7-2013

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Recommended Citation

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COMPENSATION AND PRICE DELEGATION FOR HETEROGENEOUS SALES FORCE

ABSTRACT

A heterogeneous sales force may not be as desirable as a homogeneous sales force for two reasons: premiums are required for all except from one agent type, and only the highest type would work as hard as though they were from a homogeneous sales force. This study revisits the heterogeneous sales force compensation and price delegation problem with type-dependent reservation. We find that an equilibrium separating or pooling compensation contract always exists. Different types of agents may receive premiums, and there are scenarios when no premiums are paid. Retaining centralized pricing provides a tool for regulating agent behavior. More than one or even all agent types may work as hard as though they were members of a homogeneous sales force. These findings differ from existing results and their driving force is the dynamics between the differences in reservations and agents' effort costs arising from concealing their true types.

1   Introduction

For a firm that employs a sales force to sell its products, a central problem is the alignment of its sales force incentive scheme with the objectives of the sales agents. Not surprisingly, sales force compensation problems have been accorded considerable attention, as indicated in the comprehensive reviews by Coughlan and Sen (1989), Coughlan (1993), Albers (2008), as well as Mantrala et al. (2010).

Sales agents mainly work independently. Their sales efforts are usually unobservable by management. An effective compensation scheme is thus often directly related to the sales outcome of each agent. Basu et al. (1985) showed that a non-linear compensation plan with a fixed base salary and a sales-dependent commission is optimal for a homogenous sales force under symmetric information. The salary depends on the available information, such as selling abilities. However, sales forces are usually heterogeneous. Although firms may know the differences in ability among their agents, they often do not initially know the exact ability of an individual agent. This information asymmetry between a firm and its agents renders the plan of Basu et al. (1985) ineffective. This difficulty presented by asymmetric information can be overcome by designing a compensation scheme with a menu of sales quotas (Rao 1990, Raju and Srinivasan 1996, and Albers 1996). With this scheme, an agent freely chooses the quota that best suits him and, in the process, reveals his true type/ability to the firm (Rao 1990).

In summarizing his findings on the optimal menu-based compensation contract, Rao (1990) observed that in a heterogeneous sales force: all but the agent with the lowest ability receive compensation that exceeds their reservation utility (i.e., receive a premium or information rent); and all but the agent with the highest ability expend efforts at lower levels than they would if they were members of a homogeneous sales force. The first observation is a standard result in contract theory that is applied between an uninformed principal and an agent of unknown type but whose reservation is independent of the type. However, reservation can be type-dependent for a number of reasons, such as competing principals in the market, different fixed trading/opportunity costs, and renegotiation (Fudenberg and Tirole 1991, Jullien 2000). When agent reservations are type dependent, the rationality condition has been noted to possibly bind (no information rent) at the highest type, some middle type, or for some interior range rather than at the lowest type (Lewis and Shappington 1989a, 1989b, Champsaur and Rochet 1989, Laffont and Tirole 1990a, 1990b). Lewis and Shappington (1989b) presented a suitable

example of type-dependent reservation and its impact. A type-dependent fixed cost was introduced to the standard principal-agent problem studied by Baron and Myerson (1982). This fixed cost can be interpreted either as a technology-dependent fixed production cost or as a type-dependent opportunity cost. The change of the fixed cost from type-independent to type-dependent produces a major impact on the optimal contract design, that is, the rationality condition is binding at a certain type of realization in the middle rather than at the lower end. A type-realization interval was likewise observed, in which only pooling equilibrium and no separating equilibrium exists. Clearly, type-dependent reservation is an important feature of principal-agent problems and has been studied from different perspectives in the economics literature. However, how type-dependent reservation affects sales force compensation scheme has not been studied in marketing literature.

When relying on agents to sell products, a firm also needs to decide whether to delegate or to centralize the pricing decision. Although management may prefer to retain the pricing decision, agents with a better knowledge of the market environment may be at a better position to determine prices. Mishra and Prasad (2004) found that when the private information known by agents can be revealed through contracting, centralized pricing performs as least as well as price delegation. However, when the pricing issue is considered jointly with the compensation scheme design for agents with type-dependent reservations, will centralized pricing still be preferred?

In this paper, we examine the effect of type-dependent reservation on the compensation scheme for a heterogeneous sales force and the pricing strategy of the firm. A number of questions may be asked. Is it necessary for a firm to pay information rents to all but the lowest type of agents to determine their true types? Is it possible to design a mechanism that pays information rents to fewer types or pays no information rent to any type, and still reveal the true types of all agents? Is it still true that only agents of the highest type expend the best effort as if they were members of a homogenous sales force when reservation is type-dependent? Furthermore, with type-dependent reservation, can the firm still design type-specific contracts that induce agents to reveal their true types? We likewise intend to know how pricing strategy and compensation contract design interact with each other and whether pricing can be used to regulate agents' behavior. Finally, Rao's observations imply that for a firm a homogeneous sales force works better than a heterogenous sales force. Is this always true when the heterogenous sales force has type-dependent reservation? Can we identify specific conditions under which a well managed heterogeneous sales force works at least as well as or even better than a homogeneous sales force?

We focus on a sales force with two types of agents. A sales force may have many agents, who belong to a high-type with a higher ability and a higher reservation, or to a low-type with a lower ability and a lower reservation. We later show that the main results hold for problems with many different agent types. We identify the conditions under which the equilibrium compensation contract exists under different pricing schemes with type-dependent reservations and then completely characterize the equilibrium contracts. We found that *at least* one agent type would not receive information rent, which could be the low type, the high type, or any type (when there are many types), depending on the magnitude of the difference between the reservations and its comparison with the effort cost difference when the agent would not report his true type. This finding differs from the results in the above economics literature, where for different problems, the rationality condition is binding at a different type rather than the lowest type. We also show that it is possible for the firm to design a compensation mechanism that could reveal the true types of all agents without paying any information rents. Furthermore, rather than only agents with the highest ability, different types of agents may work as hard (and even harder) as though they were members of a homogeneous sales force; and under certain conditions, all types of agents may work as hard as if they comprise a homogenous sales force. Meanwhile, under equilibrium, the firm may not always be able to design type-specific contracts, and hence cannot always identify the type of each agent. We find that pricing adds a dimension in contract design for regulating agents' behavior. Finally, comparing centralized pricing with delegated pricing with type-dependent reservations, we found that delegated pricing is preferred by the firm when centralized equilibrium separating contracts do not exist.

Basu et al. (1985) presented one of the first works on sales force compensation problems. The authors considered a homogenous and risk-averse sales force with the moral hazard and found a nonlinear optimal compensation plan. Lal and Staelin (1986) found that the nonlinear compensation contract proposed by Basu et al. (1985) is not always optimal when the sales force is heterogenous with information asymmetry. The conditions under which a menu of compensation plans of Basu et al. (1985) can be optimal for a heterogenous sales force were shown. Lal and Srinivasan (1993) discussed linear compensation plans for single and multi-product sales forces. With ex-ante symmetric information, they showed by comparative statics that improvements in alternative job opportunities would increase the salary but would not affect the commission rate, which determines the sales effort. By contrast, in our model type-dependent reservations would not only affect the agent payments, but also their

effort decisions. Rao (1990) proposed a menu of quota-based compensation plans for a heterogeneous sales force with private information and showed that only the lowest type receives no information rent, whereas only the highest type makes the first best effort. Raju and Srinivasan (1996) showed that in specific scenarios, the performance of quota-based contracts can be very close to that of the optimal plan of Basu et al. (1985). Park (1995) and Kim (1997) demonstrated that a bonus for meeting the quota may lead to the first-best effort by agents with binding participation constraints in a moral hazard problem. Under a similar setting, Oyer (2000) demonstrated that a quota-based plan may lead to the first best effort for agent types with non-binding participation constraints. Our results show that with quota-based plans, both agents with binding and non-binding participation constraints may make the first best sales effort. Kala and Shi (2001) and Murthy and Mantrala (2005) studied the use of sales contests as a relative performance-based incentive scheme. Rather than focusing on the analysis of optimal contract form, we adopt quota-based contracts under centralized pricing and linear contracts under delegating pricing for a heterogeneous sales force with asymmetric information. These contract forms in our setting are both optimal. Our work differs from those of the above studies in that we design the optimal compensation structure for heterogeneous sales force with type-dependent reservations. Our results show that the optimal contract structure depends on different reservations.

A number of authors considered variations of sales force compensation problems. Mantrala et al. (1994) considered the compensation of a heterogeneous sales force that sells multiple products. Zhang and Mahajan (1995) analyzed the compensation schemes for a heterogeneous sales force with two complementary or substitutable products. Joseph and Thevaranjan (1998) examined the roles of both sales agent monitoring and incentive in a compensation scheme. Misra et al. (2005) considered the risk attitudes of agents in heterogeneous sales force compensation. Caldieraro and Coughlan (2007) analyzed the use of spiffs in a three-level marketing system with manufacturers, represent firms and salespeople. The spiffs are incentives given directly to sales agents by the manufacturer. Caldieraro and Coughlan (2009) demonstrated how the interaction between territory allocation and sales force compensation affects the profit of the firm. Chen (2000, 2005) studied sales force incentive problems together with inventory decision. Mukhopadhyay et al. (2009) compared franchise fee and retail price maintenance contracts where a manufacturer relies on a sales agent with private information to sell the product. They found that the level of equilibrium effort and sales of both contract forms are lower than the first best, and all agents except the lowest type would receive a positive information rent. Chu and

Lai (2013) examined sales force contracting when excess demand results in lost sales and the demand information is censored by the inventory level. They found that quota-based contract is optimal and demand censorship can induce the first-best solution under certain conditions when no adverse selection exists. When agents have private information on the market, the firm has to pay information rent to agents of high type. However, we found that the firm may not need to pay information rent to the high-type agent and the low types may likewise obtain information rent.

Our research is also related to the price delegation problem: Should price decisions be made by the firm or delegated to the sales force? This question appears to have no unconditional answer. Lal (1986) showed that price delegation is appropriate when the salesperson possesses relevant private information that is not available to the firm. Meanwhile, Mishra and Prasad (2004) found that centralized pricing reaches the upper bound on the firm's expected profit if the salesperson signs the contract with the firm after the private information is revealed to him. Bhardwaj (2001) as well as Mishra and Prasad (2005) examined the delegating pricing decisions in competitive markets. In our study, the setting for our price delegation problem is the same as that of Mishra and Prasad (2004). However, we combine the firm's compensation contract design with pricing decisions under type-related reservations. The finding shows that the firm's delegation decision depends on the difference of reservations and centralized pricing is not always preferred.

