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## **A NOTE ON THE PREFERRED HEDGE INSTRUMENT**

### **ABSTRACT**

Contrary to Battermann et al.'s (2000) claims, this paper shows that risk-averse exporters may produce less with fair commodity futures than with fair put options; moreover, they may prefer the latter instrument for hedging against its exchange rate risk.

## 1. INTRODUCTION

The behavior of a risk-averse firm under exchange rate or price risk in the presence of commodity put options and/or futures has been analyzed in several recent studies. By assuming that put options are indivisible, Wong (2002) shows that a risk-averse firm produces less under uncertainty in the presence of fair indivisible put options than under certainty (or than that of a risk-neutral counterpart). Surprisingly, Battermann et al. (2000) show that the same result holds even when put options become divisible and that a risk-averse firm always prefers using fair commodity futures to using fair divisible put options for hedging.

This paper shows using a counter-example that simple risk aversion does not guarantee Battermann et al.'s (2000) conclusions. The over-hedging behavior of a risk-averse exporter using fairly priced put options renders the payoff function non-monotonic and convex in the exchange rate risk making Battermann et al.'s proofs incorrect. It is shown that the monotonicity of the payoff function in the risk concerned is a key reason why Battermann et al.'s results hold under Wong's (2002) model with indivisible (as opposed to) put options.

## 2. EXCHANGE RATE RISK AND HEDGING USING PUT OPTIONS

An exporting firm produces a single output that is sold at competitive world price  $P$  denominated in a foreign currency. The firm faces a random foreign exchange rate  $\tilde{S}$  with distribution function  $G$ , probability density function  $g$ , mean  $F$ , realization  $S$ , and support  $[S_L, S_H]$ , where  $S_H > S_L \geq 0$ .<sup>1</sup> From now on, a  $\sim$  means that the variable is a random

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<sup>1</sup>Battermann et al. (2000) does not distinguish random variable  $\tilde{S}$  from realization  $S$ ; they use  $S$  to

variable; the same variable without a  $\sim$  is its realization. The exporter's production cost  $C$  satisfies  $C' > 0$  and  $C'' > 0$ . The exporter is risk-averse with utility function  $U$  satisfying  $U' > 0$  and  $U'' < 0$ .

Suppose a commodity put option with strike price  $K$  becomes available. For each unit of the put option purchased, the firm receives 0 if  $S$  is at or above  $K$  and receives  $K - S$  if  $S$  falls below  $K$ . Following Battermann et al. (2000), assume that the put option is unbiased (or fair) with unit price

$$R = E[\max(K - \tilde{S}, 0)] = \int_{S_L}^K (K - S) dG(S), \quad (1)$$

where  $E$  is the expectation operator. After purchasing  $Z_P$  units of the put option, the producer's profit given realization  $S$  of  $\tilde{S}$  is

$$Y_P = \begin{cases} SPX_P - C(X_P) + Z_P(K - S) - Z_P R, & \text{if } S \in [S_L, K] \\ SPX_P - C(X_P) - Z_P R, & \text{if } S \in [K, S_H] \end{cases} \quad (2)$$

which is equivalently to

$$Y_P = SPX_P - C(X_P) + Z_P [\max(K - S, 0) - R]. \quad (3)$$

The producer's problem is to choose  $(X_P, Z_P)$  to maximize its expected utility

$$\int_{S_L}^{S_H} U\{SPX_P - C(X_P) + Z_P [\max(K - S, 0) - R]\} dG(S). \quad (4)$$

The first-order conditions are given by

$$X_P : \int_{S_L}^{S_H} [SP - C'(X_P^*)] U'(Y_P^*) dG(S) = E\{[\tilde{S}P - C'(X_P^*)] U'(\tilde{Y}_P^*)\} = 0 \quad (5)$$

$$Z_P : \int_{S_L}^K (K - S) U'(Y_P^*) dG(S) - \int_{S_L}^{S_H} R U'(Y_P^*) dG(S)$$

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denote both.

$$= E\left\{\left[\max(K - \tilde{S}, 0) - R\right]U'(\tilde{Y}_p^*)\right\} = 0 \quad (6)$$

where  $X_p^*$  and  $Y_p^*$  are the optimal values of  $X_p$  and  $Y_p$ , respectively. From now on, a  $*$  denotes the optimal value of a variable. As pointed out by Eeckhoudt et al. (1991), even in a non-production model, the second-order condition for the optimal choice of deductible insurance (and similarly put option) with a kinked payoff function is complicated. Therefore, following Battermann et al. (2000), it is assumed that the second-order condition for a maximum holds.

