Optimal allocation and consumption with guaranteed minimum death benefits, external income and term life insurance

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The Effect of Time and Ambiguity Preferences on Saving and Insurance

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Abstract

In this paper, we study two classical saving-insurance problems for the intertemporal version developed by Hayashi and Miao (2011) of the smooth ambiguity model of Klibanoff et al. (2005). These models put risk, ambiguity and time preferences together in a Kreps-Porteus aggregator, and disentangle the effects among risk, ambiguity and time preferences. We show that the concepts and techniques developed by Topkis (1998) and others can be used to obtain a set of simple and intuitive sufficient conditions such that risk, ambiguity and time preferences together always raise the demand for saving and self-insurance.

Key words: Precautionary saving; Self-insurance; Self-protection; Smooth ambiguity aversion; Intertemporal substitution; Risk aversion

JEL classification: D81, D91, E21, G11, D61
1 Introduction

In real life, people usually face some future risks and adopt consciously or unconsciously some ways to treat them. In theories of individual decision-making under uncertainty, there are three main instruments: saving, insurance and prevention to deal with these risks. Literature is analyzing these issues by choosing probabilities of the uncertain events and then maximizing the values with respect to those objective or subjective probabilities.

We say a consumption-saving model generates precautionary saving if a future risk increases current saving. In the framework of Expected Utility, Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972) first formally analyze precautionary saving. Kimball (1990) proposes the term of prudence to examine the properties of the precautionary saving. These traditional models study decision makers (DMs) whose preferences are additive over time and across states of nature.

There have been two lines of theoretical progress on precautionary saving problem over the last twenty years. In the first line, with Kreps-Porteus preferences, Kimball and Weil (1992, 2009), Gollier (2001) and Li and Wang (2014) address a question: Does precautionary saving depend on time preference, risk preference, or both? They try to distinguish the prudence effect and intertemporal substitution effect on precautionary saving. In the second line, with smooth ambiguity preferences, Berger (2014) and Osaki and Schlesinger (2014) examine how risk and ambiguity preferences jointly affect precautionary saving.

Self-insurance and self-protection (i.e. prevention) are two other important risk management activities. Self-insurance means the effort of one individual reduces the size of a loss, whereas self-protection means the effort of one individual reduces the probability of a loss. Ehrlich and Becker (1972)'s seminar work has led studies on the both topics. Recently, after noticing that one DM may be ambiguous about the probability of the loss, Snow (2011) and Alary et al. (2013), within a one-period framework, examine the effect of ambiguity on the both issues and derive some sufficient conditions of reduction or increase of optimal effort level in the presence of ambiguity aversion.

We know that one DM may undertake current effort which improves the stochastic distribution of future wealth. This kind of decision-making problem can be modeled in a two-period framework. Menegatti (2009) proposes a two-period self-protection model and shows the positive effect of prudence. Since then, several studies have explored this topic. Berger (2013) extends
the works of Snow (2011) and Alary et al. (2013), and shows some sufficient conditions for the effect of ambiguity in a two-period model.

It is worthy to note that the most existing works in the literature on precautionary saving, self-insurance and self-protection all assume an exogenous additively separable time preference. Because they do not consider the effect of endogenous nonlinear time preference as proposed by Kreps and Porteus (1978), they conclude that preference over time does not matter for decisions. However, this conclusion is inconsistent with our intuitions. This gap directly motivates our work.

Recently, a generalized recursive smooth ambiguity model was developed and axiomatized by Hayashi and Miao (2011). This model allows us to separate the effects among risk, ambiguity and time preferences. Ju and Miao (2012) apply this model to quantitatively calibrate a consumption-based asset-pricing problem.

In this paper, following the theory developed by Hayashi and Miao (2011), we propose two generalized two-period smooth ambiguity models that examine precautionary saving, insurance and self-protection problems. We assume that the representative DM is ambiguous about the future risk, and her preference is represented by a generalized two-period smooth ambiguity model that leads to a three-way separation among risk, ambiguity, and time preferences. With this type of preferences, the objective function may not be concave under general conditions on the aggregator/ambiguity/utility functions. It is well known that the “supermodularity technique” permits to obtain comparative statics results without relying on well behaved (concave and differentiable) objective functions. In this paper we use this technique to obtain comparative statics results.

Given risk aversion and ambiguity aversion, we qualitatively examine how prudence, strictly decreasing absolute ambiguity aversion, intertemporal substitution and degree of absolute risk aversion jointly determine precautionary saving and the demand for self-insurance (self-protection). We show that these two models share the similar intuitions. Our models nest several existing models in the literature.

Our paper is related to the literature that studies the implications of risk, ambiguity and time preferences for precautionary saving, insurance and self-protection. We contribute to the literature by deriving a set of simple and and intuitive answers for the following somewhat different

1Kimball and Weil (1992, 2009), Gollier (2001) and Li and Wang (2014) are three exceptions. However, they do not consider ambiguity.
question: Does precautionary saving (or the demand for self-insurance and self-protection) depend on risk aversion, prudence, ambiguity prudence, and intertemporal substitution separately or jointly?

The paper proceeds as follows. Section 2 analyzes the sufficient conditions for precautionary saving. Section 3 studies self-insurance and self-protection problems. Section 4 concludes this paper. All proofs are in the Appendix.