The rest of the paper is organized as follows. In section 2, we separately define the models for centralized and delegating pricing. In section 3, we show that with different reservation profits, a key property of Rao's model cannot be preserved in both models and subsequently present the optimal contract menus for two models. In section 4, we extend the results from section 3 to N agent types. We conclude the paper in section 5.

2 Model

A firm employs a heterogeneous sales force to sell a single product. The manager (she) of the firm needs to design a set of compensation contracts for the sales agents (agents). Her objective is to maximize the profit generated from each agent, agent (he), who selects a contract to maximize his own profit by exerting an appropriate level of sales effort. The abilities (types) of individual agents are unknown to the manager. However, she knows the proportions (distribution) of different agent types in the sales

force and can estimate different market responses generated by different types of agents. The manager cannot observe the sales efforts of agents. With this lack of observability, the compensation scheme must be based on the realized sales. She therefore encounters a mixture of moral hazard (that she cannot observe the sales efforts) and adverse selection (that the agent possesses superior information of his own ability). By observing the contract chosen by an agent, the manager also wishes to learn his type. Furthermore, she needs to consider whether to centraly set the price of the product or delegate the pricing decision to the agent. Thus, under centralized pricing, a contract includes the product price whereas under delegated pricing, the agent sets the price of the product.

In this sales agent compensation problem, the sequence of events is as follows: (1) the manager offers a menu of contracts to a sales agent (prices are included in the contracts under centralized pricing); (2) the agent decides whether and which contract to sign; (3) the agent makes an effort decision (and sets the price if he is delegated to do so), and proceeds with the sales; and then (4) both parties observe the realized sales, and the firm pays the sales agent according to the contract. This sequence is standard for compensation problems with asymmetric information and is commonly assumed in literature (e.g., Lal 1986, Rao 1990, and Misra and Prasad 2004).

We assume that the realized sales s of the agent is a linear function of the price p, his type (ability) θ, and his effort e,

$$s = s_0 + \theta e - bp, \tag{1}$$

where s_0 ($s_0 > 0$) is the market potential and b ($b > 0$) represents the price elasticity. Besides its tractability, a linear response function is reasonable for many market situations and is commonly used in the sales force compensation literature, such as in Holmstrom and Milgrom (1991) and Bhardwaj (2001).

For clarity of result, we first restrict the number of agent types to two: high type and low type, with abilities θ_H and θ_L, respectively, where $\theta_H > \theta_L$. We use H and L as subscripts to indicate high and low types, respectively. The sales force could have many individual agents, but they all belong to one of the two types, and the proportions ρ and $1 - \rho$ with $\rho \in (0, 1)$ of high type and low type agents in the job market are common knowledge.

The sales efforts of the agent can include a number of factors, such as time and money (including discount off the firm's price that he offers to the customer in centralized pricing). For modeling

convenience, we assume that the total effort represents a cost to the agent and denote it by $V(e)$ for effort level e. We assume as in the marketing literature that $V(e)$ is increasing in e at an increasing rate (e.g., Rees 1985). Specifically, we assume the following convex effort cost function,

$$V(e) = e^2/2. \tag{2}$$

Let $\{(q_H, t_H, p_H), (q_L, t_L, p_L)\}$ be the compensation menu under centralized pricing, where q_H and q_L are sales quotas, t_H and t_L are the compensations, and p_H and p_L are the product prices for high- and low-type agents.

Let R_i be the agent's reservation profit, x be his choice of compensation scheme, and e_{ix} be his effort for his selected compensation contract, i as well as $x = $ H or L. Here, $R_H \geq R_L$. His profit/reward $\pi_i(e, x)$ satisfies

$$\pi_i(e, x) = t_x - V(e_{ix}) \geq R_i, \quad i, \ x = \text{H or L}. \tag{3}$$

We assume that different reservations are estimated according to the prevailing industry standard and are common knowledge.

Normalizing the unit product cost to 0 (one can think of that the unit price has already netted the cost), the profit function of the firm under the compensation menu is then

$$\pi_f(q_H, t_H, p_H, q_L, t_L, p_L) = \rho(p_H \times q_H - t_H) + (1 - \rho)(p_L \times q_L - t_L). \tag{4}$$

For the quota scheme defined above, the agent is paid a fixed amount if the quota is reached and nothing if the quota is missed. With a deterministic response function, the quota selected by the agent could be precisely achieved. For the firm, this quota form is equivalent to a linear contract with a fixed salary and a commission for each unit of sales as long as the response functions remains the same as (1). This allows us to directly compare our results with those in Rao (1990).

Incorporating the compensation scheme and considering the agent reservation, the original game problem of (3) and (4) under centralized pricing becomes the following principal-agent problem:

$$\mathbf{P_c}: \max_{p_H, q_H, t_H, p_L, q_L, t_L} \{\rho(p_H \times q_H - t_H) + (1 - \rho)(p_L \times q_L - t_L)\}$$

$$(I) \quad t_H - V(e_{HH}) \geq R_H$$

$$(II) \quad t_L - V(e_{LL}) \geq R_L$$

$$s.t. \quad (III) \quad t_H - V(e_{HH}) \geq t_L - V(e_{HL}) \qquad \qquad . \qquad (5)$$

$$(IV) \quad t_L - V(e_{LL}) \geq t_H - V(e_{LH})$$

$$(V) \quad e_{ix} = \text{argmax} \ \ t_x - V(e_{ix}), \ \ i, x = \text{H or L}$$

In $\mathbf{P_c}$, (I) and (II) are individual rationality (IR) constraints; whereas (III), (IV) and (V) are incentive compatibility (IC) constraints to ensure that the agent would select the contract designed for his type and hence reveal his true type to the firm. These constraints are standard for contract design (see, for example, Bolton and Dewatripont 2005).

Under delegated pricing, the compensation depends on the price set by the agent. We use the form $t_i = \alpha_i + (p_i - y_i) * q_i$, where α_i is the firm's margin, and p_i is the product price ($i = H, L$). The commission rate under this compensation form is $(p_i - y_i)/(p_i)$ (Bhardwaj 2001). Under this scheme, the set of contracts offered by the firm is $\{(\alpha_H, y_H), (\alpha_L, y_L)\}$. The agent would choose a contract and decide on the optimal effort and price to maximize his profit. The resulting principal-agent problem under price delegation is then:

$$\mathbf{P_d} : \max_{y_H, \alpha_H, y_L, \alpha_L} \rho(y_H \times q_{HH} - \alpha_H) + (1 - \rho)(y_L \times q_{LL} - \alpha_L)$$

$$(I) \quad \alpha_H + (p_{HH} - y_H)q_{HH} - V(e_{HH}) \geq R_H$$

$$(II) \quad \alpha_L + (p_{LL} - y_L)q_{LL} - V(e_{LL}) \geq R_L$$

$$s.t. \quad (III) \quad \alpha_H + (p_{HH} - y_H)q_{HH} - V(e_{HH}) \geq \alpha_L + (p_{HL} - y_L)q_{HL} - V(e_{HL}) \quad (6)$$

$$(IV) \quad \alpha_L + (p_{LL} - y_L)q_{LL} - V(e_{LL}) \geq \alpha_H + (p_{LH} - y_H)q_{LH} - V(e_{LH})$$

$$(V) \quad p_{ix}, e_{ix} = \text{argmax} \ \ \alpha_x + (p_{ix} - y_x)q_{ix} - V(e_{ix}) \ \ i, x = \text{H or L}, y_i \geq 0$$

In $\mathbf{P_d}$, (I) and (II) are IR constraints, whereas (III), (IV) and (V) are IC constraints. Constraint (V) can be used to predict the actions of the agent.

3   Optimal Contracts

In this section, we derive the equilibrium separating contracts under centralized pricing and delegated pricing, respectively. The equilibrium separating contract does not always exist, and thus we also examine the equilibrium pooling contracts. Comparing separating and pooling contracts, we establish

a strategy for the manager to offer either a set of separating contracts or a pooling contract under different agent type realizations. Similarly, we also compare the performance of the optimal contracts under centralized and delegated pricing to determine the choice of pricing scheme.

3.1 Centralized Pricing

Let $r = \theta_H/\theta_L$. Clearly, $r > 1$ and measures the relative ability gap (or type difference) of the low type from the high type. To facilitate the presentation of the optimal decisions, we define

$$
\begin{cases}
\psi_1 = \frac{1-\rho}{1-\rho/r^2} \\
V_{c1} = \frac{1}{2}(1 - r^{-2})\left(\frac{s_0\psi_1\theta_L}{2b-\psi_1\theta_L^2}\right)^2 \\
V_{c2} = \frac{1}{2}(1 - r^{-2})\left(\frac{s_0\theta_L}{2b-\theta_L^2}\right)^2 \\
V_{c3} = \frac{1}{2}(r^2 - 1)\left(\frac{s_0\theta_H}{2b-\theta_H^2}\right)^2
\end{cases}
. \tag{7}
$$

It is easy to show that $0 < \psi_1 < 1$ and $V_{c1} < V_{c2} < V_{c3}$. Below, these V values are shown to form three distinct thresholds. When $r^2 < 1/(1-\rho)$, we have a forth threshold $V_{c4} > V_{c3}$ defined by

$$
V_{c4} = \frac{1}{2}(r^2 - 1)\left(\frac{s_0\psi_2\theta_H}{2b - \psi_2\theta_H^2}\right)^2, \tag{8}
$$

where $\psi_2 = \frac{\rho}{1-(1-\rho)r^2} > 1$.

We assume that $2b > \theta_H^2$ to ensure a positive price in a centralized system with symmetric information. For any R_H, we further assume that s_0 is sufficiently large to ensure a non-negative profit for the firm[1]. This assumption is commonly made in literature (Laffont and Tirole 1987).

We use $\Delta R = R_H - R_L$ ($\Delta R \geq 0$) to denote the reservation difference of the two agent types, and define Condition C below, noting that when C holds, then $V_{c4} > V_{c3}$.