Battermann et al. (2000) have correctly proved the following claim stating that a risk-averse exporting firm over-hedges by purchasing more put option units than what are needed for covering its foreign currency denominated revenue.

**Claim 1**  $Z_p^* > PX_p^*$ .

Notice carefully that Claim 1 implies that optimal payoff  $Y_p^*$  first decreases in  $S$  and then increases in  $S$  with a kink at  $S = K$ . Clearly,  $Y_p^*$  is non-monotonic and convex in the exchange rate risk. This turns out to be very important to the forthcoming analysis.

### 3. PRODUCTION AND HEDGING DECISIONS IN THE PRESENCE OF CURRENCY FUTURES

Now, suppose fairly priced currency futures, instead of put options, are available. The firm chooses output level  $X_f$  and  $Z_f$  units of currency futures at unbiased forward price  $F = E(\tilde{S})$  such that its profit at realization  $S$  of  $\tilde{S}$  equals

$$Y_f = SPX_f - C(X_f) + Z_f(F - S)$$

Battermann et al. (2000) have correctly proved the following claim specifying that a risk-averse exporter chooses to fully hedge against its exchange rate risk when fairly priced currency futures are available:

**Claim 2**  $Z_f^* = PX_f^*$ .

With full hedging using fairly priced currency futures, the exporter's optimal profit  $\tilde{Y}_f^*$  is non-random. Battermann et al. (2000) prove the following certainty-equivalence result stating that in the presence of fair currency futures, a risk-averse exporter chooses its output level as if it is under certainty with fixed exchange rate  $F$  :

**Claim 3**  $FP = C'(X_f^*)$ .

#### 4. PRODUCTION DECISION IN THE PRESENCE OF PUT OPTIONS

Battermann et al. (2000) show that all risk-averse exporting firms produce less in the presence of fair put options than in the presence of fair currency futures, i.e.,  $X_p^* < X_f^*$ . Unfortunately, both their claim and its proof are incorrect. The following examples show that a risk-averse exporter's optimal output level may decrease or increase when fair put options are replaced by fair currency futures:

**Example 1** A case with  $X_p^* > X_f^*$ .<sup>2</sup>

Suppose  $U$  is quadratic with  $U'(Y) = \xi_0 + \xi Y > 0$  (where  $\xi_0 > 0$  is sufficiently large) and

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<sup>2</sup> A more detailed set of computation results for this example is available on request.

$U''(Y) = \xi < 0$  for all  $Y$  in the relevant range,

$S_L = 0, S_H = 1, K = \frac{1}{2}, g(S) = \frac{2}{3K} = \frac{4}{3}, \forall S \in [0, K]$  and

$g(S) = \frac{1}{3(1-K)} = \frac{2}{3}, \forall S \in [K, 1]$ . Simple calculation gives  $R = 1/6$ .

The first step is to show that  $-(PX_p^* - Z_p^*) > PX_p^*$ . It can be checked from (6), using integration by parts and the above assumptions, that

$$\begin{aligned}
0 &= U'(Y_p^* | S = K) \int_0^K (K - S) dG(S) - U'(Y_p^* | S = 1) \int_0^1 R dG(S) \\
&\quad - \int_0^K (PX_p^* - Z_p^*) U''(Y_p^*) \left[ \int_0^S (K - \mu - R) dG(\mu) \right] dS \\
&\quad + \int_K^1 PX_p^* U''(Y_p^*) \left[ \int_0^S R dG(\mu) \right] dS \\
&= -\frac{1}{12} \xi PX_p^* - \frac{1}{36} \xi (PX_p^* - Z_p^*) + \frac{1}{24} \xi PX_p^*. \tag{7}
\end{aligned}$$