2 Precautionary saving

As in Kimball (1990), we consider a simple two-period model with a sure income $w_0$ in the first period, but a sure income $w$ plus an uncertain income in the second period. The consumer’s prior beliefs for the second-period income distributions are represented by the conditional random variable $\tilde{\epsilon}_\theta$, $\theta = 1, \cdots, n$, together with the subjective probabilities $q_\theta$ for $\tilde{\epsilon}_\theta$, $\theta = 1, \cdots, n$. Where, $\sum_1^n q_\theta = 1$. We assume $\sum_\theta q_\theta E\tilde{\epsilon}_\theta = 0$. So the DM believes that the uncertainty is a fair gamble.

Following the work of Hayashi and Miao (2011), we propose the following saving model:

$$V(s, \tilde{\epsilon}_\theta) = W(w_0 - s, v^{-1}\sum_\theta q_\theta v \circ u^{-1}(Eu(w + \tilde{\epsilon}_\theta + s))),$$

where $u$ and $v$ are strictly increasing functions. We assume the risk-free interest rate equals 1. The axiomatic foundations of model (1) were examined by Hayashi and Miao (2011). Define $\phi = v \circ u^{-1}$, we have $v^{-1}(x) = u^{-1}(\phi^{-1}(x))$, then the model (1) can be re-written:

$$V(s, \tilde{\epsilon}_\theta) = W(w_0 - s, u^{-1}[\phi^{-1}(\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s)))]),$$

where $s$ is the saving and $W(., .)$ is a time aggregator. Time preference is determined by the property of $W$. In model (2), the shape of $\phi$ characterizes the attitude toward ambiguity, while $u$ represents the attitude toward pure risk. $\phi^{-1}(\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s)))$ is the certainty equivalent of the uncertain conditional expected utility $Eu(w + \tilde{\epsilon}_\theta + s)$ in the second period (see, e.g., Klibanoff et al. 2005, 2009). We assume $\phi$ is strictly increasing. We also assume the DM is risk averse ($u'' \leq 0$) and ambiguity averse ($\phi'' \leq 0$).

We see that the life time utility function $V(s, \tilde{\epsilon}_\theta)$ is computed by four operations. First, given $\theta$, one computes the expected utility by using utility function $u$. The concavity of $u$
measures the attitude of risk aversion. Second, one evaluates the ambiguity certainty equivalent $\phi^{-1}\left[\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s))\right]$ of the future uncertain utility by using the function $\phi$. The concavity of $\phi$ measures the attitude of ambiguity aversion. Third, one calculates the wealth certainty equivalent via $u^{-1}$. These three operations are done in atemporal context. Fourth, one computes the lifetime utility by aggregating the current consumption and the future wealth certainty equivalent by using function $W$. Because all uncertainties have been removed in this fourth operation, the properties of $W$ are related to preferences for consumption smoothing over time. Therefore, model (2) permits a separation among risk aversion, ambiguity aversion, and intertemporal substitution.

Following Gierlinger and Gollier (2012), Berger (2014) and Osaki and Schlesinger (2014), we assume $Eu(\tilde{\epsilon}_\theta)$ and $Eu'(\tilde{\epsilon}_\theta)$ are anti-comonotone in $\theta$ (i.e., $\text{cov}(g(Eu(\tilde{\epsilon}_\theta)), h(Eu'(\tilde{\epsilon}_\theta))) \leq 0$ for all functions $h$ and $g$ such that $h' \geq 0$ and $g' \geq 0$). We can explain this condition as: on average, a change in the prior beliefs causes the conditional expected utility and the conditional expected marginal utility move in opposite directions. From Gierlinger and Gollier (2012), Berger (2014) and Osaki and Schlesinger (2014), we know that, (i) the derivatives of $u$ are obeying $(-1)^{i+1}u^{(i)} > 0, i = 1, 2, ..., N + 1$, together with (ii) the $\tilde{\epsilon}_\theta$ can be ranked according to $N$-th order stochastic dominance assure $Eu(\tilde{\epsilon}_\theta)$ and $Eu'(\tilde{\epsilon}_\theta)$ are anti-comonotone in $\theta$. Condition (i) states that the DM is risk apportionment of order $i$, for $i = 1, 2, ..., N + 2$ (see, Eeckhoudt and Schlesinger, 2006). Condition (ii) implies that the ambiguity is over the orders of stochastic dominance of the distribution. We call the condition (i) as the the risk preference condition and the condition (ii) as the ambiguity (subjective beliefs) condition.

Some particular cases of model (2) are worthy to mention. For example, when $W(x, y) = u(x) + \beta u(y)$, then

$$W(w_0 - s, u^{-1}[\phi^{-1}\left(\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s))\right)])$$

$$= u(w_0 - s) + \beta \phi^{-1}\left(\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s))\right).$$

We are back to the ambiguity saving model examined by Berger (2014) and Osaki and Schlesinger (2014). When there is no ambiguity (i.e. $q_1 = 1$) and $W$ is an additively separable function,

$$W(w_0 - s, u^{-1}[\phi^{-1}\left(\sum_\theta q_\theta \phi(Eu(w + \tilde{\epsilon}_\theta + s))\right)]) = m(w_0 - s) + U(u^{-1}(Eu(w + \tilde{\epsilon}_1 + s))),$$

For other sufficient conditions, please see Gierlinger and Gollier (2012) and Berger (2014).
where $U$ is an increasing function and $m$ is the utility at the first time period, model (2) degenerates to the saving model with Kreps-Porteus Preferences which is examined by Kimball and Weil (1992, 2009), Gollier (2001) and Li and Wang (2014). Thus model (2) nests the several existing models for precautionary saving.