Condition C: $r^2 < \frac{1}{1-\rho}(1 - \frac{\rho\theta_H^2}{2b})$.

Proposition 1. *A unique equilibrium separating contract exists for each agent type when $0 \leq \Delta R \leq V_{c1}$ and $V_{c2} \leq \Delta R \leq V_{c3}$, as well as when $\Delta R \geq V_{c4}$ and C holds. When $V_{c1} < \Delta R < V_{c2}$ and $V_{c3} < \Delta R < V_{c4}$, as well as when $\Delta R \geq V_{c4}$ but C does not hold, no equilibrium separating contract exists.*

The optimal decisions under the equilibrium separating contact are given in Table 1.

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[1]In a centralized system, the firm only needs to decide the optimal price and effort to maximize $p(s_0 + \theta_H e - bp) - e^2/2$ for a type H agent. The optimal price and effort are $p = s_0/(2b - \theta_H^2)$ and $e = (s_0\theta_H)/(2b - \theta_H^2)$, respectively. Obviously, to guarantee a non-negative price, we must have $2b > \theta_H^2$. With the optimal price and effort, the maximum social welfare in centralized system is $s_0^2/[2(2b - \theta_H^2)]$, and the profit of the firm is $s_0^2/[2(2b - \theta_H^2)] - R_H$.

Table 1: Optimal decisions under equilibrium separating contracts with centralized pricing

Decisions	$0 \leq \Delta R \leq V_{c1}$	$V_{c2} \leq \Delta R \leq V_{c3}$	$\Delta R \geq V_{c4}$ (if C holds)
p_H^*	$\frac{s_0}{2b-\theta_H^2}$	$\frac{s_0}{2b-\theta_H^2}$	$\frac{s_0}{2b-\psi_2\theta_H^2}$
p_L^*	$\frac{s_0}{2b-\psi_1\theta_L^2}$	$\frac{s_0}{2b-\theta_L^2}$	$\frac{s_0}{2b-\theta_L^2}$
q_H^*	$\frac{bs_0}{2b-\theta_H^2}$	$\frac{bs_0}{2b-\theta_H^2}$	$\frac{bs_0}{2b-\psi_2\theta_H^2}$
q_L^*	$\frac{bs_0}{2b-\psi_1\theta_L^2}$	$\frac{bs_0}{2b-\theta_L^2}$	$\frac{bs_0}{2b-\theta_L^2}$
t_H^*	$R_L + \frac{s_0^2\theta_H^2}{2(2b-\theta_H^2)^2} + V_{c1}$	$R_H + \frac{s_0^2\theta_H^2}{2(2b-\theta_H^2)^2}$	$R_H + \frac{s_0^2\psi_2^2\theta_H^2}{2(2b-\psi_2\theta_H^2)^2}$
t_L^*	$R_L + \frac{s_0^2\psi_1^2\theta_L^2}{2(2b-\psi_1\theta_L^2)^2}$	$R_L + \frac{s_0^2\theta_L^2}{2(2b-\theta_L^2)^2}$	$R_H + \frac{s_0^2\theta_L^2}{2(2b-\theta_L^2)^2} - V_{c4}$
e_{HH}^*	$\frac{s_0\theta_H}{2b-\theta_H^2}$	$\frac{s_0\theta_H}{2b-\theta_H^2}$	$\frac{s_0\psi_2\theta_H}{2b-\psi_2\theta_H^2}$
e_{LL}^*	$\frac{s_0\psi_1\theta_L}{2b-\psi_1\theta_L^2}$	$\frac{s_0\theta_L}{2b-\theta_L^2}$	$\frac{s_0\theta_L}{2b-\theta_L^2}$

$r > 1$ and $2b > \theta_H^2$, and thus $\frac{1}{1-\rho}(1 - \frac{\rho\theta_H^2}{2b}) < \frac{1}{1-\rho}$. For $\frac{1}{1-\rho}(1 - \frac{\rho\theta_H^2}{2b}) < r < \frac{1}{1-\rho}$, V_{c4} defined in (8) remains greater than V_{c3}. We can find a separating contract for the high type. However, the optimal quota and price in this contract are negative. Thus, the firm has no equilibrium separating contract in the region $\Delta R \geq V_{c4}$ when condition C does not hold.

The standard contract theory can usually demonstrate that an equilibrium separating contract always exists and performs better than the corresponding pooling contract for the principle, given that different types of agents have the same reservation (Bolton and Dewatripont 2005). When the reservation is type-dependent, we find that for certain ΔR, there is no separating equilibrium contract. In other words, the firm would not always be able to design type-specific contracts for agents with type-dependent reservations, and hence cannot always identify the the agent type. We should then examine the pooling contract $\{q, t, p\}$. The manager needs to solve the following principal-agent problem:

$$\mathbf{P_{cp}} : \max_{p,q,t}\{p \times q - t\}$$

$$s.t. \quad \begin{array}{ll} (I) & t - V(e_H) \geq R_H \\ (II) & t - V(e_L) \geq R_L \\ (III) & e_i = \frac{q+bp-s_0}{\theta_i}, \quad i = \text{H or L} \end{array} \quad . \tag{9}$$

Constraints (I) and (II) are the two IRs. Solving problem $\mathbf{P_{cp}}$, we arrive at the following proposition.

Table 2: Optimal decisions under equilibrium pooling contract with centralized pricing

Decisions	$0 \leq \Delta R \leq V_{c2}$	$\Delta R \geq V_{c3}$
p^*	$\frac{s_0}{2b-\theta_L^2}$	$\frac{s_0}{2b-\theta_H^2}$
q^*	$\frac{bs_0}{2b-\theta_L^2}$	$\frac{bs_0}{2b-\theta_H^2}$
t^*	$R_L + \frac{s_0^2\theta_L^2}{2(2b-\theta_L^2)^2}$	$R_H + \frac{s_0^2\theta_H^2}{2(2b-\theta_H^2)^2}$
e_H^*	$\frac{\theta_L}{\theta_H}\frac{s_0\theta_L}{2b-\theta_L^2}$	$\frac{s_0\theta_H}{2b-\theta_H^2}$
e_L^*	$\frac{s_0\theta_L}{2b-\theta_L^2}$	$\frac{\theta_H}{\theta_L}\frac{s_0\theta_H}{2b-\theta_H^2}$

Proposition 2. *A unique equilibrium pooling contract exists for any ΔR value, except for $V_{c2} < \Delta R < V_{c3}$ wherein no pooling equilibrium exists.*

The optimal decisions of the manager and the agents under the equilibrium contract are given in Table 2.

We note (from the proof of Proposition 2) that for $0 \leq \Delta R \leq V_{c2}$, the IR constraint for the low type is binding under the equilibrium. This shows that the quota in the pooling contract can be seen as designed for the low type, and thus, the high type will obtain an information rent for accepting the quota. The opposite is true when $\Delta R \geq V_{c3}$.

Propositions 1 and 2 show that the difference of the two reservations affects both the separating equilibrium and the pooling equilibrium. Importantly, when an equilibrium separating contract does not exist for a certain range of ΔR, then an equilibrium pooling contract always exists. As such, an optimal equilibrium contract strategy exists, which the firm can follow.

Proposition 3. *The optimal equilibrium contract strategy for the manager with centralized pricing changes with the difference of the two reservations, and is given in Table 3.*

We note that for $0 \leq \Delta R \leq V_{c1}$ and for $\Delta R \geq V_{c4}$ when Condition C holds, both equilibrium separating contract and equilibrium pooling contract exist. As in the standard contract theory, we can show that when the agent has a type-dependent reservation, the performance of the firm is better under the separating contract than under the pooling contract.

Let $P_i(q_x) = \pi_i(e,x) - R_i$ be the premium accrued to type i agent when he chooses contract $\{q_x, t_x, p_x\}$, i, $x = H$, L. Table 3 shows how the premiums for the two agent types change with the reservation difference under centralized pricing.

Table 3: Contract strategy, premiums, and efforts compared with those of a homogeneous sales force

Region	$0 \leq \Delta R \leq V_{c1}$	$V_{c1} < \Delta R < V_{c2}$	$V_{c2} \leq \Delta R \leq V_{c3}$	If C holds		Otherwise
				$V_{c3} < \Delta R < V_{c4}$	$\Delta R \geq V_{c4}$	$\Delta R > V_{c3}$
Contract	Separating	Pooling	Separating	Pooling	Separating	Pooling
$P_H(q_H)$	$V_{c1} - \Delta R$	$V_{c2} - \Delta R$	0	0	0	0
$P_L(q_L)$	0	0	0	$\Delta R - V_{c3}$	$\Delta R - V_{c4}$	$\Delta R - V_{c3}$
e_H	same	lower	same	same	higher	same
e_L	lower	same	same	higher	same	higher

Rao (1990) found that only the least skilled agent receives a zero premium in the optimal contract. According to Table 3, this conclusion no longer holds when the reservation is type dependent. Below, we explain how the interaction between the reservation and the cost differences causes the change. We first state the following observation.

Observation 1 *Under the separating equilibrium with $\theta_H > \theta_L$, $q_H^* > q_L^*$, the cost difference for a low-type agent to achieve a high-type quota is always greater than the cost difference for a high-type agent to achieve a low-type quota.*

We also note that by definition: V_{c1} and V_{c2} are the differences between the costs of meeting the low-type sales quotas by the low-type and the high-type agents (noting that the effort cost of the low-type is higher); whereas V_{c3} and V_{c4} are the differences between the costs of meeting the high-type sales quotas by the low-type and high-type agents (again, the effort cost of the low-type is higher). Thus, we may consider the V function as the cost difference.

Consider $0 \leq \Delta R \leq V_{c1}$. Suppose that no information rent is stipulated in the high-type contract. A high-type agent receives $e_{HH}^{*2}/2 + R_H$ if he accepts the high-type contract. The net gain is exactly R_H. If he selects the low-type contract instead, he receives $e_{LL}^{*2}/2 + R_L = e_{HL}^{*2}/2 + V_{c1} + R_H - \Delta R$, and his net gain is at least R_H. Thus, an information rent of $V_{c1} - \Delta R$ is necessary to encourage a high-type agent to select the high-type contract and reveal his type. Now, suppose that a low-type agent selects the high-type contract with information rent. He receives $e_{HH}^{*2}/2 + R_H + V_{c1} - \Delta R = e_{HH}^{*2}/2 + R_L + V_{c1}$. His net gain is $R_L + V_{c1} - (e_{LH}^{*2} - e_{HH}^{*2})/2$, which is smaller than R_L as indicated in Observation 1.