Assume by contradiction that  $-(PX_p^* - Z_p^*) \leq PX_p^*$ . Substituting this and  $\xi < 0$  into (7)

gives

$$0 \geq \left[ -\frac{1}{12} + \frac{1}{36} + \frac{1}{24} \right] \xi PX_p^* = -\frac{1}{72} \xi PX_p^* > 0. \tag{8}$$

A contradiction! Therefore,  $-(PX_p^* - Z_p^*) > PX_p^*$ .

Next, it can be checked that  $F = 5/12$ . Denote  $\Omega(S) = \int_{S_L}^S (\mu - F) dG(S)$ . Simple calculation gives

$$\int_{S_L}^K \Omega(S) dS = \int_0^{1/2} \int_0^S \left[ \mu - \frac{5}{12} \right] \frac{4}{3} d\mu dS = \frac{4}{3} \left[ \frac{S^3}{6} - \frac{5S^2}{24} \right]_0^{1/2} = -\frac{1}{24}. \tag{9}$$

Similarly, it can be checked that

$$\int_K^{S_H} \Omega(S) dS = \int_{1/2}^1 \int_0^S \left[ \mu - \frac{5}{12} \right] \frac{2}{3} d\mu dS = \frac{2}{3} \left[ \frac{S^3}{6} - \frac{5S^2}{24} \right]_{1/2}^1 = -\frac{1}{144}. \quad (10)$$

Using first-order condition (5), integrating by parts, and realizing that  $-(PX_p^* - Z_p^*) > PX_p^*$

and  $\xi < 0$ , it can be checked that

$$\begin{aligned} & - \left[ FP - C'(X_p^*) \right] \int_{S_L}^{S_H} U'(Y_p^*) dG(S) \\ &= \int_{S_L}^{S_H} (S - F) P U'(Y_p^*) dG(S) - \int_{S_L}^{S_H} [SP - C'(X_p^*)] U'(Y_p^*) dG(S) \\ &= \int_{S_L}^{S_H} (S - F) P U'(Y_p^*) dG(S) \\ &= - \int_{S_L}^K P (PX_p^* - Z_p^*) U''(Y_p^*) \Omega(S) dS - \int_K^{S_H} P^2 X_p^* U''(Y_p^*) \Omega(S) dS \\ &> P \xi P X_p^* \left[ \int_{S_L}^K \Omega(S) dS - \int_K^{S_H} \Omega(S) dS \right]. \\ &= P \xi P X_p^* \left[ -\frac{1}{24} + \frac{1}{144} \right] = -\frac{5}{144} \xi P^2 X_p^* > 0. \end{aligned} \quad (11)$$

The inequality in (11) clearly implies that  $FP - C'(X_p^*) < 0$  and hence  $X_p^* > X_f^*$  as

$FP = C'(X_f^*)$  according to Claim 3 and  $C'' > 0$ .

**Example 2** A case with  $X_p^* < X_f^*$ .<sup>3</sup>

Suppose  $U, S_H, S_L$ , and  $K$  follow the assumptions in Example 1. Assume now that  $G$  is a uniform distribution with  $g(S) = 1, \forall S \in [0,1]$ . Simple calculation gives  $R = 1/8$ .

Under the new set of assumptions, the first equality in (7) now gives

$$0 = -\frac{1}{16} \xi P X_p^* - \frac{5}{192} \xi [(PX_p^* - Z_p^*)] + \frac{3}{64} \xi P X_p^*. \quad (12)$$

Suppose by contradiction that  $-(PX_p^* - Z_p^*) \geq PX_p^*$ . Substituting this and then  $\xi < 0$  into

(12) gives

<sup>3</sup> A more detailed set of computation results for this example is available on request.



$$0 \leq \left[ -\frac{1}{16} + \frac{5}{192} + \frac{3}{64} \right] \xi P X_p^* = -\frac{1}{96} \xi P X_p^* < 0. \quad (13)$$

A contradiction! Therefore,  $-(P X_p^* - Z_p^*) \leq P X_p^*$ .