Let $\max_s V(s,0)$ denote the optimal saving problem with the ambiguity neutral DM (i.e., model (2) with a linear function $\phi$) and $\max_s V(s,1)$ denote the optimal problem with ambiguity aversion individual (i.e., model (2) with a concave function $\phi$). Define $s_0 \in \arg \max V(s,0)$ and $s_1 \in \arg \max V(s,1)$. We first examine whether the introduction of ambiguity aversion attitude increases the precautionary saving level via a comparison between $s_0$ with $s_1$. We establish the following result:

**Proposition 2.1** We have $s_1 \geq s_0$ for a strictly DAAA (Decreasing Absolute Ambiguity Averse) DM with $W_2 > 0$, $W_{12} \geq 0$ and $-\frac{W_{22}(x,y)}{W_2(x,y)} \geq -\frac{u''(y)}{u'(y)}$.

Proposition 2.1 provides a set of general sufficient conditions on whether the introduction of ambiguity aversion attitude increases the saving level. Berger (2014) defines $s_1 \geq s_0$ as ambiguity prudence. The above proposition states that ambiguity prudence is assured by the following three effects:

(i) ambiguity attitude effect: strictly DAAA;

(ii) time preference effect: $W_2 > 0$ and $W_{12} \geq 0$. $W_2$ can be defined as the subjective discount factor (Koopmans, 1960). $W_{12} \geq 0$ can be assured by decreasing marginal impatience (DMI). DMI means people are less patient at lower levels of consumption than at higher levels. Although some economists believe the condition of increasing marginal impatience (IMI), many researchers think DMI should be intuitively more plausible\(^4\) and the validity of DMI is supported by the existing empirical studies\(^5\).

(iii) intertemporal substitution and risk aversion joint effect: $-\frac{W_{22}(x,y)}{W_2(x,y)} \geq -\frac{u''(y)}{u'(y)}$. Following Kimball and Weil (2009), we can explain $-\frac{W_{22}(x,y)}{W_2(x,y)}$ as the resistance to intertemporal substitution. So this condition means that the resistance to intertemporal substitution is larger than the strength of risk aversion.

Now we let $\max_s V(s,c)$ denote the optimal saving problem without the risk ($\tilde{\epsilon}_{\theta} \equiv 0$, for $\theta = 1, \ldots, n$, i.e., $V(s,c) = W(w_0 - s, w + s)$). Define $s_c \in \arg \max V(s,c)$. We obtain:

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Proposition 2.2 We have $s_1 \geq s_c$ for a strictly DAAA DM with $u''' \geq 0$, $W_2 > 0$, $W_{12} \geq 0$ and
\[-\frac{W_{22}(x,y)}{W_{21}(x,y)} \geq -\frac{u''(y)}{u'(y)},\]
$s_1 \geq s_c$ means the DM would like to raise her wealth accumulation in order to forearm herself to face future uncertainties. We call this precautionary motive for saving. Propositions 2.1 and 2.2 state that, given risk aversion and ambiguity aversion, ambiguity prudence and prudence ($u''' \geq 0$) imply precautionary saving. One clear difference from the existing precautionary saving models in the literature is the role of prudence, ambiguity prudence, and intertemporal substitution in joint assuring the precautionary saving. Especially, Proposition 2.2 supports the following intuition: while prudence is the motivation for precautionary saving, strictly DAAA, DMI ($W_{12} \geq 0$) and a high enough resistance to intertemporal substitution (or a low enough degree of risk aversion) reinforce these motivations. Proposition 2.2 allows a complicated precautionary saving problem to be characterized via a much simple and an intuitive way.

3 Self-insurance and self-protection

We now consider another two-period model. In the first-period, the wealth is sure with $w_0$. In the second-period, the wealth may take values $w_1 < w_2 < \cdots < w_n$. For the DM, the second-period wealth distribution is ambiguous, which depends on the parameter $\theta$ and is represented by the vector $[w_1, w_2, \cdots, w_n; p_1(\theta), p_2(\theta), \cdots, p_n(\theta)]$. Assume the value of parameter $\theta$ can be characterized by a random variable $\tilde{\theta}$ with a probability distribution $F(\theta)$ defined on the set $\Theta$, that is, for one function $g(\cdot), E_g(\tilde{\theta}) = \int_{\Theta} g(\theta)dF(\theta)$.

Following Alary et al. (2013), we assume that ambiguity is concentrated on the state $i$. The ambiguous probability of the state $i$ is $p_i(\theta)$ and the ambiguous probability of the state $j$, $j \neq i$ is given by $p_j(\theta) = [1 - p_i(\theta)]\pi_j$, where, $\sum_{j \neq i} \pi_j = 1$ and $\pi_j$ is the probability of state $j$ conditional on the state is not $i$.