Thus, a low-type agent would accept the low-type contract without information rent. The analysis above leads exactly to the conclusion predicted in Table 3.

When $V_{c1} < \Delta R < V_{c2}$, the firm offers a pooling contract. The pooling contract is essentially for the low-type agent; a premium is offered to the high type agent to encourage his acceptance.

When $V_{c2} \leq \Delta R \leq V_{c3}$, separating equilibrium contracts exist and are offered. Suppose that a high-type agent chooses the low-type contract and a low-type agent chooses the high-type contract. The net gains without information rent for the high-type and low-type agents would be $e_{LL}^{*2}/2 - e_{HL}^{*2}/2 + R_L = V_{c2} + R_H - \Delta R \leq R_H$ and $e_{HH}^{*2}/2 - e_{LH}^{*2}/2 + R_H = R_L + \Delta R - V_{c3} \leq R_L$, respectively. Thus, both types of agents accept the appropriate contract without information rent.

When $V_{c3} < \Delta R < V_{c4}$ and C holds, the firm offers a pooling contract. The pooling contract is essentially for the high-type agent; a premium is offered to the low-type agent to encourage his acceptance.

When $V_{c4} \leq \Delta R$ and C holds, separating equilibrium contracts exist and are offered. The net gain without information rent for a low-type agent to choose the high-type contract would be $e_{HH}^{*2}/2 - e_{LH}^{*2}/2 + R_H = R_L - V_{c4} + \Delta R \geq R_L$. Thus, the low-type agent needs an information rent $\Delta R - V_{c4}$ to select the appropriate contract. The net gain for a high-type agent to choose a low-type contract with information rent would be $e_{LL}^{*2}/2 - e_{HL}^{*2}/2 + R_L + \Delta R - V_{c4} = e_{LL}^{*2}/2 - e_{HL}^{*2}/2 - V_{c4} + R_H < R_H$ according to Observation 1. Thus, a high-type agent accepts the appropriate contract without information rent.

Finally, when $\Delta R > V_{c3}$ and C does not hold, the firm offers a pooling contract. The pooling contract is essentially for the high-type agent; a premium is offered to the low-type agent to encourage his acceptance.

Now, we examine how the effort exerted by an agent from a heterogeneous sales force differs from that of an agent from a homogeneous sales force. Noting that the optimal sales effort of an agent in a homogeneous sales force is $s_0 \theta_i / (2b - \theta_i^2)$, the comparison is likewise given in Table 3.

Rao (1990) demonstrated that all but the agent with the highest skill expend efforts lower than what they normally would if they were members of a homogeneous sales force. This conclusion no longer holds when the reservation is type-dependent. For our model, *except* for $0 \leq \Delta R \leq V_{c1}$ when the low-type agent exerts an effort level lower than $s_0 \theta_L / (2b - \theta_L^2)$ and for $V_{c1} < \Delta R < V_{c2}$ when the high-type agent exerts an effort level lower than $s_0 \theta_H / (2b - \theta_H^2)$, both agent types exert efforts at levels at least as high as though they were from a homogeneous sales force.

Thus far, our results for the case with two agent types show that Rao's two main observations are no longer true when the reservation is type-dependent. We now discuss another result that differs from those of the principal-agent literature with type-dependent reservation.

Observation 2 *With the change of the reservation difference ΔR, there is at least one interval in which pooling equilibrium contract exists but no separating equilibrium contract exists.*

Lewis and Sappington (1989) found that the IR constraint is binding for only one realization of the type and at most one non-degenerated pooling interval exists when the agent reservation is an increasing function of the type. Our results show that at least one IR constraint is binding (Proposition 3, two IRs are binding when $V_{c2} \leq \Delta R \leq V_{c3}$) and at least one ΔR interval only has pooling contract available (Table 3). A key difference between our model and that of Lewis and Sappington (1989) is that while a single dependence relation exists between the type and reservation in their model, we only require a higher reservation for a higher type without a fixed relationship between type and reservation. In addition, the equilibrium in our model depends on the difference of reservations instead of the reservation directly as in Lewis and Sappington (1989).

Similarly interesting to examine is how the optimal price changes with ΔR as well as how the optimal quota and the optimal effort relate to the optimal price. From Table 1, we observe that the optimal price only jumps to a higher level when ΔR passes through threshold V_{c1} for the low type and threshold V_{c4} for the high type. A similar jump can be observed from Table 2 for pooling contracts. For separating contracts, we can also observe that the optimal price for the high type is always strictly higher that that for the low type. Clearly, the optimal quota can always be written as bp_i^* for $i = H, L$. The optimal effort is also proportional to the optimal price. For both low and high types, when the optimal price jumps to a higher level, the optimal effort jumps to a higher proportion of the price.

Observation 3 *When the pricing decision is kept centralized, the firm can use pricing to regulate sales quota and effort. Specifically, when ΔR is sufficiently large (no smaller than V_{c2}), the firm sets a higher price and higher quota as well as motivates the first best or even higher effort from the heterogenous sales force.*

Table 4: Equilibrium decisions under delegated pricing for separating contracts

Decisions	$0 \leq \Delta R \leq V_{d1}$	$\Delta R \geq V_{d2}$
y_H^*	0	0
y_L^*	$\frac{s_0\rho(\theta_H^2-\theta_L^2)}{b[2b(1-\rho)-(1-2\rho)\theta_H^2-\rho\theta_L^2]}$	0
α_H^*	$R_L - \frac{s_0^2}{2(2b-\theta_H^2)} + V_{d1}$	$R_H - \frac{s_0^2}{2(2b-\theta_H^2)}$
α_L^*	$R_L - \frac{(s_0-by_L^*)^2}{2(2b-\theta_L^2)}$	$R_H - \frac{s_0^2}{2(2b-\theta_H^2)}$
p_{HH}^*	$\frac{s_0}{2b-\theta_H^2}$	$\frac{s_0}{2b-\theta_H^2}$
p_{LL}^*	$y_L^* + \frac{s_0-by_L^*}{2b-\theta_L^2}$	$\frac{s_0}{2b-\theta_L^2}$
e_{HH}^*	$\frac{s_0\theta_H}{2b-\theta_H^2}$	$\frac{s_0\theta_H}{2b-\theta_H^2}$
e_{LL}^*	$\frac{(s_0-by_L^*)\theta_L}{2b-\theta_L^2}$	$\frac{s_0\theta_L}{2b-\theta_L^2}$

3.2 Delegated Pricing

We define the following parameters:

$$\begin{cases} V_{d1} = \frac{s_0^2(\theta_H^2-\theta_L^2)(2b-\theta_H^2)}{2(2b-\theta_L^2)[2b-\theta_H^2+\rho(\theta_H^2-\theta_L^2)/(1-\rho)]^2} \\ V_{d2} = \frac{s_0^2(\theta_H^2-\theta_L^2)}{2(2b-\theta_H^2)(2b-\theta_L^2)} \end{cases}, \tag{10}$$

where $V_{d1} < V_{d2}$. For problem P_d under delegated pricing, we have the following proposition.

Proposition 4. *Except for $V_{d1} < \Delta R < V_{d2}$ in which no equilibrium separating contract exists, a unique equilibrium separating contract exists under delegated pricing, and the corresponding optimal decisions are given in Table 4.*

We note from Table 4 that the per unit revenue the firm collected from an agent may be zero, that is, $y_H^* = 0$ for any ΔR and $y_L^* = 0$ for $\Delta R \geq V_{d2}$. On the other hand, when $y_i^* = 0$, we can show that the corresponding $\alpha_i^* < 0$. In other words, the equilibrium separating contract under price delegation may require the agent to initially purchase a fixed quantity and then sell on his own in the market. This type of contract is not uncommon in practice. The interesting phenomenon is that the firm may only offer the buy-out contract to the high-type agent while offering the low-type agent the regular contract. Furthermore, if the buy-out contract is offered to both agent types (i.e., when $\Delta R \geq V_{d2}$), the buy-out payments for the two agent types are identical. In other words, the buy-out contract in this case is essentially a pooling contract.

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Table 5: Equilibrium decisions under delegated pricing with pooling contracts

Decisions	$0 \leq \Delta R \leq V_{dp}$	$\Delta R \geq V_{d2}$
y^*	$\frac{s_0 \rho (\theta_H^2 - \theta_L^2)}{b[2b - (1-2\rho)\theta_H^2 - 2\rho\theta_L^2]}$	0
α^*	$R_L - \frac{(s_0 - by^*)^2}{2(2b - \theta_L^2)}$	$R_H - \frac{s_0^2}{2(2b - \theta_H^2)}$
p_H^*	$y^* + \frac{s_0 - by^*}{2b - \theta_H^2}$	$\frac{s_0}{2b - \theta_H^2}$
p_L^*	$y^* + \frac{s_0 - by^*}{2b - \theta_L^2}$	$\frac{s_0}{2b - \theta_L^2}$
e_H^*	$\frac{(s_0 - by^*)\theta_H}{2b - \theta_H^2}$	$\frac{s_0 \theta_H}{2b - \theta_H^2}$
e_L^*	$\frac{(s_0 - by^*)\theta_L}{2b - \theta_L^2}$	$\frac{s_0 \theta_L}{2b - \theta_L^2}$

Again, some ΔR intervals may have no separating equilibrium contract, and we need to consider pooling contracts. To design a pooling contract, the manager solves the following problem under delegated pricing.

$$\mathbf{P_{dp}} : \max_{y,\alpha}\{\rho(y \times q_H - \alpha) + (1 - \rho)(y \times q_L - \alpha)\}$$

$$s.t. \quad \begin{array}{ll} (I) & \alpha + (p_H - y)q_H - V(e_H) \geq R_H \\ (II) & \alpha + (p_L - y)q_L - V(e_L) \geq R_L \\ (III) & p_H, e_H = \text{argmax} \quad \alpha + (p_H - y)q_H - V(e_H) \\ (IV) & p_L, e_L = \text{argmax} \quad \alpha + (p_L - y)q_L - V(e_L) \end{array} \tag{11}$$

Let

$$V_{dp} = \frac{s_0^2(\theta_H^2 - \theta_L^2)[2b - (1-\rho)\theta_H^2 - \rho\theta_L^2]^2}{2(2b - \theta_H^2)(2b - \theta_L^2)[2b - (1-2\rho)\theta_H^2 - 2\rho\theta_L^2]^2}. \tag{12}$$

Clearly, $V_{d1} < V_{dp} < V_{d2}$. Solving problem $\mathrm{P_{dp}}$, we arrive at the following proposition.