Next, it can be checked that  $F = 1/2$  giving rise to

$$\Omega S = \int_0^S (\mu - F) dG(\mu) = \left[ \frac{(\mu - 1/2)^2}{2} \right]_0^S = \left[ \frac{(S - 1/2)^2}{2} - \frac{(1/2)^2}{2} \right]. \quad (14)$$

Similarly,

$$\Omega(S_H - S) = \int_0^{1-S} (\mu - F) dG(\mu) = \left[ \frac{(\mu - 1/2)^2}{2} \right]_0^{1-S} = \left[ \frac{(1/2 - S)^2}{2} - \frac{(1/2)^2}{2} \right] = \Omega(S). \quad (15)$$

(15) implies that

$$\int_{1/2}^1 \Omega(S) dS = \int_{1/2}^1 \Omega(1 - S) dS = \int_{1/2}^0 \Omega(T) d(1 - T) = \int_0^{1/2} \Omega(S) dS < 0. \quad (16)$$

Substituting (16) and  $-(P X_p^* - Z_p^*) < P X_p^*$  into the third equality in (11) gives

$$- [FP - C'(X_p^*)] \int_{S_H}^{S_H} U'(Y_p^*) dG(S) < P \xi P X_p^* \left[ \int_0^{1/2} \Omega(S) dS - \int_{1/2}^1 \Omega(S) dS \right] = 0. \quad (17)$$

The inequality in (17) implies that

$$FP - C'(X_p^*) > 0, \quad (18)$$

and hence  $X_p^* < X_f^*$  as  $FP = C'(X_f^*)$  and  $C'' > 0$ .

## 5. NON-MONOTONIC, CONVEX PAYOFF AND SOME INTUITION

To understand why simple risk-aversion does not guarantee  $X_p^* < X_f^*$  and why Battermann et al.'s (2000) proof is incorrect, notice that their proof relies critically on the following inequality:

$$\text{cov}[U'(\tilde{Y}_p^*), \tilde{S}P - C'(X_p^*)] < 0.^4$$

To show that the above inequality holds, Battermann et al. (2000, footnote 4) suggest that

$$\text{Sgn}\{\text{cov}[U'(\tilde{Y}_p^*), \tilde{S}P - C']\} = -\text{Sgn}\{\text{cov}[U'(\tilde{Y}_p^*), K - \tilde{S}]\}$$

such that the problem reduces to showing that  $\text{cov}[U'(\tilde{Y}_p^*), K - \tilde{S}] > 0$ . However, in the proof

of the last inequality, Battermann et al. (2000, Appendix) have only attempted to show that

$\text{cov}[U'(\tilde{S}), K - \tilde{S}] > 0$ . According to (2) and Claim 1, it can be checked that  $Y_p^*$  is not

monotonically increasing in  $S$ . In fact,  $Y_p^*$  first decreases in  $S$  and then increases in  $S$  with a

kink at  $S = K$ . Clearly,  $\text{Sgn}\{\text{cov}[U'(\tilde{Y}_p^*), K - \tilde{S}]\}$ , and  $\text{Sgn}\{\text{cov}[U'(\tilde{S}), K - \tilde{S}]\}$  are not the

same in general given only  $U'' < 0$ .

Interestingly, it turns out that the non-monotonicity of the payoff function in the presence of put options is a crucial feature that distinguishes the model in this paper from that of Machnes (1992) and Wong (2002). Wong (2002) shows that simple risk aversion is sufficient to guarantee that a competitive exporting firm always produces more under certainty than under output price risk when it is provided with a fairly priced indivisible commodity put option.<sup>5</sup> A close look at Wong's model suggests that his result relies on the fact that the payoff function is monotonically non-decreasing in output price risk. Translating Wong's model of output price risk to the model in this paper, it can be checked that in the presence of an indivisible put option with a fixed number of put option units,  $Z_0$ , equal to output level  $X_0$ ,

