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**CONGESTION PRICING AND
PUBLIC TRANSPORT**

by

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**Faculty of Social Sciences
Lingnan College
Hong Kong
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Abstract

In this paper, we study the problem of optimal road pricing, incorporating public transport into our analysis. We find that when public transport and private transport are substitutes, the welfare implications of optimal road pricing are very different from those when public transport is neglected. In particular, road pricing may lead to Pareto improvement even in the absence of redistribution of toll revenues to road users. We also explain why our result is compatible with the prevalence of opposition to an increase in road toll in everyday experience.

JEL classification: R41, R48

Congestion Pricing and Public Transport

I. Introduction

The origin of the theory of road pricing may be traced back to Pigou (1918). The basic idea is that road pricing makes road users internalize the externality of congestion so that social optimum is achieved. While economists do not dispute the potential of road pricing improving welfare, debate remains as to whether road users can be made better off if the proceeds of road tolls are not returned to road users, as is often the case in the real world. As it turns out, the assumption of uniform value of time which is implicit in the works of the earlier authors, e.g. Walters 1961, Vickrey 1967, would guarantee that all road users are made worse off (see Evans 1992, Hau 1994, and this paper). Allowing the value of time to vary across road users, later analysis (e.g. Evans 1992, Hau 1994) finds that only those with high values of time can be made better off by optimal road pricing. All diverted road users are made worse off.

In this paper, we study the interaction between private and public transport. Earlier analyses of this issue include Frankena 1982 and Ho 1986. Alluding to second best theory, Frankena (1982) maintained that public transit fares should be lower in the absence of optimal congestion pricing on roads. Ho (1986) showed that the better is public transport as a substitute for private cars, the lower will be the marginal cost of raising the road price in terms of the economic loss suffered by those driven away by road price. Thus the optimal road price should be higher. Neither author, however, took into account the

competition between private vehicles and public buses for road space.

Incorporating public transport into the analysis drastically changes the welfare implications of road pricing. In the traditional analysis, road pricing drives some road users away from the road. They must be made worse off. For those who never use the road anyway, their welfare is not affected by road pricing. This 'conventional wisdom' is no longer valid when there is public transport, because road pricing makes the road less congested. By speeding up public transport, the "loss" imposed on diverted road users may become negative. The marginal cost of increasing the road price will then be lower, justifying a heavier road price. Because of time savings, the remaining road users may also be net gainers too, despite the higher toll. We will show under what circumstances this is more likely to happen.

The paper proceeds as follows. Section II introduces the basic model with the assumptions that the value of time is uniform among road users and that private vehicles are assumed to be the only road users. Section III considers the case where private transport competes with public transport for road space. We demonstrate that, with public transport and private transport being substitutes, optimal road pricing may be Pareto improving. We also offer some explanations why our result is compatible with the everyday experience that motorists generally oppose an increase in road toll. Section IV concludes the paper. Mathematical proofs are grouped in the Appendix.

II. Basic Model

In this Section we first consider the textbook case of private vehicles being the only road users and the unit cost of time being uniform among road users. Consider a continuum of potential road users, each characterized by a gross benefit of using a road equal to b , where b is distributed with density function $f(\cdot)$. Whether a particular potential road user will indeed use the road depends on the net benefit, $b - C$, being positive, where C is the cost of using the road and is an increasing convex function of the number of actual road users. Here, the assumption that C is common to all road users implies all potential road users have the same valuation of time. We can write $C = C(1 - F(\bar{b}))$, where $C' > 0$, $C'' > 0$, $F(\cdot)$ is the cumulative distribution function of b , and \bar{b} is a 'cut-off' value of b such that those potential road users with $b > \bar{b}$ will use the road and those with $b \leq \bar{b}$ will not use the road.

Social optimum requires the maximization of the total net benefits and corresponds to solving the following problem:

$$\begin{aligned} \text{MAX}_{b^*} \int_{b^*}^{\infty} [b - C(1 - F(b^*))] f(b) db \\ = \int_{b^*}^{\infty} b f(b) db - [1 - F(b^*)] C(1 - F(b^*)). \end{aligned} \quad [1]$$

The first order condition is

$$b^* = C(1 - F(b^*)) + [1 - F(b^*)] C'(1 - F(b^*)). \quad [2]$$

The first order condition [2] has the usual marginal interpretation. At social optimum, the cut-off value of gross benefit b^* should be equal to the sum of direct cost C and the

external cost, namely, the marginal cost C' multiplied by the number of road users $1 - F(b^*)$.

