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Interchange fee rate, merchant discount rate, and retail prices in a credit card network : a game-theoretic analysis

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INTERCHANGE FEE RATE, MERCHANT DISCOUNT RATE, AND RETAIL PRICES IN A CREDIT CARD NETWORK: A GAME-THEORETIC ANALYSIS

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by

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ABSTRACT

Interchange Fee Rate, Merchant Discount Rate, and Retail Prices in a Credit Card Network: A Game-Theoretic Analysis

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Master of Philosophy

We consider two game-theoretic settings to determine the optimal values of an issuer's interchange fee rate, an acquirer's merchant discount rate, and a merchant's retail prices for multiple products in a credit card network. In the first setting, we investigate a two-stage game problem in which the issuer and the acquirer first negotiate the interchange fee rate, and the acquirer and the retailer then determine their merchant discount rate and retail prices, respectively. In the second setting, motivated by the recent U.S. bill "H.R. 2695," we develop a three-player cooperative game in which the issuer, the acquirer, and the merchant form a grand coalition and bargain over the interchange fee rate and the merchant discount rate. Following the cooperative game, the retailer makes its retail pricing decisions. We derive both the Shapley value- and the nucleolus-characterized unique rates for the grand coalition. Comparing the two game settings, we show that the participation of the merchant in the negotiation process can result in the reduction of both rates. Moreover, the stability of the grand coalition in the cooperative game setting may require that the merchant should delegate the credit card business only to the issuer and the acquirer with sufficiently low operation costs. We also find that the large, highly-specialized merchants and banks are more likely to join the cooperative negotiation whereas the small firms may prefer the two-stage game setting. Our numerical experiments demonstrate that the acquirer's and the issuer's unit operation costs more significantly impact both rates in the cooperative game setting than in the two-stage game setting.

Key words: interchange fee rate; merchant discount rate; Nash bargaining; Stackelberg game; supermodularity; Shapley value; nucleolus.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

(GUO Hangfei)

Date:

CERTIFICATE OF APPROVAL OF THESIS

INTERCHANGE FEE RATE, MERCHANT DISCOUNT RATE, AND RETAIL PRICES IN A CREDIT CARD NETWORK: A GAME-THEORETIC ANALYSIS

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1 Introduction

In today's retailing markets, many consumers consider the credit card payment as an important, dominating means to shop from merchants. As the Nilson Report indicates, 795.5 million MasterCard and Visa cards were held by the U.S. consumers in 2004 [9]. The consumers who intend to buy now and pay later are more likely to complete transactions with credit cards. Accordingly, an increasing number of merchants have been accepting consumers' payments with credit cards, in order to entice consumers and thus improve their competitiveness in the markets that they serve. In the United States, the aggregate credit card transaction amount was 1.7 trillion in 2003, and this number has been quickly growing [9].

Although shopping with credit cards may generate the risks of excessive spending and getting into debt, there are still four major benefits for both consumers and businesses, as discussed by Hartman [14]. The first (largest) advantage of credit-card payment systems, which applies to both consumers and businesses, is that credit cards give consumers an increased buying power. We learn from Hartman [14] that consumers shopping with credit cards may spend up to 2.5 times more than consumers buying with cash; and thus, the merchants who accept credit card payments may achieve a 40% sales increase. The second advantage is that consumers can more conveniently complete their purchase payments. That is, it is easier for consumers to pay a single monthly credit card bill than to write checks or to often visit banks or ATM for cash withdrawal. Moreover, for a merchant, accepting credit cards reduces the need for employees to make change and possible daily-accounting mistakes. The third advantage of the credit-card payment is to provide consumers with an added measure of security, because consumers can easily cancel lost or stolen credit cards with, e.g., a simple phone call and stores with less cash on hand are also less susceptible to losing money. The fourth advantage is that consumers using credit cards can build up a positive credit rating over time if they use credit responsibly and make timely payments, thereby making it easier to finance a home or car in the future. For more information regarding the advantages of credit cards, see Hartman [14].

A typical credit card operation in reality usually involves two steps, as shown in Figure 1. In the first step, consumers buy products from the merchant and complete their transactions by confirming their credit card payments. In the second step, consumers pay the total credit-card transaction amount—i.e., consumers' total expense that is calculated as the sum of the merchant's sales revenue (retail price times sales quantity) for all products— from their bank accounts to the issuer. Next, the issuer retains an interchange fee—that is computed as the transaction amount times an interchange fee rate f—as its revenue and transfers the remaining amount to the acquirer. The acquirer then charges the merchant a discount fee that equals the transaction amount times a merchant discount rate d. Note that the merchant discount fee includes the issuer's interchange fee. That is, for an one-dollar credit card transaction, the acquirer obtains the merchant discount d but pays the interchange fee f to the issuer. It thus follows that the acquirer's revenue generated from this credit card business is $(d - f)\times$ total transaction amount. To assure the acquirer's non-negative profit, we realistically assume that $d \geq f$. As a result, the merchant's sales revenue is $(1 - d)\times$ total transaction amount. For specific examples illustrating the credit card operation, see, e.g., Hunt [16].

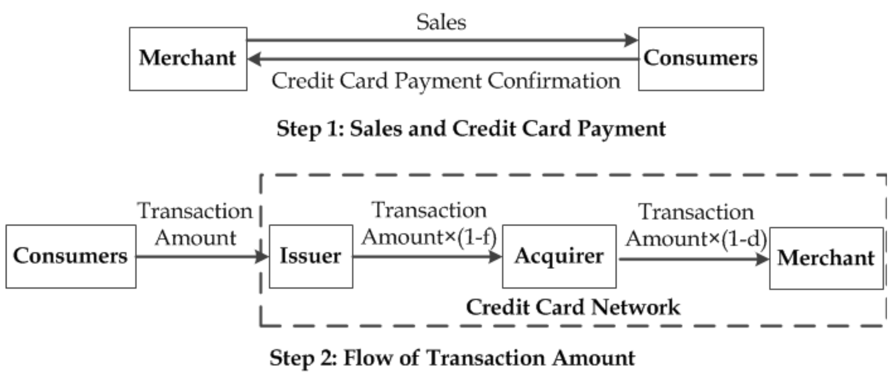

Figure 1: The two-step credit card transactions. Note that d and f denote the merchant discount rate and the interchange fee rate, respectively.

In this thesis, we consider a *three-echelon* credit card network that involves an issuer (consumers' bank), an acquirer (the merchant's bank), and a merchant that serves consumers with multiple ($n \geq 1$) products. The consumers use the credit cards (that are issued by the issuer) to buy from the merchant and pay their credit-card bills prior to the due dates. We assume that the merchant accepts the credit card payments. This assumption is reasonable because, in order to achieve satisfactory sales, most merchants are willing to accept credit cards in today's retailing industry [33]. In the credit card network, the interchange fee is used to compensate the issuer for bearing the risk of issuing credit cards, and the merchant discount fee is used to motivate the acquirer to maintain the merchant's account and process credit card transactions. The interchange and merchant discount fees have been among the largest costs of merchants who accept the credit card payments (Akers et al. [1]). If the interchange fee rate is increased, then the issuer benefits more from the credit card operation, and the acquirer may accordingly raise its merchant discount rate so as to assure its profitability. As a result, the merchant's sales revenue may be reduced, and the merchant may respond by increasing its retail prices to improve its profit. However, a higher retail price may discourage consumers from buying the product, thereby resulting in a reduction in the total transaction amount and thus a decrease in the issuer's, the acquirer's, and the merchant's profits. Following the above facts, it is important to properly determine the interchange fee rate, the merchant discount rate, and the retail prices. However, such a decision problem has not been considered in existing publications, as indicated by our literature review.

In practice, the interchange fee rate is either negotiated by the issuer and the acquirer or determined jointly by the issuer, the acquirer, and the merchant. Accordingly, we consider two game-theoretic settings to derive the optimal interchange fee rate, merchant discount rate, and product retail prices. For the first setting, we investigate a two-stage game problem in which the merchant does not participate in the negotiation of the interchange fee rate. This scenario is consistent with the fact that the interchange fee rate is normally centrally determined by the credit card companies such as Visa and MasterCard, through a committee comprising at least partially representatives of issuers and acquirers (Small and Wright [34]). That is, in the first stage, the issuer and the acquirer bargain over the interchange fee rate, as discussed by Balto [2] and Small and Wright [34]. Next, using the negotiated interchange fee rate, the acquirer determines a merchant discount rate and announces the rate to the merchant, who then makes retail pricing decisions. The second stage thus involves a sequential (Stackelberg) game in which the acquirer and the merchant act as the *leader* and the *follower*, respectively. We use backward induction to solve such a game. More specifically, we begin by solving the Stackelberg game between the acquirer and the merchant, assuming that the interchange fee rate is given. We then use the interchange fee rate-dependent Stackelberg solution to develop the issuer's and the acquirer's profit functions, and use the solution concept of Nash bargaining scheme [25] to characterize the negotiated interchange fee rate. We show that, if the price-sensitivity of each product is not high, then the acquirer and the issuer may be willing to participate in the credit card business.

For the second setting, we consider a three-player cooperative game in which the issuer, the acquirer, and the merchant bargain over the interchange fee rate and the merchant discount rate. This game is motivated by a recent U.S. legislation. In June 2009, John Conyers and Bill Shuster—who are the House Judiciary Committee Chairman Representatives—introduced the "Credit Card Fair Fee Act of 2009 (H.R. 2695)"[15] to the Senate and House of Representatives of the United States of America, which then enacted the bill to enable merchants to engage in collectively bargaining on a large scale with banks belonging to credit card networks such as Visa, MasterCard, etc. As John Conyers and Bill Shuster explained, this legislation would allow merchants to collectively negotiate with banks (i.e., acquirers and issuers) for certain credit card fees (i.e., interchange fee rate and merchant discount rate), and ultimately reduce the costs (i.e., retail prices) of everyday goods for consumers [46]. We note that, before 2009, many credit card companies (networks) did not allow merchants to directly bargain over the interchange fee rate with issuers and acquirers. But certain large merchants such as Wal-Mart could still bargain for a lower interchange fee rate (Akers et al. [1]).

We thus develop a two-step approach to find the rates for the cooperative game setting. Following the approach, we first construct a three-player cooperative game in characteristic-function form (von Neumann and Morgenstern [41, Ch. VI]). Then, we investigate whether there exist an interchange fee rate and a merchant discount rate so that the issuer, the acquirer, and the merchant are willing to cooperate for the credit card business. For similar approaches, see, e.g., Petrosjan and Zaccour [26], Sexton [30], etc. We show that, our three-player cooperative game is supermodular only when both the issuer and the acquirer can significantly contribute—i.e., generate sufficient profit surplus—to the credit card network. We also find that the grand coalition is more likely to be stable if the operation of each player is more efficient in the acquirer- and the issuer-related business. We then apply the solution concepts of Shapley value and the nucleolus to determine the interchange fee rate and the merchant discount rate that result in a fair allocation of the system-wide profit among the three players. We find that, for most cases, the Shapley value-characterized rates cannot assure the non-empty core; but, we can always find the nucleolus-characterized rates that guarantee the non-emptiness of the core and thus assure the stability of the grand coalition. Our cooperative game analysis for such a finance problem is an important focus in this thesis; for other applications of the cooperative game theory in the finance-operation interface area, see, e.g., Gow and Thomas [13] where the concepts of Shapley value and the nucleolus were used to determine interchange fees for bank ATM networks.

Comparing our *analytic* results in the two game settings, we find that the participation of the merchant in the negotiation process indeed helps reduce both rates, as expected by the U.S. bill "H.R. 2695." We show that a large merchant may have an incentive to undertake the credit card operation by itself, whereas a small merchant may need to delegate the related credit card business to the issuer and the acquirer whose unit operation costs are sufficiently low. Moreover, the participation of the large, highly-specialized banks and merchants is needed to assure the success of the bill "H.R. 2695," whereas the small merchant may be *unlikely* to join the credit card network and negotiate the rates with the issuer and the acquirer. This result may be justified by the fact that, in practice, many small merchants could be unwilling or against to accept the credit cards when the interchange fee rate and/or merchant discount rates are high (Tozzi [38] and Trichur [39]).

We then perform a sensitivity analysis to further investigate the impacts of the operation costs of the acquirer and the issuer on the optimal interchange fee rate and merchant discount rate that are obtained in our two game settings. We find that the operation costs have more significant impacts on both rates in the cooperative game setting than in the two-stage game setting. Moreover, in both the two-stage and the cooperative game settings, the merchant discount rate is increasing in the unit operation costs of both the issuer and the acquirer, c_A and c_I, and the interchange fee rate is increasing in c_I. However, the interchange fee rate is decreasing in c_A in the two-stage game setting, whereas the rate is decreasing in c_A if c_A is smaller than a cut-off level and is otherwise increasing in c_A in the cooperative game setting. Our major concluding remarks are provided in the Conclusion. The proofs of all lemmas, theorems, and corollaries in this thesis are relegated to Appendices.

The remainder of the thesis is organized as follows: In Section 2, we review major publications that are closely related to this thesis, which shows the originality of our problem. In Sections 3 and 4, we consider a two-stage game problem and a three-player cooperative game problem, respectively; and for each problem, we determine a unique interchange fee rate, a unique merchant discount rate, and a unique retail price for each product sold by the merchant. In Section 5, we perform a sensitivity analysis to examine the impacts of four important parameters on both the interchange fee rate and the merchant discount rate determined by using the two-stage game model and those determined by using the three-player cooperative game model. The thesis ends with our summary and concluding remarks in Section 6.

2 Literature Review

This thesis is associated with those concerning retail pricing-, interchange fee rate-, and merchant discount rate-related problems in credit card networks. Most existing relevant papers have been reviewed by Chakravorti [7], Hunt [16], Bolt and Chakravorti [5], and Verdier [40]. Note that the first two articles (i.e., Chakravorti [7] and Hunt [16]) mainly focused on the costs and benefits of credit cards, the third article (i.e., Bolt and Chakravorti [5]) categorized major economic models for credit card operations, and the fourth article (i.e., Verdier [40]) surveyed the recent literature about the interchange fee rate in payment card systems. Since the thesis is concerned with quantitative (game-theoretic) analysis for a credit card network, we next briefly describe Chakravorti [7], Hunt [16], and Verdier [40], and then particularly discussed several relevant publications that were reviewed by Bolt and Chakravorti [5]. But, our review differs from Bolt and Chakravorti [5] since we focus on describing the problems rather than the economic models in the publications that are significantly related to the thesis; this distinguishes the thesis from extant publications.

In Chakravorti [7], Chakravorti discussed the costs and benefits of credit cards to network participants (i.e., consumers, merchants, acquirers, issuers, etc.), and then summarized major publications that had concerned key features in credit card networks, including merchant pricing policy, interchange fees, merchant acceptance, and network competition. Though, we find that each publication reviewed by Chakravorti [7] didn't capture all of the above key features but only analyzed one or some of these features. Different from Chakravorti [7], Hunt [16] didn't conduct a review from the perspective of the operations of credit card networks but only provided a brief overview of the economics of the payment card industry. In Verdier [40], Verdier reviewed the theoretical results of recent literature, with an emphasis on the ongoing debate that opposes banks to the regulatory institutions or the competition authorities in various countries.

As the latest publication for the review of credit card-related literature, Bolt and Chakravorti [5] classified relevant economic models into the following five categories that consider different issues: (i) models focusing on interchange fees; (ii) models with price differentiation at the point of sale; (iii) models with competition between networks; (iv) models accounting for the role of credit; and (v) models with competition among payment instruments. Since the thesis focuses on how to determine the interchange fee rate, merchant discount rate, and retail prices for the merchant, we discuss some relevant papers in categories (i) and (ii). As Bolt and Chakravorti [5] reviewed, the seminal interchange fee-related publication was proposed by Baxter [3], who constructed a one-period model to investigate an interchange fee problem, assuming that the market is perfectly competitive for payment service and consumption goods. The author concluded that the interchange fee is an important and necessary tool that balances consumers' and the merchant's demand and the issuers' and the acquirers' costs.

Schmalensee [28] extended Baxter's model by assuming that issuers and acquirers have significant market powers while merchants are perfectly competitive. Similar to Baxter [3], Schmalensee [28] found that, as a balancing device, the interchange fee can increase the value of a credit card system by properly allocating costs between issuers and acquirers and thus reasonably determining the cost allocation between consumers and merchants. We note that both Baxter [3] and Schmalensee [28] ignored the strategic interactions of consumers and merchants, which is actually not negligible because of the following fact: if more consumers use credit cards, then more merchants accept credit cards, which further increases the number of consumers who use credit cards for their purchases.

As an early publication involving the strategic interactions of consumers and merchants, Rochet and Tirole [27] constructed a quantitative model where issuers have market powers but acquirers are perfectly competitive, and consumers and merchants decide rationally on whether to buy or accept a payment card. More specifically, Rochet and Tirole considered the following three-stage problem: At

Stage 1, a central planner sets an interchange fee rate to maximize total profit—that is calculated as the sum of the merchants', the acquirers' and the issuers' profits—or the issuers set an interchange fee rate to maximize their own profits. At Stage 2, the issuers set credit card-related fees for their customers, who can decide whether or not to have a card. The merchants decide whether to accept payment cards, and then determine their retail prices. At Stage 3, the customers observe the retail prices and whether cards are accepted, and then pick a merchant for their purchases. If a customer chooses a merchant that doesn't accept credit cards, then both the customer and the merchant incur opportunity costs of using an alternative payment method. Using the backward approach, Rochet and Tirole [27] obtained the equilibrium retail prices for the merchants, and then calculated the socially-optimal interchange fee for the central planner and the privately-optimal fee for the issuers. The authors found that the socially-optimal interchange fee rate for the central planner is higher than the socially-optimal interchange fee rate that was attained by Baxter [3], which happens due to the issuers' market powers.

Using the framework in Schmalensee [28] where the merchants may not accept credit cards for some strategic reasons, Wright [44] developed a model where the partial participation of heterogeneous consumers and merchants is allowed in a two-sided market (that is, the credit card market composed of cardholders and merchants). Wright found that the privately-optimal interchange fee rate may be very high, if the merchant discount fees are increasing in the interchange fees and the issuers don't charge the acquirers any additional interchange fees that are treated as rewards to customers. Our review indicates that the above publications (i.e., Baxter [3], Rochet and Tirole [27], Schmalensee [28], and Wright [44]) considered the centralized case in which the interchange fee rate is determined by a central planner or a credit card company/association such as Visa and MasterCard.

Different from the above publications, Balto [2] questioned the rationality of centrally-determined interchange fee rate, and argued that a lower interchange fee rate could be determined as a result of the bilateral negotiation between the acquirers and the issuers. During the mid-1980s, there were 10,000 to 15,000 members in the Visa network. Many practitioners believed that the negotiation cost among these members might be too high to determine the interchange fee rate according to the bilateral negotiation. However, as Balto [2] discussed, the past two decades witnesses a significant reduction of the cost of bilateral negotiation between the issuers and the acquirers, because a few large-size issuers and acquirers have dominated the issuing and acquiring markets. Even though the negotiated interchange fee problem is important, we find that very few papers have addressed the problem; thus, this is still under-researched. Assuming that the interchange fee rate is negotiated rather than centrally determined, Small and Wright [34] constructed an Nash bargaining model to characterize the bilateral bargaining over the interchange fee rate. The authors discussed the hold-up problem that results from the "honour all cards rule" (i.e., the merchants and their banks (acquirers) in a credit card network must accept all the cards issued in the network), and they found that, as a result of implementing the rule, the bilaterally-negotiated interchange fee rate may be higher than the centrally-determined interchange fee rate. Small and Wright [34] used the concept "Nash bargaining scheme" to analyze the bilateral bargaining problem; but, they didn't consider the merchant's retail pricing decision. The thesis differs from Small and Wright [34] since we also determine the retail prices for the merchant in a credit card network. Even though some publications (e.g., Rochet and Tirole [27]) considered the retail pricing decision, they didn't investigate the bargaining process for the interchange fee rate decision, which is also involved in the thesis.

As discussed in Section 1, the interchange fee rate *indirectly* affects each merchant's pricing decision because the retail price depends on the merchant discount rate that is determined by the acquirer according to the interchange fee rate. Thus, the merchants should have an incentive to participate in the negotiation of the interchange fee rate. Before 2009, many credit card companies (networks) didn't allow the merchants to directly bargain over the interchange fee rate with the issuers. But, we note that some large merchants (e.g., Wal-Mart) could still negotiate with the credit card companies for a lower interchange fee rate; see, e.g. Akers et al. [1]. In June 2009, the Senate and House of Representatives of the United States of America enacted the bill entitled "Credit Card Fair Fee Act of 2009 (H.R. 2695)" [15], which allows merchants to engage in bargaining over relevant fee rates (i.e., the interchange fee rate and the merchant discount rate) with banks belonging to credit card networks (e.g., Visa, MasterCard, etc.) in the United States. This bill would be helpful to decreasing the banking fees and ultimately reducing the retail prices of everyday goods for consumers.

According to our review, we find that Thomas [36] used the multi-player co-operative game theory to analyze the credit card-related problems. Motived by the major changes in the distribution of the credit card costs—e.g., the introduction of annual fees, the lowering of merchant service charges, etc.—in the U.K. at the end of 1980s, the author analyzed the fair allocation of the credit card-related costs among the credit card companies, the retailers who accept the credit card payments, and the cardholders. More specifically, Thomas [36] developed a credit-card cooperative game in the *linear* characteristic-value function, applied the concepts of the core, Shapley value, and the nucleolus to numerical examples, and performed a sensitivity analysis to examine the impacts of debit cards on credit-card fee structures. But, Thomas [36] did not analytically consider the interchange fee rate, merchant discount rate, and the retail prices.

Different from the aforementioned papers, we examine a two-stage game to investigate the negotiated interchange fee rate and to derive the optimal merchant discount rate and the optimal retail pricing decisions. In addition, motivated by the recent U.S. legislation H.R. 2695 [15], we consider a three-player game where the acquirer, the issuer, and the merchant negotiate the interchange fee rate and the merchant discount rate. This significantly distinguishes the thesis from the existing literature.

