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Lingnan University Hong Kong Dynamic House Prices and Trading Volumes across Quality Tiers and Upward Mobility

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# Dynamic House Prices and Trading Volumes across Quality Tiers and Upward Mobility

Lok Sang Ho<sup>\*</sup>, Yue Ma<sup>+</sup> and Donald R. Haurin<sup>^</sup>

#### Abstract

We argue that shocks to a housing market are transmitted through the hierarchy of quality tiers within a housing market. The result is the prediction of waves of house price changes accompanied by changes in transaction volume. Our study is related to existing models of spatial ripple effects across housing markets. The data are from the Hong Kong housing market. The findings from Granger causality tests strongly support the argument that ripple or domino effects within a single housing market occur in response to external shocks.

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## **1. Introduction**

Housing market dynamics are complex. Theoretical approaches include models of neighborhood change, filtering, urban growth, and housing chains. Empirical tests of these models are relatively infrequent, perhaps due to the complexity of the models or the lack of data. However, given the importance of home equity to national wealth and the importance of housing to urban and national economies, there is a need for studies of house price and transaction dynamics.

Our study focuses on the interrelationships of housing submarkets where the submarkets are defined by quality tiers. The application is to Hong Kong, this locality being chosen because their changes in housing policy form a natural experiment and data are available. One implication of the empirical findings is that houses of all quality levels are closely tied together through dynamic processes. Another finding is that price effects move through the quality continuum of a housing market with relatively great rapidity. A third is that there is a positive correlation between the turnover rate of housing and house prices, reaffirming a relationship found in recent studies of housing markets.

## 2. Review of Approaches to Housing Market Dynamics

The dynamics of housing markets is first described using a variety of approaches. We will review filtering models, then models describing spatial ripples in house price changes. Finally, we review search and equity effect models, these focusing on the short run consequences of shocks to the housing market on price and turnover rates. Each of these models plays a role in guiding our theoretical approach to housing dynamics.<sup>1</sup>

The seminal contributions to the theoretical description of the filtering model were the papers by Sweeney (1974a, 1974b). In filtering models, the housing market is separated into distinct quality levels. In equilibrium households are matched to houses of different qualities according to their income levels and willingness to pay. The model predicts the long run equilibrium distributions of the quality of housing units and prices.

In this model, if a set of households' incomes rise, the group trades up for the next higher quality homes and the vacant homes

<sup>&</sup>lt;sup>1</sup> Review articles of some aspects of house price dynamics include Gatzlaff and Tirtiroglu (1995) and Cho (1996), both focusing on housing market efficiency.

become available to a lower income group. The short run equilibrium is disturbed, causing further substitution producing a ripple effect throughout the quality continuum. One limitation of Sweeney's model is that landlords do not face a down payment constraint when purchasing housing. Thus, the dynamics caused by the interaction of the down payment constraint and household equity levels are excluded from the model.

"Ripple effects" in regional house prices have been the focus of many studies, particularly in the United Kingdom (Alexander and Barrow 1994, Meen 1999, 2002.)<sup>2</sup> It is fairly well established that house price ripples occur in the U.K. flowing from the southeastern region to the north. Meen suggests that ripple effects could be caused by four factors: migration, different reaction speeds to shocks, spatial arbitrage, and home equity effects.

Migration could cause house price ripples if households relocate in response to changes in the spatial distribution in house prices (Jones and Leishman 2006). A ripple-like effect also could be observed if regions react to economic shocks with different speeds. Meen's third explanation is based on the spatial diffusion of house prices, a manifestation of arbitrage mitigated by search costs. Pollakowski and Ray (1997) used a VAR model to test whether house price changes in one region or primary metropolitan statistical area (PMSA) predict price changes in other regions or PMSAs. Their work built on Tirtiroglu (1992) and Clapp and Tirtiroglu (1994) who found that excess returns in a housing submarket diffused to other submarkets of the same metropolitan statistical area. Pollakowski and Ray found statistically significant cross-market housing price effects at the regional level, but the pattern of results was not economically sensible. They also found significant cross-PMSA effects in the New York consolidated metropolitan statistical area, with a slight preponderance of effects being for contiguous areas.

Meen's fourth explanation is based on Stein's (1995) model where equity effects are incorporated into the housing market. Stein's notes that households who wish to own must make a down payment and pay closing costs. If house prices are rising, current owners' home equity

<sup>&</sup>lt;sup>2</sup> Oikarinen (2006) studied the Finnish housing market and finds evidence that house prices changes move from central city to surrounding area. Cook has numerous studies (2006) of ripple effects in the U.K. housing market.

rises, increasing their wealth and allowing them to make a larger down payment on another, more expensive, home. Thus, increased house prices facilitate trading-up and thus transaction volume should increase. Stein showed that this effect is enhanced if minimum down payment requirements constrain a large percentage of current owners' choice of dwelling. Similarly, if house prices fall, a household's equity falls, and this household's ability to purchase another house is reduced, perhaps greatly. Transaction volume should fall at the same time that house prices are falling. One test of Stein's hypothesis was by Lamont and Stein (1999) who found that real house prices are more sensitive to shocks to per capita income in cities where a relatively high percentage of homeowners have a high loan to value ratio.

Hort (2000) also argued that changes in the turnover rate in housing are linked with changes in house prices. The basis for this argument is a search model developed by Berkovec and Goodman (1996). Hort explained that sellers establish list prices based on their expectations of sales prices, these expectations influenced by recent market prices and the recent "ease of selling". Buyers' offers are influenced by recent prices but also are subject to demand shocks such as unexpected changes in unemployment, income, population growth, mortgage interest rates, and migration. Thus, the distribution of buyers' offers moves before that of sellers' reservation prices. The result is a rapid change in the turnover rate as the market quickly clears in upmarkets and houses remain unsold in down-markets.<sup>3</sup> Initially there is little upward price movement in the up-market because sellers have previously set list prices based on the set of price expectations at the time of listing. There is little downward price movement initially in a down-market because even though houses remain unsold, those that sell do so near their list price (skimming off the upper tail of the distribution of buyers' offers). However, as sellers become aware of the change in the expected marketing time for a home, they adjust their list prices, inducing a positive correlation between the turnover rate and house price changes. Hort (2000) found the dynamics of the Swedish housing market followed this model's predictions.

<sup>&</sup>lt;sup>3</sup> This model is further elaborated in Fisher et al. (2003) where it is applied to the commercial property market.

## 3. The Model

The literature suggests a number of reasons why external shocks to a housing market could lead to the observation of a chain of house price changes. We next present a stylized model of a housing market with multiple quality levels of housing, matching its structure to that of our data set from the Hong Kong housing market. This focus results in particular assumptions, relevant for Hong Kong, that simplify the model. More general models can be found in Ortalo-Magné and Rady (2006), and Gervais (2002). We also highlight the role of changes in wealth and the repercussions in the housing market. The role of housing as an investment good and store of wealth has been highlighted by Henderson and Ioannides (1983).

Renters live in the lowest quality tier, and there are two quality levels of owner-occupied housing (this being generalizable to many quality levels). Household wealth plays a prominent role in determining housing demand, and we include a down payment constraint. When a positive shock to the wealth of renter households occurs, it increases the demand for the lowest tier owner-occupied houses, thus increasing their price. This price increase subsequently leads to greater demand and price increases in higher quality tiers of the housing market. Similarly, the increase in transactions among lower tier housing temporally leads increases in transaction volume in higher quality markets. These responses in price and volume can be characterized as a "domino effect" flowing through the quality continuum. Negative price shocks to the lowest quality tier of housing or the rental market will have similar effects in the opposite direction, leading to downward pressure on house prices and reductions in transaction volume in higher quality tiers. We expect that the effect will be larger among households that are down payment constrained.

## 3.1 The Market Structure

The market structure we consider includes a rental housing market and two types of owner-occupied housing: low (A) and high (B) quality. We assume that housing in the rental market is lower quality than owner-occupied housing and it is provided by the government to accommodate lower income people.<sup>4</sup> All households are utility maximizers. Rental housing tenants may choose to upgrade their housing and become homeowners when it is advantageous to do so. Owner-occupiers also may buy and sell property. We also assume that dwelling units in each category of housing are homogeneous and each household occupies only one unit of housing. The model considers the response to a positive shock to the wealth of renters.