Proposition 5. *Except for $V_{dp} < \Delta R < V_{d2}$ in which no equilibrium pooling contract exists, a unique equilibrium pooling contract exists and the corresponding optimal decisions are given in Table 5.*

From Propositions 4 and 5, we can conclude that when a separating equilibrium cannot be obtained in the region $V_{d1} < \Delta R < V_{d2}$, the firm can offer an equilibrium pooling contract in a sub-region $V_{d1} < \Delta R \leq V_{dp}$. Different from the centralized pricing case, neither separating nor pooling equilibrium contract exists in the remaining sub-region $V_{dp} < \Delta R < V_{d2}$. Table 6 summarizes the firm's optimal equilibrium contract strategy under delegated pricing.

Table 6: Optimal equilibrium contract strategy under delegated pricing

Region	$0 \leq \Delta R \leq V_{d1}$	$V_{d1} < \Delta R \leq V_{dp}$	$V_{dp} < \Delta R < V_{d2}$	$\Delta R \geq V_{d2}$
Contracts	Separating	Pooling	No equilibrium	Pooling

Table 7: Premiums and efforts compared with those in a homogeneous sales force

Region	$0 \leq \Delta R \leq V_{d1}$	$V_{d1} < \Delta R \leq V_{dp}$	$\Delta R \geq V_{d2}$
P_H	$V_{d1} - \Delta R$	$V_{dp} - \Delta R$	0
P_L	0	0	$\Delta R - V_{d2}$
e_H	same	lower	same
e_L	lower	lower	same

Table 7 presents the agent premiums under price delegation. The results show that one agent type, either the low or the high type, always exists with a zero premium. However, unlike centralized pricing, the case when both agent types may receive a zero premium does not occur. Table 7 likewise presents comparisons of efforts of the two agent types in a heterogeneous sales force with the effort in a homogeneous sales force under price delegation. Recall that under centralized pricing, both agent types may exert a higher level of effort than they would if they were from a homogeneous sales force. However, this phenomenon would not occur under delegated pricing because of the assumption that $y \geq 0$. We summarize the above observations as well as those under centralized pricing in the following proposition.

Proposition 6. *In equilibrium, either agent type may receive zero premium and may exert the same effort as though he were from a homogeneous sales force under both centralized and delegated pricing schemes.*

3.3 Comparisons of Centralized Pricing and Delegated Pricing

We compare the expected profits of the firm under the two pricing strategies. For the V thresholds, we could easily verify that $V_{c2} < V_{dp} < V_{d2} < V_{c3}$. From Tables 3 and 6, we identify the firm's optimal pricing strategy and contract type, and illustrate them in Table 8. We use CS/CP and DS/DP as the abbreviations for centralized pricing with separating/pooling contracts and delegated pricing with

Table 8: The optimal pricing strategy and contract type

ΔR	$[0, V_{c1}]$	(V_{c1}, V_{c2})		$[V_{c2}, V_{c3}]$	(V_{c3}, ∞)	
Optimal Pricing and Contract Strategy	CS	If $V_{d1} \leq V_{c1}$		CS	If C Holds	
		DP			(V_{c3}, V_{c4})	$[V_{c4}, \infty)$
		If $V_{c1} < V_{d1} \leq V_{c2}$			CP/DP	CS
		$(V_{c1}, V_{d1}]$	(V_{d1}, V_{c2})		Otherwise	
		DS	DP		CP/DP	
		If $V_{c2} < V_{d1}$				
		DS				

separating/pooling contracts, respectively.

Examining Table 8, we determine that the expected profit under delegated pooling is at least as high as the profit from centralized pooling when equilibrium decisions are present under both pricing strategies.

Observation 4 *When equilibrium pooling contracts are offered under both pricing schemes, delegated pricing performs as least as well as the centralized pricing for the firm.*

When $0 \leq \Delta R \leq V_{c2}$, both centralized pooling equilibrium and delegated pooling equilibrium exist. With delegated pooling, the high-type agent would receive a higher premium than that with centralized pooling. However, the expected social welfare generated by the two agent types under delegated pooling is also higher than that under centralized pooling. The difference of social welfare from the two pricing schemes is larger than the premium difference. Thus, delegated pooling is better than centralized pooling for the firm.

When $\Delta R \geq V_{c3}$, the low-type agent receives a higher premium under delegated pooling than under centralized pooling, and delegated pooling generates a higher social welfare than does centralized pooling. However, the difference of social welfare from two pricing schemes is equal to the premium difference. Thus, delegated and centralized pooling are equivalent for the firm.

Proposition 7. *Centralized pricing is not always preferred over delegated pricing when the reservation is type-dependent and the centralized equilibrium separating contracts do not exist.*

When $\Delta R \in [0, V_{c1}]$, equilibrium contracts exist for centralized and delegated separating/pooling.

The centralized separating contract brings the highest expected profit for the firm.

When $\Delta R \in (V_{c1}, V_{c2})$, centralized and delegated pooling both have equilibriums and delegated pooling performs better than centralized pooling. The equilibrium centralized separating contract does not exist, whereas the equilibrium delegated separating contract depends on the V_{d1} value. Thus, the firm would select the price delegation scheme and offer separating (pooling) contracts if there is (is no) equilibrium separating contract.

When $\Delta R \in [V_{c2}, V_{c3}]$, the centralized separating contract is the best choice for the firm, and there is no information rent for either agent type.

When Condition C holds and $\Delta R \in (V_{c3}, V_{c4})$, centralized separating equilibrium does not exist. Pooling equilibrium exists under both pricing schemes, and the profits for the firm are identical. When $\Delta R \in [V_{c4}, \infty)$, the firm would benefit from the separating contract with centralized pricing more than the centralized or delegated pooling contract.

When condition C does not hold and $\Delta R \in (V_{c3}, \infty)$, the firm offers a pooling contract in equilibrium to both agent types under both pricing schemes.

Mishra and Prasad (2004) found that the expected profit from centralized separating is at least as high, and may even be higher, as that from delegated separating. This finding is still true for our model with type-dependent reservations. However, when the equilibrium centralized separating does not exist, delegated pricing performs as least as well as centralized pricing under pooling contract.

Observation 5 *Separating contract is always the first choice of the firm regardless of the pricing scheme.*

Our results also verify that screening agent is always better. For the firm, separating contracts are always preferred over the corresponding pooling contracts. When the equilibrium exists under all four strategies, the preference ranking for the firm from high to low is centralized separating, delegated separating, delegated pooling, and centralized pooling.

4 Discussion

In this section, we examine two issues: the benefit of knowing the difference of reservations and whether the key findings can be extended to models with multiple agent types.

4.1 Benefit of Knowing the Difference of Reservations

We have shown in previous sections that type-dependent reservation affects contract design and agent behaviors. Now, we examine the benefit of knowing the difference of reservations to the firm. Consider the case (Situation 1) of centralized pricing with reservations R_H and R_L for the high type and the low type, respectively. Suppose that the firm either does not know that the reservations differ or how they differ, and simply assumes a common reservation R (Situation 2). Under centralized pricing, we note from Table 1 that only the optimal payments t_H and t_L depends on the reservation. Thus, the optimal decisions for Situation 2 are given by the second column of Table 1, except for

$$
\begin{cases}
t_H^R = R + \frac{s_0^2 \theta_H^2}{2(2b - \theta_H^2)^2} + V_{c1} \\
t_L^R = R + \frac{s_0^2 \psi^2 \theta_L^2}{2(2b - \psi \theta_L^2)^2}
\end{cases}
. \tag{13}
$$

We then examine the decisions and performance in Situation 2 and compare them with those in Situation 1. Obviously, the assumed R must be at least as large as R_L because otherwise, the IR constraints would be violated in Situation 2.

For $0 \leq \Delta R \leq V_{c1}$, both agent types would choose the contracts for their types. The realized sales and agent efforts would be the same in both situations. Without knowing R_H and R_L, the higher compensations to the agent would reduce the expected profit of the firm.

For $V_{c1} < \Delta R < V_{c2}$, pooling contract is used in Situation 1. If $R_L \leq R < R_H - V_{c1}$, the low-type agent would accept $\{q_L^*, t_L^*\}$ whereas the high-type agent will refuse $\{q_H^*, t_H^*\}$ as his reservation is not satisfied. If $R \geq R_H - V_{c1}$, both agent types would select the contracts for their types, and the realized sales and efforts of the two types are higher than those in Situation 1. If

$$
R \leq R_L + \frac{s_0^2 b \rho (\theta_H^2 - \theta_L^2)^2}{(\theta_H^2 - \rho \theta_L^2)(2b - \theta_H^2)(2b - \theta_L^2)(2b - \psi \theta_L^2)},
$$

the profit of the firm would be higher than that in Situation 1. Otherwise, the profit of the firm would be lower.

When $V_{c2} \leq \Delta R < V_{c4}$ and Condition C holds. If $R_L \leq R < R_H - V_{c1}$, the high type would not accept contract $\{q_H^*, t_H^*\}$. If $R \geq R_H - V_{c1}$, both agent types would choose the contracts for their types. The realized sales and efforts for the high type are the same, whereas those for the low type are lower compared with those in Situation 1. Similarly, the profit of the firm is also lower.

When $\Delta R \geq V_{c4}$ and Condition C holds: Similarly, the high type would not accept the contract if $R_L \leq R < R_H - V_{c1}$. If $R \geq R_H - V_{c1}$, both types would accept the contracts for their types. The

realized sales and efforts for both agent types are lower, and the profit of the firm is also lower than that in Situation 1.