3 The Two-Stage Game Analysis

In this section, we analyze a two-stage game to determine the interchange fee rate f, merchant discount rate d, and retail prices p_i $(i = 1, \ldots, n)$ for $n \geq 1$ products in a credit card network. In the first stage, the issuer (i.e., consumers' bank) and the acquirer (i.e., the merchant's bank) bargain over the interchange fee rate. In the second stage, the acquirer determines and announces a merchant discount rate to the merchant, who then makes its retail pricing decisions for n different products. More specifically, we investigate a two-person cooperative game to find the interchange fee rate as a result of the negotiation between the issuer and the acquirer in the first stage. This cooperative-game approach reflects the following fact: In practice, when the issuer transfers to the acquirer a transaction amount that a consumer spends with his or her credit card to buy from the merchant, the issuer deducts an interchange fee (as the rate f times the transaction amount) from the amount to the acquirer. The interchange fee rate is normally centrally determined by the credit companies such as Visa and MasterCard. We learn from, e.g., Small and Wright [34] that Visa and MasterCard determine their interchange fee rates through a committee comprised at least partially of representatives of issuers and acquirers.

Accordingly, we develop a two-person cooperative game to characterize the negotiation between the acquirer and the issuer for the interchange fee rate in the first stage. The negotiation process is then followed by a non-cooperative game in the second stage, where the acquirer determines a merchant discount rate and announces the rate to the merchant, who then makes its retail pricing decisions for n different products. That is, in the non-cooperative game, the acquirer and the merchant act as the leader and the follower, respectively, which is thus regarded as a leader-follower (Stackelberg) game.

As indicated by the above, the cooperative game involving the issuer and the acquirer occurs before the non-cooperative (leader-follower) game takes place between the acquirer and the merchant. To solve our two-stage problem, we should first examine the non-cooperative game, assuming that the interchange fee rate is given; and then, we analyze the two-person cooperative game to determine the negotiated interchange fee rate. Specifically, we conduct the following sequential analysis:

1. **Analysis of the Leader-Follower Game for a Given Interchange Fee Rate**. We assume that the interchange fee rate f is given, and solve the leader-follower game to find the Stackelberg solution for the given value of f. To do so, we first find the merchant's best-response pricing decisions for n products, and then calculate the acquirer's optimal merchant discount rate. Substituting the optimal merchant discount rate into the merchant's best response function, we obtain the Stackelberg solution in terms of the given interchange fee rate f.

2. **Analysis of the Two-Person Cooperative Game**. Using the $f-$dependent Stackelberg solution that is obtained in the first step, we construct a two-person cooperative game and solve it to find Nash bargaining scheme that characterizes the interchange fee rate f^* resulting from the negotiation between the acquirer and the issuer. Substituting f^* into the $f-$dependent Stackelberg solution gives the equilibrium merchant discount rate d^* and retail prices p_i^* $(i = 1, \ldots, n)$.

3.1 The Analysis of the Stackelberg Game in the Second Stage

Assuming that the interchange fee rate f is given, we solve the leader-follower game to determine the $f-$dependent Stackelberg solution $(d^*(f), \mathbf{p}^*(f))$ where $\mathbf{p}^*(f) \equiv (p_1^*(f), \ldots, p_n^*(f))$. Next, we first calculate the merchant's best-response prices $p_i^*(d; f)$ $(i = 1, \ldots, n)$, and then find the Stackelberg solution $(d^*(f), \mathbf{p}^*(f))$.

3.1.1 The Merchant's Best-Response Pricing Decision

We now determine the merchant's best-response pricing decisions that maximize the merchant's profit when the acquirer's merchant discount rate d is given. The merchant is assumed to serve customers in a market with $n \geq 1$ products. The merchant obtains product i ($i = 1, \ldots, n$) at the acquisition cost c_i, and sells it at the retail price p_i. Since the sale price of a product usually affects the demand for the product, we assume that the demand for product i ($i = 1, \ldots, n$) is dependent on the retail price p_i. The price-dependent demand for product i is denoted by $q_i(p_i)$. This assumption has been widely used in economics, finance, and operations management; see Bertrand [4], Corbett and Karmarkar [8], Lim and Ho [21], and the references therein. In this thesis, we also assume that $q_i(p_i)$ is a deterministic, linear price-dependent demand function, which is given as,

$$q_i(p_i) = \alpha_i - \beta_i p_i, \quad \text{for } i = 1, \ldots, n, \tag{1}$$

where $\alpha_i > 0$ denotes the price-independent demand; and $\beta_i > 0$ represents the marginal impact of the price.

To assure that the demand $q_i(p_i)$ is non-negative, we assume that the parameter α_i is sufficiently large such that $\alpha_i \geq \beta_i c_i$, for $i = 1, \ldots, n$. Moreover, to generate the non-negative sales of product i, the merchant should determine the retail price such that $p_i \leq \alpha_i/\beta_i$. Thus, for product i, $(p_i - c_i)/p_i = 1 - c_i/p_i \leq (\alpha_i - \beta_i c_i)/\alpha_i$. Note that the ratio $(p_i - c_i)/p_i$ is regarded as the well-known "Lerner index" [20]. Lerner index reflects a firm's market power; that is, a firm with a higher value of Lerner index has a greater power in the market that it serves. Therefore, to assure that $q_i(p_i) \geq 0$, the merchant's market power for product i should be smaller than or equal to $(\alpha_i - \beta_i c_i)/\alpha_i$, $i = 1, \ldots, n$, which is the *relative* measure of the maximum demand for product i (i.e., $\alpha_i - \beta_i c_i$) over the constant, price-independent demand α_i.

Next, the merchant's sale revenue and total acquisition cost can be computed as $R \equiv \sum_{i=1}^{n} p_i \times q_i(p_i)$ and $c \equiv \sum_{i=1}^{n} c_i \times q_i(p_i)$, respectively. Since the acquirer charges the merchant \$$d$ per dollar of the sale revenue R, the merchant needs to pay the merchant discount fee in the amount of $T = d \times R$ to the acquirer. The merchant's profit is calculated as its sale revenue minus the sum of total acquisition cost and merchant discount fees; i.e.,

$$\pi_M = R - c - T = \sum\nolimits_{i=1}^{n} [(1-d)p_i - c_i] \times q_i(p_i), \tag{2}$$

where $[(1-d)p_i - c_i]$ can be regarded as the merchant's *net* profit per unit. That is, when the merchant sells one unit of product i, it can achieve the profit \$$[(1-d)p_i - c_i]$, which should be greater than or equal to zero. This requires that the merchant should determine the price p_i such that $p_i \geq c_i/(1-d)$. Recall from our previous discussion that the price should be smaller than or equal to α_i/β_i. Hence, the acquirer's merchant discount rate d should be determined such that $c_i/(1-d) \leq \alpha_i/\beta_i$, or, $d \leq (\alpha_i - \beta_i c_i)/\alpha_i$ for $i = 1, \ldots, n$. This requires the assumption that $\alpha_i \geq \beta_i c_i$, for $i = 1, \ldots, n$.

When the merchant makes his pricing decisions for n products, we assume that the acquirer's merchant discount rate decision d is smaller than or equal to $\kappa \equiv \min[(\alpha_i - \beta_i c_i)/\alpha_i, i = 1, \ldots, n]$, i.e., $d \leq \kappa$. Otherwise, if $d > \kappa$, then the merchant cannot achieve the nonnegative sale profit for some or all products, and is thus unwilling to join the credit card network. Moreover, for an one-dollar credit card transaction, the acquirer obtains the merchant discount rate \$$d$ but pays the interchange fee \$$f$ to the issuer. To assure the acquirer's non-negative profit, d should be greater than or equal to f. Therefore, the acquirer should determine its merchant discount rate such that $f \leq d \leq \kappa$.

To find optimal pricing decisions for n products, the merchant should maximize its profit π_M in (2) subject to $c_i/(1-d) \leq p_i \leq \alpha_i/\beta_i$ for $i = 1, \ldots, n$. The merchant's constrained maximization problem is thus developed as,

$$\begin{aligned} \max_{p_i, i=1,\ldots,n} \quad & \pi_M = \textstyle\sum_{i=1}^{n} [(1-d)p_i - c_i] \times q_i(p_i) \\ \text{s.t.} \quad & c_i/(1-d) \leq p_i \leq \alpha_i/\beta_i, \text{ for } i = 1, \ldots, n. \end{aligned} \tag{3}$$

Theorem 1 Given the acquirer's merchant discount rate d in the range $[f, \kappa]$, the merchant's best-response retail price for product i ($i = 1, \ldots, n$) is uniquely found as,

$$p_i(d) = \frac{1}{2}\left(\frac{\alpha_i}{\beta_i} + \frac{c_i}{1-d}\right). \quad \blacktriangleleft \tag{4}$$

Proof. For a proof of this theorem and the proofs for all subsequent theorems, see Appendix A. ■

We notice from (4) that the merchant's best-response price for product i—i.e., $p_i(d)$—is increasing in d. This means that, as d increases, the merchant should respond by raising its price. When the acquirer doesn't charge the merchant any merchant discount rate, i.e., $d = 0$, the merchant can reduce its price to the minimum $(\alpha_i/\beta_i + c_i)/2$.

Using the above, we can calculate the merchant's optimal quantities $q_i(d)$ and optimal credit card sale revenue $R(d)$, which is regarded as the total transaction amount in the credit card network. That is,

$$q_i(d) = \alpha_i - \beta_i p_i = \alpha_i - \frac{1}{2}\beta_i\left(\frac{\alpha_i}{\beta_i} + \frac{c_i}{1-d}\right) = \frac{1}{2}\left[\alpha_i - \frac{\beta_i c_i}{(1-d)}\right], \tag{5}$$

$$R(d) = \sum\nolimits_{i=1}^{n} p_i(d) \times q_i(d) = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right], \tag{6}$$

where

$$\omega_1 \equiv \sum\nolimits_{i=1}^{n} \frac{\alpha_i^2}{\beta_i} \quad \text{and} \quad \omega_2 \equiv \sum\nolimits_{i=1}^{n} \beta_i c_i^2. \tag{7}$$

Remark 1 When $d \leq \kappa$, we find that $\omega_1 \geq \omega_2/(1-d)^2$, and the merchant's sales revenue $R(d)$ in (6) is non-negative. This implies that, if $d \leq \kappa$, then the merchant should accept the transactions with credit cards. Moreover, we learn from our above discussion that $(\alpha_i - \beta_i c_i)/\alpha_i$ ($i = 1, \ldots, n$) represents the ratio of the maximum demand of product i (i.e., $\alpha_i - \beta_i c_i$) to the constant, price-*in*dependent demand of product i (i.e., α_i), and also note that κ is the minimum value of the ratios for all products. That is, if the demand for *each* product is *more sensitive* to the retail price, i.e., the values of α_i ($i = 1, \ldots, n$) are lower, then the value of κ is smaller and the merchant discount rate should be accordingly reduced to assure the non-negative sales. But, the acquirer and the issuer may be *less willing* to participate in the credit card business, because the acquirer's merchant discount rate d includes the interchange fee rate gained by the issuer. Thus, in order to assure the success of the credit card business, the demand of each product in the market should *not* be very sensitive to its retail price. ◀

Substituting $p_i(d)$ in (4) and $q_i(d)$ in (5) into the merchant's profit function π_M in (2), we calculate the merchant's resulting profit as,

$$\pi_M(d) = \sum\nolimits_{i=1}^{n}[(1-d)p_i(d) - c_i] \times q_i\,(p_i(d)) = \frac{1}{4}\sum\nolimits_{i=1}^{n}\frac{[(1-d)\alpha_i - \beta_i c_i]^2}{(1-d)\,\beta_i}, \quad (8)$$

which is non-negative. In addition, we notice from (8) that $\pi_M(d)$ is decreasing in d; that is, the merchant's profit is reduced when the acquirer increase its merchant discount rate d. Since $0 \leq d \leq \kappa$, we find that merchant's profit is between 0 and $\sum_{i=1}^{n}(\alpha_i - \beta_i c_i)^2/4\beta_i$.

3.1.2 The Interchange Fee Rate-Dependent Stackelberg Solution

We learn from the preceding section that the total credit card transaction amount is $R(d)$ as given in (6). When the acquirer makes a transfer to the merchant's account, the acquirer charges the merchant a merchant discount fee—that is computed as the merchant discount rate d times the transaction amount $R(d)$—so that the merchant cannot obtain the sale revenue $R(d)$ but only receives the amount $[(1-d) \times R(d)]$ from the acquirer. Since, for the transaction amount $R(d)$, the acquirer transfers to the issuer the interchange fee $f \times R(d)$ and retains the amount of $(d - f) \times R(d)$.

Assume that the acquirer incurs the unit operation cost c_A, which means that, when the acquirer processes the credit card transaction in amount of $R(d)$, its operation cost is calculated as $c_A \times R(d)$. We then develop the acquirer's profit function as,

$$\begin{aligned}\pi_A(d) &\equiv (d - f - c_A) \times R(d) \\ &= \frac{1}{4}(d - f - c_A) \times \left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right], \end{aligned} \tag{9}$$

where $(d - f - c_A)$ represents the acquirer's net profit per dollar; and the second equation is obtained by using (6). Particularly, when the acquirer processes one dollar of credit-card transaction amount, it can achieve the profit $\$[(d - f - c_A)]$, which should be greater than or equal to zero. This requires that the acquirer should determine the merchant discount rate d such that $d \geq f + c_A$. Note that, as discussed in Section 3.1.1, the merchant discount rate d should be greater than or equal to f but smaller than or equal to κ. Hence, the acquirer's merchant discount rate d must be determined such that $f + c_A \leq d \leq \kappa$. Given the value of interchange fee rate f, the acquirer should maximize its profit π_A subject to $f + c_A \leq d \leq \kappa$, in order to find the optimal merchant discount rate $d^*(f)$.

Theorem 2 Given the interchange fee rate f, the acquirer's optimal merchant discount rate $d^*(f)$ is uniquely computed as,

$$d^*(f) = \begin{cases} d_1(f), & \text{if } f \leq \hat{f}, \\ \kappa, & \text{if } \hat{f} \leq f \leq \kappa, \end{cases} \tag{10}$$

where $d_1(f)$ denotes a unique solution of the following equation,

$$\omega_1 \times [1 - d_1(f)]^3 = \omega_2 \times [1 + d_1(f) - 2f - 2c_A]; \tag{11}$$

and

$$\hat{f} \equiv \frac{(1 + \kappa - 2c_A)\omega_2 - (1-\kappa)^3\omega_1}{2\omega_2}. \quad \blacktriangleleft \tag{12}$$

Note that $d^*(f)$ given in (10) is the $f-$dependent Stackelberg merchant discount rate. In addition, Theorem 2 indicates that, if the interchange fee rate f rises, then the acquirer should respond by increasing its merchant discount rate.

But, this may discourage the merchant to accept customers' purchases with credit cards. In order to assure that the merchant is willing to trade with credit cards, the acquirer should bear a portion of the increase in the interchange fee rate. It thus follows that the increase in the merchant discount rate is smaller than the increase in f.

Corollary 1 The $f-$dependent Stackelberg merchant discount rate $d^*(f)$ is increasing in the interchange fee rate f, i.e., $\partial[d^*(f)]/\partial f \geq 0$; but, $\partial[d^*(f)]/\partial f < 1$. Moreover, we find that $d^*(f)$ is a convex function of f, i.e., $\partial^2[d^*(f)]/\partial f^2 \geq 0$. ◀

Proof. For a proof of this corollary and the proofs for all subsequent corollaries, see Appendix B. ■

Using $d^*(f)$ in (10) to replace d in the merchant's best-response retail prices $p_i(d)$ $(i = 1, \ldots, n)$ given in (4), we can find the set of n $f-$dependent Stackelberg retail prices as $\mathbf{p}^*(f) = (p_1^*(f), \ldots, p_n^*(f))$, where, for $i = 1, \ldots, n$,

$$p_i^*(f) = \begin{cases} \dfrac{1}{2}\left[\dfrac{\alpha_i}{\beta_i} + \dfrac{c_i}{(1-d_1(f))}\right], & \text{if } f \leq \hat{f}, \\ \dfrac{1}{2}\left[\dfrac{\alpha_i}{\beta_i} + \dfrac{c_i}{(1-\kappa)}\right], & \text{if } \hat{f} \leq f \leq \kappa. \end{cases} \tag{13}$$

In addition, when the acquirer and the merchant choose their Stackelberg solutions, the corresponding credit-card transaction amount in (6) is computed as,

$$R(d^*(f)) = \begin{cases} \dfrac{1}{4}\left\{\omega_1 - \dfrac{\omega_2}{[1-d_1(f)]^2}\right\} \geq 0, & \text{if } f \leq \hat{f}, \\ \dfrac{1}{4}\left[\omega_1 - \dfrac{\omega_2}{(1-\kappa)^2}\right] \geq 0, & \text{if } \hat{f} \leq f \leq \kappa, \end{cases} \tag{14}$$

where both $\omega_1 - \omega_2/[1-d_1(f)]^2$ and $\omega_1 - \omega_2/(1-\kappa)^2$ are nonnegative, because $d_1(f) \leq \kappa$ if $f \leq \hat{f}$; and as Remark 1 indicates, $\omega_1 - \omega_2/(1-d)^2 \geq 0$ when $d \leq \kappa$.

As a result, we substitute $d^*(f)$ in (10) into (9), and find that, for a given value of f, the acquirer's maximum profit is computed as,

$$\pi_A(d^*(f)) = \begin{cases} \pi_{A_1} \equiv \frac{1}{4}[d_1(f) - f - c_A]\left[\omega_1 - \frac{\omega_2}{(1-d_1(f))^2}\right] \geq 0, & \text{if } f \leq \hat{f}, \\ \pi_{A_2} \equiv \frac{1}{4}(\kappa - f - c_A)\left[\omega_1 - \frac{\omega_2}{(1-\kappa)^2}\right] \geq 0, & \text{if } \hat{f} \leq f \leq \kappa. \end{cases} \quad (15)$$

Note that, when $f = \hat{f}$, $d_1(f) = \kappa$ and $\pi_{A_1} = \pi_{A_2}$.

3.2 The Analysis of the Two-Person Cooperative Game in the First Stage

We now investigate the first-stage game problem in which the acquirer and the issuer bargain over the interchange fee rate f, assuming that the two players have complete information regarding the leader-follower game that is discussed in Section 3.1. Under the assumption, the acquirer and the issuer can "forecast" the Stackelberg solution for any given value of f. After we calculate the negotiated interchange fee rate f^*, we can substitute f^* into the $f-$dependent Stackelberg solution $(d^*(f), \mathbf{p}^*(f))$—which is obtained in Section 3.1—and obtain the Stackelberg equilibrium $(d^*, \mathbf{p}^*) = (d^*(f^*), \mathbf{p}^*(f^*))$ for the leader-follower game taking place at the second stage in our two-stage problem.

To analyze the two-player bargaining problem between the acquirer and the issuer, we can use cooperative game solution concepts such as egalitarian proposal, negotiation set, Nash bargaining scheme (a.k.a. Nash arbitration scheme), etc. Among these solutions, Nash bargaining scheme (Nash [25]) is the most useful one because it represents a unique bargaining solution that can be obtained by solving the following maximization problem: $\max_{y_1,y_2}(y_1 - y_1^0)(y_2 - y_2^0)$, s.t. $(y_1, y_2) \in \mathcal{P}$, where y_i and y_i^0 correspond to player i's profit and security level (a.k.a. *status quo* point), respectively, $i = 1, 2$; and $\mathcal{P}$ denotes the set of Pareto optimal solutions, i.e., $\mathcal{P} = \{(y_1, y_2)|y_1 \geq y_1^0, y_2 \geq y_2^0\}$. This concept has been broadly used to analyze a variety of bargaining problems in the finance field; see, e.g., Cai [6] , Ericsson and Renault [11] , etc. Next, we compute the Nash bargaining scheme to characterize the negotiated interchange fee rate.

3.2.1 Nash Bargaining Scheme

We find from Section 3.1 that, given the interchange fee rate f, the acquirer and the merchant determine their merchant discount rate and retail prices respectively in the leader-follower game setting, which results in the credit-card transaction amount $R(d^*(f))$ as given in (14). As a result, the acquirer's profit is $\pi_A(d^*(f))$ in (15); and the issuer obtains the interchange fee $f \times R(d^*(f))$ but absorbs the operation cost $c_I \times R(d^*(f))$, where c_I denotes the issuer's unit operation cost per transaction dollar. It follows that the issuer's profit $\pi_I(f)$ is calculated as, $\pi_I(f) = (f - c_I) \times R(d^*(f))$, where $(f - c_I)$ is the issuer's net profit per dollar. To assure the non-negativity of $\pi_I(f)$, the issuer only accepts the negotiated interchange fee rate that is greater than or equal to c_I. Since, as indicated by (14), the total transaction amount $R(d^*(f))$ is dependent on the value of f, the issuer's profit function is written as,

$$\pi_I(f) = \begin{cases} \pi_{I_1} \equiv \left(\dfrac{f - c_I}{4}\right) \times \left[\omega_1 - \dfrac{\omega_2}{(1 - d_1(f))^2}\right] \geq 0, & \text{if } c_I \leq f \leq \hat{f}, \\ \pi_{I_2} \equiv \left(\dfrac{f - c_I}{4}\right) \times \left[\omega_1 - \dfrac{\omega_2}{(1 - \kappa)^2}\right] \geq 0, & \text{if } \hat{f} \leq f \leq \kappa. \end{cases} \tag{16}$$

where both π_{I_1} and π_{I_2} are non-negative, as discussed in Section 3.1.2.

In our bargaining problem, we, w.l.o.g., assume that the acquirer and the issuer are player 1 and player 2, respectively. From our previous discussion, we find that the acquirer's and the issuer's profits are $y_1 = \pi_A(d^*(f))$ [as given in (15)] and $y_2 = \pi_I(f)$ [as given in (16)], respectively. We next compute the two players' security levels (y_1^0, y_2^0), which, for our problem, are defined as these two players' guaranteed profits when they don't cooperate for the credit card business. Since neither the acquirer nor the issuer will gain any profit if no transaction occurs between them, the status quo point is $(y_1^0, y_2^0) = (0, 0)$ and the set of Pareto optimal solutions is $\mathcal{P} = \{(y_1, y_2) | \pi_A(d^*(f)) \geq 0 \text{ and } \pi_I(f) \geq 0\}$.