The total stock of low-quality owner-occupied housing at the end of a period is given by:

(1)  $S_A = H_A + \theta_A (p_A - \underline{p_A}), \qquad \theta_A > 0 \text{ for } p_A > \underline{p_A}$ 

where  $H_A$  is the initial housing stock at the beginning of the period.  $\theta_A$  is a supply response coefficient that generates additional supply by the end of the period whenever housing price  $p_A > \underline{p}_A$ . Thus, any new demand  $(n_A)$ has to be met either by sales of existing houses  $(i_A)$  and/or by additional supply from developers.

Similarly, the supply of high-quality homes B depends on its housing price  $p_c$ :

(2)  $S_B = H_B + \theta_B (p_B - \underline{p_B}), \quad \theta_B > 0 \text{ for } p_B > \underline{p_B}$ 

The critical supply prices,  $\underline{p}_A$  and  $\underline{p}_B$ , are exogenous and represent prices below which developers in each of the markets would not find it profitable to supply new housing.<sup>5</sup>

In the initial state of steady state equilibrium, demand for housing is equal to the stock of housing for each market, and renters live in residences supplied by the government. For simplicity we assume depreciation is zero.

Suppose the initial equilibrium price of low-quality housing is  $\underline{p}_A$  and the number of households who own homes is  $i_A$ . Thus the initial total demand for housing stock in A is  $i_A h_A$ . Given the supply of housing A in (1), we have market A's stock equilibrium condition:

$$(3) \quad i_A h_A = H_A.$$

Similarly, suppose the initial equilibrium price of the high-quality market B before any shocks occur is  $p_B$ , and the number of households

<sup>&</sup>lt;sup>4</sup> This assumption is reasonable for our application to Hong Kong. It also provides a mechanism for introducing exogenous shocks to the rental market through changes in government policies. Shocks could also be introduced through a private sector rental market.

<sup>&</sup>lt;sup>5</sup> That is, they represent construction and land costs.

who own homes in market B is  $i_B$ . Market B's stock equilibrium condition is:

 $(4) \quad i_B h_B = H_B.$ 

Because B is the highest quality housing market, it is assumed that homeowners cannot trade up further. This assumption can be relaxed and the results inferred from activities in market A.

Additional new demand for high-quality homes  $(n_B)$  has to be met by the expansion of the supply, triggered by a price of  $p_B$  beyond  $\underline{p_B}$ . From this initial equilibrium, we examine the implications of rental tenants' wealth increasing (Ortalo-Magne and Rady 2006). When tenants in government provided housing have acquired sufficient savings, their utility maximizing behavior will lead them to bid for housing in market A and become first-time homeowners (Bardhan, et al. 2003).

## 3.2 The Model's Solution

In Appendix A, we describe the simplifying assumptions that convert a household's dynamic optimization problem into a static optimization problem. The equilibrium solution of the two housing markets is derived in Appendix B. A variable list of the model is given at the end of Appendix B. We assume all households have identical utility functions but have different amounts of lifetime wealth. The lifetime wealth of the household, K, comprises the present value of all future income plus the value of all assets. For someone who becomes a homeowner at the end of period t-1, the value of K at the end of period t is:

(5)  $K_t = K_{t-1}$  + capital appreciation through time *t*.

We assume that wealth varies among households and they can be sorted by wealth in descending order from largest to smallest with unique index *i*. For simplicity, we assume the distribution of *K* is a linear function of *i*, with the total number of households assumed to be equal to  $2\overline{K}\tau$ . Suppressing the time period subscripts, the wealth of household *i* is:

(6)  $K(i) = 2\overline{K} - i/\tau$ .

Thus, the greatest wealth of any households  $is_2\overline{K}$ , the lowest wealth is zero, and the average wealth  $is_{\overline{K}}$ . Hence, for a critical value  $K^*$ , we have:

(7)  $i^* = 2\tau \overline{K} - \tau K^*$ 

where  $i^*$  gives the number of households with wealth greater than or equal to  $K^*$ .

We next determine the demand for housing in the low-quality housing market A after a positive shock to renters' wealth has occurred. Changing wealth impacts housing choices directly as a household's lifetime wealth changes and also through the effects of the down payment constraint. Household *i* will purchase a house if and only if its wealth K(i) is greater than a critical value  $K_A^*$ ,

(8)  $K(i) > K_A^* = \varphi_A p_A - \varphi_{RA}$ 

where  $K_A^*$  defined in equation (A17) of Appendix A,<sup>6</sup> and  $\varphi_A$  and  $\varphi_{RA}$  are parameters defined also in this appendix. Otherwise, the household will not purchase a house and will remain in the rental market.

The number of renter households  $(n_A)$  that satisfy the wealth constraint and intend to change to ownership in market A is found by substituting (9) into (7), yielding:

(9) 
$$n_A' = 2\tau \overline{K}' - \tau K_A^* = 2\tau \overline{K}' - \tau \varphi_A p_A + \tau \varphi_{RA}$$

where  $p_A$  is market A's new price to be determined by the market equilibrium condition, which has now been affected with the average wealth changing from  $\overline{K}$  to  $\overline{K'}$ .

Suppose there is an additional 'liquid wealth' (W) constraint, at time *t*, following Stein's (1995) down payment hypothesis:

 $W_A \geq c.q + (1 - \varphi_L) p_A h_A$ 

where time subscript *t* is omitted for simplicity.  $W_A$  is the 'liquid wealth' at time *t* for a potential buyer in market A, this defined as anything that the household currently owns that can be sold reasonably quickly for money without incurring a loss. Also,  $h_A$  is the size of one unit of housing in market A,  $p_A$  is the price of housing in market A,  $(1 - \varphi_L)$  is the percentage of down-payment defined in Appendix A(ii),  $(1 - \varphi_L)p_Ah_A$  is the amount of down payment, and *c* and *q* are the quantity and price of the composite nonhousing goods, respectively.

Suppose at time t, only a proportion  $(\gamma_A)$  of households simultaneously satisfy the wealth constraint and the liquid wealth constraint. The number of households satisfying the lifetime wealth constraint is given by (9),  $n_A$ '. Therefore the additional number of households satisfying both the total and liquid wealth constraints and thus are able to afford a home in market A is:

<sup>&</sup>lt;sup>6</sup> The proof is provided in Appendix A.

(10)  $n_A = \gamma_A n_A' = \gamma_A (2\tau \overline{K}' - \tau \varphi_A p_A + \tau \varphi_{RA})$ 

Equation (10) implies that  $n_A$  increases with the mean wealth  $\overline{K}'$  and decreases with housing price  $p_A$ . From equation (B6) of Appendix B, we have the following result.

**Proposition 1.** A positive shock  $d\overline{K}$  to the wealth of renters, who are potential buyers in the low-quality housing market, will increase the prices of homes in market A. This is indicated by:

$$\frac{dp_A}{d\overline{K}} = \frac{2\tau h_A \gamma_A}{\Delta} > 0,$$

where  $\Delta$  is defined in equation (B5) of Appendix B and it is shown to be positive.

We now consider the transmission of the renter wealth shock through market A to the high-quality housing market B. The availability of units in market A for renters who wish to buy is subject to the willingness of existing homeowners to sell. This willingness is conditional on their utilities being higher after trading up to market B compared with staying put.

The wealth of households in market A as potential buyers in market B can be represented by:

 $K_{nB}(i, t) = K_{iA}(i,t-1) + p_A(t)h_A - p_A(t-1)h_A - \Psi_m$ where  $\Psi_m$  is the remaining mortgage. Suppressing the time indices *t* and *t*-1, and rewriting  $p_A(t-1)h_A + \Psi_m$  as  $\phi_m$ , the mean wealth  $\overline{K}_{nB}$  of the potential buyers in market B is given by:

(11)  $\overline{K}_{nB} = \overline{K}_{iA} + p_A h_A - \phi_m$ 

where  $\overline{K}_{iA}$  is the mean wealth of  $K_{iA}(i,t-1)$ . Equation (11) implies that an increase of the price in the low-quality housing market will increase the wealth of potential buyers; that is:

(12) 
$$\frac{\partial \overline{K}_{nB}}{\partial p_A} = h_A > 0.$$

Potential buyers in market B also are potential sellers in market A. But the actual number of sellers is given below by (13) after incorporating a down payment constraint similar to that in market A [cf. (10)]:

(13)  $n_B = \gamma_B (2\tau \overline{K}_{nB} - \tau K^*_B) = \gamma_B (2\tau \overline{K}_{nB} - \tau \varphi_B p_B + \tau \varphi_{BA} p_A - \tau \varphi_{B0})$ where  $p_A$  and  $p_B$  are housing prices to be determined by the market equilibrium condition,  $K^*_B$ ,  $\varphi_B$ ,  $\varphi_{BA}$  and  $\varphi_{B0}$  are defined in equation (A20) of Appendix A, and  $\gamma_B$  is the proportion of households satisfying the total wealth constraint that also satisfy the liquid wealth constraint. We assume there are no households in market B that sell their houses and thus all new demand must be met by estate developers. Thus the total demand in market B is  $n_Bh_B + i_Bh_B$ , which must equal total supply [cf. (2)]. Hence:

(14) 
$$n_B h_B + i_B h_B = \theta_B (p_B - \underline{p}_B) + H_B.$$
  
Substituting (4) and (13) into (14), we have:  
(15)  $\gamma_B (2\tau \overline{K}_{nB} h_B - \tau \varphi_B p_B h_B + \tau \varphi_{BA} p_A h_B - \tau \varphi_{B0} h_B) = \theta_B (p_B - \underline{p}_B).$   
Solving for the house price level yields:

(16) 
$$p_{B} = \frac{2\tau h_{B}\overline{K}_{nB}\gamma_{B} + p_{A}\tau \varphi_{BA}h_{B}\gamma_{B} - \tau \varphi_{B0}h_{B}\gamma_{B} + \theta_{B}\underline{p}_{B}}{\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B}}$$

From (16) we find:

(17) 
$$\frac{\partial p_B}{\partial \overline{K}_{nB}} = \frac{2\pi h_B \gamma_B}{\theta_B + \tau \varphi_B h_B \gamma_B} > 0$$

which leads to proposition 2.

**Proposition 2**. A price increase in the low-quality market will increase the price of the high-quality market through the channels of: (i) a wealth effect, and (ii) a substitution effect.

#### **Proof.**

From (16), we have: (18)  $\frac{\partial p_B}{\partial p_A} = \frac{\tau \varphi_{BA} h_B \gamma_B}{\theta_B + \tau \varphi_B h_B \gamma_B} > 0$ 

hence,

(19) 
$$\frac{dp_B}{dp_A} = \left(\frac{\partial p_B}{\partial \overline{K}_{nB}}\right) \left(\frac{\partial \overline{K}_{nB}}{\partial p_A}\right) + \frac{\partial p_B}{\partial p_A} = \frac{[2h_A + \varphi_{BA}]\tau h_B\gamma_B}{\theta_B + \tau\varphi_B h_B\gamma_B} > 0.$$

To the extent that this is a process that takes time, we expect that in our empirical test that price changes in the low quality market will lead price changes in the high quality market.

The wealth effect is given by  $\left(\frac{\partial p_B}{\partial \overline{K}_{nB}}\right) \left(\frac{\partial \overline{K}_{nB}}{\partial p_A}\right)$ . The first term,  $\frac{\partial p_B}{\partial \overline{K}_{nB}}$ , and the second term,  $\frac{\partial \overline{K}_{nB}}{\partial p_A}$ , can derived from (16) and (12), respectively. Both terms are positive, which implies that increased low-quality housing prices will raise the wealth of the homeowners in market A who are considering moving to market B.

The substitution effect is given by  $\frac{\partial p_B}{\partial p_A}$ , which is obtained directly from (18). The substitution effect is positive because the houses in the low- and high-quality markets are substitutes. Increased low-quality

housing prices will reduce the demand for the houses in this market and will induce demand for high-quality houses. As a result, the housing price in the high-quality market rises.

Combining the results in Proposition 1 and 2, we have:

**Proposition 3**. A positive shock to the wealth of potential buyers in the low-quality market will increase house prices in the high-quality market. **Proof**.

$$(20) \quad \frac{dp_B}{d\overline{K'}} = \left(\frac{dp_B}{dp_A}\right) \left(\frac{dp_A}{d\overline{K'}}\right) = \left[ \left(\frac{\partial p_B}{\partial \overline{K_{nB}}}\right) \left(\frac{\partial \overline{K_{nB}}}{\partial p_A}\right) + \frac{\partial p_B}{\partial p_A} \right] \left(\frac{dp_A}{d\overline{K'}}\right) \\ = \frac{2\tau^2 h_A h_B [2h_A + \varphi_{BA}] \gamma_A \gamma_B}{\Delta(\theta_B + \tau \ \varphi_B h_B \gamma_B)} > 0$$

where  $\Delta >0$  is defined in equation (B5) of Appendix B. This expression is derived directly from Propositions 2 and 3.

We summarize the transmission channels of a positive wealth shock  $\overline{K}$ ' initiated in the low-quality housing market A as follows:

(i) The shock raises housing prices in market A:  $\frac{dp_A}{d\overline{K}} > 0$ , [Proposition 1]. (ii) The increase in  $p_A$  increases the wealth of households in market A, some of whom then enter the next highest quality housing market B:  $\frac{\partial \overline{K}_{nB}}{\partial p_A} > 0$ , [(12)].

(iii) The house price increase in A is transmitted to house prices in the high-quality housing market via the increase of the wealth of households in market A:  $\frac{\partial p_B}{\partial \overline{K}_{rB}} > 0$ , [(17)].

(iv) The house price increase in A also increases the price of highquality housing through the substitution effect:  $\frac{\partial p_B}{\partial p_A} > 0$  [(18)].

(v) Based on (i) to (iv) above, we find that a positive shock initiated in the rental market will be transmitted throughout all levels of the housing market, eventually to the highest quality level:  $\frac{dp_B}{d\overline{K'}} = \left(\frac{dp_B}{dp_A}\right) \left(\frac{dp_A}{d\overline{K'}}\right) = \left[\left(\frac{\partial p_B}{\partial K_{nB}}\right) \left(\frac{\partial K_{nB}}{\partial p_A}\right) + \frac{\partial p_B}{\partial p_A}\right] \left(\frac{dp_A}{d\overline{K'}}\right) > 0.$ 

The model implies that the transmission channel of a positive wealth shock is unidirectional; that is, the causality goes from the lowquality market to the high-quality market. Exogenous positive shocks initiated from the high-quality market, ( $K_{BX}$ ), will not be transmitted into the low-quality market A; that is:  $\frac{\partial p_A}{\partial p_B} = 0$ .<sup>7</sup> Thus, we find that

 $\frac{dp_A}{dK_{BX}} = \left(\frac{dp_A}{dp_B}\right) \left(\frac{dp_B}{dK_{BX}}\right) = 0.$  The reason is that this shock will elicit a supply

adjustment in the high-quality market and the resulting housing price increase. Households in the next lower housing market could not "afford" a home in the high quality market prior to the shock and thus they certainly cannot afford the new price. Thus, there is no change in either demand or supply in the lower quality market; hence, its house price will not be affected by the positive shock in the higher quality market.<sup>8</sup>

Finally, from equation (B10) of Appendix B, we have the following proposition.

**Proposition 4**. Increasing the number of transactions for lower tier housing  $n_A$  will tend to increase the number of transactions for higher tier housing  $n_B$ ,

$$\frac{dn_B}{dn_A} > 0.$$

#### 4. The Hong Kong Housing Market

Hong Kong provides a unique opportunity to test this theory (see Ho (2006) for additional description of the Hong Kong market). About 40 percent of the population lived in public rental housing during the 1980s.<sup>9</sup> Public rental housing (PRH) accounts for about 70 percent of the total public housing program, the remaining 30 percent being in the form of "Home Ownership Scheme" housing (HOS). Heavy subsidies provided a significant incentive for households to remain in the public rental sector even for those who had the resources to become

<sup>&</sup>lt;sup>7</sup> Recall that we do not allow for depreciation of housing and thus we do not allow for effects through filtering as in Sweeney's (1974a, 1974b) models.

<sup>&</sup>lt;sup>8</sup> One could model a negative shock to a high quality market B. Some households in market B would move into low quality market A. Market A would benefit from the new demand as price of housing in market A rises, and thus supply rises too. The renter market could be affected in the sense that renters who otherwise would have moved to market A could find that the prices is not affordable and so they will remain in the rental market longer than otherwise. Due to space constraints, a formal analysis of these effects is deferred to future research.

<sup>&</sup>lt;sup>9</sup> Hong Kong's population was 5.7 million in 1990 and 6.7 million in 2000. In 2000, there were 2.1 million dwelling units.

homeowners.<sup>10</sup> Nevertheless, every year there is a stream of purchasers from among the PRH who buy private housing for investment purposes as well as to improve their housing conditions. In 1996 and 1997, almost 10,000 households per year moved to owned private/HOS housing from public rental housing units, probably reflecting a perceived stepping up of the policy to make richer tenants pay higher rent and a new policy to make the tenure in public rental housing not inheritable.

