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Mingming LENG
Department of Computing and Decision Sciences, Lingnan University

Mahmut PARLAR
McMaster University

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A Note on Optimal "Riskless" and "Risk" Prices for the Newsvendor Problem with an Assembly Cost[*]

Mingming Leng[†]

Department of Computing and Decision Sciences

Lingnan University

8 Castle Peak Road, Tuen Mun

Hong Kong

Mahmut Parlar[‡]

DeGroote School of Business

McMaster University

Hamilton, Ontario L8S 4M4

Canada

Abstract

For an assemble-to-order firm operating in a single period, we find that, for the multiplicative demand case the optimal riskless price may or may not be higher than the optimal risk price. However, for the additive demand case, the optimal riskless price is always greater than the optimal risk price.

Key words: Assembly/processing cost, risk price, riskless price, stocking factor, base price.

1 Introduction

Many firms adopt the assemble-to-order (ATO) operational strategy to reduce costs and increase flexibility, see, e.g., Simchi-Levi et al. [6, Ch. 6]. Dell provides one of the best-known examples where the ATO strategy is used to assemble final products only when the orders of its customers arrive. Such an ATO strategy has also been widely applied in a variety of single-period operations. For instance, during Valentine's Day, some stores trim and then pack a bunch of roses in response to a specific customer request. During Christmas shopping season, some stores may offer their customers to set up Christmas trees; other stores may provide wrapping services to their customers who buy gifts at those stores, etc.

As the above examples illustrate, a firm that operates in the ATO mode does incur an assembly, or processing, cost (hereafter, simply the "assembly cost"). In our note, we consider an ATO firm which determines the sale price p and the quantity q for a single period, and sells the products to satisfy the price-dependent random demand $D(p, \varepsilon)$ in either multiplicative or additive form as follows:

1. For the multiplicative case, the demand function is,

$$D(p, \varepsilon) = y(p)\varepsilon = ap^{-b}\varepsilon, \tag{1}$$

where $y(p) = ap^{-b}$ is the deterministic component of the random demand with $a > 0$, $b > 1$. The error term is ε with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$ taking values in the range $[A, B]$ with $A > 0$, and mean $E(\varepsilon) = \mu$ and variance $\text{Var}(\varepsilon) = \sigma^2$.

2. For the additive case, the demand function is,

$$D(p, \varepsilon) = y(p) + \varepsilon = a - bp + \varepsilon, \tag{2}$$

where $y(p) = a - bp$ is the deterministic component of the random demand with a, $b > 0$. The error term is ε with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$ taking values in the range $[A, B]$ with $A > -a$ with mean $E(\varepsilon) = \mu$ and variance $\text{Var}(\varepsilon) = \sigma^2$.

When a customer buys a product, the ATO firm incurs an assembly cost of m dollars per unit. Since the firm's realized (actual) sales is $\min[q, D(p, \varepsilon)]$, the total assembly cost is computed as $m \cdot \min[q, D(p, \varepsilon)]$. Thus, we write the ATO firm's expected profit function as follows:

$$\Pi(p, q) = (p - m)E\{\min[q, D(p, \varepsilon)]\} - cq - sE[D(p, \varepsilon) - q]^+ - hE[q - D(p, \varepsilon)]^+, \tag{3}$$

where c is the firm's unit cost of acquisition; s is the unit penalty cost for unsatisfied demand; h is the unit cost of disposing the leftovers at the end of the single period. (If $h < 0$, then it represents the unit salvage value.)