<sup>4</sup> Battermann et al. (2000) write (5) as

$$E[U'(Y_p^*)]E[\tilde{S}P - C'(X_p^*)] + \text{cov}[U'(\tilde{Y}_p^*), \tilde{S}P - C'(X_p^*)] = 0$$

Clearly

$$\text{Sgn}[FP - C'(X_p^*)] = \text{Sgn}\{E[\tilde{S}P - C'(X_p^*)]\} = -\text{Sgn}\{\text{cov}[U'(\tilde{Y}_p^*), \tilde{S}P - C'(X_p^*)]\}$$

<sup>5</sup> Wong's (2002) Proposition 2 states that in the presence of a fairly priced indivisible commodity put option, a risk-averse competitive firm always produces less compared to an otherwise identical risk-neutral firm. Clearly, the optimal output level of a risk-neutral firm equals  $X_0$  in the context of exchange rate risk.

the exporter's profit

$$Y_0 = SPX_0 - C(X_0) + [\max(K - S, 0) - R]PX_0. \quad (19)$$

The firm's first-order condition (of choosing  $x_0$ ) is given by

$$0 = \int_{S_L}^{S_H} \{SP - C'(X_0^*) + [\max(K - S, 0) - R]P\} \cdot U'(Y_0^*) dG(S)$$

which can be simplified to

$$FP - C'(X_0^*) = - \frac{P \cdot \text{cov}[\tilde{S} + \max(K - \tilde{S}, 0), U'(Y_0^*)]}{E[U'(Y_0^*)]} \quad (20)$$

Equation (20) is analogous to Wong's (2002) equation 7. A simple inspection of (19) suggests that  $Y_0$  is monotonically non-decreasing in  $S$ . Therefore, the covariance term in (20) is negative given  $U'' < 0$ , exactly as suggested by Wong (2002), such that

$$FP - C'(X_0^*) > 0 \quad (21)$$

and hence  $X_0^* < X_f^*$  according to Claim 3 as  $C'' > 0$ . The non-monotonicity of  $Y_p^*$  as discussed in Section 2 renders Wong's approach not usable when indivisible put options becomes divisible and are freely chosen by the exporter. A simple explanation of this is that the convexity of the payoff function possibly makes the overall utility of the exporter non-concave in the exchange rate risk even when its utility function is concave.

## 6. THE CHOICE BETWEEN FAIR FUTURES AND FAIR PUT OPTIONS

Battermann et al. (2000) argue that risk aversion alone is sufficient to guarantee that an exporter always prefers using fair commodity futures to using fair put options for hedging against its exchange rate risk. Unfortunately, their argument is based critically upon their incorrect claims that risk aversion alone guarantees that  $X_p^* < X_f^*$  and that (18) always holds.

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<sup>6</sup> Machnes' (1992) and Wong's (2002) papers actually deal with output price risk instead of exchange rate risk. Their model has been modified so that Wong's result can be directly compared to that in this paper.

Taking expectation on both sides of (3) gives  $E(\tilde{Y}_p) = FPX_p - C(X_p)$ . Inequality(18)and Claim 3 imply that  $E(\tilde{Y}_f^*) = \sup_x [E(\tilde{Y}_f)] = \sup_x [\tilde{Y}_p] > E(\tilde{Y}_p^*)$ . One can now use Battermann et al.'s (2000, p.88) argument to show that

$$E[U(\tilde{Y}_p^*)] < U[E(Y_p^*)] < U[E(\tilde{Y}_f^*)] = E[U(\tilde{Y}_f^*)]. \quad (22)$$

The first inequality in (22) follows from Jensen's inequality. The second inequality is due to  $E(\tilde{Y}_p^*) < E(\tilde{Y}_f^*)$ .

Notice that inequality (21) always holds such that Battermann et al.'s (2000) argument can be used to show that a risk-averse exporter prefers fair commodity futures to fair indivisible put options for hedging against its exchange rate risk. On the contrary, (18) does not hold in general such that the second inequality in (22) may be reversed. Therefore, the last conclusion may not hold when put options become divisible. Divisible put options are obviously more common than indivisible put options as suggested by Wong (2002).

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