We propose the model (5) by following Hayashi and Miao (2011):
\[V(e, \tilde{\theta}) = W(w_0 - e, v^{-1}[E_{\theta}v \circ u^{-1}(U(e, \tilde{\theta}))]),\] (5)
where
\[U(e, \theta) = p_i(e, \theta)u(w_i(e)) + [1 - p_i(e, \theta)]\sum_{j \neq i} \pi_j u(w_j),\] (6)
where $u$ is the utility function defined over the wealth and represents the attitude toward income risk and $e$ is the effort level exerted by the DM in the first period.
Again we define \( \phi \equiv v \circ u^{-1} \), then we can rewrite the model (5) as:

\[
V(e, \tilde{\theta}) = W(w_0 - e, u^{-1}[\phi^{-1}(E_{\theta}\phi(U(e, \tilde{\theta})))]).
\] (7)

We assume \( u' > 0, \phi' > 0, u'' \leq 0, \phi'' \leq 0 \), and \( U(e, \theta) \) and \( U_e(e, \theta) \) are anti-comonotone in \( \theta \). The assumption that \( U(e, \theta) \) and \( U_e(e, \theta) \) are anti-comonotone in \( \theta \) reflects the effect of ambiguity (subjective prior beliefs \( \theta \)) on \( U \) and \( U_e \) are opposite.

Using the recursive smooth ambiguity preference model of Klibanoff et al. (2009), Berger (2013) examines a two-period model for self-protection and self-insurance problem:

\[
V(e, \tilde{\theta}) = u(w_0 - e) + \phi^{-1}[E_{\theta}\phi(U(e, \tilde{\theta}))].
\] (8)

As be mentioned in Hayashi and Miao (2011), their generalized recursive smooth ambiguity preference model nests the model of Klibanoff et al. (2009) as a special example. Here it is easy to find that Berger (2013)’s model is a special case of our model (7).

Without loss of generality, we assume \( p_i(e, \theta) \) is increasing in \( \theta \). For the unfavorable state \( i \), we assume \( w_i(e) \) is increasing in \( e \).

Let \( \max_e V(e, 0) \) denote the optimal problem with the ambiguity neutral DM (i.e., model (7) with a linear function \( \phi \)) and \( \max_e V(e, 1) \) denote the optimal problem with the ambiguity averse DM (i.e., model (7) with concave function \( \phi \)). Define \( e_0 \in \arg \max V(e, 0) \) and \( e_1 \in \arg \max V(e, 1) \). Nextly, via a comparison between the size of \( e_0 \) with \( e_1 \), we provide a set of general sufficient conditions to assure that the introduction of ambiguity aversion attitude increases the effort level.

**Proposition 3.1** We have \( e_1 \geq e_0 \) for a strictly DAAA DM with \( W_2 > 0, W_{12} \geq 0 \) and

\[
\frac{W_{22}(x,y)}{W_2(x,y)} \geq -\frac{u''(y)}{u'(y)}.
\]

Comparing to the conditions discussed by Alary et al. (2013) and Berger (2013), we obtain the additional conditions on the time preference: \( W_2 > 0, W_{12} \geq 0 \) and \( \frac{W_{22}(x,y)}{W_2(x,y)} \geq -\frac{u''(y)}{u'(y)} \). There is a simple intuition to this proposition: while strictly DAAA is the motivation for the effort, DMI and a high enough resistance to intertemporal substitution (or a low enough degree of risk aversion) reinforce these motivations. Actually, Berger’s (2013) work has an implicit assumption \( W(x, y) = u(x) + \beta u(y) \), and hence \( \frac{W_{22}(x,y)}{W_2(x,y)} = -\frac{u''(y)}{u'(y)} \). Therefore, Proposition 3.1 provides a generalized result.
3.1 Self-insurance

In model (7), when $p_i(e, \theta)$ is independent of $e$, that is, $p_i(e, \theta) = p_i(\theta), \forall e$, then

$$U(e, \theta) = p_i(\theta)u(w_i(e)) + [1 - p_i(\theta)] \sum_{j \neq i} \pi_j u(w_j).$$

(9)

Hence we are back to a self-insurance model. In this setting, we have

$$U_e(e, \theta) = p_i(\theta)u'(w_i(e))w_i'(e),$$

(10)

$$U_{e\theta}(e, \theta) = p_i'(\theta)u'(w_i(e))w_i'(e)$$

(11)

and

$$U_{\theta}(e, \theta) = p_i'(\theta)(u(w_i(e)) - \sum_{j \neq i} \pi_j u(w_j)).$$

(12)

Let $u(\varphi) = \sum_{j \neq i} \pi_j u(w_j)$, that is, we define $\varphi$ as the certainty equivalent of the second period wealth conditional on the information that the state is not $i$. If $\varphi \geq w_i(e), \forall e$, we can show $sgn(U_{\theta}(e, \theta)U_{e\theta}(e, \theta)) = -1$. Then Proposition 3.1 can be used to analyze the problem of self-insurance. In particular, following Snow (2011), we can consider the second period wealth distribution with two states: a loss state $w(e)$ and a no-loss state $w$. That is,

$$U(e, \theta) = p(\theta)u(w(e)) + [1 - p(\theta)]u(w).$$

(13)

Let $w(e) = w - L(e)$, generally, we assume both $L(e) \geq 0$ and $L'(e) \leq 0, \forall e$. $L(e) \geq 0$ and $L'(e) \leq 0$ mean that the wealth value of loss-state increases as the level of self-insurance increases, but the wealth value of loss-state may not become larger than no-loss state. That is, $w(e) \leq w$ and $w'(e) \geq 0, \forall e$. Thus we have

$$U_{\theta}(e, \theta) = p_i'(\theta)[u(w(e)) - u(w)] \leq 0$$

(14)

and

$$U_{e\theta}(e, \theta) = p_i'(\theta)u'(w(e))w_i'(e) \geq 0,$$

(15)

which imply $U(e, \theta)$ and $U_{e\theta}(e, \theta)$ are anti-comonotone in $\theta$. Then, from Proposition 3.1, we have

**Corollary 3.2** When the second period wealth distribution just has two states: a loss and a no-loss, we have $e_1 \geq e_0$ for a strictly DAAA DM with $W_2 > 0$, $W_{12} \geq 0$ and $-W_{22}(x,y) W_{22}(x,y) \geq u''(y) w''(y)$.  