For many years the typical loan-to-value ratio in Hong Kong's banks was 70% of the appraised value of the property. Thus the down payment requirement posed a significant constraint to potential buyers. Most households of modest means were equity constrained and needed to get a committed buyer before moving to a better housing unit. This effect is somewhat less likely to occur among higher income homeowners because the down payment constraint was less likely to be binding.

Beginning in April 1987, the Hong Kong Housing Authority undertook a series of policy changes that decreased the incentive for households to remain in public rental housing.<sup>11</sup> First, it required tenants who had been housed for over ten years in the program and who had income exceeding twice that of the Waiting List Income Limit, (WLIL) to pay double rent.<sup>12</sup> The Housing Authority relaxed this requirement slightly in April 1993, when households with income exceeding twice the WLIL were required to pay 1.5 times the rent, while those with

<sup>&</sup>lt;sup>10</sup>With average rent set at no more than 10 per cent of the median household income of tenants, the implicit subsidies in public rental housing were substantial. Also, until quite recently Hong Kong protected the tenure of public housing tenants. Regardless of changes in financial conditions, a family and its children could remain in the unit (Housing Authority 1993). These subsidies provided a significant incentive for households to remain in the public rental sector even though they had the resources to become homeowners. In 1996 the Housing Authority recommended that household income and net asset value determine PRH household's eligibility to continue to receive public housing subsidy (Housing Authority 1996). PRH tenants whose household income and net asset value exceed the prescribed limits, or who choose not to make a declaration, have to pay market rent.

<sup>&</sup>lt;sup>11</sup>The Housing Authority is a statutory body set up by the Hong Kong government. The Authority has responsibility for housing matters and public housing development.

<sup>&</sup>lt;sup>12</sup>The Housing Authority maintains a waiting list for applicants to public housing. The total monthly income of the applicant and his/her family members must not exceed the maximum limits as stated by the Housing Authority. The incentive to invest in homes also was encouraged by high inflation and buttressed by a rapidly growing economy.

income exceeding three times the WLIL had to pay double the rent.<sup>13</sup> In April 1996, public policy changed again. Tenants of public housing whose household income and net asset value exceeded prescribed limits had to pay market rent (Housing Authority 1995, 1996). With such policies in place, and with a booming economy offering opportunities for upward mobility, it came as no surprise that a survey conducted by the Housing Authority in 1992-1993 showed that 24 per cent of home purchases in the market were by public housing tenants and that over 13 per cent of tenants owned one or more residential properties. This activity was clearly also a result of the sizable savings accumulated during the high growth and high inflation period in the 1980s and early 1990s (Watanabe, 1998).<sup>14</sup>

The final change in public policy that we consider was announced on December 8, 1997. The "Tenants Purchase Scheme" (TPS) allowed existing public rental housing tenants to purchase their own flats at up to an 88 percent discount from the estimated market price, provided that the sitting tenants committed to buy within a specified period. The units were priced from about HK\$ 70,000 (less than US\$ 10,000) to about HK\$ 300,000. The scheme was implemented in phases, with each phase covering about 25,000 tenants, starting in 1998. This policy reversed the previous incentive of tenants in public rental housing to leave their dwellings and move into the ownership market.

The announcement of the TPS program in 1997 greatly changed the incentive structure for existing tenants in the public rental program. They no longer had reason to buy expensive HOS housing or private sector homes. Even rich tenants could purchase the rental unit at discounted prices. The expected effect would be to greatly reduce the flow of these households into the private sector ownership market.

<sup>&</sup>lt;sup>13</sup>Rent in public sector rental housing is inclusive of "rates," which are similar to the U.S. property tax. Specifically, rates are a tax levied on the "ratable value" of the property as assessed by the Rating and Valuation Department.

<sup>&</sup>lt;sup>14</sup>For example, Wanatabe (1998) reports that mean monthly household savings in 1989-90 by households residing in the Housing Authority's rental flats (PRH) program was HK\$ 636 and their mean monthly expenditure was HK\$ 7,364. For PRH participants in the 50<sup>th</sup> to 74<sup>th</sup> income deciles, the mean monthly savings increased to HK\$ 2,924. For those PRH renters in the 75<sup>th</sup> to 89<sup>th</sup> income deciles, monthly savings was HK\$ 6,459 and those in the top 10% of the income distribution saved HK\$ 16,635 per month. The high amounts of savings increased in the early 1990s; for example, the amount was HK\$ 15,716 for those in the 75<sup>th</sup> to 89<sup>th</sup> income deciles and similarly greater than 1989-90 for other income levels.

Figure 1 shows a general upwards trend in property transactions (mainly residential, but also inclusive of commercial and industrial) from 1987 through 1997 (but there was a downwards trend from 1991 through 1996), but then a dramatic decline following the announcement of the TPS in December 1997. These data generally correspond to our expectations, but they report only the aggregate number of transactions while our interest focuses on ripple effects within the quality continuum.

#### [INSERT FIGURE 1]

#### **5. Data and Empirical Tests**

House price data for four classes of housing from the period 1987 to 2004 were obtained from the Hong Kong Rating and Valuation Department. The housing classes are defined according to the size of housing unit: Class A (under 40 m<sup>2</sup>), Class B (40-60 m<sup>2</sup>), Class C (70-99.9 m<sup>2</sup>), and Class D (100 m<sup>2</sup> and above).<sup>15</sup> The Ratings and Valuation Department house price indices "are designed to measure rental and price changes with quality kept at a constant" (Hong Kong Property Review 2001).<sup>16</sup> We expect that the above described changes in housing policy would first affect house prices in Class A units, followed by B, C, and D.

Our second data set relates to monthly transaction volume data for 1995 to 2004. These data pertain to existing private housing transactions and were obtained from the Land Registry: Centaline Databank Management Department. We separate properties by price and observe the time series of transactions. House price categories are: Home Ownership Scheme housing (HOS), 1-2 million \$HK, 2-3 million, 3-5 million, and 5-10 million.

#### 5.1 Housing Price Dynamics Test

The first empirical test is a causality test of whether house price changes in smaller properties lead those in larger properties throughout

<sup>&</sup>lt;sup>15</sup>All units are private domestic units, this defined as independent dwellings with separate cooking facilities and bathroom/lavatory. Our class D merges class D and class E data from the official definition.

<sup>&</sup>lt;sup>16</sup>The house price index is derived using all transactions that occurred in a particular period in a particular class of dwelling. The index accounts for variations in property size and quality by comparing transaction price to the ratable value of the property. Thus, observed variations in nominal house prices are compared with a time invariant value, yielding measures of the change in price.

the distribution of property sizes. Under the standard Granger test (1969), if X causes Y, then changes in X should precede changes in Y. That is, lagged values of X can help improve the prediction of current values of Y. First, we estimate the unrestricted model:

(20) 
$$Y_t = \mu_t + \sum_{i=1}^k \alpha_i Y_{t-1} + \sum_{i=1}^k \beta_i X_{t-1} + \varepsilon_t$$
.

Next we estimate the restricted model:

(21) 
$$Y_t = \mu_t + \sum_{i=1}^{\kappa} \alpha_i Y_{t-1} + \varepsilon_t.$$

The null hypothesis that X does not Granger-cause Y ( $H_o: \beta_1 = \beta_2 = \cdots = \beta_k = 0$ ) can be tested by the reported F-statistics. If the coefficients on the lagged values of X are jointly and significantly different from zero, the null hypothesis can be rejected. Y is said to be Granger-caused by X.<sup>17</sup>

We begin the analysis by checking the stationarity of the price series using the Augmented Dickey Fuller (ADF) test. This check is necessary because the Granger Causality tests require all time series be stationary. Table 1 shows that the null hypothesis that the price series contain a unit root cannot be rejected for any series, indicating they are non-stationary. Each variable was transformed into a first-difference form to achieve stationarity. In implementing equation (1), in order to avoid the ambiguity in choosing the lag lengths, we use the Akaike's final prediction error criterion to determine the optimal lag specifications.

#### [INSERT TABLE 1]

Six sets of the Granger test results are reported in Table 2. Although we cannot reject the null hypothesis that "the price movements of Class A do not cause those in Class B," we can reject the null that price movements in Class A do not cause those in Class C (1% level) and Class D (1% level). Also, we can reject the null hypotheses that "price movements in Class B do not cause those in Class C "(1% level) and "price movements in Class B do not cause those in Class D" (1% level). Similarly, we can reject the null that "price movements in

<sup>&</sup>lt;sup>17</sup>The VECM approach (standard Granger causality test augmented with error-correction terms) is not applicable here because these variables are not cointegrated during the period 1987 to 2000. The absence of cointegration notwithstanding, a short run causal relation may still exist, which can be determined using the conventional Granger causality test (VAR approach).