From the first order condition [2], the optimal toll t that should be levied on the actual road users is equal to the externality term $[1 - F(b^*)] C' (1 - F(b^*))$, which will result in an optimal number of road users, $1 - F(b^*)$. The cut-off value of gross benefit, b^* , is equal to $C(1 - F(b^*)) + t$.

Consider the *laissez-faire* case where no toll is levied on road users. Representing the cut-off value of gross benefit as \hat{b} , we must have $\hat{b} = C(1 - F(\hat{b}))$. We can prove the following:

Lemma 1: $\hat{b} < b^*$.

[Proof] See Appendix A.1. ■

Proposition 1: Potential road users with $b \leq \hat{b}$ are indifferent between the *laissez-faire* case and the optimal toll situation. Potential road users with $b > \hat{b}$ are strictly worse off in the optimal toll situation when compared to the *laissez-faire* case.

[Proof] See Appendix A.2. ■

In other words, under the assumption of uniform valuation of time, in the absence of a return of the collected tolls to road users, the optimal toll makes no one better off (apart from the road authority) and someone worse off. It is straightforward to demonstrate that with variable values of time among road users, some road users are made better off by the optimal toll provided that their value of time is high enough.¹

III. Model with Public Transport

The last section assumes that there are no alternatives to private transport. It follows that all those potential users that use the road under laissez-faire but do not under optimal toll must be made worse off by the optimal toll. In the real world, however, public buses often provide an alternative to private transport. Incorporating public transport into our analysis, the welfare implications are drastically altered.

To facilitate the analysis we now introduce some further notations. Suppose, in the absence of a road toll, the net benefit of a user of private transport is $b - vh - C$, where C is that part of the cost common to all road users, consisting of fuel costs and depreciation, etc., while vh , the time cost component, is the only component specific to the road user. v is the valuation of unit time of the potential road user and is assumed to be distributed with density function $g(\cdot)$. h is the time spent on the road and is assumed to be an increasing function of the number of road users of private transport, or, $h = h(1 - G(\tilde{v}))$, where $G(\cdot)$ is the cumulative distribution function of v , \tilde{v} is a 'cut-off' value of v , and $h', h'' > 0$. Therefore, vh is the total time cost spent on the road and is specific to the road user, while C is 'other cost' which is common to all road users. Note that the numbers of road users of private and public transport are $1 - G(\tilde{v})$ and $G(\tilde{v})$, respectively. In this specification, the time spent on private transport depends only on the amount of private traffic but not on the amount of public traffic.² Here we assume that the gross benefit b is constant across individuals. It is done without

loss of generality. Letting b depend on v will not change the results.

If public transport is used, the net benefit is $b - vk - D$, where $k = k(1 - G(\tilde{v}))$ is the time spent on public transport and D is the fixed out-of-pocket cost of using public transport.³ (See footnote 2 also.) We assume that both k' and k'' are positive and that public transport is slower than private transport, i.e., $k(1 - G(\tilde{v})) > h(1 - G(\tilde{v}))$ for all \tilde{v} .

Social optimum solves the following problem:

$$\begin{aligned} \text{MAX}_{v^*} & \int_{v^*}^{\infty} [b - v \cdot h(1 - G(v^*)) - C(1 - G(v^*))] g(v) dv \\ & + \int_0^{v^*} [b - vk(1 - G(v^*)) - D] g(v) dv \\ & = b - h(1 - G(v^*)) \int_{v^*}^{\infty} v g(v) dv - [1 - G(v^*)] C(1 - G(v^*)) \\ & \quad - k(1 - G(v^*)) \int_0^{v^*} v g(v) dv - G(v^*) D. \end{aligned}$$

[3]

The first order condition is

$$\begin{aligned} & v^* h(1 - G(v^*)) + C(1 - G(v^*)) + [1 - G(v^*)] C'(1 - G(v^*)) \\ & + h'(1 - G(v^*)) \int_{v^*}^{\infty} v g(v) dv + k'(1 - G(v^*)) \int_0^{v^*} v g(v) dv = v^* k(1 - G(v^*)) + D. \end{aligned}$$

[4]

The optimum toll t is such that

$$v^* h(1 - G(v^*)) + C(1 - G(v^*)) + t = v^* k(1 - G(v^*)) + D.$$

Those with $v > v^*$ use private transport and those with $v \leq v^*$ use public transport. Under laissez-faire where no toll is levied, we have

$$\hat{v} h(1 - G(\hat{v})) + C(1 - G(\hat{v})) = \hat{v} k(1 - G(\hat{v})) + D,$$

where \hat{v} is the cut-off value of v under laissez-faire.