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6$U_e(e, \theta)$ denotes $\frac{\partial U_e(e, \theta)}{\partial e}$. We follow the same subscript convention for the derivatives $U_{\theta}(e, \theta)$ and $U_{e\theta}(e, \theta)$. 

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3.2 Self-protection

In model (7), when $w_i(e) = w_i$, but $p_j(e, \theta)$ is dependent on $e$, that is,

$$U(e, \theta) = p_i(e, \theta)u(w_i) + [1 - p_i(e, \theta)] \sum_{j \neq i} \pi_j u(w_j).$$

(16)

Model (7) degenerates to a self-protection problem. We have

$$U_e(e, \theta) = \frac{\partial p_i(e, \theta)}{\partial e} (u(w_i) - \sum_{j \neq i} \pi_j u(w_j)),$$

(17)

$$U_{e\theta}(e, \theta) = \frac{\partial^2 p_i(e, \theta)}{\partial e \partial \theta} (u(w_i) - \sum_{j \neq i} \pi_j u(w_j))$$

(18)

and

$$U_{\theta}(e, \theta) = \frac{\partial p_i(e, \theta)}{\partial \theta} (u(w_i) - \sum_{j \neq i} \pi_j u(w_j)).$$

(19)

We know that, if $\frac{\partial^2 p_i(e, \theta)}{\partial e \partial \theta}$ and $\frac{\partial p_i(e, \theta)}{\partial \theta}$ have opposite sign, then $U(e, \theta)$ and $U_e(e, \theta)$ are anti-comonotone in $\theta$. Since we assume that $\frac{\partial p_i(e, \theta)}{\partial \theta} > 0$, then, from Proposition 3.1, we have,

**Corollary 3.3** If effort decreases the degree of ambiguity of state $i$ (i.e. $\frac{\partial p_i(e, \theta)}{\partial e}$ is decreasing in $\theta$) or does not affect the degree of ambiguity of state $i$, then, we have $e_1 \geq e_0$ for a strictly DAAA DM with $W_2 > 0$, $W_{12} \geq 0$ and $-\frac{W_{22}(x,y)}{W_{21}(x,y)} \geq -\frac{u''(y)}{u'(y)}$.

The condition that “effort decreases the degree of ambiguity of state $i$” is consistent to the assumption of Snow (2011). Snow (2011, p. 35) uses a positive and decreasing protection function $\rho(e)$ to capture the effect of the self-protect effort on the ambiguity degree of the loss state and shows ambiguity aversion increases the self-protection effort. Following his work, we assume that $p_i(e, \theta) = p_i(\theta)\rho(e), \forall (e, \theta)$ where $\rho(e) > 0, \rho'(e) < 0$. Then since $p'_i(\theta) > 0$, we obtain $\frac{\partial p_i(e, \theta)}{\partial e \partial \theta} = p'_i(\theta)\rho'(e) < 0$ which satisfies the condition in Corollary 3.3. Thus, in this situation, we conclude that ambiguity aversion increases the self-protection effort via Corollary 3.3.

4 Conclusion

Given risk aversion and ambiguity aversion, the paper qualitatively analyzes how prudence, strictly DAAA, intertemporal substitution and degree of absolute risk aversion jointly determine precautionary saving. It then characterizes the impact of risk, ambiguity and time attitudes on the demand for self-insurance and self-protection. The saving and insurance problems are
the fundamental problems in economics. Although there are many papers that analyze the role of intertemporal substitution, risk preference and ambiguity preference, as the authors acknowledge, none of them can completely separate the effect of these elements. The contribution of the paper is to show, under the framework of the generalized smooth ambiguity model, we can completely disentangle the effect of these elements via the concepts and techniques developed by Topkis (1998) and others.

5 Appendix

5.1 The proof of Proposition 2.1

This proof is essentially from a result in Topkis’s Monotonicity Theorem\(^7\). Wang and Li (2015) provide an intuitive explanation for the result in Topkis’s Monotonicity Theorem. We provide the detail of proof here for the sake of completeness.

Suppose \(s_0 > s_1\). Since

\[
V(s_0, 1) - V(s_1, 1) > V(s_0, 0) - V(s_1, 0) \quad (20)
\]

\[
\Leftrightarrow W(w_0 - s_0, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))]) - W(w_0 - s_1, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s_1))])
\]

\[
> W(w_0 - s_0, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s))) - W(w_0 - s_1, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s_1)))
\]

\[
\Leftrightarrow W(w_0 - s_0, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))]) - W(w_0 - s_0, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s_0)))
\]

\[
> W(w_0 - s_1, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s_1)))]) - W(w_0 - s_1, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s_1)))
\]

\[
\Leftrightarrow W(w_0 - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))]) - W(w_0 - s, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s)))
\]

is increasing in \(s\), therefore,

\[
W_1(w_0 - s, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s))) \geq W_1(w_0 - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))]) \quad (22)
\]

and

\[
\frac{W_2(w_0 - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))])}{u'(w^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_\theta + s)))])} \sum_{\theta} q_{\theta} \phi'(Eu(w + \tilde{\epsilon}_\theta + s)) E_u'(w + \tilde{\epsilon}_\theta + s)
\]

\[
= \frac{W_2(w_0 - s, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s)))}{u'(u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_\theta + s)))} \sum_{\theta} q_{\theta} Eu'(w + \tilde{\epsilon}_\theta + s) > 0 \quad (23)
\]

\(^7\)For Topkis’s Monotonicity Theorem, please see, Sundaram (1996, Theorem 10.6), Topkis (1998, Theorem 2.8.5) and Vives (2000, Theorem 2.3).
imply (20).