In [1], Karlin and Carr assumed price-dependent demand $D(p, \varepsilon)$ in multiplicative form (1), and investigated joint pricing and stocking decisions for a firm which operates in the make-to-stock mode in the newsvendor setting. Different from an ATO firm, the make-to-stock firm assembles final products before customers' orders arrive. Thus, Karlin and Carr [1] didn't consider the assembly cost m when the make-to-stock firm sells a product in a single period, and proved that, for the multiplicative case, the make-to-stock firm's optimal riskless price is always smaller than the optimal risk price. This result was also demonstrated by Petruzzi and Dada [4]. [The optimal "riskless" price is computed

when ε is deterministic, i.e., $E(\varepsilon) = \mu$ and $\text{Var}(\varepsilon) = 0$; and the optimal "risk" price is obtained when the error term ε in the demand function $D(p, \varepsilon)$ is random as given above, i.e., $E(\varepsilon) = \mu$ and $\text{Var}(\varepsilon) = \sigma^2$]. Zabel [9] again analyzed the problem in Karlin and Carr [1] but considered general cost functions and variations in initial inventory levels. Zabel proved that the make-to-stock firm's optimal riskless price may be greater than its optimal risk price, if the firm's production cost is strictly convex and its production quantity is sufficiently low.

In [3], Mills assumed price-dependent demand $D(p, \varepsilon)$ in additive form (2), and investigated joint pricing and quantity decisions for a make-to-stock firm in the newsvendor setting. Mills proved that, for the additive demand case, the optimal riskless price is always greater than the optimal risk price. This result was also shown by Karlin and Carr [1] and Petruzzi and Dada [4].

Young [8] developed a generalized demand model that subsumes both the multiplicative and the additive models. Such a demand model was specified as $D(p, \varepsilon) = y_1(p)\varepsilon + y_2(p)$ where $y_1(p)$ and $y_2(p)$ are two general functions of the price p, and ε is the error term with $E(\varepsilon) = \mu$ and $\text{Var}(\varepsilon) = \sigma^2$. Young computed the coefficient of variation of demand as $y_1(p)\sigma/[y_1(p)\mu + y_2(p)]$ and the variance of demand as $y_1(p)^2\sigma^2$. For a make-to-stock firm, Young showed that, if the coefficient of variation of demand is non-increasing in p while the variance is decreasing in p, then $p^* > p^0$; if the variance of demand is non-decreasing in p while the coefficient of variation of demand is increasing in p, then $p^* < p^0$.

As we discussed previously, in practice a firm may use the ATO strategy and thus incur an assembly cost when a product is sold. In light of this observation, we pose the following question: Do the above results (on the comparison between the optimal riskless and risk prices for the make-to-stock firm) still hold for the ATO operation? To address the question, in this note we maximize $\Pi(p, q)$ in (3) to find the optimal riskless and risk prices, and compare these two prices for both the multiplicative and the additive demand cases. We show that when the assembly cost is incorporated into the ATO firm's expected profit function, under certain conditions, for the multiplicative demand case the optimal riskless price is higher than the optimal risk price; this result differs from the one found in Karlin and Carr [1] and Petruzzi and Dada [4]. However, for the additive demand case even with the assembly cost, the result for the make-to-stock firm remains valid, i.e., the optimal riskless price is always greater than the optimal risk price.

2 Comparison between Optimal Riskless and Risk Prices

In this section, we present an analytical comparison of the optimal riskless and risk prices for the multiplicative and additive demand cases. Our results are illustrated by some numerical examples.

2.1 Comparison for the Multiplicative Demand Case

We now show that when the assembly cost is incorporated into the ATO firm's expected profit function, under certain conditions, for the multiplicative demand case the optimal riskless price is higher than the optimal risk price.

Theorem 1 For the multiplicative case with random demand and the positive unit assembly cost m, the optimal "risk" price p^* and optimal order quantity q^* are *finite* solutions that satisfy the conditions,

$$z[1 - \xi(p^*)] + \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\mu = \left[1 - (p^* - m + h + s)\frac{b}{p^*}\right]\int_z^B xf(x)\ dx, \qquad (4)$$

$$q^* = y(p^*)z, \qquad (5)$$

where,

$$\xi(p^*) \equiv \frac{p^* - m + s - c}{p^* - m + s + h}, \quad z = F^{-1}(\xi(p^*)), \qquad (6)$$

and z represents the stocking factor, as defined by Petruzzi and Dada [4].