We need the following assumption to prove results:

Assumption 1: For all v , $\frac{dk(1-G(v))}{d(1-G(v))} \leq \frac{dh(1-G(v))}{d(1-G(v))}$.

Assumption 1 says that the $h(\cdot)$ -schedule is steeper than the $k(\cdot)$ -schedule, or, when there is more traffic, the time spent on private transport increases more than the time spent on public transport does. One consequence of Assumption 1 is that when \hat{v} increases, the difference $k(\cdot) - h(\cdot)$ increases. This is consistent with everyday experience that there is a huge time difference between using public transport and using private transport when traffic is light and only a minor time difference when traffic is heavy. This assumption is especially realistic in jurisdictions (e.g. in Hong Kong) that provide bus-only lanes on congested roads.

We can now prove the following:

Lemma 2: $\hat{v} < v^*$.

[Proof] See Appendix A.3. ■

Define $\Delta B(v)$ as the difference between the net benefits under laissez-faire and under optimal toll of a road user with v . In other words, $\Delta B(v) \equiv B_{LF} - B_T$, where B_{LF} and B_T are the net benefits under laissez-faire and under optimal toll, respectively. Then, the following can be proved:

Proposition 2: (a) Those with $v \leq \hat{v}$ are better off when the optimal toll is levied. (b) (i) In the case of $\Delta B(v^*) > 0$ (i.e., the welfare of the "marginal motorist" under optimal toll being reduced by the toll), there exist $v^o \in (\hat{v}, v^*)$ and $\bar{v} > v^*$ such that when the optimal toll is levied, those with $v \in (v^o, \bar{v})$ are

worse off, those with $v \in (\hat{v}, v^0)$ or $v > \bar{v}$ are better off, and those with $v = v^0$ or $v = \bar{v}$ are indifferent. (ii) In the case of $\Delta B(v^*) < 0$, all those with $v > \hat{v}$ are better off when the optimal toll is levied. (iii) In the case of $\Delta B(v^*) = 0$, those with $v \in (\hat{v}, v^*)$ or $v > v^*$ are better off and those with $v = v^*$ are indifferent.

[Proof] See Appendix A.4. ■

The normal case is $\Delta B(v^*) > 0$, where those whose time value is very high ($v > \bar{v}$) or very low ($v < v^0$) are better off and those with intermediate time value ($v^0 < v < \bar{v}$) are worse off. (See Figure 1.) But it is possible that the optimal toll makes everyone better off (in the case of $\Delta B(v^*) < 0$). Those public transport users under laissez-faire are better off when the toll is introduced because there is less traffic. In contrast to many other studies of road using, with public transport as a substitute to private transport, those road users who are driven away from using private transport by the toll are not necessarily worse off. When the value of time is low enough so that one uses public transport even without the toll, she is made better off. For those with very high value of time and use private transport regardless of whether a toll is imposed, the toll makes the road less congested and so they are better off. All these are quite obvious. The difficult part to see is why those with value of time around v^* can also be better off when the toll is levied. In Section II, those with value of time around v^* must be worse off with the toll because under laissez-faire, they use the road and derive positive net benefit. With the toll, those with v

slightly less than v^* stop using the road and get zero net benefit. So, they are worse off. By continuity argument, those with v slightly bigger than v^* are also worse off. But, now, with public transport, those with v slightly less than v^* and driven to use public transport by the toll are not necessarily worse off because public transport is faster now. The net benefit derived from faster public transport may be higher than that of the private transport under laissez-faire. By continuity argument, those with v slightly bigger than v^* are also better off. Since the net benefit is increasing in v when $v > v^*$, those with $v > v^*$ are all better off.

 Insert Figure 1 here.