Since ambiguity aversion \((\phi'' \leq 0)\) implies that

\[
\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s))) \leq \sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_{\theta} + s),
\]

then we have,

\[
u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s)))] \leq \sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_{\theta} + s),
\]

thus \(W_{12} \geq 0\) together with \(\phi'' \leq 0\) imply (22).

We notice that

\[
\begin{align*}
W_{2}(w_{0} - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s)))] & = \frac{\sum_{\theta} q_{\theta} \phi'(Eu(w + \tilde{\epsilon}_{\theta} + s))}{\phi'(\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s))))} \sum_{\theta} q_{\theta} Eu'(w + \tilde{\epsilon}_{\theta} + s) \\
+ W_{2}(w_{0} - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s)))] & = \frac{\sum_{\theta} q_{\theta} \phi'(Eu(w + \tilde{\epsilon}_{\theta} + s))}{\phi'(\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s))))} \sum_{\theta} q_{\theta} Eu'(w + \tilde{\epsilon}_{\theta} + s)
\end{align*}
\]

Again we note that \(\phi'' \leq 0\) implies \(u^{-1}(\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s)))) \leq u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_{\theta} + s))\) (i.e., (25)). Because \(-\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)}\) means that \(W_{2}(x,y)\) declines more quickly than \(\ln(u'(y))\) in the variable \(y\), \(\frac{W_{22}(x,y)}{W_{2}(x,y)}\) is decreasing in \(y\). Hence

\[
W_{2}(w_{0} - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s)))] \geq W_{2}(w_{0} - s, u^{-1}(\sum_{\theta} q_{\theta} Eu(w + \tilde{\epsilon}_{\theta} + s)))
\]

(27)

If \(Eu(w + \tilde{\epsilon}_{\theta} + s)\) and \(Eu'(w + \tilde{\epsilon}_{\theta} + s)\) are anti-comonotone in \(\theta\), then \(\phi'' \leq 0\) implies

\[
cov(\phi'(Eu(w + \tilde{\epsilon}_{\theta} + s)), Eu'(w + \tilde{\epsilon}_{\theta} + s)) \geq 0.
\]

(28)

Let \(\alpha(s) = \frac{\sum_{\theta} q_{\theta} \phi'(Eu(w + \tilde{\epsilon}_{\theta} + s))}{\phi'(\phi^{-1}(\sum_{\theta} q_{\theta} \phi(Eu(w + \tilde{\epsilon}_{\theta} + s))))}\), from an inequality in Gollier (2001, the inequality (20.14)), we know that that strictly DAAA implies \(\alpha(s) > 1\).

Hence, recalling both (27) and (28), we can show that strictly DAAA, \(W_{2} > 0\), \(\phi'' \leq 0\) and

\[-\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)}\] assure that (26) holds, i.e., (23) holds.

Therefore \(W_{2} > 0\), \(W_{12} \geq 0\), \(-\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)}\), \(\phi'' \leq 0\) and strictly DAAA imply

\[
V(s_{0}, 1) - V(s_{1}, 1) > V(s_{0}, 0) - V(s_{1}, 0).
\]

(29)

Since \(s_{0} \in \arg\max V(s, 0)\) implies

\[
V(s_{0}, 0) - V(s_{1}, 0) \geq 0,
\]

(30)
combining (29) and (30) generates \( V(s_0,1) - V(s_1,1) > 0 \) which contradicts \( s_1 \in \text{arg max } V(s,1) \).

Hence, we have \( s_0 \leq s_1 \).

5.2 The proof of Proposition 2.2

Suppose \( s_c > s_1 \). Since

\[
V(s_c,1) - V(s_1,1) > V(s_c,c) - V(s_1,c)
\]

(31)

\[
\Leftrightarrow W(w_0 - s_c, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s_c)))] - W(w_0 - s_1, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s_1)))]
\]

\[
> W(w_0 - s_c, w + s_c) - W(w_0 - s_1, w + s_1)
\]

\[
\Leftrightarrow W(w_0 - s_c, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s_c)))] - W(w_0 - s_1, w + s_1)
\]

\[
> W(w_0 - s_1, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s_1)))] - W(w_0 - s_1, w + s_1)
\]

\[
\Leftrightarrow W(w_0 - s, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))]) - W(w_0 - s, w + s)
\]

(32)

is increasing in \( s \),

hence

\[
W_1(w_0 - s, w + s) \geq W_1(w_0 - s, , u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))]
\]

(33)

and

\[
\frac{W_2(w_0 - s, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] - \sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s))}{u'(u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] - \phi'[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))]})
\]

\[
- W_2(w_0 - s, w + s) > 0
\]

(34)

imply (31).