Proof. For the multiplicative demand case, we write the firm's expected profit function as

$$\Pi(p, q) = -(c + h)q + (p - m + h)y(p)\mu + (p - m + h + s)\int_{q/y(p)}^B [q - y(p)x]f(x)\ dx$$

where, as Petruzzi and Dada [4] assumed, the demand function is $D(p, \varepsilon) = y(p)\varepsilon$; the deterministic term is $y(p) = ap^{-b}$ ($a > 0$, $b > 1$) and the error term ε takes values in the range $[A, B]$ with $A > 0$ and $B < \infty$.

The first-order partial derivatives of $\Pi(p, q)$ w.r.t. p and q are

$$\frac{\partial \Pi}{\partial p} = q\left[1 - F\left(\frac{q}{y(p)}\right)\right] + \left[1 - (p - m + h)\frac{b}{p}\right]y(p)\int_A^{q/y(p)} xf(x)dx + s\frac{b}{p}y(p)\int_{q/y(p)}^B xf(x)\ dx, \quad (7)$$

and,

$$\frac{\partial \Pi}{\partial q} = -(c + h) + (p - m + h + s)\left[1 - F\left(\frac{q}{y(p)}\right)\right]. \qquad (8)$$

We now show the existence of a finite optimal solution for the ATO firm which incurs the assembly cost when the demands are satisfied. We set $\partial \Pi(p, q)/\partial q$ in (8) to zero, and find that $F(q/y(p)) = \xi(p)$, where $\xi(p)$ is given as in (6). Then, we compute

$$q(p) = y(p)z, \qquad (9)$$

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where $z = F^{-1}(\xi(p))$ represents the stocking factor as in [4]. Note that $\xi(p) < 1$ for following reasons: When we maximize the expected profit, the optimal sale price p^* must be greater than the sum of the acquisition cost c and the assembly cost m, i.e., $p^* > c + m$; otherwise, there would be a loss of $\$(c + m - p)$ for each unit sold. Thus, the optimal price p^* (that maximizes the firm's profit) must satisfy the inequality $0 < \xi(p^*) < 1$, which implies that $A < z < B$ because $F(A) = 0$ and $F(B) = 1$. As a result, we find that the optimal quantity q^* must assume a finite value satisfying the condition (9). Substituting (9) into (7) gives (4).

As we argued above, p^* must be greater than $c + m$. In order to maximize the expected profit, one must assure that the customer demand is nonzero. To guarantee nonzero demand, the optimal price must be finite; otherwise, if p approaches infinity, then demand will be zero since, from (1) we would have $\lim_{p \to \infty} D(p, \varepsilon) = \lim_{p \to \infty} a p^{-b} \varepsilon = 0$, where $a > 0$, $b > 1$ and $\varepsilon \in [A, B]$ with $A > 0$. We thus conclude that the optimal price p^* must take a finite value satisfying the first-order condition (4).

In conclusion, for the ATO firm, a *finite* optimal price p^* and a *finite* optimal quantity q^* always exist for the multiplicative case, and the optimal solution (p^*, q^*) must satisfy the first-order optimality conditions (4) and (5). ∎

It is possible to compute the second-order partial and mixed derivatives and form the Hessian in order to examine the concavity of the expected profit function in (3) and hence the uniqueness of the solution. However, this is not necessary for our analysis as our objective is to compare the optimal riskless and risk prices in two different problems. Even for the special case of $m = 0$, i.e., for the make-to-stock firm, Petruzzi and Dada [4] did not examine the concavity properties but only demonstrated that a *finite* optimal solution (p^*, q^*) can be found by solving first-order conditions. In our note, since the optimal solution is finite (as shown above), similar to Petruzzi and Dada [4], we can use the first-order conditions to compare the riskless and risk prices for the ATO firm.

It is easy to find a simple expression for the optimal riskless prices for the multiplicative case, as shown in the following theorem.