Class C do not cause those in Class D" (5% level). All of these results support the hypotheses that price changes in low quality properties lead those of higher quality properties during this period. A ripple effect of prices upwards through the quality continuum is apparent.

We also find that the Granger tests do not support reverse causality. Price changes in Class B, C, and D do not Granger cause those in Class A. Neither do price changes in Class C or D Granger cause those in Class B. Finally, we find that price changes in Class D do not Granger cause those in Class C. Thus, all of the tests support a single direction of the price ripple as predicted by the model.

### [INSERT TABLE 2]

#### 5.2 Housing Transaction Dynamics Test

In a market where many households are down paymentconstrained, the model predicts patterns in the number of transactions, again beginning with low quality housing and continuing upwards through the quality continuum. Our base level is the publicly subsidized Homeownership Scheme housing (HOS). Typically, the households who are HOS owners were previously tenants in public rental housing and they were given preference as "green form applicants", although the public may apply as "white form applicants" by satisfying specified income and asset level tests. As described above, after December 1997 when the TPS was announced, public rental housing tenants' demand for HOS housing fell dramatically because they could purchase their rental unit at as low as 12% of the estimated market price. This drop in demand for HOS housing should reduce transaction volume, price, and equity among HOS owners. Our model predicts that negative effects on HOS transactions would be transmitted upwards through the quality continuum.

#### [INSERT TABLE 3]

Our results are reported in Table 3. The total number of transactions over the period from 1995 to 2004 by house value category ranges from 182,510 in the one to two million \$HK category, to 94,341 for two to three million \$HK, to 75,702 for three to five million \$HK, to 42,430 for five to ten million \$HK. We find that changes in the

transaction volume of HOS housing Granger-caused changes in transaction volume in all higher valued housing. This is convincing evidence that the lowest tier of housing quality was the leading submarket in terms of changing transaction volume. There is no evidence of reverse causality from higher to lower quality levels.

In addition, changes in the transaction volume of private homes valued between one and two million \$HK Granger-caused changes in transaction volume for all groups of houses with higher prices. Thus, booms or busts in the submarket for low quality housing are passed on with a lag to the next highest quality housing, and subsequently from this level to higher valued levels. However, no clear causality in transactions was found among the pairs 2 to 3 million versus 3 to 5 million; 2 to 3 million versus 5 to 10 million; and 3 to 5 million versus 5 to 10 million. Causality from the lowest category and HOS housing to higher categories is clear, while causality from the intermediate price categories to higher price categories is less clear.

As shown in table 3, the AIC criterion identifies 1 lag for cases (1), (2), (3) and (4); 3 lags for case (7). (8), (9) and (10); and 5 lags for case (5) and (6). Thus, the evidence suggests that the ripple effect on transaction, when it exists, takes one to five months to transmit to the higher tiers.

## 5.3 Tenant Purchase Scheme Effects

The above evidence characterizes the Hong Kong housing market in general form. Next, we present a specific test of the impact of the Tenants Purchase Scheme (TPS), which was announced on December 8, 1997. This program allowed sitting tenants in the public rental housing program to purchase their unit at a deep discount from market price. Our dependent variable is the monthly number of purchases of existing homes in the period surrounding the announcement of the TPS program. The first test is of the impact of the announcement of the TPS. We create a dummy variable that takes the value 1 during this period and is 0 before December 1997. We expect the coefficient on this variable to be negative.

Confounding the analysis of the impact of the TPS was the advent of the Asian financial crisis (AFC), this beginning in the third quarter of 1997 (Tsang and Ma, 2002). We expect that increased uncertainty about the HK-US dollar link would discourage purchase of HK dollar denominated assets, particularly homes, and thus reduce turnover. We use the difference between the spot exchange rate and the one-year forward rate to measure the AFC<sup>18</sup>. This measure does not require subjective judgment as to when the crisis began and when it phased out. It also is highly sensitive to the swings of market confidence during financial crises. The variable is normalized and constrained to be between zero and unity. We also include in the regression a measure of the rate of house price appreciation during the previous year. This value is measured by the average price appreciation during the past six months as compared to one year earlier. As discussed above, greater house price appreciation relaxes households' down payment constraints and thus increases the ability of homeowners to trade up the housing quality continuum.

#### [INSERT TABLE 4]

The results of the estimation are in Table 4.<sup>19</sup> We find that the Tenants Purchase Scheme had a significant and large negative impact on the volume of residential property transactions. Because our dependent variable measures the volume of transactions in the secondary market (private and public housing), we are capturing all of the ripple effects described above. There is some evidence that the Asian Financial Crisis dampened transaction volume, and some evidence that high rates of house price appreciation increase transaction volume, but neither coefficient is statistically significant.

#### **6.** Conclusions

While a number of empirical studies have analyzed the spatial dispersion of shocks to house prices, we focus on modeling the short run dynamic changes of house prices and transaction volume in a single market. Our model highlights the role of wealth shocks to the housing market and the resulting domino effect in prices and transactions, working through the housing quality continuum.

An application of the model is to data from Hong Kong. Shocks to the rental housing market were due to changes in the Hong Kong Housing Authority's policies. The first policy change was one that

<sup>&</sup>lt;sup>18</sup>This method was first used by Ho (2000), Chapter 18.

<sup>&</sup>lt;sup>19</sup>Both the transaction volume and price appreciation series are stationary.

increased the flow of public housing renters to the private ownership market, the second being one that dramatically decreased this flow. We find strong evidence that these policy changes resulted in price and transactions volume changes throughout the quality continuum, spreading from low quality unit to high quality tiers. These results are consistent with the predictions of our model of short run housing market dynamics and they highlight the interconnectedness of the housing market within a locality. It is important to note that our results do not prove that changes in wealth were the precise cause of the changes in Hong Kong's housing market, but the observed changes in house prices and transaction volumes closely follow the model's predictions.

## Appendix

#### A: The Dynamic Optimization Model

Households maximize the expected present value of utility:

(A1) Max E(U) = 
$$\sum_{t=0}^{\infty} \delta^t u(h, c_t)$$

where  $\delta$  is the discount factor,  $c_t$  is the composite consumption of nonhousing goods and services at time t, h is housing consumption which is represented by a composite index of "housing service" of the house. In our two-market model, we assume that housing services are available in set discrete units with  $h_R < h_A < h_B$ , where R indicates rental housing, and A and B indicate low- and high-quality owner-occupied housing.

Homes are purchased only for owners' occupation. All rented units are assumed to be owned by the government, this assumption relevant for our particular empirical application, but otherwise dispensable. The choice variables for a household are  $c_t$  and h.  $c_t$  is a continuous variable which is optimized according to its first-order condition. h is a discrete choice variable that is chosen among  $h_R$ ,  $h_A$ , or  $h_B$ , whichever gives the highest present-value of utility, U.

For simplicity, we assume:

(A2)  $u(h,c_t) \equiv \beta_1 ln(h) + \beta_2 ln(c_t)$ 

#### A (i) Optimization for Renter Households

Assume that  $p_R$  represents the present value of rent to be paid per unit of housing if the household remains a tenant indefinitely. Thus,  $p_Rh_R$  is the present value cost of renting a house in market R. For such a tenant, the relevant budget constraint is:

$$(A3) \quad \sum_{t=0}^{\infty} \delta^t c_t q_t + p_R h_R = K$$

where  $q_t$  is the price of the nonhousing composite good, *K* includes the present value of labor and non-labor income and the value of all assets. Because our focus is on the housing market, we simplify by assuming that the price of consumption  $q_t$ , and wealth *K* are exogenous.

To solve the household dynamic optimization problem, we write the Lagrangian of the problem as follows (Dixit 1990):

(A4) 
$$L = \sum_{t=0}^{\infty} \delta^{t} u(h, c_{t}) - \lambda \left[ \sum_{t=0}^{\infty} \delta^{t} c_{t} q_{t} + h_{R} p_{R} - K \right]$$
$$= \sum_{t=0}^{\infty} \delta^{t} \left[ \beta_{1} \ln(h) + \beta_{2} \ln(c_{t}) \right] - \lambda \left[ \sum_{t=0}^{\infty} \delta^{t} c_{t} q_{t} + h_{R} p_{R} - K \right].$$

The first-order conditions are:

(A5) 
$$\partial L/\partial c_t = \delta^t \frac{\beta_2}{c_t} - \lambda q_t \delta^t = 0$$
  
(A6)  $\partial L/\partial \lambda = \sum_{t=0}^{\infty} \delta^t c_t \cdot q_t + p_R h_R = K$ .  
Simplifying (A5) yields:

(A7)  $c_t = \frac{\beta_2}{\lambda q_t}$ .