Since ambiguity aversion (\( \phi'' \leq 0 \)) and risk aversion (\( u'' \leq 0 \)) imply that \( \phi^{-1}[\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s))] \leq \sum_\theta q_\theta Eu(w + \bar{\epsilon}_\theta + s) \leq u(w + s) \) (where, the second inequality is gotten by \( \sum_\theta q_\theta E \bar{\epsilon}_\theta = 0 \)), we obtain

\[
u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] \leq u^{-1}(\sum_\theta q_\theta Eu(w + \bar{\epsilon}_\theta + s)) \leq w + s,
\]

(35)

thus, \( W_{12} \geq 0 \) implies (33).

We notice that

\[
\Leftrightarrow W_2(w_0 - s, u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] - \sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s))}{u'(u^{-1}[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] - \phi'[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))]})
\]

\[
+ W_2(w_0 - s, \phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))] - \sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s))}{\phi'[\phi^{-1}(\sum_\theta q_\theta (Eu(w + \bar{\epsilon}_\theta + s)))]})
\]

\[
> W_2(w_0 - s, w + s).
\]

(36)
Again ambiguity aversion and risk aversion imply that 
\[ u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta}\phi(Eu(w + \tilde{\theta} + s)))] \leq u^{-1}(\sum_{\theta} q_{\theta}Eu(w + \tilde{\theta} + s)) \leq w + s \] (i.e., (35)). Because 
\[ -\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u'(y)}{u''(y)} \] means that \( \ln(W_{2}(x,y)) \) declines more quickly than \( \ln(u'(y)) \) in the variable \( y \), \( \frac{W_{22}(x,y)}{u'(y)} \) is decreasing in \( y \). Therefore, we have

\[
\begin{align*}
\frac{W_{2}(w_{0} - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta}\phi(Eu(w + \tilde{\theta} + s)))])}{u'(w + s)} & \geq \frac{W_{2}(w_{0} - s, w + s)}{u'(w + s)} \\
& \geq \frac{W_{2}(w_{0} - s, w + s)}{\sum_{\theta} q_{\theta}Eu'(w + \tilde{\theta} + s)},
\end{align*}
\]

(37)

where the second inequality in (37) is gotten by noting that \( u'' \geq 0 \Rightarrow u'(w + s) \leq \sum_{\theta} q_{\theta}Eu'(w + \tilde{\theta} + s). \) This means

\[
\begin{align*}
& \frac{W_{2}(w_{0} - s, u^{-1}[\phi^{-1}(\sum_{\theta} q_{\theta}\phi(Eu(w + \tilde{\theta} + s)))])}{u'(w + s)} - \sum_{\theta} q_{\theta}Eu'(w + \tilde{\theta} + s) \\
& \geq W_{2}(w_{0} - s, w + s).
\end{align*}
\]

(38)

If \( Eu(w + \tilde{\theta} + s) \) and \( Eu'(w + \tilde{\theta} + s) \) are anti-comonotone in \( \theta \), then \( \phi'' \leq 0 \) implies

\[
\text{cov}(\phi'(Eu(w + \tilde{\theta} + s)), Eu'(w + \tilde{\theta} + s)) \geq 0.
\]

(39)

Let \( \alpha(s) = \frac{\sum_{\theta} q_{\theta}\phi'(Eu(w + \tilde{\theta} + s))}{\phi^{-1}(\sum_{\theta} q_{\theta}\phi(Eu(w + \tilde{\theta} + s)))} \), from an inequality in Gollier (2001, the inequality (20.14)), we know that strictly DAAA implies \( \alpha(s) > 1 \).

So recalling (38) and (39), we can show that \( W_{2} \geq 0, \phi'' \leq 0, u'' \leq 0, u''' \geq 0, -\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)} \) and strictly DAAA assure that (36) holds.

Therefore, \( W_{2} \geq 0, W_{12} \geq 0, \phi'' \leq 0, u'' \leq 0, u''' \geq 0, -\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)} \) and strictly DAAA imply

\[
V(s_{c},1) - V(s_{1},1) > V(s_{c},c) - V(s_{1},c);
\]

(40)

and \( s_{c} \in \arg \max V(s, c) \) implies

\[
V(s_{c},c) - V(s_{1},c) \geq 0;
\]

(41)

thus, combining (40) with (41) generates \( V(s_{c},1) - V(s_{1},1) > 0 \) which contradicts \( s_{1} \in \arg \max V(s,1) \).

Hence, we have \( s_{c} \leq s_{1} \).

5.3 The proof of Proposition 3.1

Suppose \( e_{0} > e_{1} \). Since

\[
V(e_{0},1) - V(e_{1},1) > V(e_{0},0) - V(e_{1},0)
\]
\[ W(w_0 - e_0, u^{-1}[\phi^{-1}(E_\theta \phi(U(e_0, \tilde{\theta})))]) - W(w_0 - e_1, u^{-1}[\phi^{-1}(E_\theta \phi(U(e_1, \tilde{\theta})))]) \]

\[ > W(w_0 - e_0, u^{-1}[\phi^{-1}(E_\theta \phi(U(e_0, \tilde{\theta})))]) - W(w_0 - e_1, u^{-1}(E_\theta \phi(U(e_1, \tilde{\theta}))) \]

\[ W(w_0 - e_0, u^{-1}[\phi^{-1}(E_\theta \phi(U(e_0, \tilde{\theta})))]) - W(w_0 - e_0, u^{-1}(E_\theta \phi(U(e_0, \tilde{\theta}))) \]