But why do we often not observe that everyone endorses an increase of road toll in the real world?⁴ Does this imply that it is rare that $\Delta B(v^*) < 0$? To answer these questions, we assume that there already exists a certain level of road toll t . Then,

$$\tilde{v} h(1 - G(\tilde{v})) + C(1 - G(\tilde{v})) + t = \tilde{v} k(1 - G(\tilde{v})) + D,$$

where \tilde{v} is the cut-off value of v . It is true for all t . Differentiate w.r.t. t and we get

$$[\tilde{h} - (\tilde{v}h' + \tilde{C}') \cdot \tilde{g}] \frac{d\tilde{v}}{dt} + 1 = (\tilde{k} - \tilde{v}k'\tilde{g}) \frac{d\tilde{v}}{dt}.$$

Rearranging, we have

$$\frac{d\tilde{v}}{dt} = \frac{1}{\tilde{v}(\tilde{h}' - \tilde{k}')\tilde{g} + \tilde{C}'\tilde{g} + (\tilde{k} - \tilde{h})},$$

which is positive. For road users with $v > \tilde{v}$, the net benefit of using the road is $NB = B - v h(1 - \tilde{G}) - C(1 - \tilde{G}) - t$.

Suppose there is a small increase in t . Most road users with $v > \bar{v}$ still stick to private transport. Then,

$$\begin{aligned} \frac{dNB}{dt} &= (v\bar{h} + \bar{c}') \bar{g} \cdot \frac{d\bar{v}}{dt} - 1 \\ &= \frac{v\bar{h}'\bar{g} + \bar{c}'\bar{g} - \bar{v}\bar{h}'\bar{g} + \bar{v}\bar{k}'\bar{g} - \bar{c}'\bar{g} - (\bar{k} - \bar{h})}{\bar{v}(\bar{h}' - \bar{k}')\bar{g} + \bar{c}'\bar{g} + (\bar{k} - \bar{h})} \\ &= \frac{(v - \bar{v})\bar{h}'\bar{g} + \bar{v}\bar{k}'\bar{g} - (\bar{k} - \bar{h})}{\bar{v}(\bar{h}' - \bar{k}')\bar{g} + \bar{c}'\bar{g} + (\bar{k} - \bar{h})} \end{aligned}$$

[5]

The sign of dNB/dt is indeterminate. However, if $\bar{k} - \bar{h}$ is close to zero, dNB/dt is likely to be positive. In other words, when the traffic is heavily congested so that using private transport and using public transport do not make much difference in terms of time (and so $\bar{k} - \bar{h}$ is close to zero), road users are likely to support an increase in toll. Moreover, when \bar{h}' and \bar{k}' are large, i.e., when traffic time is highly responsive to the number of users of private transport, dNB/dt is also likely to be positive. This happens when there is much underutilization of public transport, or when the per-capita road surface occupied by users of public transport is small compared to that of private public. Therefore, we expect, under these conditions, people are more likely to support an increase in road toll.

In general, when v is large, so is $v - \bar{v}$ and dNB/dt is positive. But for a small increase in t , there always exist some road users with v slightly above \bar{v} who stick to using private transport. For them, $v - \bar{v}$ is close to zero, and so if $\bar{k} - \bar{h}$ is large, dNB/dt is likely to be negative. They are thus worse off. That explains why there is always an opposition to a small

increase in road toll. But when the increase in road toll is big enough, the $v - \bar{v}$ term of those who stick to using private transport cannot be arbitrarily close to zero. It follows that it is possible that the first term in the numerator dominates the third term. As a result, dNB/dt becomes positive. The moral is that in order to bring about Pareto improvement, **(the increase in toll has to be big enough to divert sufficiently many users of private transport to using public transport so that there is a drastic decline in road congestion.)** If the toll increase is small and thus improves road congestion only marginally, some road users must oppose the increase. We may visualize this seemingly paradoxical result by treating a big increase in toll as a series of successive small increases. Each small increase in road toll makes someone worse off. But further increases may improve the welfare of those who were made worse off earlier. When compared with the initial situation without any increase in toll, they are better off and should support the big increase in toll. But in each step of a small increase, there are always someone who are made worse off and oppose the increase.

Given our analysis, the likely scenario for our Pareto improvement result to hold when private transport and public transport are substitutes is: (1) the traffic is heavily congested; (2) there is much underutilization of public transport; (3) the per-capita area of road surface occupied by users of public transport is small when compared to that of private transport; and/or (4) the increase of road toll is large enough.