According to the above discussion, the Nash bargaining scheme (NBS) model for our bargaining problem can be written as: $\max_f \Lambda(f) \equiv \pi_A(d^*(f)) \times \pi_I(f)$, s.t. $\pi_A(d^*(f)) \geq 0$ and $\pi_I(f) \geq 0$. Using (15) and (16), we specify the NBS function $\Lambda(f)$ as,

$$\Lambda(f) = \begin{cases} \Lambda_1(f) \equiv & \dfrac{[d_1(f) - f - c_A]\,(f - c_I)}{16} \\ & \times \left[\omega_1 - \dfrac{\omega_2}{(1 - d_1(f))^2}\right]^2, \quad \text{if } c_I \leq f \leq \hat{f}, \\ \Lambda_2(f) \equiv & \dfrac{(\kappa - f - c_A)\,(f - c_I)}{16} \\ & \times \left[\omega_1 - \dfrac{\omega_2}{(1 - \kappa)^2}\right]^2, \quad \text{if } \hat{f} \leq f \leq \kappa. \end{cases} \tag{17}$$

We learn from (15) and (16) that, when $c_I \leq f \leq \kappa$, $\pi_A(d^*(f)) \geq 0$ and $\pi_I(f) \geq 0$. That is, in order to find the NBS solution, we need to consider the following maximization problem: $\max_{f \in [c_I, \kappa]} \Lambda(f)$, where the objective function $\Lambda(f)$ is given as in (17).

Note that $\Lambda(f)$ in (17) may be $\Lambda_1(f)$ or $\Lambda_2(f)$, which depends on the value of f. Thus, in order to find the optimal interchange fee rate f^*, we should consider the following two scenarios: $c_I \leq f \leq \hat{f}$ and $\hat{f} \leq f \leq \kappa$; and maximize $\Lambda(f)$ for each scenario. Accordingly, we next (i) maximize $\Lambda_1(f)$ subject to $c_I \leq f \leq \hat{f}$, and find the optimal solution $f_1^* \equiv \arg\max(\Lambda_1(f), c_I \leq f \leq \hat{f})$ and calculate $\Lambda_1(f_1^*)$; (ii) maximize $\Lambda_2(f)$ subject to $\hat{f} \leq f \leq \kappa$, and find the optimal solution $f_2^* \equiv \arg\max(\Lambda_2(f), \hat{f} \leq f \leq \kappa)$ and calculate $\Lambda_2(f_2^*)$; and (iii) compare $\Lambda_1(f_1^*)$ and $\Lambda_2(f_2^*)$ to find the NBS-based interchange fee rate f^*.

The NBS Solution under the Constraint that $c_I \leq f \leq \hat{f}$ We now maximize the function $\Lambda_1(f)$ in (17) under the constraint that $c_I \leq f \leq \hat{f}$, and find the corresponding NBS-based interchange fee rate f_1^*. We begin by analyzing the property of the function $\Lambda_1(f)$, which is given in the following lemma.

Lemma 1 The function $\Lambda_1(f)$ in (17) is strictly log-concave in the interchange fee rate f; that is, $\ln[\Lambda_1(f)]$ is a strictly concave function of f. ◀

Proof. see Appendix C. ■

The log-concavity of the function $\Lambda_1(f)$ in (17) implies that $\Lambda_1(f)$ is a quasi-concave (unimodal) function of f with a *unique* maximizing value $\tilde{f}_1 \equiv \arg\max \Lambda_1(f)$. Next, we determine the optimal interchange fee rate f_1^* by maximizing $\Lambda_1(f)$ subject to $c_I \leq f \leq \hat{f}$.

Theorem 3 The optimal interchange fee rate f_1^*—that maximizes $\Lambda_1(f)$ in (17) subject to $c_I \leq f \leq \hat{f}$—is uniquely found as $f_1^* = \min(\tilde{f}_1, \hat{f})$, where $\tilde{f}_1$ is a unique solution that satisfies the following equation,

$$\begin{aligned}
&\left\{[d_1(f_1^*) - 2f_1^* - c_A] + c_I + (f_1^* - c_I)\frac{\partial[d_1(f_1^*)]}{\partial f}\right\}\left[\omega_1 - \frac{\omega_2}{[1 - d_1(f_1^*)]^2}\right] \\
= \; & 4(f_1^* - c_I)[d_1(f_1^*) - f_1^* - c_A]\left[\frac{\omega_2}{(1 - d_1(f_1^*))^3} \times \frac{\partial[d_1(f_1^*)]}{\partial f}\right], \qquad (18)
\end{aligned}$$

where $\partial[d_1(f_1^*)]/\partial f = 2\omega_2/\{3[1 - d_1(f_1^*)]^2\omega_1 + \omega_2\}$. ◄

The NBS Solution under the Constraint that $\hat{f} \leq f \leq \kappa$ When $\hat{f} \leq f \leq \kappa$, then the optimal merchant discount rate is κ, the corresponding transaction amount is $[\omega_1 - \omega_2/(1-\kappa)^2]$, and the NBS function is $\Lambda_2(f)$ as given in (17). We next maximize $\Lambda_2(f)$ subject to $\hat{f} \leq f \leq \kappa$, to find the optimal interchange fee rate f_2^*.

Theorem 4 The optimal interchange fee rate f_2^* is uniquely determined as,

$$f_2^* = \begin{cases} \tilde{f}_2 \equiv (\kappa - c_A + c_I)/2, & \text{if } r_1 \leq r_2, \\ \hat{f}, & \text{if } r_1 \geq r_2, \end{cases} \qquad (19)$$

where $\hat{f}$ is given as in (12); and,

$$r_1 \equiv (1 - c_A - c_I)/(1-\kappa)^3 \quad \text{and} \quad r_2 \equiv \omega_1/\omega_2. \quad ◄ \qquad (20)$$

The NBS-Based Interchange Fee Rate In order to find the interchange fee rate f^* maximizing $\Lambda(f)$ in (17), we need to compare $\Lambda_1(f_1^*)$ and $\Lambda_2(f_2^*)$, where

f_1^* and f_2^* are given as in Theorems 3 and 4, respectively.

Theorem 5 When the acquirer and the issuer bargain over the interchange fee rate, we find that the NBS-characterized rate f^* is uniquely determined as follows:

1. If $r_1 \leq r_2$ where r_i $(i = 1, 2)$ are given as in (20), then we obtain f^* as,

$$f^* = \begin{cases} f_1^*, & \text{if } \Lambda_1(f_1^*) \geq \Lambda_2(\tilde{f}_2), \\ \tilde{f}_2, & \text{if } \Lambda_1(f_1^*) \leq \Lambda_2(\tilde{f}_2), \end{cases} \tag{21}$$

   where $\tilde{f}_2$ is given as in (19).

2. If $r_1 \geq r_2$, then the interchange fee rate f^* is obtained as f_1^*, which falls in the range $[c_I, \hat{f}]$. ◀

As the above theorem indicates, the negotiated interchange-fee rate may be greater than or may be smaller than $\hat{f}$, which depends on the values of r_1 and r_2. We learn from (20) that, if the acquirer's and/or the issuer's per dollar operation costs (i.e., c_A and c_I) are sufficiently small such that $r_1 \geq r_2$, then the two players would determine an interchange fee rate smaller than or equal to $\hat{f}$, that is, $f_1^* \in [c_I, \hat{f}]$. Otherwise, the two players may choose a high rate in the range $[\hat{f}, \kappa]$. The result happens simply because of the following reason: If the issuer incurs a high operation cost, then the player attempts to charge a high interchange fee rate to offset its cost; if the acquirer absorbs a high operation cost, then the acquirer may increase the merchant discount rate; this raises the acquirer's profit margin (i.e., the merchant discount rate minus the interchange fee rate) if the interchange fee rate is unchanged. However, by observing this, the issuer would accordingly bargain with the acquirer to increase its interchange fee rate.

3.2.2 Stackelberg Equilibrium

After the acquirer and the issuer determine their NBS-characterized interchange fee rate f^* given in Theorem 5, the acquirer and the merchant behave in a leader-follower game in which the acquirer determines the merchant discount rate and announces it to the merchant who then make the retail pricing decision. In Section 3.1, we obtain the $f-$dependent Stackelberg solution $(d^*(f), \mathbf{p}^*(f))$, for a given value of f. Substituting the NBS-based rate f^* (given in Theorem 5) into $(d^*(f), \mathbf{p}^*(f))$, we have the Stackelberg equilibrium $(d^S, \mathbf{p}^S)$—where $\mathbf{p}^S \equiv (p_1^S, \ldots p_n^S)$ with p_i^S $(i = 1, \ldots, n)$ denoting the Stackelberg retail price for the ith product—as,

$$(d^S, \mathbf{p}^S) = \begin{cases} (\kappa, \mathbf{p}^*(\tilde{f}_2)), & \text{if } r_1 \leq r_2 \text{ and } \Lambda_1(f_1^*) \leq \Lambda_2(\tilde{f}_2), \\ (d^*(f_1^*), \mathbf{p}^*(f_1^*)), & \text{otherwise,} \end{cases} \tag{22}$$

where $d^*(f)$ and $\mathbf{p}^*(f)$ are given in Section 3.1.2; and f_1^* and $\tilde{f}_2$ are obtained as given in Theorems 3 and 4.

Remark 2 We find from the above that the negotiated interchange fee rate and the Stackelberg merchant discount are $\tilde{f}_2$ and κ, respectively, if $r_1 \leq r_2$ and $\Lambda_1(f_1^*) \leq \Lambda_2(\tilde{f}_2)$; and they are f_1^* and $d^*(f_1^*)$, otherwise. Hence, the acquirer and the issuer should both obtain positive profits. However, we note from (16) that π_{I_2} is strictly increasing in f. That is, if the issuer determines the interchange fee rate by maximizing its own profit rather than by bargaining with the acquirer, then the optimal rate should be either a value maximizing π_{I1} in the range $[c_I, \hat{f}]$ or κ. Note that, if $f = \kappa$, then the acquirer's merchant discount rate d is also equal to κ, because the acquirer should determine its discount rate such that $d \leq \kappa$, as discussed in Section 3.1.1. It thus follows that, if the interchange fee rate is determined by the issuer itself rather than by the negotiation, then the acquirer's profit would possibly be zero. This means that the negotiation may help increase the acquirer's profit and keep it to stay in the credit card network. $\triangleleft$

To illustrate our above analysis, we next provide three examples which correspond to the single-product, two-product, and three-product cases, in which the merchant serves the market with one, two and three products, respectively.

Example 1 We first consider a simple, single-product case, where the merchant's unit acquisition cost of the product is $c_1 = \$100$ per unit, and the parameters in the demand function in (1) are assumed to be $\alpha_1 = 80$ and $\beta_1 = 0.7$. We also assume that the issuer's and the acquirer's operation costs per dollar are \$0.006 and \$0.005, i.e., $c_I = 0.006$ and $c_A = 0.005$. According to Theorem 5, we find that the NBS-characterized interchange fee rate is $f^* = 3.57\%$. Then, using (22), we obtain the Stackelberg merchant discount rate and retail price for the product as $d^S = 8.43\%$ and $p^S = \$111.75$, respectively.

We then consider a two-product case in which the parameters for product 1 are the same as the above for the single-product case, and the parameters for product 2 are specified as follows: $\alpha_2 = 85$, $\beta_2 = 0.70$, and $c_2 = 99$. Similarly, we find the NBS interchange fee rate as $f^* = 4.43\%$, the Stackelberg merchant discount rate as $d^S = 10.51\%$, and the Stackelberg retail prices for products 1 and 2 as $p_1 = \$113.01$ and $p_2 = \$116.03$, respectively.

Next, we analyze a three-product case in which the merchant sells three products—that is, products 1, 2, and 3. Assuming that the parameter values for products 1 and 2 are the same as those in the above two-product case, we set the parameter value for product 3 as follows: $\alpha_3 = 78$, $\beta_3 = 0.65$, and $c_3 = 102$. The issuer's and the acquirer's operation costs per dollar are the same as those for the single- and two-product cases. We compute the NBS-based interchange fee rate as $f^* = 4.37\%$, the Stackelberg merchant discount rate as $d^S = 10.37\%$, and the Stackelberg retail prices of the three products as $p_1 = \$112.93$, $p_2 = \$115.94$, and $p_3 = \$116.90$, respectively. ♦

4 The Cooperative Game Analysis

We learn from our discussion in Section 1 that, in practice, the merchant may bargain with the acquirer over the merchant discount rate; see, e.g., MasterCard

[23]. Moreover, if the merchant has a sufficiently strong power, then it may also intend to negotiate the interchange fee rate with the issuer, because, as discussed in Section 3, the interchange fee rate affects the merchant discount rate which then impacts the merchant's retail pricing decision; for details, see Akers et al. [1]. As discussed in Section 1, the Senate and House of Representatives of the United States of America, which then enacted the bill "Credit Card Fair Fee Act of 2009 (H.R. 2695)" [15] in June 2009. This legislation allows merchants to collectively *bargain* with banks (i.e., the acquirers and issuers) over the cost of certain credit card fees (i.e., the interchange fee rate and the merchant discount rate), and ultimately reduce the costs (i.e., retail prices) of everyday goods for consumers. For more discussions on the bill, see, e.g., Zywicki [46] .

Motived by the above fact, we now allow the merchant to negotiate with the acquirer and the issuer for the interchange fee rate and the merchant discount rate. This is different from Section 3 in which the merchant cannot bargain with other players but only accept the rates and respond by determining its retail prices. Thus, in order to determine the interchange fee rate and the merchant discount rate, we need to develop a three-player cooperative game that characterizes the bargaining among the issuer, the acquirer, and the merchant. In the theory of Cooperative Games with side payments, which was introduced by von Neumann and Morgenstern who assumed that (i) players can communicate with each other and form coalitions with other players, and (ii) players can make side payments to other players; for more discussion, see, for example, Straffin [35].

The cooperative game theory with $n \geq 3$ players concerns the allocation of profit surplus or cost savings among the n players who cooperate to achieve the surplus or savings. For our credit card network involving a merchant, an acquirer, and an issuer, we find that, when the three firms cooperate for the credit card business, they can jointly achieve the profit generated by the business. But, two critical questions arise as follows: how can the profit be divided among the three firms in a fair manner? and how can the three firms implement a fair allocation scheme? To address the former question, we shall construct a three-player cooperative game and solve it for a fair allocation scheme. Since the merchant pays the merchant discount fee to the acquirer who pays the interchange fee to the issuer, we find that an interchange fee rate and a merchant discount rate corresponds to an allocation scheme. For example, a higher value of the interchange fee rate results in more allocation to the issuer; similarly, increasing the merchant discount rate may raise the allocation to the acquirer. That is, given specific values of the two rates, we can compute the corresponding allocation scheme.

Next, we provide an approach to find the rates assuring that the grand coalition is stable; for similar approaches, see, e.g., Petrosjan and Zaccour [26], Sexton [30], etc. Specifically, in order to find the negotiated rates for the credit card network, we should consider the following two steps:

Step 1: Construct a cooperative game given the rates in the grand coalition. In this step, we construct a three-player cooperative game with the characteristic values in terms of the interchange fee rate and the merchant discount rate in the grand coalition, which are assumed to be given.

Step 2: Search for the rates that assure the stability of the grand coalition. We solve the cooperative game to find a fair allocation scheme and the corresponding rates, assuring that the grand coalition is stable. We first derive the conditions for the non-empty core, in which each point represents a fair allocation scheme. In order to find a *unique* solution, we then compute Shapley value and the corresponding rates for the cooperative game. Note that (i) Shapley value-based rates may not satisfy the conditions that assure the non-emptiness of the core, and (ii) Shapley value may not be in the core even if the core is non-empty. Thus, we need to examine whether or not Shapley value can be used to find the rates that result in the stability of the grand coalition. If Shapley value cannot assure that the grand coalition is stable, then we consider another important—and common—solution concept of "nucleolus," which must exist in a non-empty core. We shall develop an algorithm to find the rates satisfying the conditions for the non-emptiness of the core.

Following the above two steps, we next develop a three-player cooperative game, and derive an interchange fee rate and a merchant discount rate that result in a fair allocation scheme assuring the stability of the grand coalition.

4.1 The Three-Player Cooperative Game Given the Rates in the Grand Coalition

To construct a three-player cooperative game where any two or all of three credit card members (i.e., the issuer, the acquirer, and the merchant) can negotiate with each other and form various coalitions, we should compute the characteristic values of all possible coalitions. Note that the characteristic value of a coalition is the *minimum* profit that all members in the coalition can jointly achieve by using their own efforts only; see von Neumann and Morgenstern [41] . For our credit card network, the characteristic value of a coalition is defined as the total profit that all players in the coalition could jointly achieve when they are involved in the credit card business. All possible coalitions for our three-player cooperative game involves (i) the empty coalition $\{\varnothing\}$, which means that no player considers the credit card business; (ii) three single-player coalitions $\{i\}$ $(i = M, A, I)$, which mean that only the firm i is involved in the business; (iii) three two-player coalitions $\{ij\}$ $(i, j = M, A, I,\ i \neq j)$, which represent the coalitions each involving the two firms i and j for the credit card business; and (iv) the three-player (grand) coalition $\{MAI\}$ in which all of three players cooperate to implement the credit card business.

According to the above, our three-player cooperative game in the characteristic-function form includes the following characteristic values: $v(\varnothing)$; $v(M)$, $v(A)$, $v(I)$; $v(MA)$, $v(MI)$, $v(AI)$; and $v(MAI)$. Next, we calculate these values to specify our cooperative game, and solve it to find a fair allocation scheme. Since the profit generated by the empty coalition $\varnothing$ is naturally zero, we have $v(\varnothing) = 0$. We also note from the practice that, as usual, the acquirer and the issuer—which are two banks—cannot participate in the retailing business and sell any products to consumers. Since the acquirer and the issuer profit from the merchant discount fee and the interchange fee, respectively, their profits are both zero when the merchant doesn't accept the credit card payments of its customers. Therefore, if the merchant doesn't join the credit card network, then either the acquirer or the issuer or both of them cannot achieve any profit from the credit card business; that is, $v(A) = v(I) = v(AI) = 0$.

However, we find that any coalition involving the merchant may gain the profit induced by customers' purchases with their credit cards. More specifically, in the coalition $\{M\}$, the merchant (e.g., JUSCO) itself issues its store credit cards (e.g., the AEON credit card by JUSCO), and thus acts as the "acquirer" and "issuer" besides operating normally as a retailing system. In addition, the merchant may only cooperate with the issuer to form the two-player coalition $\{MI\}$ for the credit card business; for example, as an issuer, the Citibank issues the store-labeled credit card for the Sears, which is one of the largest retailer in the North America. Similarly, the merchant may form the two-player coalition $\{MA\}$ with the acquirer for the credit card business. For example, the General Electric Money Bank is the bank of the Wal-Mart, and is thus considered as the acquirer, who also issues the Wal-Mart Discover credit card to individual customers. This means that the bank does not only play the role of the acquirer but also acts as the issuer. The most common credit card network in reality is that involving a merchant, an acquirer, and an issuer. That is, the grand coalition $\{MAI\}$ is a common form adopted by banks and retailers to implement their credit card business. For example, in addition to the Wal-Mart Discover credit card, the Wal-Mart also accepts other credit cards (e.g., Visa, MasterCard, etc.) issued by other banks such as the Citibank. For a further discussion on the above, see, e.g., Akers et al. [1] that considered three networks including (i) $\{M\}$; (ii) $\{MA\}$ and $\{MI\}$; and (iii) $\{MAI\}$.

Next, we compute the characteristic values $v(M)$, $v(MA)$, $v(MI)$, and $v(MAI)$, in order to develop our three-player cooperative game model.

4.1.1 The Characteristic Value $v(M)$

We now calculate the characteristic value of the coalition $\{M\}$ where the merchant itself operates the credit card business by also acting as an "issuer" and an "acquirer." As discussed previously, this case may happen in reality. For example, the Japan United Stores Company (JUSCO) is a well-known chain of "general merchandise stores" in Japan; and the various JUSCO companies are subsidiaries of AEON Japan Group. In 1981, the AEON group established the AEON Credit Japan, which has become one of the largest credit card issuers in Japan. For over 12 years, the AEON Credit Japan and the JUSCO companies have jointly launched a variety of co-branded credit cards that are labeled with "AEON", "JUSCO", and a credit card company/network (e.g., Visa, MasterCard, American Express, etc.) which the AEON Credit Japan and JUSCO companies join. Note that, even though the AEON and JUSCO join credit card networks such as Visa and MasterCard, they still don't cooperate with any external acquirers and issuers but process credit card transactions by themselves. This helps attract more customers to buy at the JUSCO stores. For more information regarding the AEON JUSCO credit cards, see, e.g., a report by the Kenanga Research company [17].

As the above indicates, a merchant (e.g., JUSCO) may operate its own credit card service, which means that the merchant also acts as an issuer and an acquirer. To calculate $v(M)$ for our three-player cooperative game, we should observe the merchant's operation cost induced by taking the roles of the issuer and the acquirer. Recall from Section 3 that an external acquirer and an external issuer incur the per dollar operation costs c_A and c_I, respectively, when they process credit card transactions. We assume that, when the merchant operates the business by itself, it absorbs the per dollar operation cost δc_A for its "acquirer" role and the per dollar cost γc_I for its "issuer" role. It is also assumed that the parameters δ and γ are both greater than or equal to 1; this is justified as follows: In practice, acquirers and issuers are usually financial banks that are specialized with the operational process of credit card transactions. Moreover, those banks process a very large transaction amount, which can reduce their operation costs because of the economics of scales. Thus, acquirers and issuers should be able to process the transactions more efficiently than the merchant operating its own credit card program; and it is reasonable to assume that $\delta, \gamma \geq 1$.

Next, we compute the merchant's maximum profit $v(M)$ in the coalition $\{M\}$, where the merchant operates the credit card program by itself and thus doesn't need to consider the merchant discount fee and the interchange fee. Since, as discussed above, the merchant absorbs the per dollar operation cost $\delta c_A + \gamma c_I$ (which happens because the merchant takes the roles of the acquirer and the issuer), the merchant can attain the profit $(1 - \delta c_A - \gamma c_I)$ from an one-dollar transaction. The merchant sells n products with the retail price $\mathbf{p} \equiv (p_1, \ldots, p_n)$ to customers whose aggregate (price-dependent) demand for product i is $q_i(p_i) = \alpha_i - \beta_i p_i$, as given in (1). Hence, the merchant's profit from the credit card transaction is computed as $(1-\delta c_A-\gamma c_I) \sum_{i=1}^{n} [p_i \times q_i(p_i)]$. Because the merchant's unit acquisition cost is c_i for product i, its total acquisition cost is $\sum_{i=1}^{n} [c_i \times q_i(p_i)]$.