\[ > W(w_0 - e_1, u^{-1}[\phi^{-1}(E_\theta \phi(U(e_1, \tilde{\theta})))]) - W(w_0 - e_1, u^{-1}(E_\theta \phi(U(e_1, \tilde{\theta}))) \]

\[ W(w_0 - e, u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) - W(w_0 - e, u^{-1}(E_\theta \phi(U(e, \tilde{\theta}))) \]

is increasing in \( e \),

therefore,

\[ W_1(w_0 - e, u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) \leq W_1(w_0 - e, u^{-1}(E_\theta \phi(U(e, \tilde{\theta}))) \]

and

\[ \frac{W_2(w_0 - e, u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) - E_\theta \phi(U(e, \tilde{\theta}))) \]

\[ \frac{w'[u^{-1}(\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))])} {\phi'[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) \]

\[ \frac{W_2(w_0 - e, u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} {w'(u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} > 0 \]

(42)

imply (42).

Since ambiguity aversion \( (\phi'' \leq 0) \) implies that

\[ \phi^{-1}[E_\theta \phi(U(e, \tilde{\theta}))] \leq E_\theta \phi(U(e, \tilde{\theta})) \]

which implies that

\[ u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) \leq u^{-1}(E_\theta \phi(U(e, \tilde{\theta})) \]

(43)

thus \( W_{12} \geq 0 \) together with \( \phi'' \leq 0 \) imply (43).

We notice that

\[ \frac{W_2(w_0 - e, u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) - E_\theta \phi(U(e, \tilde{\theta}))) \]

\[ \frac{w'[u^{-1}(\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))])} {\phi'[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) \]

\[ \frac{W_2(w_0 - e, u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} {w'(u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} \]

(46)

Again recalling that \( \phi'' \leq 0 \) implies \( u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]) \leq u^{-1}(E_\theta \phi(U(e, \tilde{\theta})) \) i.e. (45). Because \( \frac{W_2(x,y)}{W_{12}(x,y)} \geq \frac{w''(y)}{w'(y)} \) means that \( \ln(W_2(x,y)) \) declines more quickly than \( \ln(u'(y)) \) in the variable \( y \), \( \frac{W_2(x,y)}{w'(y)} \) a decreasing function in \( y \). Hence

\[ \frac{W_2(w_0 - e, u^{-1}[\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))])}{u'[u^{-1}(\phi^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} \geq \frac{W_2(w_0 - e, u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))]} {u'(u^{-1}(E_\theta \phi(U(e, \tilde{\theta})))} \]

(47)
If $U(e, \theta)$ and $U_e(e, \theta)$ are anti-comonotone in $\theta$, then the $\phi'' \leq 0$ implies that

$$\text{cov}(\phi'(U(e, \tilde{\theta})), U_e(e, \tilde{\theta})) \geq 0.$$  

(48)

Let $\alpha(e) = \frac{E_\theta[\phi'(U(e, \tilde{\theta}))]}{\phi^{-1}(E_\theta[\phi(U(e, \tilde{\theta}))])}$, from an inequality in Gollier (2001, the inequality (20.14)), we know that strictly DAAA implies $\alpha(e) > 1$.

Hence, recalling both (47) and (48), we can show that strictly DAAA, $W_2 > 0, \phi'' \leq 0$ and

$$\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)}$$

mean that (46) holds.

Therefore $W_2 > 0, W_{12} \geq 0, \phi'' \leq 0, -\frac{W_{22}(x,y)}{W_{2}(x,y)} \geq -\frac{u''(y)}{u'(y)}$ and strictly DAAA imply

$$V(e_0, 1) - V(e_1, 1) > V(e_0, 0) - V(e_1, 0).$$  

(49)

Since $e_0 \in \arg \max V(e, 0)$ implies

$$V(e_0, 0) - V(e_1, 0) \geq 0;$$  

(50)

combining (49) and (50) generates $V(e_0, 1) - V(e_1, 1) > 0$ which contradicts $e_1 \in \arg \max V(e, 1)$. Hence, we have $e_0 \leq e_1$.

6 References


Gierlinger, J., Gollier, C., 2012. Socially efficient discounting under ambiguity averse-
sion. University of Toulouse IDEI working paper no. 561.
Hayashi, T., Miao, J., 2011. Intertemporal Substitution and Recursive Smooth Am-
biguity Preferences. Theoretical Economics 6, 423-475.
559-591.
Li, J., Wang, J., 2014, Precautionary saving with Selden/Kreps-Porteus preferences
revisited. Working paper.
Kimball, M., 1990. Precautionary savings in the small and in the large, Econometrica
58, 53-73.
Kimball, M.S., Weil, P., 1992. Precautionary saving and consumption smoothing
across time and possibilities. NBER Working Paper 3976, Cambridge, MA.
Kimball, M.S., Weil, P., 2009. Precautionary saving and consumption smoothing
across time and possibilities. Journal of Money, Credit and Banking 41, 245-284.
Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A smooth model of decision making
under ambiguity. Econometrica 78, 1849-1892.
Klibanoff, P., Marinacci, M., Mukerji, S., 2009. Recursive smooth ambiguity prefer-
Koopmans, T. C., 1960. Stationary ordinal utility and impatience, Econometrica 28,
287-309.
Koopmans, T. C., 1986. Representation of preference ordering over time, in Decision
and Organization, 2nd edn (Eds) C. B. Mcguire and R. Radner, University of
Minnesota Press, Minneapolis, MN.