IV. Concluding Remarks

This paper studies a long-neglected element in road pricing analysis, which is public transport being a substitute to private transport. The conclusions are quite different from the traditional ones. It now becomes possible that the diverted road users as well as existing public transport users are made better off. While the remaining road users are likely to become worse off there is a possibility that roads become so uncongested that they also become better off. This case represents a reversal of the case of the Appendix article in Mishan (1967).⁵

Note that the welfare analysis in this paper is done in terms of the number of road users affected, while in conventional analysis, the welfare analysis is done in terms of total benefits and costs. There is a novelty in our way of doing welfare analysis since in many policy issues, it is the relative numbers of supporters of and opponents to the policy that matters, rather than the intensities of how people are affected. It can be seen that our framework of analysis can be adopted easily to provide answers about total costs and benefits as well.

Appendix

A.1. Proof of Lemma 1.

Suppose the contrary, i.e., $\hat{b} \geq b^*$. It follows that $F(\hat{b}) \geq F(b^*)$ and $C(1 - F(\hat{b})) \leq C(1 - F(b^*))$. Therefore, $\hat{b} \geq b^* = C(1 - F(b^*)) + t > C(1 - F(b^*)) \geq C(1 - F(\hat{b})) = \hat{b}$, which is a contradiction. ■

A.2. Proof of Proposition 1.

It is obvious that potential road users with $b \leq \hat{b}$, are indifferent between the situations with and without optimal toll, since they do not use the road in either case. For potential road users with $b \in (\hat{b}, b^*]$, they use the road under laissez-faire and do not use the road under optimal toll. Let the net benefits under laissez-faire and under optimal toll be B_{LF} and B_T respectively. Then, $B_{LF} = b - C(1 - F(\hat{b})) = b - \hat{b} > 0$, and $B_T = 0$. Therefore, they are worse off when the optimal toll is levied. For potential road users with $b > b^*$, they use the road in both regimes. We then have $B_{LF} = b - C(1 - F(\hat{b})) = b - \hat{b}$, and $B_T = b - [C(1 - F(b^*)) + t] = b - b^* < b - \hat{b} = B_{LF}$. Therefore, they are also worse off when the optimal toll is levied. ■

A.3. Proof of Lemma 2.

Suppose not. Then, $\hat{v} \geq v^*$. Let $\hat{v} h(1 - G(\hat{v})) + C(1 - G(\hat{v})) = \hat{v} k(1 - G(\hat{v})) + D$. Then by Assumption 1, $v^*[k(1 - G^*) - h(1 - G^*)] \leq \hat{v}[k(1 - \hat{G}) - h(1 - \hat{G})] = C(1 - \hat{G}) - D \leq C(1 - G^*) - D$, which is a contradiction, since $v^*h(1 - G^*) + C(1 - G^*) + t = v^*k(1 - G^*) + D$. ■

A.4. Proof of Proposition 2.

Since there is less private traffic after the toll is levied,

those with $v \leq \hat{v}$ must be better off with the toll. Next consider those with $v \in (\hat{v}, v^*]$. They use private transport under laissez-faire and use public transport with the optimal toll. Then,

$$\begin{aligned} B_{LF} &= b - v\hat{h} - \hat{C} = b - v\hat{h} - \hat{v}\hat{h} - \hat{v}\hat{k} - D \\ &= b - (v - \hat{v})\hat{h} - \hat{v}\hat{k} - D, \\ B_T &= b - vk^* - D. \end{aligned}$$

So, we have

$$\Delta B(v) \equiv B_{LF} - B_T = vk^* - \hat{v}\hat{k} - (v - \hat{v})\hat{h}.$$