According to the above, we calculate the merchant's net profit for the coalition $\{M\}$ as $\Pi_{M_1}(\mathbf{p}) = \sum_{i=1}^{n} \{[(1 - \delta c_A - \gamma c_I) \times p_i - c_i] \times q_i(p_i)\}$, which, using (1), can be re-written as,

$$\Pi_{M_1}(\mathbf{p}) = \sum_{i=1}^{n} \{[(1 - \delta c_A - \gamma c_I)p_i - c_i] \times (\alpha_i - \beta_i p_i)\}. \tag{23}$$

In order to assure the non-negativity of $\Pi_{M_1}(\mathbf{p})$ in (23), we assume that $1 \geq \delta c_A + \gamma c_I$ and the merchant determines its prices such that $\alpha_i/\beta_i \geq p_i \geq c_i/(1 - \delta c_A - \gamma c_I)$, for $i = 1, \ldots, n$. Next, we maximize $\Pi_{M_1}(\mathbf{p})$ in (23) under the constraint that $\alpha_i/\beta_i \geq p_i \geq c_i/(1 - \delta c_A - \gamma c_I)$ to find the merchant's optimal pricing decision for the case that the merchant doesn't cooperate with other firms but joins the single-player coalition $\{M\}$.

Theorem 6 In the coalition $\{M\}$, the merchant's optimal pricing decision is $\mathbf{p}^{M_1} = (p_1^{M_1}, \ldots, p_n^{M_1})$, where $p_i^{M_1}$ $(i = 1, \ldots, n)$ is the optimal retail price for product i and can be obtained as a unique solution of the following equation:

$$p_i^{M_1} = \frac{1}{2}\left(\frac{\alpha_i}{\beta_i} + \frac{c_i}{1 - \delta c_A - \gamma c_I}\right), \quad \text{for } i = 1, \ldots, n. \quad \blacktriangleleft \tag{24}$$

Comparing $p_i(d)$ in Theorem 1 with $p_i^{M_1}$ in the above theorem, we find that the merchant's optimal retail price $p_i^{M_1}$ ($i = 1, \ldots, n$) in the coalition $\{M\}$ is *greater* than the Stackelberg equilibrium price p_i^S for our two-stage game problem in Section 3, if the merchant's credit card operation cost $\delta c_A + \gamma c_I$ is *less* than the Stackelberg equilibrium merchant discount fee d^S that is regarded as the "cost" of the merchant in the two-stage game setting. That is, when $\delta c_A + \gamma c_I < d^S$, the merchant should have an incentive to undertake the credit card business instead of "outsourcing" such a business to the acquirer and the issuer and being thus engaged in the two-stage game problem. Since $c_A + c_I < d^S$, as discussed in Section 3, the parameters δ and γ play an important role in affecting the merchant's willingness to hold the credit card business. Specifically, if the merchant is sufficiently efficient in the credit card business, then both δ and γ should be small enough to assure that $\delta c_A + \gamma c_I < d^S$, and the merchant should be inclined to operate the credit card business rather than to subcontract the financial service out to the acquirer and the issuer. Noting that, in practice, the majority of merchants are specialized in the retailing service rather than the financial service, we conclude that *only* the large-scale merchants—which include, e.g., JUSCO, as an example for the coalition $\{M\}$—*may* consider the credit card business, whereas the other merchants (especially, small-scale merchants) may have to rely on the acquirer and the issuer as in the two-stage game setting.

Substituting $p_i^{M_1}$ in (24) into the demand function $q_i(p_i) = \alpha_i - \beta_i p_i$ and simplifying it we have,

$$q_i(p_i^{M_1}) = \frac{1}{2}\left(\alpha_i - \frac{\beta_i c_i}{1 - \delta c_A - \gamma c_I}\right), \quad \text{for } i = 1, \ldots, n. \tag{25}$$

Next, we compute the characteristic value of the coalition $\{M\}$. Letting $\mathbf{p}^{M_1} \equiv$

$(p_1^{M_1}, \ldots, p_n^{M_1})$ and using the optimal retail price $p_i^{M_1}$ in (24) and $q_i(p_i^{M_1})$ in (25), we find that

$$v(M) = \Pi_{M_1}(\mathbf{p}^{M_1}) = \frac{1}{4}\sum\nolimits_{i=1}^{n} \frac{[\alpha_i(1-\delta c_A - \gamma c_I) - \beta_i c_i]^2}{\beta_i(1-\delta c_A - \gamma c_I)} \geq 0. \qquad (26)$$

4.1.2 The Characteristic Value $v(MI)$

In the coalition $\{MI\}$, the merchant cooperates with the issuer to process consumers' payments with the merchant-labeled credit cards issued by the issuer. For this coalition, the issuer also acts as an "acquirer", since, as the merchant's bank, the issuer now also processes the cardholders' (consumers) bills and directly deposits their payments (excluding the interchange fee) into the merchant's account. For example, the Citibank has issued the Sears credit cards since the Sear sold its retail credit card operations to the Citibank in 2003 [43]. As the Citibank has taken over the Sears's retail credit card operations, the Sears is no more involved in the financial process of any credit card transactions. The Citibank now acts as an issuer as well as an acquirer to transfer relevant transaction amount to the Sears' corporate account at the Citibank. For more discussions and other examples (e.g., Macy's), see DeGennaro [9].

Since a single financial bank takes the roles of the issuer and the acquirer, we do not need to consider the merchant discount rate for the coalition $\{MI\}$ because the issuer charges the merchant (rather than an acquirer) for the interchange fee. Thus, the issuer and the merchant negotiate the interchange fee rate f, and the merchant then makes its retail pricing decisions. The above is actually a two-stage game problem, which is similar to that in Section 3. To solve the problem and calculate the characteristic value $v(MI)$, we next (i) find the best-response retail prices for the merchant, assuming that the interchange fee rate f is given; (ii) develop the merchant's and the issuers' profit functions, construct an NBS model, and calculate the NBS-characterized interchange fee rate f^{MI}; and (iii) substitute f^{MI} into the merchant's best-response function, and then calculate two players' corresponding profit functions and sum them to find the value $v(MI)$.

Merchant's Best-Response Pricing Decision in the Coalition $\{MI\}$ We now find the merchant's best-response pricing decision for a given value of the interchange fee rate f. When the merchant joins the coalition $\{MI\}$, it has the sale revenue $\sum_{i=1}^{n}[p_i \times q_i(p_i)]$ but incurs the interchange fee $f \times \sum_{i=1}^{n}[p_i \times q_i(p_i)]$ and the acquisition cost $\sum_{i=1}^{n}[c_i \times q_i(p_i)]$. Thus, the merchant's net profit for the coalition $\{MI\}$ is calculated as $\Pi_{M_2}(\mathbf{p}) = \sum_{i=1}^{n}\{[(1-f)p_i - c_i] \times q_i(p_i)\}$. Similar to Section 3, we assume that, in order to assure the non-negativity of $\Pi_{M_2}(\mathbf{p})$, the merchant determines its prices such that $p_i \geq c_i/(1-f)$, for $i = 1, \ldots, n$. Recalling from Section 3.1.1 that $p_i \leq \alpha_i/\beta_i$ $(i = 1, \ldots, n)$, we find that the merchant's price p_i $(i = 1, \ldots, n)$ should be in the range $[c_i/(1-f), \alpha_i/\beta_i]$.

Using (1), we re-write the merchant's profit function $\Pi_{M_2}(\mathbf{p})$ as,

$$\Pi_{M_2}(\mathbf{p}) = \sum_{i=1}^{n}\{[(1-f)\alpha_i + \beta_i c_i] \times p_i - (1-f)\beta_i p_i^2 - \alpha_i c_i\}.$$

Next, we maximize the above under the constraint that $c_i/(1-f) \leq p_i \leq \alpha_i/\beta_i$ (for $i = 1, \ldots, n$) to find the merchant's best-response pricing decision.

Theorem 7 Given the interchange fee rate f, the merchant's best-response retail pricing decision is $\mathbf{p}^{M_2}(f) \equiv (p_1^{M_2}(f), \ldots, p_n^{M_2}(f))$, where $p_i^{M_2}(f)$ is the best-response price for product i $(i = 1, \ldots, n)$ and is computed as,

$$p_i^{M_2}(f) = \frac{1}{2}\left[\frac{\alpha_i}{\beta_i} + \frac{c_i}{(1-f)}\right], \quad \text{for } i = 1, \ldots, n. \quad \blacktriangleleft \tag{27}$$

The NBS-Based Interchange Fee Rate for the Coalition $\{MI\}$ Using the best-response retail price $\mathbf{p}^{M_2}(f)$ (as given in Theorem 7), we calculate the merchant's corresponding sale revenue $R(\mathbf{p}^{M_2}(f))$ and profit $\Pi_{M_2}(\mathbf{p}^{M_2}(f))$ as,

$$R(\mathbf{p}^{M_2}(f)) = \sum_{i=1}^{n}\left[p_i^{M_2}(f) \times q_i(p_i^{M_2}(f))\right] = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-f)^2}\right], \tag{28}$$

$$\begin{aligned}\Pi_{M_2}(\mathbf{p}^{M_2}(f)) &= \frac{1}{4}\left[\omega_1(1-f) + \frac{\omega_2}{1-f} - 2\sum_{i=1}^{n}(\alpha_i c_i)\right] \\ &= \frac{1}{4}\sum_{i=1}^{n}\frac{[\alpha_i(1-f) - \beta_i c_i]^2}{\beta_i(1-f)}.\end{aligned} \tag{29}$$

where ω_1 and ω_2 are given as in (7).

Theorem 8 If and only if $f \leq \delta c_A + \gamma c_I$, then $\Pi_{M_2}(\mathbf{p}^{M_2}(f)) \geq v(M)$, where $v(M)$ is given as in (26). ◄

As the above theorem indicates, the merchant should have an incentive to leave the coalition $\{M\}$ for the coalition $\{MI\}$, when the merchant's per dollar operation cost $\delta c_A + \gamma c_I$ is no less than the negotiated interchange fee rate f. This happens because of the following fact: Recall from Section 4.1.1 that, if the merchant operates the credit card business by its own effort rather than with the cooperation of the issuer, then the merchant incurs the per dollar operation cost $\delta c_A + \gamma c_I$. Otherwise, if the merchant joins the coalition $\{MI\}$ to cooperate with the issuer, then it doesn't absorb the cost $\delta c_A + \gamma c_I$ but needs to pay the rate f to the issuer. Therefore, in order to assure the stability of the coalition $\{MI\}$, the negotiated rate f must be smaller than or equal to $\delta c_A + \gamma c_I$.

Next, for the coalition $\{MI\}$, we assume that the issuer incurs a per dollar operation cost c_I for the issuer-related operations and a per dollar operation cost δc_A (where $\delta \geq 1$) for the acquirer-related operations. The parameter δ helps distinguish the issuer acting as an acquirer and the external acquirer specialized in the relevant operations. Similar to Section 3.2.1, the issuer attains the interchange fee $f \times R(\mathbf{p}^{M_2}(f))$ but absorbs the per dollar operation cost $c_I \times R(\mathbf{p}^{M_2}(f))$ (incurred for the issuer-related operations) and also the per dollar operation cost $\delta \times c_A \times R(\mathbf{p}^{M_2}(f))$ (incurred for the acquirer's operations). Hence, the issuer's profit is calculated as,

$$\begin{aligned} \Pi_{I_2}(f) &= (f - c_I - \delta c_A) \times R(\mathbf{p}^{M_2}(f)) \\ &= \frac{1}{4}(f - c_I - \delta c_A) \times \left[\omega_1 - \frac{\omega_2}{(1-f)^2}\right], \end{aligned} \tag{30}$$

which is non-negative if and only if $c_I + \delta c_A \leq f \leq 1 - \sqrt{\omega_2/\omega_1}$, because both $(f - c_I - \delta c_A)$ and $R(\mathbf{p}^{M_2}(f))$ must be non-negative. Therefore, to assure that $\Pi_{I_2}(f) \geq 0$, we assume that $c_I + \delta c_A \leq 1 - \sqrt{\omega_2/\omega_1}$.

We then construct the NBS model to characterize the bargaining process between the issuer and the merchant. Note that, if the merchant doesn't cooperate with the issuer, then it launches the credit card business and gains the profit $v(M)$, as discussed in Section 4.1.1. Therefore, the merchant's security level (status quo point) is $v(M)$. But, if the issuer doesn't join the coalition $\{MI\}$, then its profit $v(I)$ is zero and its security level is zero. Using the above, similar to Section 3.2.1, the NBS model for the coalition $\{MI\}$ is developed as,

$$\begin{array}{rl}\max_f & \Lambda^{MI} = [\Pi_{M_2}(\mathbf{p}^{M_2}(f)) - v(M)] \times \Pi_{I_2}(f) \\ \text{s.t.} & \Pi_{M_2}(\mathbf{p}^{M_2}(f)) \geq v(M) \text{ and } \Pi_{I_2}(f) \geq 0.\end{array} \quad (31)$$

We learn from Theorem 8 that $\Pi_{M_2}(\mathbf{p}^{M_2}(f)) \geq v(M)$ iff $f \leq \delta c_A + \gamma c_I$. We also find from our above discussion that $\Pi_{I_2}(f) \geq 0$ iff $c_I + \delta c_A \leq f \leq 1 - \sqrt{\omega_2/\omega_1}$. Thus, we can re-write the constraints in the maximization problem (31) as, $c_I + \delta c_A \leq f \leq \min(\delta c_A + \gamma c_I, 1 - \sqrt{\omega_2/\omega_1})$; and as a result, our maximization problem can re-written as,

$$\begin{array}{rl}\max_f & \Lambda^{MI} = [\Pi_{M_2}(\mathbf{p}^{M_2}(f)) - v(M)] \times \Pi_{I_2}(f) \\ \text{s.t.} & c_I + \delta c_A \leq f \leq \min(\delta c_A + \gamma c_I, 1 - \sqrt{\omega_2/\omega_1}).\end{array} \quad (32)$$

Theorem 9 There is a unique solution for the maximization problem in (32). That is, for the coalition $\{MI\}$, the NBS-characterized interchange fee rate f^{MI} uniquely exists. ◀

Calculation of the Characteristic Value $v(MI)$ Replacing the rate f in $p_i^{M_2}(f)$ $(i = 1, \ldots, n)$ and $\Pi_{M_2}(\mathbf{p}^{M_2}(f))$ [given by (27) and (28), respectively] with the negotiated rate f^{MI} (given in Theorem 9) and simplifying them, we find the merchant's optimal retail pricing decision and its maximum profit as,

$$\begin{aligned} p_i^{M_2}(f^{MI}) &= \frac{1}{2}\left[\frac{\alpha_i}{\beta_i} + \frac{c_i}{(1-f^{MI})}\right], \text{ for } i = 1, \ldots, n; \\ \Pi_{M_2}(\mathbf{p}^{M_2}(f^{MI})) &= \frac{1}{4}\sum_{i=1}^{n}\left\{\frac{\left[\alpha_i(1-f^{MI}) - \beta_i c_i\right]^2}{\beta_i(1-f^{MI})}\right\}. \end{aligned} \quad (33)$$

Similarly, using f^{MI}, we calculate the issuer's corresponding maximum profit as,

$$\Pi_{I_2}(f^{MI}) = \frac{f^{MI} - c_I - \delta c_A}{4} \left[\omega_1 - \frac{\omega_2}{(1 - f^{MI})^2}\right]. \tag{34}$$

Following along the lines similar to those in Theorem 6, we find that, if the issuer can *efficiently* act as an "acquirer," then the merchant and the issuer *may* decide to operate the credit card business *without* subcontracting the acquirer-related operations out to the acquirer as in the two-stage game setting—that is discussed in Section 3. We compute the characteristic value $v(MI)$ as $\Pi_{M_2}(\mathbf{p}^{M_2}(f^{MI})) + \Pi_{I_2}(f^{MI})$; that is,

$$\begin{aligned} v(MI) &= \frac{1}{2}\sum_{i=1}^{n}\left\{\left(\alpha_i - \frac{\beta_i c_i}{1 - f^{MI}}\right)\left[\frac{1 - c_I - \delta c_A}{2}\left(\frac{\alpha_i}{\beta_i} + \frac{c_i}{1 - f^{MI}}\right) - c_i\right]\right\} \\ &\geq v(M). \end{aligned} \tag{35}$$

4.1.3 The Characteristic Value $v(MA)$

In the coalition $\{MA\}$, the merchant and the acquirer cooperate for the credit card business. This coalition exists in practice. For example, as the bank where Wal-Mart opens its account, the General Electric (GE) Money Bank issues the Wal-Mart Discover credit card, acting as both the acquirer and the issuer. The game for this coalition is described as follows: The merchant and the acquirer first negotiate the merchant discount rate. Then, the merchant makes its own pricing decisions. Similar to the above, we still use the backward induction approach to find the value $v(MA)$.

Given the merchant discount rate d, the merchant's optimal retail prices can be found by maximizing its profit $\Pi_{M_3}(p) = \sum_{i=1}^{n}[(1-d)\times p_i - c_i] \times (\alpha_i - \beta_i p_i)$. We learn from Theorem 1 that the optimal retail price of product i in the coalition $\{MA\}$ is $p_i^{M_3}(d) = [\alpha_i/\beta_i + c_i/(1-d)]/2$, for $i = 1, \ldots, n$. Letting $\mathbf{p}^{M_3}(d) \equiv (p_1^{M_3}(d), \ldots, p_n^{M_3}(d))$, we have,

$$R(d) = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right] \text{ and } \Pi_{M_3}(\mathbf{p}^{M_3}(d)) = \frac{1}{4}\sum_{i=1}^{n}\frac{[(1-d)\alpha_i - \beta_i c_i]^2}{\beta_i(1-d)}. \tag{36}$$

Theorem 10 $\Pi_{M_3}(\mathbf{p}^{M_3}(d)) \geq v(M)$ if and only if $d \leq \delta c_A + \gamma c_I$.

The above theorem indicates that, when the merchant's per dollar operation cost $\delta c_A + \gamma c_I$ is no less than the negotiated merchant discount rate d, the merchant should have an incentive to leave the coalition $\{M\}$ and join the coalition $\{MA\}$. The justification for this theorem is similar to that for Theorem 7. For the coalition $\{MA\}$, the acquirer is assumed to incur a per dollar operation cost c_A for the acquirer-related operations and a per dollar operation cost γc_I (where $\gamma \geq 1$) for the issuer-related operations. The acquirer's profit $\Pi_{A_3}(d)$ is thus calculated as,

$$\Pi_{A_3}(d) = (d - c_A - \gamma c_I) \times R(d) = \frac{d - c_A - \gamma c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right], \qquad (37)$$

which is non-negative iff $c_A + \gamma c_I \leq d \leq 1 - \sqrt{\omega_2/\omega_1}$.

Then, we use the NBS to determine the merchant discount rate as a result of the negotiation between the acquirer and the merchant; that is, we can find the NBS-based solution by maximizing $\Lambda^{MA} = [\Pi_{M_3}(\mathbf{p}^{M_3}(d)) - v(M)] \times \Pi_{A_3}(d)$ subject to $\Pi_{M_3}(\mathbf{p}^{M_3}(d)) \geq v(M)$ and $\Pi_{A_3}(d) \geq 0$, where, using our arguments for the coalition $\{MA\}$, we find that the security levels for the merchant and the acquirer are $v(M)$ and zero, respectively. Using $\Pi_{A_3}(d)$ in (37) and Theorem 10, we can re-write the NBS optimization problem as

$$\begin{aligned} \max_d \quad & \Lambda^{MA} = [\Pi_{M_3}(\mathbf{p}^{M_3}(d)) - v(M)] \times \Pi_{A_3}(d) \\ \text{s.t.} \quad & c_A + \gamma c_I \leq d \leq \min(\delta c_A + \gamma c_I, 1 - \sqrt{\omega_2/\omega_1}). \end{aligned} \qquad (38)$$

Theorem 11 Λ^{MA} is quasi-concave in d; thus, a unique NBS-characterized merchant discount rate d^{MA} must exist for the coalition $\{MA\}$.

Using similar lines as those in Theorem 6, we find that whether or not the acquirer and the merchant are willing to undertake the credit card business by themselves depends on the acquirer's *efficiency* in acting as an "issuer" in the coalition $\{MA\}$. Specifically, only when the negotiated rate d^{MA}

is less than the Stackelberg equilibrium merchant discount fee d^S in the two-stage game setting, the merchant and the acquirer should have an incentive to cooperate in the coalition $\{MA\}$. Substituting d^{MA} into the merchant's and the acquirer's profit functions gives the characteristic value of the coalition $\{MA\}$ as $v(MA) = \Pi_{M_3}(\mathbf{p}^{M_3}(d^{MA})) + \Pi_{A_3}(d^{MA})$; that is,

$$\begin{aligned} v(MA) &= \frac{1}{2}\sum_{i=1}^{n}\left\{\frac{1-c_A-\gamma c_I}{2}\left[\frac{\alpha_i^2}{\beta_i}+\frac{\beta_i c_i^2}{(1-d^{MA})^2}\right]\right. \\ &\quad \left. -\beta_i c_i\left[\frac{\alpha_i}{\beta_i}+\frac{c_i(d^{MA}-c_A-\gamma c_I)}{(1-d^{MA})^2}\right]\right\} \\ &\geq v(M). \end{aligned} \tag{39}$$

4.1.4 The Characteristic Value $v(MAI)$

We now calculate the characteristic value of the grand coalition $\{MAI\}$, where the merchant, the acquirer and the issuer cooperate for the credit card business. This coalition is the most common in practice; for instance, Wal-Mart—who opens its account at the GE Money Bank—accepts the payments by the customers using the credit cards issued by the Citibank. In this example, Wal-Mart, the GE Money Bank and the Citibank are the merchant, the acquirer and the issuer, respectively. After joining the grand coalition, the three firms bargain over both the interchange fee rate f and the merchant discount rate d. Observing the negotiated rates f^{MAI} and d^{MAI}, the merchant determines optimal retail prices $\mathbf{p}^{M_4}(d^{MAI}) = (p_1^{M_4}(d^{MAI}), \ldots, p_n^{M_4}(d^{MAI}))$.

One may note that the grand coalition $\{MAI\}$ is similar to the two-stage game problem that we analyze in Section 3. Even though, in the coalition $\{MAI\}$, the three firms also cooperate to operate the credit card business, we consider a different approach to determine the rates f and d. More specifically, in our two-stage game problem, f is negotiated only by the acquirer and the issuer, and d is determined by the acquirer, whereas, in the coalition $\{MAI\}$, both f and d are dependent on the negotiation among the merchant, the acquirer, and the issuer.