Substituting (A7) into (A6), we have:

(A8) 
$$\lambda = \frac{\sum_{t=0}^{\infty} \delta^t \frac{\beta_2}{q_t}}{K - p_R h_R} = \frac{\varphi_1}{K - p_R h_R}.$$

The maximized utility of a renter household is thus:

(A9) 
$$U(h_R) = \sum_{t=0}^{\infty} \delta^t u(h_R, c_t) = \sum_{t=0}^{\infty} \delta^t (\beta_1 \ln h_R + \beta_2 \ln c_t)$$
  
=  $\sum_{t=0}^{\infty} \delta^t \left( \beta_1 \ln h_R + \beta_2 \ln \frac{\beta_2}{\varphi_1 q_t} + \beta_2 \ln(K - p_R h_R) \right).$ 

## A (ii) Optimization for Renters Moving to Market A

Assume the current price of one unit of housing is  $p_h$ . That is,  $p_A h_A$  is the purchase price of a house in market A. Assume that households must pay a fraction of the housing price as a down-payment  $(1-\phi_L)p_hh$ , where  $\phi_L$  is the loan-to-value ratio. Their mortgage loan  $\phi_L p_h h$  is characterized by a fixed interest rate of r and a fixed monthly repayment schedule. Then the period fixed-payment (hF) is obtained by solving the following present-value formula (Mishkin 2003):

Loan amount = 
$$\phi_L p_h h = \sum_{t=1}^{N} \frac{F h}{(1+r)^t} = F h \sum_{t=1}^{N} \frac{1}{(1+r)^t} = F h f(r).$$

This equation states that the value of the loan equals to the sum of the present value of the period payments. Hence,

 $(A10)F = \phi_L p_h / f(r).$ 

The household's present-value, life-long, wealth K, constraint is:

$$\sum_{t=0}^{\infty} \delta^{t} c_{t} q_{t} + (1 - \phi_{L}) p_{h} h + \sum_{t=1}^{N} \delta^{t} h F = K + p_{h} h.$$

That is,

$$(A11)\sum_{t=0}^{\infty}\delta^{t}c_{t}q_{t}-\phi_{L}p_{h}h+\sum_{t=1}^{N}\delta^{t}hF=K.$$

*K* includes the present-value of life-long, labor and non-labor income of the household and other assets, evaluated before the decision of whether to buy a house. If the decision is to buy, then the *ex post K* becomes K'' = K + the present value of the house net of outstanding mortgage liabilities. If the decision is to remain a renter, then *K* remains unchanged.

To solve the household dynamic optimization problem, we write the Lagrangian of the problem as follows (Dixit 1990):

$$(A12)L = \sum_{t=0}^{\infty} \delta^{t} u(h,c_{t}) - \lambda \left[ \sum_{t=0}^{\infty} \delta^{t} c_{t} \cdot q_{t} - \phi_{L} \cdot h \cdot p_{h} + \sum_{t=1}^{N} \delta^{t} h F - K \right]$$
$$= \sum_{t=0}^{\infty} \delta^{t} \left[ \beta_{1} \ln(h) + \beta_{2} \ln(c_{t}) \right] - \lambda \left[ \sum_{t=0}^{\infty} \delta^{t} c_{t} \cdot q_{t} - \phi_{L} \cdot h \cdot p_{h} + \sum_{t=1}^{N} \delta^{t} h \cdot \phi_{L} \cdot p_{h} / f(r) - K \right].$$

The first-order conditions are:

(A13)
$$\partial L/\partial c_t = \delta^t \frac{\beta_2}{c_t} - \lambda q_t \delta^t = 0$$
, which gives:  $c_t = \frac{\beta_2}{\lambda q_t}$   
(A14) $\partial L/\partial \lambda = \sum_{k=1}^{\infty} \delta^t c_k q_k - \phi_k p_k h + \sum_{k=1}^{N} \delta^t \frac{\phi_L p_k h}{\delta t} = K$ .

$$(A14)\partial L/\partial \lambda = \sum_{t=0} \delta^t c_t q_t - \phi_L p_h h + \sum_{t=1} \delta^t \frac{\psi_L p_h n}{f(r)} = K .$$

Substituting (A13) into (A14) yields:

$$(A15)\lambda = \frac{\sum_{t=0}^{\infty} \delta^t \frac{\beta_2}{q_t}}{K - \left[-\phi_L + \sum_{t=1}^N \delta^t \frac{\phi_L}{f(r)}\right] p_h h} = \frac{\varphi_1}{K - \varphi_{2h} p_h}$$

The maximized utility of a household that bought a house in market A is thus:

$$(A16)U(h_A) = \sum_{t=0}^{\infty} \delta^t u(h_R, c_t) = \sum_{t=0}^{\infty} \delta^t \left(\beta_1 \ln h_A + \beta_2 \ln c_t\right)$$
$$= \sum_{t=0}^{\infty} \delta^t \left(\beta_1 \ln h_A + \beta_2 \ln \frac{\beta_2}{\varphi_1 q_t} + \beta_2 \ln(K - \varphi_{2A} p_A)\right).$$

If the utility of a household as an owner in market A (A16) is greater than that as a renter (A9),  $U(h_A) > U(h_R)$ , then the household will enter market A and buy a house; otherwise, it will rent. A home purchase occurs if and only if:

$$\beta_1 \ln(h_A / h_R) + \beta_2 \ln\left(\frac{K - \varphi_{2A} p_A}{K - p_R h_R}\right) > 0.$$

Define

(A17) 
$$K_{A}^{*} = \frac{\varphi_{2A}p_{A} - (h_{A}/h_{R})^{-\beta_{1}/\beta_{2}}}{1 - (h_{A}/h_{R})^{-\beta_{1}/\beta_{2}}} = \varphi_{A}p_{A} - \varphi_{RA}.$$

where  $\varphi_A >0$  since  $h_A / h_R > 1$ . If and only if  $K > K^*_A$ , will  $U(h_A) > U(h_R)$ . In this case the household will enter market A to buy a new house; otherwise, it will remain a renter.

#### A (iii) Optimization for Homeowners in Market A Moving to Market B

The availability of units in market A for new buyers is subject to the willingness of the existing homeowners in that market to sell. This willingness depends on their utilities being higher after trading up to market B compared with remaining in their home in market A.

The wealth of households in market A who are potential buyers in market B is:

$$K_{nB}(i, t) = K_{iA}(i, t-1) + p_A(t)h_A - p_A(t-1)h_A - \Psi_m$$

where  $\Psi_m$  is the remaining mortgage. Suppressing the time indices *t* and *t*-1, and rewriting  $p_A(t-1)h_A + \Psi_m$  as  $\phi_m$ , the mean wealth  $\overline{K}_{nB}$  of the potential buyers in market B is given by:

$$(A18)\overline{K}_{\scriptscriptstyle nB}=\overline{K}_{\scriptscriptstyle iA}+p_{\scriptscriptstyle A}h_{\scriptscriptstyle A}-\phi_{\scriptscriptstyle m}$$

where  $\overline{K}_{iA}$  is the mean wealth of  $K_{iA}(i,t-1)$ . The lifetime wealth constraint is:

(A19) 
$$\sum_{t=0}^{\infty} \delta^t c_t \cdot q_t + (1 - \phi_L) p_B \cdot h_B + \sum_{t=1}^{N} \delta^t h_B F = \overline{K}_{nB} + p_B \cdot h_B \cdot h_B$$

Similar to the result of sub-section (ii), we have  $U(h_B) > U(h_A)$  if and only if:

$$\beta_1 \ln(h_B / h_A) + \beta_2 \ln\left(\frac{\overline{K}_{nB} - \varphi_{2B} p_B}{\overline{K}_{iA} - \varphi_{2A} p_A}\right) > 0.$$

Define:

(A20)  $K_B^* = \varphi_{2B} p_B - (h_B / h_A)^{-\beta_1 / \beta_2} \varphi_{2A} p_A + (h_B / h_A)^{-\beta_1 / \beta_2} \overline{K}_{iA} = \varphi_B p_B - \varphi_{BA} p_A + \varphi_{B0}$ . If and only if  $K_{nB}(i, t) > K_B^*$  will  $U(h_B) > U(h_A)$  and the household switches homes from market A to B. Otherwise, it will remain in market A.