When $\Delta R \geq V_{c2}$ and Condition C does not hold: The comparisons for this case are the same as the corresponding comparisons for the case when $V_{c2} \leq \Delta R < V_{c4}$ and Condition C holds.

In summary, the firm benefits from knowing the two different reservations for most values of ΔR. The firm may not benefit only when separating contracts do not exist in Situation 1 and the assumed R is relatively low.

4.2   Generalization to N Types

In this subsection, we consider the compensation problem of a heterogenous sales force with N agent types under centralized pricing. Our purpose is to examine whether the finding, Proposition 6, is generally true. The equivalent principal-agent problem formulation is (for $\sum_i \rho_i = 1, 0 < \rho_i < 1$)

$$\textbf{P0}: \max_{t_i, q_i, p_i} \sum_i \rho_i(p_i \times q_i - t_i)$$

$$s.t. \quad \begin{array}{ll} (I) & t_i - V(e_{ii}) \geq R_i \quad for \;\; all \;\; i \in [1, 2, ..., N] \\ (II) & t_i - V(e_{ii}) \geq t_j - V(e_{ij}) \quad for \;\; all \;\; i, j \in [1, 2, ..., N] \;\; and \;\; i \neq j \;\; \cdot \\ (III) & e_{ix} = argmax \;\; t_x - V(e_{ix}) \quad for \;\; all \;\; i, x \in [1, 2, ..., N] \end{array} \qquad (14)$$

Here, (I) includes the individual rationality constraints, whereas (II) and (III) are the incentive compatibility constraints.

The proof of Proposition 6' follows directly from the arguments in the proof of Proposition 6 and is outlined in the appendix.

Proposition 6' *Under the equilibrium centralized pricing contract, any of the N agent types may receive zero premium and may exert the same effort as though they were members of a homogeneous sales force depending on the reservation difference.*

In contrast to the finding of Rao (1990), we determine that with type-dependent reservations, paying information rent to all agent types, except the lowest type, in order to reveal the true types of all agents is not always necessary. Paying no information rent to any agent type and determine true agent types is possible. Furthermore, a contract may be designed to induce any agent type to extend his/her best efforts. Under certain conditions, mobilizing all the agents in a heterogeneous sales force to exert the same effort as though they were from a homogeneous sales force is possible without

paying any information rent. All these imply that a heterogeneous sales force may work as well as a homogeneous sales force for a firm.

5   Conclusion

In an early work on quota-based sales force compensation plan, Rao (1990) observed that only agents with the highest ability in a heterogeneous sales force exert their best sales effort and that the firm that employs a heterogeneous sales force has to pay information rents to all but the agent with the lowest ability in the optimal compensation plan. This observation implies that as far as a firm is concerned, a heterogeneous sales force does not work as well as a homogeneous sales force. In this paper, we revisit the same problem but with type-dependent reservation. We also incorporate the issue of pricing decision scheme in our model. By first using a model with two agent types and then considering N agent types, we show results opposite to Rao: at least one agent type does not receive information rent, whereas at least one agent type exerts the best sales effort under the optimal compensation plan. Furthermore, retaining centralized pricing brings the benefit of a tool for regulating agent behavior. Moreover, in some cases, designing a compensation plan under which a firm can learn the private informationof agents without paying any information rent is possible, and motivate all agents to exert their best efforts. Interestingly, when no agent receives a positive premium, all agents exert their best sales efforts. This is an ideal situation for a firm and duplicates the performance of a homogeneous sales force. We find that the outcomes depend on the interplay between agent reservation differences and the cost differences for achieving quotas not intended for their own types. This finding under scores the importance of a good understanding of the goals and effort costs of different agents when a heterogeneous sales force is employed. Our findings also show that although simplifying analysis, the commonly used assumption of identical reservation can lead to distorted conclusions.

We also find that screening the agents is always better for the firm, that is, separating contracts are always preferred over the corresponding pooling contracts. Although centralized pricing is preferred when separating contracts offered, delegated pricing provides greater benefits for the firm when only pooling contracts are available.

We use discrete agent types in the model. When the agent type parameter is continuous as in Rao (1990), we could still show that Rao's observation on information rent and effort level is no longer true

when reservation is type dependent.

We use a deterministic sales response function to refine the analysis. We believe that the general conclusion would still hold under an uncertain sales response function. The analysis can be much more complicated, but is worthy of further study.

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Appendix

Proof of Proposition 1: We introduce the Kuhn-Tucker Lagrangian function of $\mathbf{P_c}$ as follows:

$$
\begin{aligned}
L(p_H, q_H, t_H, p_L, q_L, t_L) \quad &= \rho(p_H \times q_H - t_H) + (1 - \rho)(p_L \times q_L - t_L) \\
&\quad -\lambda_1[R_H - (t_H - V(e_{HH}))] - \lambda_2[R_L - (t_L - V(e_{LL}))] \\
&\quad -\lambda_3[t_L - V(e_{HL}) - (t_H - V(e_{HH}))] - \lambda_4[t_H - V(e_{LH}) - (t_L - V(e_{LL}))],
\end{aligned}
$$

where $\lambda_i \geq 0$ and $i = 1, 2, 3, 4$ are the Lagrangian multipliers. The Kuhn-Tucker conditions (KTC) are

$$\frac{\partial L}{\partial q_H} = 0, \quad \frac{\partial L}{\partial t_H} = 0, \quad \frac{\partial L}{\partial q_L} = 0, \quad \frac{\partial L}{\partial t_L} = 0,$$

$$\lambda_1[R_H - (t_H - V(e_{HH})] = 0, \quad \lambda_2[R_L - (t_L - V(e_{LL}))] = 0,$$

$$\lambda_3[t_L - V(e_{HL}) - (t_H - V(e_{HH}))] = 0, \quad \text{and} \quad \lambda_4[t_H - V(e_{LH}) - (t_L - V(e_{LL}))] = 0.$$

From $\partial L/\partial t_H = -\rho + \lambda_1 + \lambda_3 - \lambda_4 = 0$ and $\partial L/\partial t_L = -(1 - \rho) + \lambda_2 - \lambda_3 + \lambda_4 = 0$, we have $\lambda_1 + \lambda_2 = 1$. Given that $\lambda_1 \geq 0$, we have either $\lambda_1 = 0$ or $\lambda_1 > 0$.

1) $\lambda_1 = 0$. We have $\lambda_2 = 1$; thus, Constraint (II) is binding. From $\partial L/\partial t_H = 0$, we have $\lambda_3 = \lambda_4 + \rho$; hence, $\lambda_3 > 0$ and Constraint (III) is binding. Given that (III) is binding, Constraint (I) becomes $t_L - V(e_{HL})) \geq R_H$. With this inequality and (II) being binding, we conclude that for $\lambda_1 = 0$ a Kuhn-Tucker point (KTP) may be found if $\Delta R \leq V(e_{LL}) - V(e_{HL}) = V_{c1}$. Here, e_{LL} and e_{HL} are the efforts needed by type L and type H to satisfy quota q_L of the KTP ($\{p_H, q_H, t_H, p_L, q_L, t_L\}$) with $\lambda_1 = 0$ and $\lambda_2 = 1$, provided that the KTP exists.

2) $\lambda_1 > 0$. In this case, Constraint (I) is binding. For λ_2, if $\lambda_2 > 0$, Constraint (II) is binding. With both (I) and (II) binding, Constraints (III) and (IV) become

$$R_H \geq R_L + V(e_{LL}) - V(e_{HL}) \quad \text{and} \quad R_L \geq R_H + V(e_{HH}) - V(e_{LH}),$$

respectively. By these two inequalities, a KTP can be be found when $\lambda_1 > 0$ and $\lambda_2 > 0$ if $V_{c2} \leq \Delta R \leq V_{c3}$, where $V_{c2} = V(e_{LL}) - V(e_{HL})$ and $V_{c3} = V(e_{LH}) - V(e_{HH})$. Here, if a KTP exists when $\lambda_1 > 0$ and $\lambda_2 > 0$, e_{LL} and e_{HL} are the efforts needed for type L and type H to satisfy q_L while e_{LH} and e_{HH} are the efforts needed for type L and type H to satisfy q_H of the KTP.

If $\lambda_2 = 0$, we have from $\partial L/\partial t_L = 0$, $\lambda_4 = \lambda_3 + 1 - \rho$; hence, $\lambda_4 > 0$ and Constraint (IV) is binding. Given that (IV) is binding, Constraint (II) can be rewritten as $t_H - V(e_{LH}) \geq R_L$. With this inequality and Constraint (I) binding, a K-T point can be found with $\lambda_1 > 0$ and $\lambda_2 = 0$ if

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$\Delta R \geq V(e_{LH}) - V(e_{HH}) = V_{c4}$. Here, e_{LH} and e_{HH} are the efforts needed for type L and type H to satisfy the q_H in the KTP when $\lambda_1 > 0$ and $\lambda_2 = 0$.

To summarize, there are three possible KTPs, corresponding to the following three cases: (1) $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 > 0$, $\lambda_4 \geq 0$; (2) $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 \geq 0$, $\lambda_4 \geq 0$; and (3) $\lambda_1 = 1$, $\lambda_2 = 0$, $\lambda_3 \geq 0$, $\lambda_4 > 0$.

First solving P_c in case (1) $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 > 0$, $\lambda_4 \geq 0$ (where $0 \leq \Delta R \leq V_{c1}$). P_c can be simplified as P_1.

$$\mathbf{P_1}: \max_{\mathbf{p_H, q_H, t_H, p_L, q_L, t_L}} \{\rho(p_H \times q_H - t_H) + (1-\rho)(p_L \times q_L - t_L)\}$$

$$s.t. \begin{cases} t_L - V(e_{LL}) = R_L \\ t_H - V(e_{HH}) = t_L - V(e_{HL}) \\ t_L - V(e_{LL}) \geq t_H - V(e_{LH}) \end{cases}$$

We need to consider two cases for P_1: $\lambda_4 > 0$ and $\lambda_4 = 0$.