Theorem 2 The optimal riskless price p^0 for the multiplicative case is computed as $p^0 = b(m + c)/(b-1)$. ◀

Next, we compare the optimal riskless and risk prices for the multiplicative demand case.

Theorem 3 For the multiplicative demand case, the optimal risk price p^* may be greater than, may

be equal to or may be less than the optimal riskless price p^0. More specifically,

$$\begin{cases} p^* > p^0, & \text{if } \kappa > 1/b, \\ p^* = p^0, & \text{if } \kappa = 1/b, \\ p^* < p^0, & \text{if } \kappa < 1/b, \end{cases}$$

where $\kappa \equiv (p^* - m - c)/p^*$ is the well-known "Lerner index" (i.e., the ratio of the unit profit to the price; see Lerner [2]); and b, a parameter in the multiplicative demand function (1), is the "price elasticity of demand" (see, e.g., Wang *et al.* [7]).

Proof. As Theorem (1) indicates, the optimal price p^* satisfies the following equation

$$F^{-1}(\xi(p^*))[1 - \xi(p^*)] + \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\mu = \left[1 - (p^* - m + h + s)\frac{b}{p^*}\right]\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx,$$

which can be re-written as

$$\frac{1}{p^*} = \frac{b-1}{(m+c)b} + \frac{\omega + (c+h)b\mu}{p^*(m+c)b\mu} = \frac{1}{p^0} + \frac{\omega + (c+h)b\mu}{p^*(m+c)b\mu},$$

with

$$\omega \equiv p^*\left\{\left[1 - (p^* - m + h + s)\frac{b}{p^*}\right]\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx - F^{-1}(\xi(p^*))[1 - \xi(p^*)]\right\}. \qquad (10)$$

To determine whether p^* is greater or smaller than p^0, we need to examine the sign of the term $[\omega + (c+h)b\mu]/[p^*(m+c)b\mu]$, or simply, the sign of the term $[\omega + (c+h)b\mu]$. Now, the RHS of (4) can be re-written as,

$$\left[1 - (p^* - m + h + s)\frac{b}{p^*}\right]\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx$$

$$= \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx - s\frac{b}{p^*}\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx.$$

Moving the first term to the LHS of (4), we have

$$F^{-1}(\xi(p^*))[1 - \xi(p^*)] + \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\mu - \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\int_{F^{-1}(\xi(p^*))}^{B} xf(x)\,dx$$

$$= F^{-1}(\xi(p^*))[1 - \xi(p^*)] + \left[1 - (p^* - m + h)\frac{b}{p^*}\right]\int_{A}^{F^{-1}(\xi(p^*))} xf(x)\,dx,$$

because $\mu = \int_{A}^{B} xf(x)\,dx$.

As a result, the equality (4) can be re-written as

$$F^{-1}(\xi(p^*))[1 - \xi(p^*)] + \left[1 - (p^* - m + h)\frac{b}{p^*}\right] \int_A^{F^{-1}(\xi(p^*))} x f(x) \, dx = -s\frac{b}{p^*} \int_{F^{-1}(\xi(p^*))}^B x f(x) \, dx,$$

which is negative. Since $F^{-1}(\xi(p^*))[1 - \xi(p^*)] > 0$ and $\int_A^{F^{-1}(\xi(p^*))} x f(x) \, dx$, we find that

$$[1 - (p^* - m + h)b/p^*] < 0,$$

and thus, $\omega < 0$.

Even though $\omega < 0$, we cannot immediately determine that $\omega + (c + h)b\mu < 0$, because, if $(c+h)b\mu$ is sufficiently large, the term $[\omega + (c+h)b\mu]$ could be greater than zero. More specifically, if $\omega + (c+h)b\mu < 0$, then $1/p^* < 1/p^0$, and consequently, $p^* > p^0$; if $\omega + (c+h)b\mu = 0$, then $1/p^* = 1/p^0$, and so, $p^* = p^0$; if $\omega + (c + h)b\mu > 0$, then $1/p^* > 1/p^0$, and thus, $p^* < p^0$.