It follows that

$$\Delta B'(v) = k(1 - G^*) - h(1 - \hat{G}).$$

A priori, we do not know the sign of $k^* - \hat{h}$. Note that $\Delta B(\hat{v}) = \hat{v}(k^* - \hat{k}) < 0$. If $\Delta B'(v) = k^* - \hat{h} \leq 0$, then $\Delta B(v) < 0$, for all those with $v \in (\hat{v}, v^*]$. Therefore, road users with $v \in (\hat{v}, v^*]$ are better off under optimal toll. Suppose that $\Delta B'(v) = k^* - \hat{h}$ is positive. It says that the time spent on public transport with optimal toll is longer than that spent on private transport under laissez-faire. It is plausible because we should not expect an optimal toll can improve the traffic so much that it takes less time to travel by public transport with the toll than to travel by private transport under laissez-faire. Now, $\Delta B(v^*) = v^*k^* - \hat{v}\hat{k} - (v^* - \hat{v})\hat{h} = v^*(k^* - \hat{h}) - \hat{v}(\hat{k} - \hat{h})$, and the sign of $\Delta B(v^*)$ is indeterminate. It can be positive, negative, or zero. In the case of $\Delta B(v^*) < 0$, all those with $v \in (\hat{v}, v^*]$ are better off under optimal toll. In the case of $\Delta B(v^*) > 0$, under optimal toll, there exists $v^0 \in (\hat{v}, v^*)$ such that those with $v \in (\hat{v}, v^0)$ are better off, those with $v \in (v^0, v^*)$ are worse off, and those

with $v = v^0$ are indifferent. In the case of $\Delta B(v^*) = 0$, all those with $v \in (\hat{v}, v^*)$ are better off under optimal toll and those with $v = v^0$ are indifferent.

Consider those with $v > v^*$. They always use private transport regardless of whether the toll is levied. Analogous to what we have done above,

$$B_{LF} = b - v\hat{h} - \hat{C} = b - (v - \hat{v})\hat{h} - \hat{v}\hat{k} - D \quad \text{and}$$

$$B_T = b - vh^* - C^* - t = b - vh^* + v^*h^* - v^*k^* - D,$$

and therefore

$$\Delta B(v) \equiv B_{LF} - B_T = (v - v^*)h^* + v^*k^* - \hat{v}\hat{k} - (v - \hat{v})\hat{h}.$$

It follows that

$$\Delta B'(v) = h^* - \hat{h} < 0.$$

Now,

$$\Delta B(v^*) = v^*(k^* - \hat{h}) - \hat{v}(\hat{k} - \hat{h}).$$

As shown above, the sign of $\Delta B(v^*)$ is indeterminate. In the case of $\Delta B(v^*) \leq 0$, all those with $v > v^*$ are better off under optimal toll. In the case of $\Delta B(v^*) > 0$, under optimal toll, there exists $\bar{v} > v^*$ such that those with $v > \bar{v}$ are better off, those with $v \in (v^*, \bar{v})$ are worse off, and those with $v = \bar{v}$ are indifferent. ■

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Footnotes

- ¹ The proof is available from the authors on request.
- ² When there is much underutilization of public transport, i.e., more buses are not needed when there are more users of public transport, the number of public vehicles is fixed and hence the number of users of public transport does not affect the traffic time. But it is more correct to write $h = h(1 - G(\bar{v}), G(\bar{v}))$, where $h_1, h_2 > 0$. In general we expect that the effect of the number of motorists (the first argument) on h dominates that of the number of public transport passengers (the second argument) on h because the per-capita road surface occupied by users of public transport is much smaller when compared to that of private transport. As a result, when there are less users of private transport and more users of public transport, on net, h decreases. Then, we write $h = h(1 - G(\bar{v}))$ as a reduced form specification. The above discussion applies also to the specification of the time used in public transport, $k(\cdot)$, with the complication that it is also possible that the frequency of service is increased as a result of an increase in the number of passengers. Therefore the average waiting time may be shortened. It follows that when there are more users of public transport, k may decrease. But that only enhances our assumption that when there are less users of private transport, the traffic time is less.
- ³ As pointed out by the referees, it may be more correct to treat D , the non-time component of the cost of using public

transport, as endogenous. This can be easily done by specifying D as a decreasing convex function of $G(\bar{v})$. The justification is that there is a break-even constraint in running public transport. So, the more people are using public transport, the less is the per capita public transport fare. But it is easy to see that such a specification will not alter the basic results. For simplicity reason, this more realistic assumption is not adopted.

⁴ We thank the editor for bringing up this issue.

⁵ Mishan (1967) described the eventual deterioration of welfare for all travellers as public transport passengers first opted for private transport for the greater comfort and time savings achieved, and later, by necessity, roads got increasingly congested. Under the dynamics he described, all travellers became worse off compared to the initial situations because buses running on uncongested roads would have required less time than driving on congested roads. We thank a referee for suggesting that the insight resembles the Edgeworth tax paradox that taxing one good may lead to a fall in the price of both goods and may be traced back to Hotelling 1932.