Similar to Sections 4.1.1–4.1.3, we first calculate the merchant's best-response pricing decision $\mathbf{p}^{M_4}(d)$ for any given values of f and d, which is obtained by solving the following constrained maximization problem:

$$\max_{p_i,\ i=1,\ldots,n} \Pi_{M_4}(\mathbf{p}) = \sum\nolimits_{i=1}^{n} [(1-d)p_i - c_i] \times q_i(p_i), \quad \text{s.t., } c_i/(1-d) \leq p_i \leq \alpha_i/\beta_i.$$

Using Theorem 1, we find that $\mathbf{p}^{M_4}(d) = (p_1^{M_4}(d), \ldots, p_n^{M_4}(d))$ where $p_i^{M_4}(d) = [\alpha_i/\beta_i + c_i/(1-d)]/2$, for $i = 1, \ldots, n$.

We learn from Section 3 that the merchant's, the acquirer's, and the issuer's profit functions are,

$$\pi_{M_4}(d) = (1-d) \times R(d) - \sum\nolimits_{i=1}^{n} [c_i \times q_i(p_i^{M_4}(d))],$$
$$\pi_{A_4}(d) = (d - f - c_A) \times R(d) \quad \text{and} \quad \pi_{I_4}(f) = (f - c_I) \times R(d),$$

where $R(d) = \sum_{i=1}^{n} [p_i^{M_4}(d) \times q_i(p_i^{M_4}(d))]$ is the sale revenue realized by the merchant. Thus, the characteristic value $v(MAI)$—which is defined as the three firms' total profit—is computed as,

$$\begin{aligned} v(MAI) &= \pi_{M_4}(d) + \pi_{A_4}(d) + \pi_{I_4}(f) \\ &= \sum\nolimits_{i=1}^{n} \left(\alpha_i - \frac{\beta_i c_i}{1-d}\right) \left[\frac{1 - c_A - c_I}{4} \left(\frac{\alpha_i}{\beta_i} + \frac{c_i}{1-d}\right) - \frac{c_i}{2}\right]. \quad (40) \end{aligned}$$

For the grand coalition $\{MAI\}$, the interchange fee rate and the merchant discount rate are negotiated by the merchant, the acquirer, and the issuer. Thus, even though $v(MAI)$ [as given in (40)] is dependent on merchant discount rate d, we cannot maximize $v(MAI)$ to find the merchant discount rate but have to calculate it by solving a three-player cooperative game model that characterizes the bargaining process arising among the three firms. Next, we use the theory of cooperative games to find the rates d and f for the grand coalition $\{MAI\}$.

4.2 The Analysis of the Three-Player Cooperative Game for the Rates Assuring the Stability of the Grand Coalition

For our credit card network, we have developed the three-player cooperative game $\mathcal{G} = (N, v)$ where $N = \{M, A, I\}$ and the characteristic values of all possible coalitions are computed as follows: $v(\varnothing) = v(A) = v(I) = v(AI) = 0$, and $v(M)$, $v(MI)$, $v(MA)$, and $v(MAI)$ are given as in (26), (35), (39), and (40), respectively. In this section, we solve this game to obtain a unique allocation scheme and compute the corresponding interchange fee rate and merchant discount rate. Note that, in this thesis, the merchant, the acquirer and the issuer implement the allocation scheme by determining the interchange fee rate and merchant discount rate. Prior to finding an allocation scheme, we next discuss whether or not our cooperative game in the characteristic-function form is *superadditive* and *convex.* The cooperative game $\mathcal{G}$ with 3 players is superadditive if $v(C_1 \cup C_2) \geq v(C_1) + v(C_2)$ for any two *disjoint* coalitions C_1 and C_2 in the three-player game; for details, see, for example, Straffin [35]. Moreover, the game is convex if it is supermodular. According to Shapley [32], we find that our three-player cooperative game's characteristic function is supermodular if $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$, for all S, $T \subseteq N$. Note that a cooperative game is convex and also superadditive if its characteristic function is *supermodular*; for more information, see, e.g., Driessen [10], Topkis [37], etc. Next, we examine the *supermodularity* of the characteristic function for our game.

4.2.1 Supermodularity and the Core

We now derive the sufficient conditions under which our three-player cooperative game is supermodular, and also find the sufficient conditions that assure the existence of a non-empty core for our game.

Theorem 12 If the merchant discount rate d is negotiated such that $v(MAI)+$

$v(M) \geq v(MA) + v(MI)$, then our three-player cooperative game is supermodular. ◀

In the sufficient condition given by the above theorem, $v(MI) - v(M)$ means the "additional" profit generated when the issuer participates in the credit card business by undertaking both the issuer- and the acquirer-related operations. Moreover, $v(MAI) - v(MA)$ represents the "additional" profit generated by the participation of the issuer who only operates the issuer-related business while the acquirer exists in the credit card network. That is, the sufficient condition in Theorem 12 implies that the issuer, the acquirer, and the merchant can jointly achieve a higher profit from the credit card business (i.e., the game is supermodular and thus superadditive), if the issuer's "contribution" is *greater* when the acquirer serves the credit card network than when the acquirer is not involved in such a business. Furthermore, one may note that the sufficient condition that $v(MAI) - v(MA) \geq v(MI) - v(M)$ can be equivalently re-written as $v(MAI) - v(MI) \geq v(MA) - v(M)$. Using our above argument, we can also conclude that the three firms can enjoy a *higher* system-wide profit, if the acquirer's contribution is *greater* when the issuer exists in the credit card network than when the issuer is not involved.

In addition to the above, we find that the merchant should be *always* involved in the credit card business, even though both the issuer and the acquirer are important to the profitability—and efficiency—of the credit card network. The reason is given as follows: if the merchant does not join the network, then both the issuer and the acquirer cannot gain any profit from the credit card business since they, as two financial firms, could not play as the role of an "merchant" to realize the sale revenue in the retailing market.

Remark 3 We find that the merchant's participation in the credit card network is *significantly* important to the success of the network. Moreover, the issuer and the acquirer may need to have *sufficient* expertise in the issuer- and the acquirer-related business, respectively, in order to make the network-wide profit higher than when the merchant operates the credit card business by itself or delegates the business to either the issuer or the acquirer. We also learn from Theorem 12 that the issuer and the acquirer should both operate in the credit card network, in order to improve the profitability of the network. This implies that the popularity of the credit card service in the financial market could be important to the success of the recent U.S. bill "Credit Card Fair Fee Act of 2009 (H.R. 2695)," in which the merchant is encouraged to participate in the credit card network for the negotiation of the interchange fee rate and the merchant discount rate. ◄

When our cooperative game is supermodular, it has a non-empty core where there exist a set of fair allocation schemes each assuring the stability of the grand coalition $\{MAI\}$ [12]. Letting y_i denote the profit allocated to firm i, $i \in N = \{M, A, I\}$, we can call a proper allocation scheme $y \equiv (y_M, y_A, y_I)$—where $y_M = \Pi_{M_4}(d)$, $y_A = \Pi_{A_4}(d)$, and $y_I = \Pi_{I_4}(f)$—an imputation for our game $\mathcal{G} = (N, v(.))$, if the scheme satisfies the following two properties: (i) *individual rationality*, i.e., $y_i \geq v(i)$, for all $i \in N$; and (ii) *collective rationality*, i.e., $\sum_{i \in N} y_i = v(MAI)$ [35]. The core is the set of all *undominated imputations* (fair allocation schemes) (y_M, y_A, y_I) such that $\sum_{i \in T} y_i \geq v(T)$ for all coalitions $T \subseteq N = \{M, A, I\}$.

Even though a supermodular game must have a non-empty core, we cannot conclude that the core is empty for the non-supermodular game. In the following theorem, we provide necessary and sufficient conditions under which our cooperative game has a non-empty core.

Theorem 13 The core of our cooperative game is non-empty if and only if $f^{MI} - \delta c_A \leq f$ and $f + d^{MA} - \gamma c_I \leq d \leq \min(d^{MA}, f^{MI})$.

The above theorem implies that, as the merchant joins the grand coalition $\{MAI\}$ and bargains with the acquirer and the issuer over the merchant discount rate and the interchange fee rate, the negotiated interchange fee rate and merchant discount rate are no more than d^{MA} and f^{MI}, respectively. That is, the two rates are reduced as a result of the negotiation among the issuer, the acquirer, and the merchant.

As Theorem 13 indicates, the interchange fee rate f must be greater than or equal to $f^{MI} - \delta c_A$. We learn from Section 4.1.2 that f^{MI} is the rate paid by the merchant to the issuer in the coalition $\{MI\}$, where the issuer absorbs the unit cost δc_A in undertaking the acquirer-related operations. This implies that $f^{MI} - \delta c_A$ represents the issuer's *unit* "net gain" resulting from its own (issuer-related) operations. Thus, the condition that $f^{MI} - \delta c_A \leq f$ assures that the issuer is willing to join the grand coalition $\{MAI\}$. Similarly, in the two-player coalition $\{MA\}$, the acquirer's *unit* "net gain" resulting from its own (acquirer-related) operations is $d^{MA} - \gamma c_I$. To assure that the acquirer has an incentive to join the grand coalition $\{MAI\}$, we should determine the merchant discount rate d such that the acquirer's unit net gain $(d-f)$ in $\{MAI\}$ is no less than $d^{MA}-\gamma c_I$, i.e., $f+d^{MA}-\gamma c_I \leq d$. In addition, we find that the merchant pays the rates d^{MA} and f^{MI} in the two-player coalitions $\{MA\}$ and $\{MI\}$, respectively. In order to entice the merchant to stay in the grand coalition, we should choose the merchant discount rate that is lower than both d^{MA} and f^{MI}, i.e., $d \leq \min(d^{MA}, f^{MI})$. According to the above discussion, we draw the following insights.

Remark 4 The grand coalition is more likely to be stable, if each player (i.e., the merchant, the acquirer, or the issuer) is larger and more specialized in its *own* operation for the credit card business. The justification is given as follows: Theorem 13 implies that, in the credit card network, all of the three players are willing to join the grand coalition, if and only if the deviation of any one player from the grand coalition shall make the remaining two players worse off. That is, if each two-player coalition can achieve a *significantly* high profit (characteristic value), then the grand coalition is *unlikely* to be stable. Recall that the profit of the coalition $\{AI\}$ is zero. From Sections 4.1.2 and 4.1.3, we learn that the profits of the coalitions $\{MI\}$ and $\{MA\}$ *could* be small if the values of the scale parameters $\delta \geq 1$ and $\gamma \geq 1$ are significantly high, respectively. Therefore, in the credit card network with the high values of δ and γ, neither the two-player coalition $\{MI\}$ nor $\{MA\}$ would be stable and instead, the grand coalition is likely to be stable. Note that the scale parameter δ reflects the operation efficiency of the issuer or the merchant in acting as an "acquirer;" and, the scale parameter γ reflects the operation efficiency of the acquirer or the merchant in acting as an "issuer." That is, if the merchant, the acquirer, and the issuer are larger and more specialized in their own operations for the credit card business, then it should be more costly for the issuer and the merchant (the acquirer and the merchant) to take the role of an "acquirer" ("issuer"), the value of δ (γ) is higher, and the grand coalition is more likely to be stable. ◄

The above remark implies that the recent U.S. bill "Credit Card Fair Fee Act of 2009 (H.R. 2695)" should be more effective, if, in credit card networks, the banks and the merchants are larger and more specialized in their own operations. This result could be justified by the fact that, in the United States, the participants in the negotiation of the merchant discount rate and the interchange fee rate are usually the large, highly-specialized banks (e.g., Citibank) and merchants (e.g., Wal-Mart). In fact, many small merchants in practice may be unwilling or against to accept the credit cards; see Tozzi [38] and Trichur [39]. Letting

$$\Theta \equiv \{(d, f) | f + d^{MA} - \gamma c_I \leq d \leq \min(d^{MA}, f^{MI}) \text{ and } f^{MI} - \delta c_A \leq f\}, \quad (41)$$

we have the following corollary.

Corollary 2 If $\gamma > 1$, then the set Θ is non-empty and thus, the interchange fee rate and the merchant discount rate in Θ can assure that the corresponding allocation scheme is in the core and the grand coalition $\{MAI\}$ is stable. However, if $\gamma = 1$, then both the set Θ and the core are empty.

In this thesis, γ is assumed to be greater than 1, because, as discussed previously, the acquirer/merchant incurs a higher operation cost when acting as "an issuer." We also note that, if $\delta = 1$, then $c_A + c_I \leq f^{MI} \leq c_A + \gamma c_I = d^{MA}$. Thus,

$d = f^{MI}$ and $f = f^{MI} - \delta c_A$ should belong to the set Θ.

4.2.2 The Unique Fair Allocation Scheme

Since the core includes many allocation schemes rather than only a unique scheme, one may need to make a decision on which imputation in the core to be chosen to allocate the profit $v(MAI)$. Shapley value [31] and the nucleolus [29] are the two most commonly-used solutions in cooperative game theory. We thus apply both of them to derive a unique allocation solution and its corresponding rates d and f.

We now find a unique fair allocation scheme for our cooperative game, which is in a non-empty core of our game. We learn from Corollary 2 that there must exist (d, f) satisfying the conditions in (41), which assures that our game has a non-empty core. Note that any imputation in the core corresponds to a fair allocation scheme. Next, we find a unique merchant discount rate d and a unique interchange fee rate f to assure that the core is non-empty and the corresponding allocation scheme is the non-empty core.

In Leng and Parlar [18], Leng and Parlar described major concepts in cooperative game theory, and concluded that Shapley value (Shapley [31]) and nucleolus (Schmeidler [29]) are the two commonly-used solutions each representing a unique allocation scheme. Shapley value can be computed easily by using a formula regardless of whether or not the core is empty, and also, it is a unique, monotonic solution; see, Megiddo [24] and Young [45]. The monotonicity of a solution means that, if the profit achieved by each possible coalition increases, then the profit allocation to each player should be also increased. But, Shapley value has the following weakness: it may not be in the core even though the core is non-empty. Compared with Shapley value, the nucleolus also exists in the non-empty core; but, this solution concept has been found by solving a series of linear problems. Thus, for the purpose of easy computation, we prefer Shapley value to the nucleolus for the calculation of the interchange fee rate and the merchant discount rate. But, if Shapley value is not in the core, then we should consider the nucleolus.

Shapley Value For our three-player cooperative game, the unique Shapley value (ϕ_M, ϕ_A, ϕ_I) is computed as $\phi_i = \sum_{i\in T}(|T|-1)!(n-|T|)![v(T)-v(T-\{i\})]/(n!)$, where T denotes a possible coalition that the firm i $(i = M, A, I)$ joins, and $|T|$ is the size of T. More specifically, the unique allocation scheme in terms of Shapley value is given as,

$$\begin{cases} \phi_M = \dfrac{2v(MAI) + 2v(M) + v(MA) + v(MI)}{6}; \\ \phi_A = \dfrac{2v(MAI) - v(M) + v(MA) - 2v(MI)}{6}; \\ \phi_I = \dfrac{2v(MAI) - v(M) + v(MI) - 2v(MA)}{6}. \end{cases} \tag{42}$$

As discussed previously, the allocation to the merchant, that to the acquirer, and that to the issuer are $\pi_{M_4}(d)$, $\pi_{A_4}(d)$ and $\pi_{I_4}(f)$, respectively, because the three firms implement the allocation scheme by determining d and f. That is, we can solve the equations $\{\pi_{M_4}(d) = \phi_M$ and $\pi_{A_4}(d) = \phi_A\}$—where $\pi_{M_4}(d)$ and $\pi_{A_4}(d)$ are given in Section 4.1.4—to find the Shapley value-based merchant discount rate d^{SP} and the Shapley value-based interchange fee rate f^{SP}. Note that we don't need to condition the equation that $\pi_{I_4}(f^{SP}) = \phi_I$, because $\pi_{M_4}(d^{SP}) + \pi_{A_4}(d^{SP}) + \pi_{I_4}(f^{SP}) = \phi_M + \phi_A + \phi_I = v(MAI)$.

However, when we use Shapley value-characterized allocation scheme to find d^{SP} and f^{SP}, we cannot assure that the values of d^{SP} and f^{SP} are in the set Θ in (41). If the Shapley value-based d^{SP} and f^{SP} doesn't belong to the set Θ, then the core may be empty and the grand coalition $\{MAI\}$ may not be stable. Therefore, if Shapley value cannot assure the stability of the grand coalition, then we should apply the nucleolus to our calculation.

To illustrate the above, we provide two numerical examples including one with empty core and the other with non-empty core.

Example 2 We use the parameter values for the three-product case in Example 1. Moreover, we assume that the parameters δ and γ are 10 and 9, respectively. Note from Section 4.1 that, in our three-player cooperative game model, $v(\varnothing) = v(A) = v(I) = v(AI) = 0$; and we need to calculate the characteristic values $v(M)$, $v(MA)$, $v(MI)$, and $v(MAI)$. According to Theorem 24, we find that, in the coalition $\{M\}$, the merchant's optimal prices for three products are $p_1 = 112.946$, $p_2 = 115.960$, and $p_3 = 116.920$. Then, using (26) we compute the characteristic value $v(M)$ as 25.409. Next, we solve the NBS models in (32) and (38), and find the NBS-characterized interchange fee rate for the coalition $\{MI\}$ and merchant discount rate for the coalition $\{MA\}$ as $f^{MI} = 7.507\%$ and $d^{MA} = 7.704\%$, respectively. It thus follows that the characteristic values of the coalitions $\{MI\}$ and $\{MA\}$ are calculated as $v(MI) = 76.389$ and $v(MA) = 72.410$, respectively. Using the above, our three-player cooperative game is constructed as follows:

$$v(\varnothing) = 0; \qquad v(A) = v(I) = 0, \quad v(M) = 25.409;$$
$$v(AI) = 0, \quad v(MI) = 76.389, \quad v(MA) = 72.410; \qquad v(MAI),$$

where $v(MAI)$ is a function of the merchant discount rate d, as given in (40). Next, we solve the equation set $\{\pi_{M_4}(d) = \phi_M$ and $\pi_{A_4}(d) = \phi_A\}$, where $\pi_{M_4}(d)$ and $\pi_{A_4}(d)$ are given in Section 4.1.4, and ϕ_M and ϕ_A are given as in (42); and find the Shapley value-characterized merchant discount rate and interchange fee rate as $d^{SP} = 4.973\%$ and $f^{SP} = 2.003\%$, respectively. The corresponding characteristic value of the grand coalition $\{MAI\}$ is $v(MAI) = 145.817$. We easily find that $4.307\% \leq d^{SP} \leq \min(d^{MA}, f^{MI}) = \min(7.507\%, 7.704\%)$ but $f^{SP} < 2.507$, which means that (d^{SP}, f^{SP}) is not in the set Θ in (41). Therefore, as Corollary 2 indicates, the core must be empty and the grand coalition $\{MAI\}$ must be thus unstable. ♦

Next, we provide another example to show that the core may be non-empty if the Shapley value is used to determine the merchant discount rate and the interchange fee rate for the credit card network.

Example 3 We re-consider Example 2 but assume that the acquirer's and the issuer's per dollar operation costs c_A and c_I are both equal to 0.5%, i.e., $c_A =$

$c_I = 0.005$; and the parameter $\gamma = 15$, which is greater than $\delta = 10$. Similarly, we calculate the characteristic values $v(M)$, $v(MA)$, and $v(MI)$; and thus find the three-player cooperative game as,

$$v(\varnothing) = 0; \qquad v(A) = v(I) = 0, \quad v(M) = 12.184;$$
$$v(AI) = 0, \quad v(MI) = 76.479, \quad v(MA) = 46.406; \qquad v(MAI).$$

Then, we solve the equations $\pi_{M_4}(d) = \phi_M$ and $\pi_{A_4}(d) = \phi_A$, and find the Shapley value-characterized merchant discount rate and interchange fee rate as $d^{SP} = 5.720\%$ and $f^{SP} = 2.895\%$, respectively. It follows that $v(MAI) = 143.362$. Since $5.031\% \leq d^{SP} \leq \min(d^{MA}, f^{MI}) = \min(9.636\%, 7.885\%)$ and $f^{SP} \geq 2.885\%$, we find that d^{SP} and f^{SP} are in the set Θ in (41) and, as Corollary 2 indicates, the core must be non-empty. That is, if the merchant, the acquirer and the issuer choose d^{SP} and f^{SP}, then the grand coalition $\{MAI\}$ must be stable and all of the three players are willing to cooperate for the credit card business. ♦

The Nucleolus Since the Shapley value-characterized rates (d^{SP}, f^{SP}) may not be in the set Θ, we use another common concept "the nucleolus" to find a unique pair of the rates. For our three-player cooperative game, the nucleolus solution is defined as a 3-tuple imputation $\mathbf{x} = (x_M, x_A, x_I)$ such that the excess ("unhappiness") $e_S(\mathbf{x}) = v(S) - \sum_{i \in S} x_i$ of any possible coalition S cannot be lowered without increasing any other greater excess; see, Schmeidler [29]. According to this definition, we find that the nucleolus is a solution concept that makes the largest unhappiness of the coalitions as small as possible, or, equivalently, minimizes the worst inequity. A most common approach to find the nucleolus is the sequential LP method that is based on lexicographic ordering (Maschler et al. [22]). Using the LP approach, we should first reduce the largest excess $\max\{e_S(x)$, for all $S \subseteq N\}$ as much as possible, then decrease the second largest excess as much as possible, and continue this process until the 3-tuple imputation $\mathbf{x}$ is determined.

Since the interchange fee rate and the merchant discount rate must be in the set Θ [given in (41)], we should add the constraints in the set Θ (i.e., $f + d^{MA} - \gamma c_I \leq d \leq \min(d^{MA}, f^{MI})$ and $f^{MI} - \delta c_A \leq f$) when we solve the sequence of LP problems to find the nucleolus. This assures that the allocation scheme in terms of the nucleolus must be in the non-empty core. We let d^n and f^n respectively denote the merchant discount rate and the interchange fee rate that correspond to the nucleolus. For our game, the LP algorithm is develop as follows:

1. We solve the first LP model, which is developed as,

$$\begin{aligned} \min_{d,f} \quad & u \\ \text{s.t.} \quad & \text{(i) } v(T) - \textstyle\sum_{i \in T} x_i \leq u, \quad \text{for any } T \subseteq \{M, A, I\}; \\ & \text{(ii) } x_M + x_A + x_I = v(MAI); \\ & \text{(iii) } x_M = \pi_{M_4}(d) \quad \text{and} \quad x_A = \pi_{A_4}(d); \\ & \text{(iv) } f + d^{MA} - \gamma c_I \leq d \leq \min(d^{MA}, f^{MI}) \\ & \qquad \text{and} \quad f^{MI} - \delta c_A \leq f, \end{aligned} \tag{43}$$

   where u denotes the "unhappiness" of the most unhappy coalition. In the above LP problem, the constraint (i) assures that the unhappiness of all coalitions are smaller than or equal to the maximum unhappiness (upper bound) u; the constraint (ii) assures that $v(MAI)$ is completely allocated among M, A and I; the constraint (iii) assures that the three firms implement the allocation by determining d and f, as discussed previously; and the constraint (iv) assures that the game has a non-empty core incorporating the nucleolus. Because, as Corollary 2 indicates, Θ is non-empty when $\gamma > 1$, the solution of the above linear problem must exist under the assumption that $\gamma > 1$.