#### **B.** Equilibrium Solutions of the Two Housing Market Prices

The relationship between  $p_A$  and  $p_B$  can be obtained by substituting (11) into (16) in the main text:

$$(B1) \quad p_{B} = \frac{2\tau \ h_{B}\gamma_{B}(K_{iA} + p_{A}h_{A} - \phi_{m}) + p_{A}\tau \ \varphi_{BA}h_{B}\gamma_{B} - \tau \ \varphi_{B0}h_{B}\gamma_{B} + \theta_{B}\underline{p_{B}}}{\theta_{B} + \tau \ \varphi_{B}h_{B}\gamma_{B}}$$
$$= \frac{p_{A}(2h_{A} + \varphi_{BA})\tau \ h_{B}\gamma_{B} + 2\tau \ h_{B}\gamma_{B}(\overline{K}_{iA} - \phi_{m}) - \tau \ \varphi_{B0}h_{B}\gamma_{B} + \theta_{B}\underline{p_{B}}}{\theta_{B} + \tau \ \varphi_{B}h_{B}\gamma_{B}}$$

To find the model's solution, we also need the market equilibrium condition in the low-quality market A. The number of new buyers  $n_A$ [see (10)] in market A plus the initial house owners  $i_A$  should be equal to the developers' supply plus the number of sellers in market A. The number of sellers in market A in turn equals the number of new house buyers  $n_B$  [see (13)] in the high-quality housing market B. That implies: (B2)  $h_A n_A + h_A i_A = h_A n_B + \theta_A (p_A - \underline{p_A}) + H_A$ . Substituting (3), (10), (11), and (13) into (B2) yields: (B3)  $\gamma_A (2\tau h_A \overline{K} - \tau \varphi_A p_A h_A + \tau \varphi_{PA} h_A)$ 

$$= \left[ 2\tau (\overline{K}_{iA} + p_A h_A - \phi_m) - \tau \varphi_B p_B + \tau \varphi_{BA} p_A - \tau \varphi_{B0} \right] h_A \gamma_B + \theta_A p_A - p_A \theta_A.$$

Substituting (B1) into (B3), we have:

$$\begin{array}{l} \gamma_A \left( 2\tau \ \overline{K}' h_A - \tau \ \varphi_A \ p_A \ h_A + \tau \ \varphi_{RA} \ h_A \right) \\ = p_A \left[ 2\tau \ h_A^2 \gamma_B + \tau \ \varphi_{BA} \ h_A \gamma_B + \theta_A \right] + 2\tau ( \ \overline{K}_{iA} - \phi_m) h_A \gamma_B - \tau \ \varphi_{B0} h_A \gamma_B - \frac{p_A}{\rho_A} \ \theta_A \end{array}$$

$$-\tau \gamma_{B} \varphi_{B} h_{A} \frac{p_{A} (2h_{A} + \varphi_{BA})\tau h_{B} \gamma_{B} + 2\tau h_{B} \gamma_{B} (\overline{K}_{iA} - \phi_{m}) - \tau \varphi_{B0} h_{B} \gamma_{B} + \theta_{B} \underline{p_{B}}}{\theta_{B} + \tau \varphi_{B} h_{B} \gamma_{B}}$$

Hence,

$$p_{A} = \frac{\tau h_{A}(2\gamma_{A}\overline{K}' + \gamma_{A}\varphi_{RA} + \gamma_{B}\varphi_{B0}) + \theta_{A} \underline{p_{A}} - 2\tau h_{A}\gamma_{B}(\overline{K}_{iA} - \phi_{m})}{\Delta}$$
$$+ \frac{\tau \gamma_{B}\varphi_{B}h_{A}}{\frac{2\tau h_{B}\gamma_{B}(\overline{K}_{iA} - \phi_{m}) - \tau \gamma_{B}\varphi_{B0}h_{B} + \theta_{B} \underline{p_{B}}}{\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B}}}{\Delta}$$

(B4) 
$$p_A = \frac{2\tau h_A \gamma_A}{\Delta} \overline{K}' + \alpha_A$$

where

$$(B5) \quad \Delta = \tau \ h_{A} [2h_{A}\gamma_{B} + \varphi_{BA}\gamma_{B} + \varphi_{A}\gamma_{A}] + \theta_{A} - \tau^{2}\gamma_{B}^{2}\varphi_{B}h_{A}h_{B} \frac{2h_{A} + \varphi_{BA}}{\theta_{B} + \tau \ \varphi_{B}h_{B}\gamma_{B}}$$
$$= \frac{\tau \ h_{A}\theta_{B} [2h_{A}\gamma_{B} + \varphi_{BA}\gamma_{B} + \varphi_{A}\gamma_{A}] + \theta_{B}\theta_{A} + \tau \ \varphi_{B}h_{B}(\theta_{A} + \tau \ \varphi_{A}h_{A}\gamma_{A})\gamma_{B}}{\theta_{B} + \tau \ \varphi_{B}h_{B}\gamma_{B}} > 0$$

and

$$\alpha_{A} = \frac{\tau h_{A}(\varphi_{RA}\gamma_{A} + \varphi_{B0}\gamma_{B}) + \theta_{A}\underline{p}_{A} - 2\tau h_{A}\gamma_{B}(\overline{K}_{iA} - \phi_{m}) + \tau\gamma_{B}\varphi_{B}h_{A}\frac{2\tau h_{B}\gamma_{B}(\overline{K}_{iA} - \phi_{m}) - \tau \gamma_{B}\varphi_{B0}h_{B} + \theta_{B}\underline{p}_{B}}{\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B}}}{\Delta}$$

Thus a positive shock to the wealth of the potential buyers in the lowquality housing market will increase its price; that is,

(B6) 
$$\frac{dp_A}{d\overline{K}'} = \frac{2\pi h_A \gamma_A}{\Delta} > 0.$$

This is Proposition 1 of the main text, which is derived directly from (B4). To solve for  $p_B$ , we rewrite (B1) as:

(B7) 
$$p_B = p_A \frac{[2h_A + \varphi_{BA}]\tau h_B \gamma_B}{\theta_B + \tau \varphi_B h_B \gamma_B} + \alpha_B$$
  
where  $\alpha_A = \frac{2\tau \gamma_B h_B (\overline{K}_{iA} - \phi_m) - \tau \gamma_B \varphi_{B0} h_B + \theta_B \underline{p}_B}{1 + \alpha_B - \tau \gamma_B \varphi_{B0} h_B}$ 

where 
$$\alpha_B = \frac{\theta_B + \tau \, \varphi_B h_B \gamma_B}{\theta_B + \tau \, \varphi_B h_B \gamma_B}$$

Substituting (B4) into (B7):

$$(B8) \quad p_{B} = \frac{[2h_{A} + \varphi_{BA}]\tau \gamma_{B}h_{B}}{\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B}} \left(\frac{2\tau h_{A}\gamma_{A}}{\Delta}\overline{K}' + \alpha_{A}\right) + \alpha_{B}$$
$$= \frac{2\tau^{2}h_{A}h_{B}\gamma_{A}\gamma_{B}[2h_{A} + \varphi_{BA}]}{\Delta(\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B})}\overline{K}' + \alpha_{A}\frac{[2h_{A} + \varphi_{BA}]\tau h_{B}\gamma_{B}}{\theta_{B} + \tau \varphi_{B}h_{B}\gamma_{B}} + \alpha_{B}.$$

This expression gives the total effect of the wealth shock on house prices in the high quality market,  $p_B$ .

Finally, we prove Proposition 4 that transactions for lower tier housing  $n_A$  will tend to increase transactions for higher tier housing  $n_B$ . Substituting (3) and (10) of the main text into (B2) gives:

(B9) 
$$h_A n_A = h_A n_B + \theta_A \frac{2\tau \gamma_A K' + \tau \gamma_A \varphi_{RA} - n_A}{\tau \gamma_A \varphi_A} - \theta_A \underline{p_A}.$$

Hence,

(B10) 
$$\frac{dn_B}{dn_A} = 1 + \frac{\theta_A}{\tau \gamma_A \varphi_A h_A} > 0.$$

This proves Proposition 4.

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70000.0 60000.0 50000.0 40000.0 30000.0 20000.0 10000.0 0.0 1986.1 1988.1 1990.1 1992.1 1994.1 1996.1 2000.1 1998.1 1984.1

Figure 1: Property Transaction Volume in Hong Kong (1984Q1 – 2001Q4)

Notes:

1) Property transaction volume is measured by the "Agreement for sale and purchase of a building unit".