We conclude from the above that the ATO firm's optimal risk price p^* may be greater than, may be equal to or may be smaller than the firm's optimal riskless price p^0. However, since the expression (10) for ω is very complicated, we cannot provide any meaningful managerial explanation for the term $[\omega + (c + h)b\mu]$. In order to find the meaningful condition for the comparison between p^* and p^0, we compute $(1/p^* - 1/p^0)$ as,

$$\frac{1}{p^*} - \frac{1}{p^0} = \frac{1}{p^*} - \frac{b - 1}{(m + c)b} = \frac{p^* - (p^* - m - c)b}{p^*(m + c)b} = \frac{1}{p^0} + \frac{1/b - \kappa}{m + c}.$$

where $\kappa \equiv (p^* - m - c)/p^*$ is the ratio of the firm's unit profit (i.e., $p - m - c$) to the price p. (Note that, in this note, p, m and c represent the firm's price, unit assembly cost and unit acquisition cost, respectively.) Such a ratio, first introduced in 1934 by the economist Lerner [2], was named as "Lerner index", which is commonly used to measure a firm's market power and has been widely applied to investigate a large number of business- and economics-related research problems.

According to our previous analysis, we learn that p^* may or may not be greater than than p^0; this implies that the Lerner index κ may be greater than, may be equal to or may be less than $1/b$, (i.e., the inverse of the price elasticity of demand). Hence, we can draw the following conclusions: if $\kappa > 1/b$, then $1/p^* < 1/p^0$, or, $p^* > p^0$; if $\kappa = 1/b$, then $1/p^* = 1/p^0$, or, $p^* = p^0$; otherwise, if $\kappa < 1/b$, then $1/p^* > 1/p^0$, or, $p^* < p^0$. This proves the theorem. ∎

From Theorem 3 we find that the ATO firm's optimal risk price p^* may be greater than, equal to, or smaller than its optimal riskless price p^0. This is *different* from the result of Karlin and Carr [1] where p^* is always greater than p^0 when the firm is a make-to-stock operation and the assembly cost

m is not involved. Moreover, from Theorem 3 we draw an interesting and important conclusion, as described in the following remark.

Remark 1 We learn from Theorem 3 that whether or not an ATO firm's optimal risk price p^* is greater than its optimal riskless price p^0 depends on the comparison between κ (i.e., the Lerner index) and $1/b$ (i.e., the inverse of the price elasticity of demand). It is interesting to note that many researchers have used the comparison between the Lerner index and the inverse of the price elasticity of demand to analyze their problems. For example, in [2], Lerner found that the Lerner Index is equal to the inverse of the price elasticity of demand when a firm adopts the optimal price that maximizes its profit (i.e., sale revenue minus acquisition cost).

As Lerner [2] showed, the Lerner Index reflects a firm's market power, and the firm with a higher value of Lerner index has a greater market power. Hence, Theorem 3 implies that, when an ATO firm adopts its optimal risk (riskless) price that maximizes its random (deterministic) profit involving the assembly cost,

1. if the ATO firm's market power (measured by its Lerner index) is greater than the inverse of the price elasticity of demand, then its optimal risk price p^* is greater than its optimal riskless price p^0;

2. if the ATO firm's market power (measured by its Lerner index) is equal to the inverse of the price elasticity of demand, then its optimal risk price p^* is equal to its optimal riskless price p^0;

3. if the ATO firm's market power (measured by its Lerner index) is smaller than the inverse of the price elasticity of demand, then its optimal risk price p^* is smaller than its optimal riskless price p^0.

As shown above, when an ATO firm incurs the assembly cost in the newsvendor setting, the firm's market power (Lerner index) significantly affects the impact of demand randomness on its pricing decision. ♦

2.2 Comparison for the Additive Demand Case

We now consider the comparison between optimal riskless and risk prices for the additive demand case.