   Solving the first LP, we can find an optimal solution that minimizes the most unhappy coalition's unhappiness. Next, we should determine whether or not the optimal solution for the first LP problem is the constrained nucleolus solution (d^n, f^n). We find the nucleolus (d^n, f^n) and terminate our search if one of the following two things occurs:

(a) If all constraints are binding, then we can terminate our search and the optimal solution for the first LP problem is the nucleolus solution (d^n, f^n). This is justified as follows: If the players in the coalition T are the most unhappy and their total unhappiness (i.e., $v(T) - \sum_{i \in T} x_i$) is minimized by solving the first linear problem, then, in the constraint set (i), the constraint corresponding to the coalition T must be binding, i.e., $\sum_{i \in T} x_i + u = v(T)$; and vice versa. Therefore, when all constraints are binding, all coalitions' unhappiness are minimized and the optimal solution (for the first LP problem) is the nucleolus.

(b) If one or more constraints are not binding for the first LP problem but the binding constraints in the first LP problem can *uniquely* determine an optimal solution, then we can still terminate our search and the optimal solution (for the first LP problem) is the nucleolus solution (d^n, f^n). This is explained as follows: When we minimize the second most unhappiness, we shall still use the constrained minimization model in (43) *but*, in the constraint set (i), the inequality "$\leq$" is changed to the equality "$=$" for each binding constraint that results from the minimization of the largest unhappiness in the first LP problem. Since the equalities can *uniquely* determine an optimal solution, the optimal solution for the second LP problem must be the same as that for the first LP problem. For example, if the constraints correspond to the coalitions $\{M\}$, $\{A\}$ and $\{I\}$ are binding, then we can find a unique optimal solution by solving the equations $\{v(M) - x_M = u$, $v(A) - x_A = u$ and $v(I) - x_I = u\}$, where u is the minimum value found by solving the first LP problem.

Otherwise, if neither of the above two things occurs, then we should continue with the second step.

2. For the second LP problem, we solve the constrained minimization model in (43) where, in the constraint set (i), the inequality "$\leq$" is replaced with the equality "$=$" for each binding constraint in the first LP problem. Similar to the above first step, after solving the second LP problem, we should examine (i) what constraints are binding and (ii) whether or not the binding constraints can determine a unique optimal solution. If all constraints are binding and/or the binding constraints can uniquely determine an optimal solution, then we can terminate our search and the solution for the second LP problem is the constrained nucleolus solution (d^n, f^n); otherwise, we should continue with the third step.

3. We repeat the above steps until all constraints are binding and/or the binding constraints uniquely determine an optimal solution; and the corresponding optimal solution is the constrained nucleolus solution (d^n, f^n).

For a detailed discussion of the LP approach for the nucleolus solution, see, e.g., Leng and Parlar [19], Wang [42], etc. Next, we provide a numerical example to illustrate the calculation of d^n and f^n when the nucleolus solution is used to allocate the system-wide profit $v(MAI)$ among the merchant, the acquirer, and the issuer.

Example 4 We consider the parameter values in Example 2 where the merchant discount rate and the interchange fee rate corresponding to Shapley value *cannot* assure the stability of the grand coalition $\{MAI\}$. As Example 2 indicates, $d^{MA} = 7.704\%$ and $f^{MI} = 7.507\%$; and $v(M) = 25.409$, $v(MA) = 72.410$, and $v(MI) = 76.389$. Recall from Section 4.1 that $v(\varnothing) = v(A) = v(I) = v(AI) = 0$.

To find d^n and f^n in terms of the nucleolus solution, we next follow our above procedure to solve a series of LP problems. We start with the first linear problem, which is specified as follows:

$$
\begin{aligned}
\min_{d,f} \quad & u \\
\text{s.t.} \quad & 25.409 - x_M \le u, \quad -x_A \le u, \quad -x_I \le u, \\
& 72.410 - x_M - x_A \le u, \quad 76.389 - x_M - x_I \le u, \quad -x_A - x_I \le u; \\
& x_M + x_A + x_I = v(MAI); \\
& x_M = \pi_{M_4}(d) \quad \text{and} \quad x_A = \pi_{A_4}(d); \\
& f - 2.304\% \le d \le 7.507\% \quad \text{and} \quad 2.507\% \le f.
\end{aligned} \tag{44}
$$

Solving the constrained minimization problem in (44), we find that $(d, f) = (5.764\%, 2.932\%)$; the minimum value of the most unhappiness is $u = -32.656$; and $x_M = 76.389$, $x_A = 32.656$, and $x_I = 32.656$. One can easily find that the constraints that $-x_A \le u$, $-x_I \le u$ and $v(MI) - x_M - x_I \le u$ are binding, which means that the most unhappy coalitions are $\{A\}$, $\{I\}$ and $\{MI\}$. Although the constraints corresponding to other coalitions are not binding, we can solve the equations $\{-x_A = -32.656, -x_I = -32.656, 76.389 - x_M - x_I = -32.656\}$ to determine the *unique* merchant discount rate and the unique interchange fee rate as $d = 5.764\%$ and $f = 2.932\%$, respectively. According to our above discussion, we can terminate our search with the nucleolus-characterized merchant discount rate as $d^n = 5.764\%$ and the nucleolus-characterized interchange fee rate as $f^n = 2.932\%$. Since the solution (d^n, f^n) satisfies the constraints that $f^n + d^{MA} - \gamma c_I \le d^n \le \min(d^{MA}, f^{MI})$ and $f^{MI} - \delta c_A \le f^n$, we conclude from Corollary 2 that the allocation scheme in terms of (d^n, f^n) must be in a non-empty core and the grand coalition $\{MAI\}$ must be thus stable. That is, if the nucleolus-based solution (d^n, f^n) is implemented for the credit card network, then the merchant, the acquirer and the issuer should be willing to join the network and cooperate for the credit card business. This is different from the Shapley value-based solution given in Example 2 since the latter cannot assure the stability of the grand coalition $\{MAI\}$.

We compare the nucleolus-characterized interchange fee and merchant discount rates with those (i.e., $f^* = 4.37\%$ and $d^S = 10.37\%$) found in Example 1 where we used the two-stage approach specified in Section 3 to find the interchange fee rate and the merchant discount rate. It follows from the comparison that, if the merchant is allowed to bargain with the issuer and the acquirer, then both the merchant discount and the interchange fee rate are reduced. ♦

In the following example, we consider the parameter values in Example 4—where the Shapley value-based allocation scheme is located in a non-empty core—to compute the merchant discount rate and the interchange fee rate that determine the nucleolus solution.

Example 5 We learn from Example 4 that $d^{MA} = 9.636\%$ and $f^{MI} = 7.885\%$; and $v(M) = 12.184$, $v(MA) = 46.406$, and $v(MI) = 76.479$. Moreover, $v(\varnothing) = v(A) = v(I) = v(AI) = 0$, as discussed in Section 4.1. We can thus specify the first LP problem in (43) as,

$$\begin{aligned}
\min\nolimits_{d,f} \quad & u \\
\text{s.t.} \quad & 12.184 - x_M \le u, \quad -x_A \le u, \quad -x_I \le u, \\
& 46.406 - x_M - x_A \le u, \quad 76.479 - x_M - x_I \le u, \quad -x_A - x_I \le u; \\
& x_M + x_A + x_I = v(MAI); \\
& x_M = \pi_{M_4}(d) \quad \text{and} \quad x_A = \pi_{A_4}(d); \\
& f - 2.136\% \le d \le 7.885\% \quad \text{and} \quad 2.885\% \le f.
\end{aligned} \tag{45}$$

Solving the above minimization problem, we find that $(d, f) = (5.764\%, 2.885\%)$; the minimum value of the most unhappiness is $u = -33.312$; and $x_M = 76.395$, $x_A = 33.123$, and $x_I = 33.396$. It is easy to find that the constraints that $-x_A \le u$ and $v(MI) - x_M - x_I \le u$ are binding, which means that the most unhappy coalitions are $\{A\}$ and $\{MI\}$. Even though the constraints corresponding to other coalitions are not binding, we can solve the equations $\{-x_A = -33.312, 76.479 - x_M - x_I = -33.312\}$ to *uniquely* determine the merchant discount rate and the interchange fee rate as $d = 5.764\%$ and $f = 2.885\%$, respectively. Thus, as discussed above, we can terminate our search and find the nucleolus-characterized merchant discount rate as $d^n = 5.764\%$ and the nucleolus-characterized interchange fee rate as $f^n = 2.885\%$. As Corollary 2 indicates, the allocation scheme in terms of (d^n, f^n) must be in a non-empty core and the grand coalition $\{MAI\}$ must be stable, because $(d^n, f^n) \in \Theta$. Hence, the merchant, the acquirer and the issuer should be willing to accept the nucleolus-based solution (d^n, f^n) for the credit card network.

Compare the nucleolus-based solution (d^n, f^n) with the Shapley value-based solution $(d^{SP}, f^{SP}) = (5.720\%, 2.895\%)$ that is given in Example 4. We find that the nucleolus solution concept suggests a higher merchant discount rate but a lower interchange fee rate for the credit card network. Given this result, the acquirer should prefer the nucleolus to the Shapley value, whereas the issuer should be inclined to the Shapley value. ♦

5 Sensitivity Analysis and Managerial Implications

In this section, we examine the impacts of the acquirer's and the issuer's operation costs on the interchange fee rate and the merchant discount rate that are determined by solving the two-stage game model (given in Section 3) and the three-player cooperative game model (given in Section 4). According to a large number of numerical examples, we find that the Shapley value-characterized rates can assure the non-emptiness of the core only when γ is in a small range (e.g., $[14.91, 15.1]$ for other parameter values as given in Examples 2 and 4). That is, for most cases, the Shapley value cannot result in the stability of the grand coalition $\{MAI\}$. Thus, for the sensitivity analysis in the three-player cooperative game setting, we don't consider the Shapley value but only use the nucleolus solution to compute the interchange fee rate and the merchant discount rate.

We note from Sections 3 and 4 that the acquirer's and the issuer's per dollar operation costs (i.e., c_A and c_I) affect the interchange fee rate and the merchant discount rate in both the two-stage and the cooperative game settings. But, the parameters δ and γ only impact our cooperative game analysis. Recall from Section 4 that the parameter δ distinguishes the operation cost of the merchant or issuer who acts as an "acquirer" from that of the acquirer in the credit card network; and the parameter γ discriminates the operation cost of the merchant or acquirer who acts as an "issuer" from that of the issuer in the credit card network. To that end, we discuss the impacts of the parameters c_A and c_I on the solutions obtained from both two-stage and cooperative game analysis, but investigate the effects of the parameters δ and γ on the solution only given by the cooperative game analysis.

Next, we perform our sensitivity analysis by using the parameter values in Example 2 as base values. According to our results from the sensitivity analysis, we provide our discussion and draw managerial implications.

5.1 The Impacts of the Unit Operating Costs c_A and c_I

We now investigate the effects of the parameters c_A and c_I on the Stackelberg interchange fee and merchant discount rates (f^S, d^S) for the two-stage game setting, and also on the nucleolus-characterized rates (f^n, d^n) for the three-player game setting. The parameters c_A and c_I are the acquirer's and the issuer's per dollar operation costs, respectively. We begin by examining the impacts of the acquirer's per dollar operation cost c_A. In this sensitivity analysis, we increase the value of c_A from 0.20% to 0.65% in increments of 0.05%, and compute the Stackelberg solution (f^S, d^S) and the nucleolus-based solution (f^n, d^n). For our computational results, see Table 1 in Appendix D. Using the data in Table 1, we plot two graphs (given in Figure 2) to help discuss managerial insights.

We find from Figure 2(a) that the nucleolus-based interchange fee and merchant discount rates f^n and d^n are significantly smaller than the Stackelberg rates f^S and d^S, respectively. That is, when the merchant is allowed to bargain over

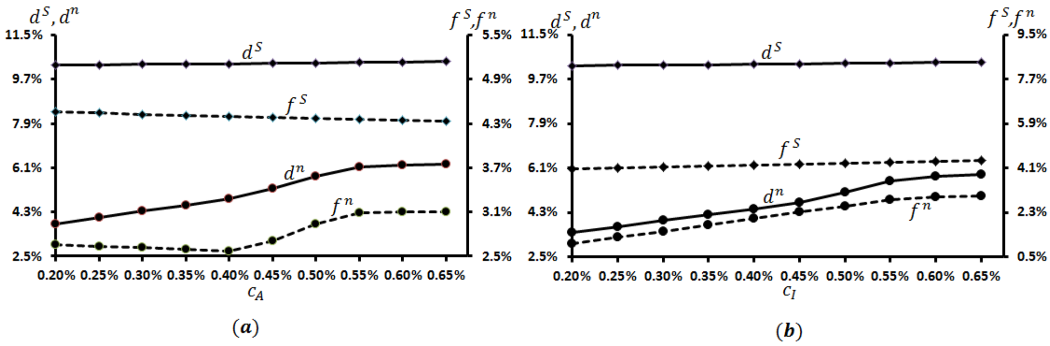

Figure 2: The impacts of the parameters c_A and c_I on the Stackelberg solution (f^S, d^S) for the two-stage game setting and the nucleolus-based solution (f^n, d^n) for the three-player cooperative game setting.

the rates with the issuer and the acquirer, the rates should be decreased, as expected by, e.g., the U.S. recent bill "H.R. 2695". Moreover, we also learn from Figure 2(a) that, as c_A increases, the merchant discount rates d^S and d^n are both increased; this is justified as follows: When the acquirer's per dollar operation cost c_A is higher, the merchant discount rate would be increased to assure the acquirer's profit. Otherwise, the acquirer may be unwilling to participate in the credit card business. In addition, we note that the Stackelberg rate d^S (when the merchant doesn't bargain over the rate) is increased at a lower rate than the nucleolus-characterized rate d^n (when the merchant bargains over the rate).

Figure 2(a) indicates that, as c_A increases, the two-stage game analysis and the three-player cooperative game analysis result in different patterns for the interchange fee rate. More specifically, from our two-stage game analysis, we find that, if the merchant is not involved in the negotiation for the interchange fee rate and the merchant discount rate, then the interchange fee rate is reduced at a low rate. This occurs because, in the two-stage game setting, when only the issuer and the acquirer bargain over the interchange fee rate, the issuer is willing to reduce the rate to offset the acquirer's increasing operation cost c_A. However, we learn from the cooperative game analysis that, if the merchant bargains with the issuer and the acquirer, then the interchange fee rate is reduced at a low rate when c_A is smaller than a cutoff level (e.g., 0.40) but is increased when c_A is greater than the cutoff level. This result is explained as follows: When c_A is sufficiently small (e.g., $c_A \leq 0.40$), increasing c_A leads to a higher operation cost for the acquirer, who may thus bargain with the issuer and the merchant to reduce the interchange fee rate. Since the reduction of the interchange fee rate is smaller as c_A is small, the issuer and the merchant may agree to decrease the rate so as to entice the acquirer to stay in the grand coalition $\{MAI\}$. When c_A is greater than the cutoff level, the issuer may not agree to reduce the interchange fee rate because a further reduction of the rate would largely lower the issuer's profit. But, increasing the rate may discourage the acquirer to cooperate with the issuer and the merchant, who may thus suggest to increase the merchant discount rate at a higher rate. Noting that the acquirer's per dollar profit margin is $[(d-f)-c_A]$, we find that, in order to offset the acquirer's increasing operation cost c_A, the acquirer should have an increasing revenue $(d-f)$. Using the data in Table 1 (given in Appendix D), we have,

c_A	0.20%	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%	0.65%
$d^n - f^n$	1.15%	1.43%	1.72%	2.00%	2.28%	2.56%	2.83%	3.04%	3.10%	3.16%
$(d^n - f^n) - c_A$	0.95%	1.18%	1.42%	1.65%	1.88%	2.11%	2.33%	2.49%	2.50%	2.51%

From the above table, we learn that the acquirer's profit margin increases, which means that the acquirer would be willing to stay in the grand coalition $\{MAI\}$.

Next, we discuss the impacts of the issuer's per dollar operation cost c_I on the the Stackelberg solution (f^S, d^S) for the two-stage game setting and the nucleolus-based solution (f^n, d^n) for the three-player cooperative game setting. In this sensitivity analysis, similar to our above analysis for c_A, we increase c_I from 0.20% to 0.65% in increments of 0.05%, and compute the Stackelberg solution (f^S, d^S) and the nucleolus-based solution (f^n, d^n) for each value of c_I. We find from Figure

2(b) that, if c_I increases, then the issuer absorbs a higher operation cost and thus bargains with the acquirer and the merchant to increase the interchange fee rate. This fact is reflected by Figure 2(b), which indicates that both the Stackelberg rate f^S and the nucleolus-based rate f^n are increased as c_I is increased. But, a higher interchange fee rate may prevent the acquirer from joining the coalition $\{MAI\}$. In order to entice the acquirer to cooperate with the issuer and the merchant, all players agree to accordingly increase the merchant discount rate, as shown by Figure 2(b). Moreover, we find that the nucleolus solutions f^n and d^n change at a larger rate than the Stackelberg solutions f^S and d^S.

5.2 The Impacts of the Scale Parameters δ and γ

We now examine the effects of δ and γ on the nucleolus-characterized solution (f^n, d^n). We increase the value of either δ or γ from 2 to 11 in increments of 1, and compute the nucleolus-based solution (f^n, d^n). [Note that, as discussed previously, the Stackelberg solution (f^S, d^S) is independent of both δ and γ.] Similar to Section 5.1, our computational results are presented in Table 1 that is given in Appendix D; and we use the data in Table 1 to plot two graphs in Figure 3.

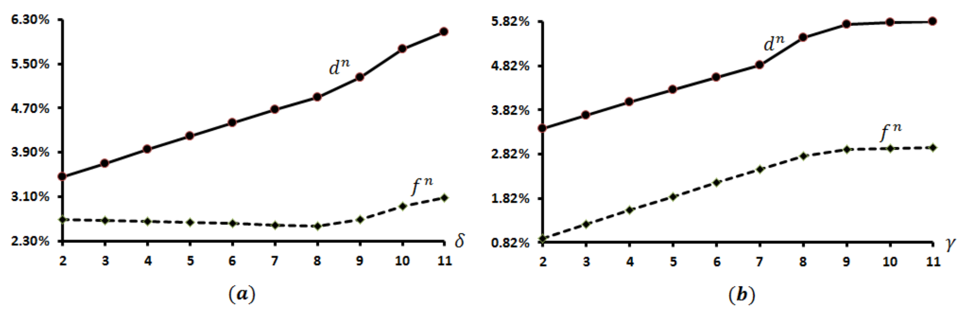

Figure 3: The impacts of the parameters δ and γ on the nucleolus-characterized interchange fee and merchant discount rates (i.e., f^n and d^n) that are determined as a result of the negotiation among the issuer, the acquirer and the merchant.

We begin by examining the impacts of the parameter δ, which is the scale parameter measuring the per dollar operation cost absorbed by either the merchant or the issuer, when the acquirer doesn't join the credit card network and either the merchant or the issuer instead acts as an "acquirer". Figure 3(a) indicates that f^n decreases in δ when δ is smaller than a cutoff level, and increases in δ, otherwise. That is, when δ is small, increasing its value can generate a higher operation cost for the player—the merchant or the issuer—who acts as an acquirer. Therefore, the player may be unwilling to take this role, and would suggest a lower interchange fee rate so as to entice the acquirer to join the credit card network. Meanwhile, the merchant discount rate d^n increases, thereby assuring that the acquirer's profit margin $[(d - f) - c_A]$ increases. However, when δ is sufficiently high, the issuer disagrees to further reduce its interchange fee rate as its profit would be reduced to a very low level. As a result, the interchange fee rate increases when $\delta \geq 8$, see Figure 3(a). But, in order to assure the profitability of the acquirer, the three players in the credit card network may agree to increase the merchant discount rate at a greater rate. Similar to Section 4.1, we use the data in Table 1 to calculate the acquirer's profit margin (with $c_A = 0.5\%$) as follows:

δ	2	3	4	5	6	7	8	9	10	11
$d^n - f^n$	0.77%	1.03%	1.29%	1.56%	1.82%	2.07%	2.33%	2.58%	2.83%	2.99%
$(d^n - f^n) - c_A$	0.27%	0.53%	0.79%	1.06%	1.32%	1.57%	1.83%	2.08%	2.33%	2.49%

The above table shows that the acquirer should have an incentive to join the grand coalition $\{MAI\}$.

The parameter γ is the scale factor that measures the per dollar operation cost absorbed by the merchant/acquirer when it acts as an issuer. We also vary γ from 2 to 11 in increments of 1 to compute the corresponding nucleolus-characterized solution (f^n, d^n), see Figure 3(b) and Table 1. Figure 3(b) indicates that, as γ increases, both the interchange fee rate and the merchant discount rate increase, which is justified as follows: when γ increases, the merchant and the acquirer lose the incentive to take the role of an issuer, and they are thus more willing to increase the interchange fee rate so as to attract the issuer to join the grand coalition $\{MAI\}$. However, raising the interchange fee rate hurts the acquirer's profit; thus, the three players in the network may also agree to increase the merchant discount rate.

6 Summary and Concluding Remarks

In this thesis, we consider a credit card network involving an issuer, an acquirer, and a merchant who serves consumers in a market with n products. In the network, the issuer serves as the bank of consumers and the acquirer serves as the bank of the merchant. After the consumers buy from the merchant using credit cards issued by the issuer, the issuer should transfer to the acquirer the amount that is calculated as total transaction amount (i.e., the consumers' total expenditure) minus the interchange fee. The interchange fee—which is calculated as the interchange fee rate times the total transaction amount—is regarded as the issuer's revenue generated from the credit card business. When the acquirer receives the amount transferred from the issuer, it retains an amount as its revenue from the credit card transaction and deposit the remaining to the merchant's account. The amount that the acquirer retains is also proportional to the total transaction amount, and it is computed as the total transaction amount times the difference between the merchant discount rate and the interchange fee rate.