2) Source: Monthly Digest of Statistics, various issues, Hong Kong Census and Statistics Department.

Property Type	Test on	No Trend	Trend	Conclusion
Class A	Level 1 <sup>st</sup> difference	-1.739 -4.153**	-1.014 -4.370*	I (1)
Class B	Level 1 <sup>st</sup> difference	-1.729 -4.562**	-0.981 -4.753*	I (1)
Class C	Level 1 <sup>st</sup> difference	-1.527 -3.900**	-0.822 -4.206*	I (1)
Class D	Level 1 <sup>st</sup> difference	-1.519 -4.206**	-0.339 -4.909**	I (1)

#### Table 1. Augmented Dickey-Fuller Unit Root Test

Notes:

1) 95% critical values for the ADF statistic with and without the trend are -2.90 and -3.46 respectively,

2) \* Indicates significance at the 5 % level, \*\* indicates significance at the 1% level.

3) The optimal lag is determined by AIC criteria.

#### Table 2. Granger Causality Tests of House Price Changes: Period 1982Q1 to 2004Q2

	Null hypothesis:	F-Statistic	Probability	Causality
(1)	CLASS B does not Granger cause CLASS A CLASS A does not Granger cause CLASS B	1.565 1.163	0.214 0.284	
(2)	CLASS C does not Granger cause CLASS A CLASS A does not Granger cause CLASS C	0.001 5.716	0.977 0.019*	$A \rightarrow C$
(3)	CLASS D does not Granger cause CLASS A CLASS A does not Granger cause CLASS D	0.678 5.616	0.412 0.020*	A → D
(4)	CLASS C does not Granger cause CLASS B CLASS B does not Granger cause CLASS C	0.252 9.223	0.874 0.003**	$B \rightarrow C$
(5)	CLASS D does not Granger cause CLASS B CLASS B does not Granger cause CLASS D	0.006 10.113	0.940 0.002*	B → D
(6)	CLASS D does not Granger cause CLASS C CLASS C does not Granger cause CLASS D	0.303 4.302	0.583 0.041*	$C \rightarrow D$

Notes:

1) Akaike's final prediction error criterion identifies 1 lag for all cases.

2) \* Indicates 5% significance level and \*\* indicates 1% significance level.

	Null Hypothesis	Probability	Causality
(1)	PR 1m < < 2m does not Granger cause HOS	0.321	
	HOS does not Granger cause PR 1m <<2m	0.000***	HOS $\rightarrow$ PR 1m<<2m
(2)	PR 2m < 3m does not Granger cause HOS	0.948	
	HOS does not Granger cause PR 2m <<3m	0.002***	HOS $\rightarrow$ PR 2m<<3m
(3)	PR 3m << 5m does not Granger cause HOS	0.950	
	HOS does not Granger cause PR 3m <<5m	0.002**	HOS $\rightarrow$ PR 3m<<5m
(4)	PR 5m <<10m does not Granger cause HOS	0.789	
	HOS does not Granger cause PR 5m <<10m	0.013***	HOS $\rightarrow$ PR 5m<<10m
(5)	PR 2m<<3m does not Granger cause 1m<<2m	0.396	
	PR 1m<<2m does not Granger cause 2m<<3m	0.017**	$PR 1m < < 2m \rightarrow PR 2m < < 3m$
(6)	PR 3m<<5m does not Granger cause 1m<<2m	0.571	
	PR 1m<<2m does not Granger cause 3m<<5m	0.066*	PR 1m<<2m $\rightarrow$ PR 3m<<5m
(7)	PR 5m<<10m does not Granger cause 1m<<2m	0.981	
. ,	PR 1m<<2m does not Granger cause 5m<<10m	0.116*	PR 1m<<2m $\rightarrow$ PR 5m<<10m
(8)	PR 3m<<5m does not Granger cause 2m<<3m	0.686	
	PR 2m<<3m does not Granger cause 3m<5m	0.979	No causality
(9)	PR 5m<<10m does not Granger cause 2m<<3m	0.545	
	PR 2m<<3m does not Granger cause 5m<<10m	0.656	No causality
(10)	PR 5m<<10m does not Granger cause PR 3m<<5m	0.675	
	PR 3m<<5m does not Granger cause PR 5m<<10M	0.253	No causality

#### Table 3. Granger Causality Tests of Transaction Volume Changes: July 1995 to Jan 2004

Notes:

1) All series are in Log level and are stationary I (0).

Akaike's final prediction error criterion identifies 1 lag for cases (1), (2), (3) and (4); 3 lags for case (7), (8), (9) and (10) and 5 lags for case (5) and (6)

3) Significance levels are \*\* 1%, \* 5%.

4) The price ranges are in millions of \$HK.

5) Variable Names:

HOS - Registrations of Home Ownership Scheme Flats

PR1M<..<2M - Registrations of Private Residential: 1 to 2 million

PR2M<..<3M - Registrations of Private Residential: 2 to 3 million

 $PR3M{<}..{<}5M$  - Registrations of Private Residential: 3 to 5 million

PR5M<..<10M - Registrations of Private Residential: 5 to 10 million

Variable	Coefficient	t-ratio
Constant	9.265	88.179**
TPS Dummy	-0.843	-5.892**
Asian Financial Crisis Dummy	-0.166	-0.935
House Price Appreciation Rate	0.002	0.606
R-bar squared	0.80	

#### Table 4. Explaining the Log of Existing Housing Transaction Volume

Notes:

Estimation period is for July 1995 to September 2001 using monthly data.
 \*\*Indicates a 1% significance level.

3) DW-statistic = 2.05.

4) An AR (1) procedure was used to correct for autocorrelation.

<u>Variable list</u>	
А	symbol for low-quality market
В	symbol for high-quality market
$\alpha_A$ , $\alpha_B$	parameters in equilibrium price formulas
$\beta_{c}, \beta_{l}, \beta_{2}$	fixed parameters.
$C_t$	quantity of composite consumption at time period t
$\delta$	discount factor of a consumer
F	fraction of the total housing expenditure that requires a fixed-payment
	mortgage loan
$\Psi_m$	remaining mortgage of a household
$\phi_m$	$=p_A(t-1)h_A+\Psi_m$
$\varphi_{l}$ , $\varphi_{2h}$ , $\varphi_{2A}$ , $\varphi_{2B}$	fixed parameters.
$\varphi_{B}, \varphi_{BA}, \varphi_{B0}$	parameters in the decision rule of switching from market A to B
$\varphi_A, \varphi_{RA}$	parameters in the decision rule of switching from rental market to market A
$\gamma_B, \gamma_B$	proportions of households satisfying the lifetime wealth constraint that
	also satisfy the liquid wealth constraint in market A and B, respectively
h	fixed size of a house
$h_R$	fixed size of a rental house
$h_A, h_B$	fixed size of a house in market A and B, respectively
$H_A, H_B$	stock of market A and B, respectively
Î 	individual <i>i</i> in housing market
$l_A$ *, $l_B$ *	number of buyers in market A and B, respectively
$l_A, l_B$	initial number of house owners in market A and B, respectively
К <i>V(</i> ;)	present-value of lifetime wealth of the nousehold
$\frac{K(l)}{\overline{M}}$	
K =	initial mean lifetime wealth of the renters
<u>K'</u>	new mean lifetime wealth of renters (with $K' > K$ )
$\underline{K}_{iA}$	initial mean lifetime wealth of the home-owners in market A
$K_{nB}$	new mean lifetime wealth of the home-owners in market A who are
	potential buyers in market B (with $\overline{K}_{nB} > \overline{K}_{iA}$ )
K*4	critical values of mean wealth switching from rental market to market A
$K^{*_{B}}$	critical values of mean wealth switching from market A to B
λ	shadow price of household's dynamic optimization problem
Ν	term of the mortgage loan
$n_A$	number of new buyers in market A
$n_B$	number of sellers in market A, i.e., new buyers in market B
$\underline{p_A}$ , $\underline{p_B}$	initial prices of market A and B, respectively
$p_{A}$ , $p_{B}$	price of market A and B, respectively
$p_h$	housing price
$p_R$	rental price of housing
$q_t$	price of composite consumption at time period t
r	interest rate of the mortgage loan
$S_A$ , $S_B$	supply of market A and B, respectively
τ	slope parameter in the household wealth distribution function
$ heta_{\!A}$ , $ heta_{\!B}$	parameters of supply function for market A and B, respectively
u(.,.)	utility function
U(.)	present-value of the utility
W	liquid wealth of a household