Theorem 4 For the additive demand case, the finite optimal risk price p^* is determined by solving,

$$2bp^* + \int_A^z F(x)dx - z = a + b(m + c), \tag{11}$$

and the finite optimal production quantity q^* is found as $q^* = y(p^*) + z$, where $\xi(p^*)$ and z are both

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defined as in (**6**). ◄

We omit the proof of the above theorem because it is similar to the proof of Theorem 5 in Karlin and Carr [1]. When we assume deterministic demand, the optimal riskless price for the additive case can be calculated easily.

Theorem 5 For the additive demand case, the optimal riskless price is $p^0 = [a + b(m + c) + \mu]/2b$. ◄

By using the results in Theorems 4 and 5, we compare the optimal riskless and risk prices below.

Theorem 6 For the additive demand case, the globally-optimal solution p^* is always less than the optimal riskless price p^0, i.e., $p^* < p^0$.

Proof. For the additive demand case, we solve equation (11) for p^*, and find that $p^* = p^0 - (2b)^{-1} \int_z^B (x - z)f(x) \, dx$, which implies $p^* < p^0$. ∎

In Theorem 6 we have found the same result for the additive case as that in Mills [3]. This means that the ATO and make-to-stock operations have the same insights regarding the price comparison; that is, for any type of operation, the firm should always set the optimal riskless price (for the deterministic demand) greater than the optimal risk price (for the random demand).

3 Other Discussions: Stocking Factor and Base Price

Petruzzi and Dada [4, Section 1.3] calculated the "stocking factor" and "base price", and obtained unified results for the multiplicative and the additive demand cases. In this section, we compute the two concepts for the ATO firm, in order to examine whether or not the unified results for the make-to-stock firm still hold for the ATO firm.

We begin by investigating the stocking factor z, which was defined as in (6). For the make-to-stock firm, Petruzzi and Dada [4, Theorem 3] found that, for both the multiplicative and the additive cases, z can be computed as $z = \mu + SF \times \sigma$, where SF denotes the safety factor that was defined by Silver and Peterson [5] as $SF \equiv \{q - E[D(p, \varepsilon)]\}/\sqrt{\text{Var}[D(p, \varepsilon)]}$.

Theorem 7 For both the multiplicative and the additive cases, the stocking factor z for the ATO firm can be expressed as $z = \mu + SF \times \sigma$. That is, the unified result for the make-to-stock firm holds for the ATO firm. ◄

The proof of the above theorem is omitted because it is similar to the proof of Theorem 3 in [4]. In addition to the stocking factor z, Petruzzi and Dada [4] also calculated the "base price", and

obtained a unified result for the multiplicative and the additive demand cases. More specifically, it was defined in [4] that, *for a given value of z*, the base price $p_B(z)$ is the price that maximizes the expected sales contribution excluding the cost of disposing the leftovers and the shortage cost. Note that, for the make-to-stock firm in [4], the assembly cost is not involved and the sales contribution is thus computed as $J(z, p) = (p - c)E\{\min[q, D(p, \varepsilon)]\}$. Petruzzi and Dada showed that, for the multiplicative demand case, the base price $p_B(z)$ is equal to the optimal riskless price p^0 (which is smaller than p^*), i.e., $p_B(z) = p^0 \leq p^*$; but for the additive demand case, the base price $p_B(z)$ is equal to the optimal risk price p^*, i.e., $p_B(z) = p^*$. The unified result for the make-to-stock firm is that, for both the multiplicative and the additive cases, $p^* \geq p_B(z)$, as shown in [4, Theorem 4].

Next, we compute the ATO firm's base price that maximizes the following sales contribution

$$J(z, p) = (p - m - c)E\{\min[q, D(p, \varepsilon)]\}. \tag{12}$$

For this problem, we find the base price as follows: for a given value of z in (6), $p_B(z) = p^0$ for the multiplicative case and $p_B(z) = p^*$ for the additive case. It follows that, for both the multiplicative and the additive cases, the formula used to calculate the base price for the ATO firm is the same as that for the make-to-stock firm. However, we *cannot* conclude that the ATO and make-to-stock firms have the same base price because p^* and p^0 for the ATO firm are different from those for the make-to-stock firm.