We perform the game-theoretic analysis to determine the interchange fee rate for the issuer, the merchant discount rate for the acquirer, and the retail prices for the merchant. In practice, the interchange fee rate results from the negotiation between the issuer and the acquirer; for more discussion, see, e.g., Akers et al. [1], Balto [2], etc. However, as our literature review indicates, very few publications (e.g., Small and Wright [34]) determine the interchange fee rate as a result of the negotiation between the issuer and the acquirer. Furthermore, starting from June 2009, the Senate and House of Representatives of the United States of

America have been implementing the bill "Credit Card Fair Fee Act of 2009 (H.R. 2695)" [15], in which merchants are encouraged to participate in bargaining over relevant fee rates (i.e., the interchange fee rate and the merchant discount rate) with banks in credit card networks (e.g., Visa, MasterCard, etc.) in the United States. According to our review, we find that no publication has considered the negotiation among three players (i.e., the issuer, the acquirer and the merchant) to determine the interchange fee rate and the merchant discount rate.

Motived by the above, we considered two approaches to determine the interchange fee rate and the merchant discount rate and retail prices for the credit card network. The first approach is to develop a two-stage game model. In the first stage, the issuer and the acquirer bargain over the interchange fee rate; and in the second stage, the acquirer first determines its merchant discount rate and announces it to the merchant, who then makes its retail pricing decisions. The first approach corresponds to the credit card operations where the merchant doesn't participate in the negotiation for the interchange fee rate and the merchant discount rate. To solve the two-stage game problem, we used the backward method for our calculation. At first, we determined the optimal merchant discount rate and retail prices for a given value of interchange fee rate. More specifically, we maximized the merchant's profit to obtain its best response decision (i.e., optimal retail prices), given the interchange fee rate and the merchant discount rate. Then, using the merchant's best response decision, we developed the acquirer's profit function and maximizes it to find the optimal (Stackelberg) merchant discount rate, which was proved to be an increasing, convex function of the interchange fee rate. Substituting the optimal merchant discount rate into the merchant's best response function gave the Stackelberg retail prices that is dependent on the interchange fee rate. Next, we used the interchange fee rate-dependent Stackelberg solution to develop the issuer's profit, and applied the solution concept of Nash bargaining scheme to determine the interchange fee, which was then used to compute the Stackelberg solution for the acquirer and the merchant.

The second approach is to construct a three-player cooperative game in the characteristic function form. This is motived by the fact that the merchant is allowed (and encouraged) to bargain with the issuer and the acquirer over both the interchange fee and the merchant discount rates, as suggested by, e.g., the recent U.S. bill entitled "Credit Card Fair Fee Act of 2009" (H.R. 2695). We calculated the characteristic values of all possible coalitions for our three-player cooperative game. We then derived a sufficient condition for the supermodularity, and attained a sufficient and necessary condition under which the core of our cooperative game is non-empty. We proved that there must exist a pair of the interchange fee rate and the merchant discount rate such that the core is non-empty. Noting that a merchant discount rate and an interchange fee rate determine a unique allocation scheme, we apply the solution concepts of Shapley value and the nucleolus to find a fair allocation scheme under which the merchant, the acquirer and the issuer are all better off by joining the credit card network. Then, using the fair allocation scheme, we calculate the corresponding interchange fee rate and merchant discount rate. We found that the Shapley value may or may not exist in a non-empty core, which were demonstrated by two numerical examples. In order to assure the stability of the grand coalition, we developed an algorithm to compute the nucleolus solution-characterized interchange fee rate and merchant discount rate.

In order to find more managerial insights, we perform a sensitivity analysis to examine the impacts of credit card operation costs on the interchange fee rate and the merchant discount rate. Our major analytic managerial insights are summarized as follows:

1. In the two-stage game setting, the demand of each product in the market should not be very sensitive to its retail price, in order to assure the success of the credit card business. Otherwise, if the demand for *each* product is *more sensitive* to the retail price, then the acquirer and the issuer may attain lower rates and may thus be *less willing* to participate in the credit

card business.

2. In the two-stage game setting, if the interchange fee rate is determined by the issuer itself rather than by the negotiation between the issuer and the acquirer, then the acquirer would not benefit from the credit card business. This implies that such a two-player negotiation can help entice the acquirer to stay in the credit card network.

3. In the cooperative game setting, *only* the large merchants (e.g., JUSCO) *may* have an incentive to undertake the credit card operations by itself, whereas the other merchants (especially, small merchants) may have to only focus on the retailing service with the financial services from the issuer and the acquirer as in the two-stage game setting.

4. We show that the merchant's participation in the credit card network is *significantly* important to the profitability of the network. Moreover, the issuer and the acquirer may need to have *sufficient* expertise in the issuer- and the acquirer-related business, respectively, in order to make the network-wide profit higher than when the merchant operates the credit card business by itself or delegates the business to either the issuer or the acquirer.

   Moreover, the popularity of the credit card service in the financial market would be important to the success of the recent U.S. bill "Credit Card Fair Fee Act of 2009 (H.R. 2695)," in which the merchant is encouraged to participate in the credit card network for the negotiation of the interchange fee rate and the merchant discount rate.

5. The interchange fee rate and the merchant discount rate in the cooperative game setting are smaller than those in the two-stage game setting. This means that, as a result of implementing the U.S. bill "H.R. 2695," both rates are reduced, which shows that the bill is effective.

6. The U.S. bill "H.R. 2695" should be more effective, if the issuer, the acquirer, and the merchant are all more specialized in their own operations.

That is, the success of the bill may also require that the participants in the negotiation of the merchant discount rate and the interchange fee rate are the large, highly-specialized banks (e.g., Citibank) and merchants (e.g., Wal-Mart). This insight could be justified by the fact that, in practice, many small merchants may be unwilling or against to accept the credit cards; see Tozzi [38] and Trichur [39].

In addition to the above, we obtain some important managerial insights from the numerical study as follows:

1. The interchange fee rate and the merchant discount rate in the cooperative game are *significantly* smaller than the Stackelberg equilibrium rates in the two-stage game setting.

2. The acquirer's and the issuer's unit operation costs more significantly impact the interchange fee rate and the merchant discount rate in the cooperative game setting than in the two-stage game setting.

3. The interchange fee rate in the two-stage game setting is always decreasing in the acquirer's unit operation cost c_A, whereas the interchange fee rate in the cooperative game setting is decreasing in c_A if c_A is smaller than a cutoff level and is otherwise increasing in c_A. But, the merchant discount rate is always increasing in c_A in both the two-stage and the cooperative game settings.

4. In both the two-stage and the cooperative game settings, the interchange fee rate and the merchant discount rate are always increasing in the issuer's unit operation cost c_I.

5. In the cooperative game setting, the interchange fee rate is decreasing in δ—which measures the per dollar operation cost absorbed by the merchant/issuer when it acts as an acquirer, if δ is smaller than a cutoff level, and is increasing in δ, otherwise. But, the merchant discount rate is always increasing in δ.

Moreover, the interchange fee rate and the merchant discount rate are always increasing in γ, which measures the per dollar operation cost absorbed by the merchant/acquirer when it acts as an issuer.

In conclusion, we find that the recent U.S. bill "H.R. 2695" should be useful to reduce both the interchange fee rate and the merchant discount rate. We have identified some important managerial insights that could help improve the efficiency of the credit card operations.

Appendices

Appendix A Proofs of Theorems

Proof of Theorem 1. Temporarily ignoring the constraint that $c_i/(1-d) \leq p_i \leq \alpha_i/\beta_i$, we differentiate π_M in (2) once and twice w.r.t. p_i, and have,

$$\frac{d\pi_M}{dp_i} = (1-d)\alpha_i + \beta_i c_i - 2\beta_i(1-d)p_i \quad \text{and} \quad \frac{d^2\pi_M}{dp_i^2} = -2\beta_i(1-d) < 0,$$

which implies that π_M is a strictly concave function of p_i. Equating π_M/dp_i to zero and solving it for p_i, we obtain the optimal price $p_i(d)$ as given in (4). Since $d \leq \kappa = \min[(\alpha_i - \beta_i c_i)/\alpha_i, i = 1, 2, \ldots, n]$, we find that $c_i/(1-d) \leq \alpha_i/\beta_i$, which implies that $p_i(d)$ in (4) must be between $c_i/(1-d)$ and α_i/β_i. That is, $p_i(d)$ in (4) satisfies the constraint in the maximization problem in (3). ■

Proof of Theorem 2. Temporarily ignoring the constraint that $f + c_A \leq d \leq \kappa$, we calculate the first- and second-order derivatives of $\pi_A(d)$ in (9) w.r.t. d, which are given as,

$$\frac{\partial \pi_A(d)}{\partial d} = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right] - \frac{\omega_2(d-f-c_A)}{2(1-d)^3}, \tag{46}$$

and

$$\frac{\partial^2 \pi_A(d)}{\partial d^2} = -\frac{\omega_2}{2(1-d)^3} - \frac{1}{4}\left[\frac{2\omega_2}{(1-d)^3} + \frac{6\omega_2(d-f-c_A)}{(1-d)^4}\right],$$

which cannot be immediately determined to be positive or negative. At the point(s) that satisfy the first-order condition (i.e., $\partial\pi_A(d)/\partial d = 0$), we re-write the second-order derivative $\partial^2\pi_A(d)/\partial d^2$ as,

$$\begin{aligned} \left.\frac{\partial^2 \pi_A(d)}{\partial d^2}\right|_{\partial\pi_A(d)/\partial d=0} &= -\frac{\omega_2}{2\,(1-d)^3} - \frac{1}{4}\left\{\frac{2\omega_2}{(1-d)^3} + \frac{3\omega_1}{1-d} - \frac{3\omega_2}{(1-d)^3}\right\} \\ &= -\frac{1}{4(1-d)}\left[3\omega_1 + \frac{\omega_2}{(1-d)^2}\right], \end{aligned}$$

which is negative for any point satisfying $\partial\pi_A(d)/\partial d = 0$. This implies that the acquirer's profit $\pi_A(d)$ in (9) is a quasi-concave function of the merchant discount rate d. That is, $\pi_A(d)$ in (9) is a unimodal function of d with a unique maximizing value $d_1(f)$, which, by solving the first-order condition $\partial\pi_A(d)/\partial d = 0$, can be obtained as the unique solution of the equation (11).

Next, we compute the acquirer's optimal merchant discount rate under the constraint that $f + c_A \leq d \leq \kappa$. When $d = f + c_A$, we calculate the first-order derivative $\partial\pi_A(d)/\partial d$ in (46) as,

$$\left.\frac{\partial\pi_A(d)}{\partial d}\right|_{d=f+c_A} = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-f-c_A)^2}\right],$$

which is greater than or equal to zero because $\omega_1 \geq \omega_2/\left(1-d\right)^2$, as indicated by Remark 1. This implies that $f + c_A \leq d_1(f)$ because $\pi_A(d)$ is a quasi-concave function of d with the unique optimal solution $d_1(f)$. As a result, $d_1(f)$ must satisfy the constraint that $d \geq f + c_A$, which is thus redundant.

When the acquirer sets its merchant discount rate as the largest value κ, i.e., $d = \kappa$, we re-write $\partial\pi_A(d)/\partial d$ in (46) as,

$$\left.\frac{\partial\pi_A(d)}{\partial d}\right|_{d=\kappa} = \frac{1}{4}\left[\omega_1 - \frac{\omega_2}{(1-\kappa)^2}\right] - \frac{\omega_2(\kappa - f - c_A)}{2(1-\kappa)^3}, \tag{47}$$

which may be positive or negative, depending on the value of the interchange fee f. It is easy to find that $\partial\pi_A(d)/\partial d|_{d=\kappa}$ in (47) is strictly increasing in the interchange fee f, and it is equal to zero if and only if $f = \hat{f}$, where $\hat{f}$ is given as in (12).

As a result, when $f \leq \hat{f}$, then $\partial\pi_A(d)/\partial d|_{d=\kappa}$ must be less than or equal to zero. Because of the quasi-concavity of $\pi_A(d)$, κ must be greater than or equal to $d_1(f)$, and it thus follows that the constraint that $d \leq \kappa$ is satisfied. Hence, when $f \leq \hat{f}$, the optimal merchant discount rate is $d^*(f) = d_1(f)$. On the other hand, if $\hat{f} \leq f \leq \kappa$, then $\partial\pi_A(d)/\partial d|_{d=\kappa}$ is nonnegative and κ is thus smaller than or equal to $d_1(f)$. This means that $d^*(f) = \kappa$. We thus prove this theorem. ■

Proof of Theorem 3. As Lemma 1 implies, the function $\Lambda_1(f)$ in (17) is a unimodal function of f with a *unique* maximizing value $\tilde{f}_1$ that satisfies the first-order condition, i.e., $\partial[\Lambda_1(f)]/\partial f = 0$, which can be re-written as (18). Because of the quasi-concavity of $\Lambda_1(f)$, we can easily attain f_1^* as given in this theorem.

■

Proof of Theorem 4. Temporarily ignoring the constraint that $\hat{f} \leq f \leq \kappa$, we differentiate the function $\Lambda_2(f)$ in (17) once and twice w.r.t. f, which are given as,

$$\frac{\partial[\Lambda_2(f)]}{\partial f} = \frac{1}{16}\left[\kappa - 2f - c_A + c_I\right] \times \left[\omega_1 - \frac{\omega_2}{(1-\kappa)^2}\right]^2,$$

and

$$\frac{\partial^2[\Lambda_2(f)]}{\partial f^2} = -\frac{1}{8} \times \left[\omega_1 - \frac{\omega_2}{(1-\kappa)^2}\right]^2 < 0,$$

which implies that the function $\Lambda_2(f)$ is a strictly concave function of f. Equating $\partial\Lambda_2(f)/\partial f$ to zero and solving it for f, we find the interchange fee rate $\tilde{f}_2$—that maximizes $\Lambda_2(f)$ with no constraint—as $\tilde{f}_2 = (\kappa - c_A + c_I)/2$.

Next, we consider the constraint that $\hat{f} \leq f \leq \kappa$, and obtain the optimal interchange-fee rate f_2^*. We first prove that $\tilde{f}_2 \leq \kappa$. Since $\kappa \geq d \geq f + c_A$—as discussed in Section 3.1—and $f \geq c_I$, we find that $\kappa \geq c_A + c_I$, or, $c_I \leq \kappa - c_A$; and thus,

$$\tilde{f}_2 = (\kappa - c_A + c_I)/2 \leq (\kappa - c_A + \kappa - c_A)/2 = \kappa - c_A \leq \kappa.$$

It then follows that the constrain that $f \leq \kappa$ is redundant. We then consider the constraint that $\hat{f} \leq f$. Because of the concavity of $\Lambda_2(f)$, we find that the optimal interchange-fee rate $f_2^* = \max(\hat{f}, \tilde{f}_2)$. At the point that $f = \hat{f}$, the first-order derivative $\partial[\Lambda_2(f)]/\partial f$ is computed as,

$$\left.\frac{\partial[\Lambda_2(f)]}{\partial f}\right|_{f=\hat{f}} = \frac{1}{16}\left[\frac{(c_A + c_I - 1)\,\omega_2 + (1-\kappa)^3\,\omega_1}{\omega_2}\right] \times \left[\omega_1 - \frac{\omega_2}{(1-\kappa)^2}\right]^2,$$

which has the same sign as the term $(c_A + c_I - 1)\,\omega_2 + (1-\kappa)^3\,\omega_1$. More specifically, we find that, if $r_1 \leq r_2$ where r_i $(i = 1, 2)$ are given as in (20), then

$\partial[\Lambda_2(f)]/\partial f$ is non-negative at the point that $f = \hat{f}$. Because of the concavity of $\Lambda_2(f)$, we find that $\hat{f} \leq \tilde{f}_2$ and thus, $f_2^* = \tilde{f}_2$. If $r_1 \geq r_2$, then $\partial[\Lambda_2(f)]/\partial f$ is non-positive at the point that $f = \hat{f}$, which means that $\hat{f} \geq \tilde{f}_2$ and $f_2^* = \hat{f}$. This theorem is thus proved. ■

Proof of Theorem 5. According to Theorems 3 and 4, we find that $f_1^* = \min(\tilde{f}_1, \hat{f})$, where $\tilde{f}_1$ is a unique solution satisfying (18); and f_2^* is given as in (19), which depends on the comparison between r_1 and r_2. If $r_1 \leq r_2$, then $f_2^* = \tilde{f}_2$, and the NBS-characterized rate f^* is dependent on the comparison between $\Lambda_1(f_1^*)$ and $\Lambda_2(\tilde{f}_2)$. Noting that $\tilde{f}_1$—which satisfies the complicated equation (18)—cannot determined analytically, we have to write f^* as in (21). If $r_1 \geq r_2$, then $f_2^* = \hat{f}$. Since $\Lambda_1(\hat{f}) = \Lambda_2(\hat{f})$, we find that the interchange fee rate f^* is obtained as f_1^*. This means that, when $r_1 \geq r_2$, the acquirer and the issuer shall choose their interchange fee rate in the range $[c_I, \hat{f}]$. This theorem is thus proved. ■

Proof of Theorem 6. The first- and second-order partial derivatives of $\Pi_{M_1}(\mathbf{p})$ in (23) w.r.t p_i $(i = 1, \ldots, n)$ are computed as,

$$\frac{\partial[\Pi_{M_1}(\mathbf{p})]}{\partial p_i} = -2\beta_i p_i(1 - \delta c_A - \gamma c_I) + \alpha_i(1 - \delta c_A - \gamma c_I) + \beta_i c_i,$$

and $\partial^2[\Pi_{M_1}(\mathbf{p})]/\partial p_i^2 = -2(1 - \delta c_A - \gamma c_I)\beta_i \leq 0$, because $(1 - \delta c_A - \gamma c_I) \geq 0$. Thus, $\Pi_{M_1}(\mathbf{p})$ is a concave function of p_i. Equating $\partial[\Pi_{M_1}(\mathbf{p})]/\partial p_i$ to zero and solving it for p_i gives the merchant's optimal price $p_i^{M_1}$, as given in (24). It is easy to show that $p_i^{M_1}$ in (24) falls in the range $[c_i/(1 - \delta c_A - \gamma c_I), \alpha_i/\beta_i]$. We thus prove this theorem. ■

Proof of Theorem 7. Temporarily ignoring the condition that $c_i/(1 - f) \leq p_i \leq \alpha_i/\beta_i$, we take the first- and second-order partial derivatives of $\Pi_{M_2}(\mathbf{p})$ w.r.t p_i, and find,

$$\frac{\partial[\Pi_{M_2}(\mathbf{p})]}{\partial p_i} = -2(1-f)\beta_i p_i + [(1 - f)\alpha_i + \beta_i c_i] \text{ and } \frac{\partial^2[\Pi_{M_2}(\mathbf{p})]}{\partial p_i^2} = -2(1-f)\beta_i < 0,$$

which implies the concavity of the function $\Pi_{M_2}(\mathbf{p})$. Equating $\partial[\Pi_{M_2}(\mathbf{p})]/\partial p_i$ to zero and solving it for p_i gives the merchant's best-response retail price $p_i^{M_2}(f)$ as shown in (27). Noting from Section 3.1.1 that $c_i/(1-d) \leq \alpha_i/\beta_i$, $i = 1, \dots, n$, we find that $c_i/(1-f) \leq \alpha_i/\beta_i$ since $f \leq d$. This means that $p_i^{M_2}(f)$ in (27) satisfies the constraint that $c_i/(1-f) \leq p_i \leq \alpha_i/\beta_i$, which is thus redundant. ■

Proof of Theorem 8. Using (26), we re-write the value $v(M)$ as,

$$v(M) = \frac{1}{4}\sum_{i=1}^{n}\left\{\left(\alpha_i - \frac{\beta_i c_i}{1-\delta c_A - \gamma c_I}\right)\left[\frac{\alpha_i(1-\delta c_A - \gamma c_I)}{\beta_i} - c_i\right]\right\};$$

and using (29), we re-write $\Pi_{M_2}(\mathbf{p}^{M_2}(f))$ as,

$$\Pi_{M_2}(\mathbf{p}^{M_2}(f)) = \frac{1}{4}\sum_{i=1}^{n}\left\{\left(\alpha_i - \frac{\beta_i c_i}{1-f}\right)\left[\frac{\alpha_i(1-f)}{\beta_i} - c_i\right]\right\}.$$

Comparing the right-hand sides of the above two equations, we prove this theorem. ■

Proof of Theorem 9. Using (29) and (30), we re-write the function Λ^{MI} as,

$$\begin{aligned}\Lambda^{MI} &= \frac{f - c_I - \delta c_A}{16} \times \left[\omega_1(1-f) + \frac{\omega_2}{1-f} - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)\right] \\ &\quad \times \left[\omega_1 - \frac{\omega_2}{(1-f)^2}\right].\end{aligned}$$

Similar to the proof of Lemma 1, we analyze the property of the logarithm of Λ^{MI}. Taking the first- and second-order derivatives of $\ln(\Lambda^{MI})$ w.r.t. f gives,

$$\begin{aligned}\frac{\partial[\ln(\Lambda^{MI})]}{\partial f} &= \frac{1}{f - c_I - \delta c_A} - \frac{\omega_1 - \omega_2/(1-f)^2}{\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)} \\ &\quad - \frac{2\omega_2/(1-f)^3}{\omega_1 - \omega_2/(1-f)^2}, \\ \frac{\partial^2[\ln(\Lambda^{MI})]}{\partial f^2} &= -\frac{1}{(f - c_I - \delta c_A)^2} - \frac{[2\omega_2/(1-f)^3]^2}{[\omega_1 - \omega_2/(1-f)^2]^2} - \frac{6\omega_2/(1-f)^4}{\omega_1 - \omega_2/(1-f)^2} \\ &\quad - \frac{[\omega_1 - \omega_2/(1-f)^2]^2}{\left[\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)\right]^2} \\ &\quad + \frac{2\omega_2/(1-f)^3}{\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)}.\end{aligned}$$