1) For $\lambda_4 > 0$, Constraint (IV) becomes

$$(A1) \quad t_L - V(e_{LL}) = t_H - V(e_{LH}),$$

by which Constraint $t_H - V(e_{HH}) = t_L - V(e_{HL})$ in $P1$ can be rewritten to $V(e_{HH}) + V(e_{LL}) = V(e_{LH}) + V(e_{HL})$. By demand response function $s = s_0 + \theta e - bp$ and assumption $V(e) = e^2/2$, we have

$$\frac{1}{2}(\frac{q_H - s_0 + bp_H}{\theta_H})^2 + \frac{1}{2}(\frac{q_L - s_0 + bp_L}{\theta_L})^2 = \frac{1}{2}(\frac{q_H - s_0 + bp_H}{\theta_L})^2 + \frac{1}{2}(\frac{q_L - s_0 + bp_L}{\theta_H})^2,$$

leading to $q_L + bp_L = q_H + bp_H$ and $t_L = t_H$.

With all three constraints in P_1 binding, it is optimal for the firm to offer the same quotas and payments to the two types of agents. Replacing p_H, q_H and t_H by p_L, q_L and t_L, respectively, in the objective function of P_1, and further replacing t_L by $V(e_{LL}) + R_L$ according to (A1), P_1 is reduced to

$$(A2) \quad \mathbf{P_1(1)}: \max_{\mathbf{p_L, q_L}} \{p_L \times q_L - V(e_{LL}) - R_L\}.$$

Applying demand response function (1) and assumption (2) again, (A2) becomes

$$\mathbf{P_1(1)}: \max_{\mathbf{p_L, q_L}} \pi_f(p_L, q_L) = p_L \times q_L - \frac{1}{2}(\frac{q_L - s_0 + bp_L}{\theta_L})^2 - R_L.$$

The Hessian matrix of $\pi_f(p_L, q_L)$ is negative definite everywhere. We obtain $p_{L1}^*(1) = p_{H1}^*(1), q_{L1}^*(1) = q_{H1}^*(1)$ from the first-order conditions of $\pi_f(p_L, q_L)$:

$$p_{L1}^*(1) = \frac{s_0}{2b - \theta_L^2}, q_{L1}^*(1) = \frac{bs_0}{2b - \theta_L^2}$$

Under this quota, type H and type L exert the following efforts

$$e_{HH1}^*(1) = \frac{\theta_L}{\theta_H} \frac{s_0 \theta_L}{2b - \theta_L^2}, e_{LL1}^*(1) = \frac{s_0 \theta_L}{2b - \theta_L^2}$$

and the optimal payments are $t_{L1}^*(1) = t_{H1}^*(1) = R_L + \frac{1}{2} e_{LL1}^{*2}(1)$. Then, $\{p_{i1}^*(1), q_{i1}^*(1), t_{i1}^*(1)\}$ $(i = H, L)$ is a KTP.

2) For $\lambda_4 = 0$, Constraint (IV) is slack and can be eliminated. P_1 now becomes

$$\mathbf{P_1(2)} : \max_{\mathbf{p_H, q_H, t_H, p_L, q_L, t_L}} \{\rho(p_H \times q_H - t_H) + (1 - \rho)(p_L \times q_L - t_L)\}$$

$$s.t. \begin{cases} t_L - V(e_{LL}) = R_L \\ t_H - V(e_{HH}) = t_L - V(e_{HL}) \end{cases}.$$

Substituting the two binding constraints into the objective function, $P_1(2)$ becomes

$$\mathbf{P_1(2)} : \max_{\mathbf{p_H, q_H, p_L, q_L}} \{\rho(p_H \times q_H - (V(e_{LL}) + V(e_{HH}) - V(e_{HL}) + R_L)) + (1 - \rho)(p_L \times q_L - (V(e_{LL}) + R_L))\}$$

without any constraints. Again, we can write $P1(2) : \max_{\mathbf{p_H, q_H, p_L, q_L}} \pi_f(p_H, q_H, p_L, q_L)$ where

$$\pi_f(p_H, q_H, p_L, q_L) = \rho[p_H \times q_H - \frac{1}{2}[(\frac{q_L - s_0 + bp_L}{\theta_L})^2 + (\frac{q_H - s_0 + bp_H}{\theta_H})^2 - (\frac{q_L - s_0 + bp_L}{\theta_H})^2]]$$
$$+ (1 - \rho)[p_L \times q_L - \frac{1}{2}(\frac{q_L - s_0 + bp_L}{\theta_L})^2] - R_L.$$

The Hessian matrix of $\pi_f(p_H, q_H, p_L, q_L)$ is negative definite everywhere. We obtain $p_{H1}^*(2), q_{H1}^*(2)$ and $p_{L1}^*(2), q_{L1}^*(2)$ from the first-order conditions,

$$(A3) \quad \begin{cases} p_{H1}^*(2) = \frac{s_0}{2b - \theta_H^2}, q_{H1}^*(2) = \frac{bs_0}{2b - \theta_H^2} \\ p_{L1}^*(2) = \frac{s_0}{2b - \psi\theta_L^2}, q_{L1}^*(2) = \frac{bs_0}{2b - \psi\theta_L^2} \end{cases}$$

where $\psi_1 = \frac{1 - \rho}{1 - \rho/r^2}$. The optimal efforts for the two agents are

$$e_{HH1}^*(2) = \frac{s_0 \theta_H}{2b - \theta_H^2}, e_{LL1}^*(2) = \frac{s_0 \psi_1 \theta_L}{2b - \psi_1 \theta_L^2}$$

The optimal payments are

$$\begin{cases} t_{H1}^*(2) = \frac{1}{2}(e_{HH1}^*(2))^2 + (1 - \frac{1}{r^2})\frac{1}{2}(e_{LL1}^*(2))^2 + R_L \\ t_{L1}^*(2) = \frac{1}{2}(e_{LL1}^*(2))^2 + R_L \end{cases}$$

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$\{p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2)\}(i = H, L)$ is another KTP for P$_1$. Comparing the profits of the firm at the two KTPs, we find

$$\pi_f(p_{i1}^*(1), q_{i1}^*(1), t_{i1}^*(1)) < \pi_f(p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2)) \quad (i = H, L).$$

In other words, the KTP given by (A3) should be kept for P$_1$. Because $e_{HL} = \frac{q_L + bp_L - s_0}{\theta_H}$, taking the value of $q_{L1}^*(2)$ in P$_1$'s optimal solution, we have $V_{c1} = \frac{1}{2}(1 - \frac{1}{r^2})(e_{LL1}^*(2))^2$.

We still need to justify whether KTP $\{p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2)\}$ $(i = H, L)$ is the global maximum point of P$_c$ when $0 \leq \Delta R \leq V_{c1}$. We use the theorem of KKT Second-order Sufficient Conditions.

Rearranging Constraints (II) and (III) in P$_c$, we have

$$\begin{cases} h_1(p_H, q_H, t_H, p_H, q_L, t_L) = R_L - t_L + \frac{(q_L - s_0 + bp_L)^2}{2\theta_L^2} \leq 0 \\ h_2(p_H, q_H, t_H, p_H, q_L, t_L) = t_L - \frac{(q_L - s_0 + bp_L)^2}{2\theta_H^2} - t_H + \frac{(q_H - s_0 + bp_H)^2}{2\theta_H^2} \leq 0 \end{cases}.$$

Notice that constraints (II) and (III) are binding at the KTP for $0 \leq \Delta R \leq V_{c1}$. We can obtain the gradients of $h_i(,)$ $(i = 1, 2)$ at the KTP:

$$\begin{cases} \nabla h_1(,)^t = (0, 0, 0, \frac{bs_0\psi_1}{2b - \psi_1\theta_L^2}, \frac{s_0\psi_1}{2b - \psi_1\theta_L^2}, -1) \\ \nabla h_2(,)^t = (\frac{bs_0}{2b - \theta_H^2}, \frac{s_0}{2b - \theta_H^2}, -1, -\frac{bs_0\psi_1}{r^2(2b - \psi_1\theta_L^2)}, -\frac{s_0\psi_1}{r^2(2b - \psi_1\theta_L^2)}, 1) \end{cases}.$$

Define the cone $C = \{d \neq 0 : \nabla h_1(,)^t d = 0 \ and \ \nabla h_2(,)^t d = 0\}$. Then vector d is given by

$$\begin{vmatrix} x_1 \\ x_2 \\ (bx_1 + x_2)\frac{s_0}{2b - \theta_H^2} + (bx_3 + x_4)(1 - \frac{1}{r^2})\frac{s_0\psi_1}{2b - \psi_1\theta_L^2} \\ x_3 \\ x_4 \\ (bx_3 + x_4)\frac{s_0\psi_1}{2b - \psi_1\theta_L^2} \end{vmatrix},$$

where $x_i \epsilon R$, $i = 1, 2, 3, 4$ and not every x_i can be zero at the same time. The Hessian of the Lagrangian function $L(p_H, q_H, t_H, p_L, q_L, t_L)$ at the KTP is

$$\nabla^2 L(,) = \begin{vmatrix} -\frac{\rho b^2}{\theta_H^2} & \rho(1 - \frac{b}{\theta_H^2}) & 0 & 0 & 0 & 0 \\ \rho(1 - \frac{b}{\theta_H^2}) & -\frac{\rho}{\theta_H^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b^2(\frac{\rho}{\theta_H^2} - \frac{1}{\theta_L^2}) & 1 - \rho + b(\frac{\rho}{\theta_H^2} - \frac{1}{\theta_L^2}) & 0 \\ 0 & 0 & 0 & 1 - \rho + b(\frac{\rho}{\theta_H^2} - \frac{1}{\theta_L^2}) & \frac{\rho}{\theta_H^2} - \frac{1}{\theta_L^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}.$$

We have $d^t \nabla^2 L(p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2))d < 0$ $(i = H, L)$. Thus, KTP $\{p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2)\}$, $(i = H, L)$ is the strict local maximum for P when $0 \leq \Delta R \leq V_{c1}$. Because P_c is continuous in the six variables, $\{p_{i1}^*(2), q_{i1}^*(2), t_{i1}^*(2)\}$, $(i = H, L)$ must be the global maximum of P_c for $0 \leq \Delta R \leq V_{c1}$.

The solution procedures for the other two cases are completely parallel and are omitted.