Even though the formula for the base price of the ATO firm is the same as that for the make-to-stock firm, in our case it is not possible to duplicate Petruzzi and Dada's [4] unified result $p^* \geq p_B(z)$. This is so, because as Theorem 3 indicates, for the multiplicative case, p^* may or may not be greater than p^0. Thus, it may not be true that $p_B(z) = p^0 \leq p^*$ for the multiplicative case. The next theorem summarizes our discussion above.

Theorem 8 The unified result—i.e., $p^* \geq p_B(z)$ for both the multiplicative and the additive cases— for the make-to-stock firm does not hold for the ATO firm. ◄

4 Conclusion

Previous publications related to the current research (e.g., [1], [3], [4]) implicitly assumed that a firm adopts the make-to-stock strategy to meet the price-sensitive demand. They obtained the following important managerial insights: For the make-to-stock firm which doesn't incur the assembly cost when demand is satisfied, the optimal riskless price is always smaller than the optimal risk price for the multiplicative case whereas the former is always greater than the latter for the additive case.

Many firms have implemented the ATO strategy to assemble final products in response to customers' specific requests. In this note we compared the optimal riskless and risk prices for the ATO firm in both the multiplicative and the additive demand cases, and examined whether or not the relevant results for the make-to-stock firm still hold for the ATO firm which incurs the assembly cost when its customers' orders arrive. Our comparisons revealed the following two useful results: In the multiplicative demand case, for the ATO firm, the optimal riskless price p^0 may be greater than, equal to or smaller than the optimal risk price p^*, which depends on the comparison between the Lerner index and the inverse of the price elasticity of demand. This result differs from that for the make-to-stock firm, (i.e., p^0 is always smaller than p^*). In the additive demand case, for the ATO firm, p^0 is always greater than p^*. This is the same result as that for the make-to-stock firm.

In addition to the comparison between the riskless and risk prices for the ATO firm, we found that there is a unified result regarding the stocking factor for both the multiplicative and the additive demand cases, whereas there is no unified result regarding the base price for the two demand cases. Our result for the base price is different from Petruzzi and Dada [4] in which, for both stocking factor and safety stock, there are unified results for the two demand cases.

References

[1] S. Karlin and C. R. Carr. Prices and optimal inventory policy. In K. J. Arrow, S. Karlin, and H. Scarf, editors, *Studies in Applied Probability and Management Science*, pages 159–172. Stanford University Press, Stanford, California, 1962.

[2] A. P. Lerner. The concept of monopoly and the measurement of monopoly power. *The Review of Economic Studies*, 1(3):157–175, June 1934.

[3] E. S. Mills. Uncertainty and price theory. *The Quarterly Journal of Economics*, 73(1):116–130, February 1959.

[4] N. C. Petruzzi and M. Dada. Pricing and the newsvendor problem: A review with extensions. *Operations Research*, 47(2):183–194, 1999.

[5] E. A. Silver and R. Peterson. *Decision Systems for Inventory Management and Production Planning*. John Wiley, New York, 2nd edition, 1985.

[6] D. Simchi-Levi, P. Kaminsky, and E. Simchi-Levi. *Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies*. McGraw-Hill, New York, third edition, 2008.

[7] Y. Wang, L. Jiang, and Z. Shen. Channel performance under consignment contract with revenue sharing. *Management Science*, 50(1):34–47, January 2004.

[8] L. Young. Price, inventory and the structure of uncertain demand. *New Zealand Operations Research*, 6:157–177, 1978.

[9] E. Zabel. Monopoly and uncertainty. *The Review of Economic Studies*, 37(2):205–219, April 1970.