At the point(s) satisfying the first-order condition that $\partial[\ln(\Lambda_1^{MI})]/\partial f = 0$, we have,

$$\frac{1}{f - c_I - \delta c_A} = \frac{\omega_1 - \omega_2/(1-f)^2}{\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)} + \frac{2\omega_2/(1-f)^3}{\omega_1 - \omega_2/(1-f)^2},$$

or,

$$\begin{aligned} & \frac{2\omega_2/(1-f)^3}{[f - c_I - \delta c_A][\omega_1 - \omega_2/(1-f)^2]} - \frac{[2\omega_2/(1-f)^3]^2}{[\omega_1 - \omega_2/(1-f)^2]^2} \\ = \; & \frac{2\omega_2/(1-f)^3}{\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)}. \end{aligned}$$

It thus follows that $\partial^2[\ln(\Lambda_1^{MI})]/\partial f^2$ can be re-written as,

$$\begin{aligned} \frac{\partial^2[\ln(\Lambda_1^{MI})]}{\partial f^2} \; = \; & -\frac{1}{(f - c_I - \delta c_A)^2} - \frac{[2\omega_2/(1-f)^3]^2}{[\omega_1 - \omega_2/(1-f)^2]^2} \\ & + \frac{2\omega_2/(1-f)^3}{[f - c_I - \delta c_A][\omega_1 - \omega_2/(1-f)^2]} \\ & - \frac{[2\omega_2/(1-f)^3]^2}{[\omega_1 - \omega_2/(1-f)^2]^2} - \frac{6\omega_2/(1-f)^4}{\omega_1 - \omega_2/(1-f)^2} \\ & - \frac{[\omega_1 - \omega_2/(1-f)^2]^2}{[\omega_1(1-f) + \omega_2/(1-f) - 2\sum_{i=1}^{n}(\alpha_i c_i) - 4v(M)]^2}, \end{aligned}$$

which is negative because

$$\frac{1}{(f - c_I - \delta c_A)^2} + \frac{[2\omega_2/(1-f)^3]^2}{[\omega_1 - \omega_2/(1-f)^2]^2} > \frac{2\omega_2/(1-f)^3}{[f - c_I - \delta c_A][\omega_1 - \omega_2/(1-f)^2]}.$$

Therefore, we conclude that $\ln(\Lambda_1^{MI})$ is a quasi-concave function, which means that Λ_1^{MI} is also a quasi-concave function, because of the following fact: Since $\partial[\ln(\Lambda_1^{MI})]/\partial f = (\partial\Lambda_1^{MI}/\partial f)/(\Lambda^{MI})$, we find that $\partial[\ln(\Lambda_1^{MI})]/\partial f$ and $\partial\Lambda_1^{MI}/\partial f$ can be zero at the same point(s). Therefore, at the points that satisfies $\partial[\ln(\Lambda_1^{MI})]/\partial f = 0$, $\partial\Lambda_1^{MI}/\partial f$ must be also equal to zero, i.e., $\partial\Lambda_1^{MI}/\partial f = 0$; and thus, $\partial^2[\ln(\Lambda_1^{MI})]/\partial f = (\partial^2\Lambda_1^{MI}/\partial f)/(\Lambda_1^{MI}) - (\partial\Lambda_1^{MI}/\partial f)^2/(\Lambda_1^{MI})^2 = (\partial^2\Lambda_1^{MI}/\partial f)/(\Lambda_1^{MI})$. This means that $\partial^2[\ln(\Lambda_1^{MI})]/\partial f$ and $\partial^2\Lambda_1^{MI}/\partial f$ have the same sign at the points satisfying $\partial[\ln(\Lambda_1^{MI})]/\partial f = 0$. As shown above, $\partial^2[\ln(\Lambda_1^{MI})]/\partial f|_{\partial^2[\ln(\Lambda_1^{MI})]/\partial f = 0} < 0$; so, we can find that $\partial^2 \Lambda_1^{MI}/\partial f|_{\partial \Lambda_1^{MI}/\partial f=0} < 0$. ■

Proof of Theorem 10. This proof is similar to the proof of Theorem 8. ■

Proof of Theorem 11. This is similar to the proof of Theorem 9. ■

Proof of Theorem 12. We learn from Driessen [10] that, in order to show the supermodularity, we need to prove that $v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T)$, for all $S \subseteq T \subseteq N \backslash \{i\}$, $i = M, A, I$. If $S = T$, then the above condition for the supermodularity must be satisfied. Next, we assume that $S \subset T \subseteq N \backslash \{i\}$, and examine if the above condition is satisfied.

1. If $S = \{\varnothing\}$ and $T = \{M\}$, then player i is either A or I, and we find that $v(S \cup \{i\}) - v(S) = v(i) - v(\varnothing) = 0$, because $v(\varnothing) = v(A) = v(I) = 0$; and $v(T \cup \{i\}) - v(T) = v(Mi) - v(M)$, which must be greater than or equal to zero if $f \leq d \leq \delta c_A + \gamma c_I$, as indicated by Theorems 8 and 10. Therefore, for this case (i.e., $S = \varnothing$ and $T = M$), the supermodularity condition must be satisfied.

2. If $S = \{\varnothing\}$ and $T = \{A\}$, then player i is either M or I. If $i = I$, then $v(S \cup \{I\}) - v(S) = v(I) - v(\varnothing) = v(T \cup \{I\}) - v(T) = v(AI) - v(A) = 0$. If $i = M$, then we find that

$$v(S \cup \{M\}) - v(S) = v(M) \leq v(T \cup \{M\}) - v(T) = v(MA),$$

which results from our discussion in Section 4.1.3.

3. If $S = \{\varnothing\}$ and $T = \{I\}$, then player i is either M or A. Similar to the second case, we find that the supermodularity condition is satisfied, according to Section 4.1.2.

4. If $S = \{A\}$ and $T = \{AI\}$, then player i is M. As a result, $v(S \cup \{M\}) - v(S) = v(MA)$ and $v(T \cup \{M\}) - v(T) = v(MAI)$. From Sections 4.1.3 and 4.1.4, we find that, for the coalitions $\{MA\}$ and $\{MAI\}$, the merchant's optimal retail price is $[\alpha_i/\beta_i + c_i/(1-d)]/2$, for $i = 1, \ldots, n$, where, but,

the merchant discount rate d is negotiated by M and A in the coalition $\{MA\}$ whereas d results from the negotiation among M, A and I in the grand coalition $\{MAI\}$. We re-write the characteristic values of the two coalitions as,

$$\begin{aligned} v(MA) &= \frac{1-c_A-\gamma c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d^{MA})^2}\right] - \sum_{i=1}^{n}\left\{\frac{c_i}{2}\left[\frac{\alpha_i}{\beta_i} + \frac{c_i}{1-d^{MA}}\right]\right\}, \\ v(MAI) &= \frac{1-c_A-c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right] - \sum_{i=1}^{n}\left\{\frac{c_i}{2}\left[\frac{\alpha_i}{\beta_i} + \frac{c_i}{1-d}\right]\right\}. \end{aligned}$$

From the above, we find that, if $d \le d^{MA}$, then $v(MAI) \ge v(MA)$.

5. If $S = \{A\}$ and $T = \{MA\}$, then player i is I, and it thus follows that $v(S \cup \{I\}) - v(S) = 0$ and $v(T \cup \{I\}) - v(T) = v(MAI) - v(MA)$, which is non-negative if $d \le d^{MA}$, as discussed for the fourth case.

6. If $S = \{I\}$ and $T = \{AI\}$, then player i is M. As a result, $v(S \cup \{M\}) - v(S) = v(MI)$ and $v(T \cup \{M\}) - v(T) = v(MAI)$. Similar to the fourth case, we find that, if $d \le f^{MI}$, then $v(MAI) \ge v(MI)$. Note that, in the coalition $\{MI\}$, f^{MI} is equivalent to the merchant discount rate because the issuer also acts as an "acquirer".

7. If $S = \{I\}$ and $T = \{MI\}$, then player i is A. Hence, $v(S \cup \{A\}) - v(S) = 0$ and $v(T \cup \{A\}) - v(T) = v(MAI) - v(MI)$. Similar to the sixth case, we find that, if $d \le f^{MI}$, then $v(MAI) \ge v(MI)$.

8. If $S = \{M\}$ and $T = \{MA\}$, then player i is I. As a result, $v(S \cup \{I\}) - v(S) = v(MI) - v(M)$ and $v(T \cup \{I\}) - v(T) = v(MAI) - v(MA)$. For this case, we need to find whether or not $v(MAI) + v(M) \ge v(MA) + v(MI)$. Because of the intractable complexity, we cannot compare $v(MAI) + v(M)$ and $v(MA) + v(MI)$ analytically. But, we note that, if $v(MAI) + v(M) \ge v(MA) + v(MI)$, then $v(MAI)$ must be greater than or equal to both $v(MA)$ and $v(MI)$.

9. If $S = \{M\}$ and $T = \{MI\}$, then player i is A. Similar to the eighth case,

we cannot determine if $v(MAI) + v(M)$ is greater than $v(MA) + v(MI)$ analytically.

In conclusion, we prove this theorem. ■

Proof of Theorem 13. We use the definition of the core (given by Gillies [12].) to derive the sufficient conditions. That is, we should analyze the inequalities that $\sum_{i \in T} y_i \geq v(T)$, for all coalitions $T \subseteq N = \{M, A, I\}$. Note that, as discussed previously, $y_M = \pi_{M_4}(d)$, $y_A = \pi_{A_4}(d)$, and $y_I = \pi_{I_4}(f)$, where the functions $\pi_{i_4}(d)$ $(i = M, A, I)$ are given as in Section 4.1.4.

1. When $T = \{A\}$, $v(T) = v(A) = 0$ and

$$\sum_{i \in T} y_i = y_A = \pi_{A_4}(d) = (d - f - c_A)R(d) = \frac{d - f - c_A}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right].$$

   If $f + c_A \leq d \leq 1 - \sqrt{\omega_2/\omega_1}$, then $y_A \geq v(A)$.

2. When $T = \{I\}$, $v(T) = v(I) = 0$ and

$$\sum_{i \in T} y_i = y_I = (f - c_I)R(d) = \frac{f - c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right].$$

   If $f \geq c_I$ and $d \leq 1 - \sqrt{\omega_2/\omega_1}$, then $y_I \geq v(I)$.

3. When $T = \{M\}$, $v(T) = v(M)$—which is given as in (26)—and

$$\sum_{i \in T} y_i = y_M = \frac{1}{4}\sum_{i=1}^{n} \frac{[\alpha_i(1-d) - \beta_i c_i]^2}{\beta_i(1-d)}.$$

   Comparing the above with (26), we find that, if $d \leq \delta c_A + \gamma c_I$, then $y_M \geq v(M)$.

4. When $T = \{A, I\}$, $v(T) = v(AI) = 0$ and

$$\sum_{i \in T} y_i = y_A + y_I = \frac{d - c_A - c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right],$$

   which is greater than or equal to $v(AI)$ if $c_A + c_I \leq d \leq 1 - \sqrt{\omega_2/\omega_1}$.

5. When $T = \{M, I\}$, $v(T) = v(MI)$, which, using (33) and (34), can be re-written as,

$$v(MI) = \frac{1}{4}\sum_{i=1}^{n}\left[\frac{\left[\alpha_i(1-f^{MI}) - \beta_i c_i\right]^2}{\beta_i(1-f^{MI})}\right] + \frac{f^{MI} - c_I - \delta c_A}{4}\left[\omega_1 - \frac{\omega_2}{(1-f^{MI})^2}\right],$$

Moreover, we have,

$$\sum_{i\in T} y_i = y_M + y_I = \frac{1}{4}\sum_{i=1}^{n}\frac{\left[\alpha_i(1-d) - \beta_i c_i\right]^2}{\beta_i(1-d)} + \frac{f - c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right],$$

which is greater than or equal to $v(MI)$ if $f^{MI} - \delta c_A \leq f \leq d \leq f^{MI}$.

6. When $T = \{M, A\}$, $v(T) = v(MA)$; that is,

$$v(MA) = \frac{1}{4}\sum_{i=1}^{n}\frac{\left[(1-d^{MA})\alpha_i - \beta_i c_i\right]^2}{\beta_i(1-d^{MA})} + \frac{d^{MA} - c_A - \gamma c_I}{4}\left[\omega_1 - \frac{\omega_2}{(1-d^{MA})^2}\right].$$

We also have,

$$\sum_{i\in T} y_i = y_M + y_A = \frac{1}{4}\sum_{i=1}^{n}\frac{\left[\alpha_i(1-d) - \beta_i c_i\right]^2}{\beta_i(1-d)} + \frac{d - f - c_A}{4}\left[\omega_1 - \frac{\omega_2}{(1-d)^2}\right],$$

which is greater than or equal to $v(MA)$ if $f + d^{MA} - \gamma c_I \leq d \leq d^{MA}$.

7. When $T = N = \{M, A, I\}$, $v(T) = v(MAI) = \sum_{i\in T} y_i$.

Summarizing the above, we find the conditions in (41) that assures that the core is non-empty. In addition, we note from (41) that the negotiated merchant discount rate and interchange fee rate are no more than d^{MA} and f^{MI}, respectively. We can also easily find that, if the conditions in (41) are not satisfied, then the core must be empty. Thus, we arrive to this theorem. ■

Appendix B Proofs of Corollaries

Proof of Corollary 1. We learn from (10) that $d^*(f)$ is equal to either $d_1(f)$—given by (11)—or κ which is constant. In order to prove this corollary, we only need to investigate the impact of f on $d_1(f)$. Taking the first-order derivative of $d_1(f)$ w.r.t. f at both sides of (11), we have,

$$\frac{\partial[d_1(f)]}{\partial f} = \frac{2\omega_2}{3[1-d_1(f)]^2\omega_1 + \omega_2} > 0,$$

which implies that $d_1(f)$ is strictly increasing in f. Moreover, we find that

$$\frac{\partial[d_1(f)]}{\partial f} - 1 = -\frac{3\omega_1 - \omega_2/[1-d_1(f)]^2}{3[1-d_1(f)]^2\omega_1 + \omega_2} < 0,$$

where $\omega_1 - \omega_2/[1-d_1(f)]^2 > 0$, because $d_1(f) \leq \kappa$ if $f \leq \hat{f}$; and as Remark 1 indicates, $\omega_1 - \omega_2/(1-d)^2 \geq 0$ when $d \leq \kappa$. In order to show the convexity of $d_1(f)$, we compute the second-order derivative of $d_1(f)$ w.r.t. f as follows:

$$\frac{\partial^2[d_1(f)]}{\partial f^2} = \frac{12[1-d_1(f)]\omega_1\omega_2}{\{3[1-d_1(f)]^2\omega_1 + \omega_2\}^2}\frac{\partial d_1(f)}{\partial f} > 0,$$

which means that $d_1(f)$ is convex in f. This corollary is thus proved. ■

Proof of Corollary 2. Using the above discussion, we find that the set Θ in (41) must be non-empty if $\gamma > 1$. Moreover, if $(d, f) \in \Theta$, then we learn from the proof of Theorem 13 that the allocation scheme in terms of d and f must be in the core. That is, the allocation scheme assures that all players are better off by joining the grand coalition $\{MAI\}$, which is thus stable. But, as discussed above, the core must be empty if $\gamma = 1$. ■

Appendix C Proof of Lemma 1

We calculate the logarithm of $\Lambda_1(f)$ in (17) as $\ln[\Lambda_1(f)] = (\xi_1 + \xi_2 + \xi_3)/16$, where

$$\xi_1 \equiv \ln(f - c_I), \quad \xi_2 \equiv \ln[d_1(f) - f - c_A] \quad \text{and} \quad \xi_3 \equiv 2\ln\left[\omega_1 - \frac{\omega_2}{(1 - d_1(f))^2}\right].$$

In order to prove the concavity of $\ln[\Lambda_1(f)]$, we should analyze ξ_i $(i = 1, 2, 3)$ as follows:

1. The first- and second-order derivatives of ξ_1 w.r.t. f are computed as,

$$\frac{\partial \xi_1}{\partial f} = \frac{1}{f - c_I} \quad \text{and} \quad \frac{\partial^2 \xi_1}{\partial f^2} = -\frac{1}{[f - c_I]^2} < 0,$$

   which means that ξ_1 is strictly concave in f.

2. The first- and second-order derivatives of ξ_2 w.r.t. f are found as,

$$\begin{aligned} \frac{\partial \xi_2}{\partial f} &= \frac{1}{d_1(f) - f - c_A}\left(\frac{\partial[d_1(f)]}{\partial f} - 1\right), \\ \frac{\partial^2 \xi_2}{\partial f^2} &= -\frac{1}{[d_1(f) - f - c_A]^2}\left(\frac{\partial[d_1(f)]}{\partial f} - 1\right)^2 + \frac{1}{d_1(f) - f - c_A}\frac{\partial^2[d_1(f)]}{\partial f^2}, \end{aligned}$$

   which cannot be immediately determined to be positive or negative because $\partial^2[d_1(f)]/\partial f^2 > 0$, as shown by Corollary 1. According to Theorem 2, we find that $d_1(f)$ is determined by (11), which is obtained by simplifying (46). This means that $d_1(f)$ satisfies (46); that is,

$$\frac{1}{d_1(f) - f - c_A} = \frac{2\omega_2}{(1 - d_1(f))^3[\omega_1 - \omega_2/(1 - d_1(f))^2]}.$$

   It thus follows that the second term on the right-hand side of $\partial^2\xi_2/\partial f^2$ can be re-written as,

$$\frac{1}{d_1(f) - f - c_A}\frac{\partial^2[d_1(f)]}{\partial f^2} = \frac{2\omega_2}{(1 - d_1(f))^3[\omega_1 - \omega_2/(1 - d_1(f))^2]}\frac{\partial^2[d_1(f)]}{\partial f^2}.$$

3. The first- and second-order derivatives of ξ_3 w.r.t. f are calculated as,

$$\begin{aligned}
\frac{\partial \xi_3}{\partial f} &= -\frac{4\omega_2}{(1-d_1(f))^3[\omega_1-\omega_2/(1-d_1(f))^2]}\frac{\partial [d_1(f)]}{\partial f}, \\
\frac{\partial^2 \xi_3}{\partial f^2} &= -\left\{\frac{4(\omega_2)^2}{(1-d_1(f))^6[\omega_1-\omega_2/(1-d_1(f))^2]^2}\right. \\
&\quad \left. +\frac{12\omega_2}{(1-d_1(f))^4[\omega_1-\omega_2/(1-d_1(f))^2]}\right\} \\
&\quad \times\left(\frac{\partial [d_1(f)]}{\partial f}\right)^2-\frac{4\omega_2}{(1-d_1(f))^3[\omega_1-\omega_2/(1-d_1(f))^2]}\frac{\partial^2 [d_1(f)]}{\partial f^2},
\end{aligned}$$

which is negative; this means that ξ_3 is strictly concave in f.

Using the above, we calculate the sum of $\partial^2\xi_2/\partial f^2$ and $\partial^2\xi_3/\partial f^2$ as,

$$\begin{aligned}
\frac{\partial^2 \xi_2}{\partial f^2}+\frac{\partial^2 \xi_3}{\partial f^2} &= -\frac{1}{[d_1(f)-f-c_A]^2}\left(\frac{\partial [d_1(f)]}{\partial f}-1\right)^2 \\
&\quad -\left\{\frac{4(\omega_2)^2}{(1-d_1(f))^6[\omega_1-\omega_2/(1-d_1(f))^2]^2}\right. \\
&\quad \left. +\frac{12\omega_2}{(1-d_1(f))^4[\omega_1-\omega_2/(1-d_1(f))^2]}\right\} \\
&\quad \times\left(\frac{\partial [d_1(f)]}{\partial f}\right)^2-\frac{2\omega_2}{(1-d_1(f))^3[\omega_1-\omega_2/(1-d_1(f))^2]}\frac{\partial^2 [d_1(f)]}{\partial f^2},
\end{aligned}$$

which is negative. Since $\partial^2\xi_1/\partial f^2 < 0$, we can conclude that $\ln[\Lambda_1(f)] = (\xi_1 + \xi_2 + \xi_3)/16$ is strictly concave in f. This lemma is thus proved.

Appendix D Results of the Sensitivity Analysis in Section 5

$c_A(\%)$	$f^S(\%)$	$d^S(\%)$	$f^n(\%)$	$d^n(\%)$	δ	$f^n(\%)$	$d^n(\%)$
0.20	4.4557	10.2737	2.6549	3.8047	2	2.6886	3.4544
0.25	4.4419	10.2903	2.6358	4.0702	3	2.6725	3.7029
0.30	4.4281	10.3068	2.6148	4.3323	4	2.6549	3.9487
0.35	4.4142	10.3234	2.5917	4.5905	5	2.6358	4.1911
0.40	4.4004	10.3399	2.5663	4.8443	6	2.6148	4.4299
0.45	4.3866	10.3565	2.7052	5.2605	7	2.5917	4.6644
0.50	4.3728	10.3730	2.9323	5.7646	8	2.5663	4.8940
0.55	4.3590	10.3896	3.0873	6.1245	9	2.6802	5.2605
0.60	4.3452	10.4062	3.0978	6.1957	10	2.9323	5.7646
0.65	4.3314	10.4228	3.1077	6.2654	11	3.0883	6.0765
$c_I(\%)$	$f^S(\%)$	$d^S(\%)$	$f^n(\%)$	$d^n(\%)$	γ	$f^n(\%)$	$d^n(\%)$
0.20	4.0834	10.2407	1.0417	3.5016	2	0.9177	3.3947
0.25	4.1195	10.2572	1.3000	3.7458	3	1.2336	3.6934
0.30	4.1557	10.2737	1.5572	3.9877	4	1.5473	3.9881
0.35	4.1919	10.2903	1.8132	4.2271	5	1.8585	4.2781
0.40	4.2281	10.3068	2.0677	4.4636	6	2.1667	4.5625
0.45	4.2642	10.3234	2.3206	4.6967	7	2.4712	4.8424
0.50	4.3004	10.3399	2.5725	5.1449	8	2.7737	5.4474
0.55	4.3366	10.3565	2.8243	5.5987	9	2.9323	5.7646
0.60	4.3728	10.3730	2.9323	5.7646	10	2.9465	5.7931
0.65	4.4090	10.3896	2.9921	5.8341	11	2.9599	5.8198

Table 1: The impacts of the parameters c_A and c_I on the Stackelberg interchange fee rate and merchant discount rate (f^S, d^S) for the two-stage game setting and the nucleolus-characterized rates (f^n, d^n) for the three-player cooperative game setting; and the impacts of δ and γ on the nucleolus-characterized rates (f^n, d^n) for the three-player cooperative game setting.

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