Proof of Proposition 2: We introduce the Kuhn-Tucker Lagrangian function of $\mathbf{P_{cp}}$ as follows:

$$L(p, q, t) = p \times q - t - \lambda_1(R_H - (t - V(e_H))) - \lambda_2(R_L - (t - V(e_L)))$$

where $\lambda_i \geq 0$ and $i = 1, 2$ are the Lagrangian multipliers. The Kuhn-Tucker conditions (KTC) are

$$\frac{\partial L}{\partial p} = 0, \quad \frac{\partial L}{\partial t} = 0, \quad \frac{\partial L}{\partial q} = 0$$
$$\lambda_1[R_H - (t - V(e_H)] = 0, \quad \lambda_2[R_L - (t - V(e_L))] = 0$$

From $\frac{\partial L}{\partial t} = -1 + \lambda_1 + \lambda_2 = 0$, we have $\lambda_1 + \lambda_2 = 1$. Given $\lambda_1 \geq 0$, we have $\lambda_1 = 0$ or $\lambda_1 > 0$.

1) $\lambda_1 = 0$, $\lambda_2 = 1$; thus, Constraint $t - V(e_L) \geq R_L$ is binding. Constraint $t - V(e_H) \geq R_H$ becomes $V(e_L) - V(e_H) \geq R_H - R_L$. So we conclude that for $\lambda_1 = 0$, a KPT may be found if $\Delta R \leq V(e_L) - V(e_H)$. In this case, P_{cp} becomes,

$$\mathbf{P_{cp}(1)} : \max_{\mathbf{p,q}} \pi_f(p, q) = p \times q - \frac{1}{2}(\frac{q - s_0 + bp}{\theta_L})^2 - R_L.$$

The Hessian matrix of $\pi_f(p, q)$ is negative definite everywhere. We obtain the optimal p and q from the first-order conditions and $V(e_L) - V(e_H) = V_{c2}$.

2) $\lambda_1 = 1$, $\lambda_2 = 0$; Case 2) is similar to Case 1). We just need to interchange "H" and "L". The KPT may be found if $\Delta R \geq V(e_L) - V(e_H)$.

$$\mathbf{P_{cp}(2)} : \max_{\mathbf{p,q}} \pi_f(p, q) = p \times q - \frac{1}{2}(\frac{q - s_0 + bp}{\theta_H})^2 - R_H.$$

The optimal p and q can be get from the FOCs and $V(e_L) - V(e_H) = V_{c3}$ in this case.

3) $\lambda_1 > 0$, $\lambda_2 > 0$; thus, two constrains will be binding, which leads to $R_H - R_L = V(e_L) - V(e_H)$. So Case 3) will be at the endpoint of Case 1) or at the initial point of Case 2) for different ΔR.

The proofs of Proposition 4 and Proposition 5 are similar to that of Proposition 1 and Proposition 2 and are omitted.

Proof of Premiums in Table 3: We show the conclusions in five cases according to the value of ΔR.

1) When $0 \leq \Delta R \leq V_{c1}$, type H and type L agents choose contracts $\{p_{H1}^*, q_{H1}^*, t_{H1}^*\}$ and $\{p_{L1}^*, q_{L1}^*, t_{L1}^*\}$ (second column of Table 1) in equilibrium, respectively. The premiums they receive are $P_H(q_{H1}^*) = t_{H1}^* - V(e_{HH1}^*) - R_H = V_{c1} - \Delta R \geq 0$ and $P_L(q_{L1}^*) = t_{L1}^* - V(e_{LL1}^*) - R_L = 0$.

2) When $V_{c1} < \Delta R < V_{c2}$, type H and type L agents will accept the only contract $\{p_L^*, q_L^*, t_L^*\}$ in equilibrium (second column of Table 2). Their premiums are $P_H(q_L^*) = V_{c2} - \Delta R$ and $P_L(q_L^*) = 0$.

3) When $V_{c2} \leq \Delta R \leq V_{c3}$, type H and type L agents choose contracts $\{p_{H2}^*, q_{H2}^*, t_{H2}^*\}$ and $\{p_{L2}^*, q_{L2}^*, t_{L2}^*\}$ in equilibrium(third column of Table 1), respectively. Their premiums are $P_H(q_{H2}^*) = 0$ and $P_L(q_{L2}^*) = 0$.

4) When $V_{c3} < \Delta R < V_{c4}$, type H and type L agents will accept the only contract $\{p_H^*, q_H^*, t_H^*\}$ in equilibrium (third column of Table 2). Their premiums are $P_H(q_L^*) = 0$ and $P_L(q_L^*) = \Delta R - V_{c3}$.

5) When $V_{c4} \leq \Delta R$, if condition C holds, type H and type L agents choose contracts $\{p_{H3}^*, q_{H3}^*, t_{H3}^*\}$ and $\{p_{L3}^*, q_{L3}^*, t_{L3}^*\}$ (fourth column of Table 1) in equilibrium, respectively. Their premiums are $P_H(q_{H3}^*) = 0$ and $P_L(q_{L3}^*) = \Delta R - V_{c4} \geq 0$. If condition C is not satisfied, the two agent types receive the same premium as given in (4).

Proof of Observation 1: The observation can be drawn directly from Proposition 1. If condition C holds, for $0 \leq \Delta R \leq V_{c1}$, $V_{c2} \leq \Delta R \leq V_{c3}$, and $V_{c4} \leq \Delta R$, we have p_H^*, q_H^* and p_L^*, q_L^* from the following equations, respectively:

$$\begin{cases} p_{H1}^* = \frac{s_0}{2b - \theta_H^2}, & q_{H1}^* = \frac{bs_0}{2b - \theta_H^2} \\ p_{L1}^* = \frac{s_0}{2b - \psi_1\theta_L^2}, & q_{L1}^* = \frac{bs_0}{2b - \psi_1\theta_L^2} \end{cases}$$

$$\begin{cases} p_{H2}^* = \frac{s_0}{2b - \theta_H^2}, & q_{H2}^* = \frac{bs_0}{2b - \theta_H^2} \\ p_{L2}^* = \frac{s_0}{2b - \theta_L^2}, & q_{L2}^* = \frac{bs_0}{2b - \theta_L^2} \end{cases}$$

$$\begin{cases} p_{H3}^* = \frac{s_0}{2b - \psi_2\theta_H^2}, & q_{H3}^* = \frac{bs_0}{2b - \psi_2\theta_H^2} \\ p_{L3}^* = \frac{s_0}{2b - \theta_L^2}, & q_{L3}^* = \frac{bs_0}{2b - \theta_L^2} \end{cases}$$

Given that

$$\psi_1 = \frac{1 - \rho}{1 - \rho/r^2} < 1 \quad \text{and} \quad \psi_2 = \frac{\rho}{1 - (1 - \rho)r^2} > 1,$$

we have $p_H^* > p_L^*$, $q_H^* > q_L^*$ for all three cases. The cost difference required for type H and type L agents to achieve q_H^* is

$$V(e_{LH}^*) - V(e_{HH}^*) = \frac{1}{2}(q_H^* - s_0 + bp_H^*)^2(\frac{1}{\theta_L^2} - \frac{1}{\theta_H^2}).$$

The cost difference required for type H and type L agents to achieve q_L^* is

$$V(e_{LL}^*) - V(e_{HL}^*) = \frac{1}{2}(q_L^* - s_0 + bp_L^*)^2(\frac{1}{\theta_L^2} - \frac{1}{\theta_H^2}).$$

Given that $p_H^* > p_L^*$ and $q_H^* > q_L^*$, we have $V(e_{LH}^*) - V(e_{HH}^*) > V(e_{LL}^*) - V(e_{HL}^*)$.

Proof of Proposition 6: We only need to show the optimal effort of an agent in a homogeneous sales force. For this, we solve the following problem:

$$\max_{\mathbf{p},\mathbf{q},\mathbf{t}}\{p \times q - t\}$$

$$s.t. \quad t - \frac{1}{2}(\frac{q - s_0 + bp}{\theta})^2 \geq R.$$

Similar to the proof of Proposition 1, we can show that $p^* = \frac{s_0}{2b-\theta^2}$ and $q^* = \frac{bs_0}{2b-\theta^2}$ The optimal effort the agent has to exert to achieve quota q^* is $e^* = \frac{s_0\theta}{2b-\theta^2}$. The optimal payment is $t^* = \frac{1}{2}e^{*2} + R$.

Proof of Proposition 6':

The firm needs to solve the optimization problem P0 with N^2 constraints. We introduce the Kuhn-Tucker Lagrangian function for Problem P0:

$$\begin{aligned}
L(p_i, q_i, t_i) &= \sum_i \rho_i(p_i \times q_i - t_i) \\
&- \sum_i \lambda_{ii}[R_i - (t_i - V(e_{ii}))] \\
&- \sum_i \sum_j \lambda_{ij}[t_j - V(e_{ij}) - (t_i - V(e_{ii}))]
\end{aligned}$$

where $\lambda_{ii} \geq 0, \lambda_{ij} \geq 0$, $i, j \in [1, 2, ..., N]$ and $i \neq j$, $\lambda_{ii}, \lambda_{ij}$ are the Lagrangian multipliers.

$$\begin{cases} \frac{\partial L}{\partial p_i} = 0, \frac{\partial L}{\partial q_i} = 0, \frac{\partial L}{\partial t_i} = 0 \\ \lambda_{ii}[R_i - (t_i - V(e_{ii}))] = 0, \quad \lambda_{ij}[t_j - V(e_{ij}) - (t_i - V(e_{ii}))] = 0 \end{cases}.$$

From

$$\frac{\partial L}{\partial t_i} = -\rho_i + \lambda_{ii} + \sum_j(\lambda_{ij} - \lambda_{ji}) = 0 \quad for \quad all \quad i \neq j$$

we have $\sum_i \lambda_{ii} = 1$. Because $\lambda_{ii} \geq 0$, the KTCs require $\lambda_{ii} = 0$ or $\lambda_{ii} > 0$. λ_{ii} is the Lagrangian multiplier for type i's IR constraint. Thus, similar to the proof of Proposition 1, any λ_{ii} could be Zero, but could not all be Zero at the same time. At least one λ_{ii} must be positive, and any λ_{ii} could be

positive. Further, all λ_{ii} could be positive at the same time. With these properties, we can conclude that any type agent's IR could be binding and thus any type can get Zero premium. Following the analysis of Proof for Proposition 1, we have Proposition 6'.