$

Therefore, if $1 - \exp(-b_2 + b_2 r(q^*)) \geq 0$, or, the condition in (7) is satisfied, then $D(\bar{p}^*) \geq D(p^*, r(q^*))$. Otherwise, $D(\bar{p}^*) \geq D(p^*, r(q^*))$. This theorem is thus proved. ■

Proof of Proposition 4. We can prove this proposition, using similar arguments as for Propositions 1 and 2. ■

Proof of Theorem 3. The expected sales under a partial refund policy and those under the full refund policy are given as follows

$$\bar{D}_1 = (1-\tau)D(\bar{p}^*, r(q^*)) \text{ and } D_1 = (1 - \tau\beta(f^*))D(p^*, r(q^*)).$$

The difference between $\bar{D}_1$ and D_1 is computed as

$$\begin{aligned} \bar{D}_1 - D_1 &= (1-\tau)D(\bar{p}^*, r(q^*)) - (1-\tau\beta(f^*))D(p^*, r(q^*)) \\ &= a\exp(\frac{b_2^2}{b_1} - (2+b_2))[(1-\tau)\exp(-\frac{b_1(c-\tau s)}{1-\tau} - \frac{\tau b_2^2}{b_1}) - (1-\tau\beta(f^*)) \\ &\quad \times \exp(\frac{-b_1(c + W(-\tau\gamma z \exp(-\gamma z)))}{\gamma})]. \end{aligned}$$

We find that if and only if the following condition is satisfied,

$$\beta(f^*) \geq \frac{1}{\tau}\left[1-(1-\tau)\exp\left(\frac{b_1\tau(s-c)}{1-\tau}-\frac{\tau b_2^2}{b_1}+\frac{b_1}{\gamma}W\left(-\tau\gamma z\exp\left(-\gamma z\right)\right)\right)\right],$$

then $\bar{D}_1 > D_1$, viz., the consumers' sensitivity to the restocking fee can reduce the realized sales. Otherwise, the realized sales rise. This theorem is thus proved.

■