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Joint lead time and price quotation : dynamic or static?

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by

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ABSTRACT

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Master of Philosophy

Intuitively, quoting dynamic lead time and price to customers based on real-time system state provides more efficient capacity utilization and increases revenue compared with quoting static lead time and price. However, dynamic quotation may require higher operational costs for the firm and it is often inconvenient to customers. This study aims to compare dynamic and static lead time and price quotations under fixed capacity and different potential demand rates. We hypothesize that there exists a potential demand rate under which the additional costs of dynamic quotation and the additional profit from dynamic quotation are equal. Thus static quotation may yield better performance under certain potential demand rates. We use an M/M/1 queuing model to model the supply system of a firm and formulate profit maximization models in an average reward criterion under both static and dynamic lead time and price quotations. Numerical analyses are presented to illustrate performances of both static and dynamic lead time and price quotation and thus find the threshold potential demand rate. Besides, we study performance of two different kinds of dynamic lead time quotation and find that when firm can decide their price, performance of dynamic lead time quotation is good enough and when firm can not decide their price, the dynamic lead time quotation is good only when lead time sensitive factor is small and potential demand rate is big.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

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CERTIFICATE OF APPROVAL OF THESIS

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1. Introduction

Increased competition in today's global marketplace is pressing companies to compete in many dimensions other than price to differentiate themselves from competitors. Setting lead time and/or delivery guarantees have been adopted by more and more make-to-order manufacturing/ service/online retail companies as an essential strategy for customer acquisition and retention. For example, JD.com, Inc., one of the largest online direct sales companies in China, offers a wide selection of authentic products at competitive prices which are delivered in a speedy and reliable manner. By 'speedy and reliable manner', they mean a short lead time with delivery guarantees. In fact, their delivery services like Next-Day Delivery, Night Delivery and Three-Hour Delivery are proved to play a equally important role as competitive price to attract customers and in some cases their delivery services are order earners.

We define the lead time as the difference between the finish of an order and its arrival time. Firms make quotation on lead time and price in the first place to attract customers and due to work congestion, actual lead time in the end can differ from the quoted lead time. It is common sense that a short and reliable lead time and a competitive price are the key features to customers' preference. And yet as many firms move from mass production to customized production as well as the booming of online retailing, lead time becomes more important. A shorter quoted lead time can attract more customers but burden operational department more. In the worse case, firms will not be able to meet the promised lead times and suffer from the loss of future potential customers and monetary penalties. A longer quoted lead time on the other hand might release the burden of the operational department but lose the lead time competitiveness. Firms are more likely to meet the quoted lead time as it is relatively longer but may lose customers who are time sensitive. To be conclusive, the fundamental problem in making lead time quotation decisions is to quote a reasonable lead time that balance customers' expectation and companies' operational constraints. Price quotation also requires balance of customers' preference and firms' revenue. In this thesis we mainly study the jointly lead time and price quotation, which quite fit the reality where a great number of make-to-order manufacturing/service firms are quoting lead time and price jointly to customers.

In a real market, companies' policy in lead time and price quotation can be classified into two types: dynamic quotation and static quotation. In dynamic quotation, companies quote a lead time and price to each arriving customer according to the real-time system state (i.e. capacity, backlog, etc.). In static quotation however companies quote lead time and price to customers which is pre-determined and fixed. There are also two types of quotation modes under static model based on whether firms provide single quotation or not. A popular approach is to quote a single uniform lead time and price to customers while in some cases with heterogeneous customers, different lead times and prices are provided to different types of customers.

Both static and dynamic quotation modes have their own advantages and disadvantages. Consider a make-to-order company facing both lead time and price sensitive customers. When backlog in the company is less, a new coming order can be done more quickly than usual while in static quotation mode, a predetermined relatively longer lead time will anyhow be quoted to new customers and thus lose those customers who prefer a shorter lead time. Different from that in static quotation mode, sales department knows that backlog is less after communicating with operational department and thus quotes a shorter lead time to attract more customers in dynamic quotation mode. Meanwhile, price quotation can also be decided dynamically based on system state. Empirically, without considering additional cost like information searching cost, dynamic lead time and price quotation leads to more efficient capacity utilization thus it can bring a higher profit than static quotation. In this thesis, we further compare performance of dynamic model and static model numerically and see how exactly does dynamic quotation performs better than static model.

Though intuitively dynamic lead time and price quotation performs better than static quotation, static quotation is not a less common approach in reality. It requires less operational cost for both firms and customers than dynamic quotation. From the firm's perspective, sales department getting real-time system state from operational department is costly and making quotation decisions every time a customer coming is also costly. From the customers' perspective, dynamic quotation mode will incur customers' information searching cost thus cause decreases on demand.

As competition between firms becomes severer in our global market, one of the negative side of dynamic lead time and price quotation, the information searching cost for customers are playing a more significant role in customers purchase process. We focus on this information cost as a disadvantage of dynamic model. Differentiated quotation, one of the static quotation modes, also requires information searching cost for customers. Hence, only uniform quotation are considered in our static model. After the discussion about advantages and disadvantages of dynamic model and static model, it is natural to come up with a trade-off where the additional profit and additional cost from the dynamic quotation mode are equal. We hypothesize that there exists a demand rate under which the trade-off exists, thus when demand rate is lower than this threshold rate, firms should choose static quotation to gain more revenue and choose dynamic quotation only when potential demand rate is higher than this threshold rate.

Note that in our dynamic quotation mode, lead time and price are both quoted dynamically over time. However in reality many firms only quote lead time dynamically while price is fixed. And there are also two kinds of this 'dynamic lead time quotation'. Some firms can decide an optimal price as their fixed price while some firms can not change the original price in the market. Specifically, we denote the quotation mode in which firms can decide an optimal price and quote lead time dynamically as DLQ_1 and the quotation mode in which firms have to follow a price in the market and quote lead time dynamically as DLQ_2. In our study we use price in the static quotation mode as the original price. We know reward from DLQ_1 and DLQ_2 is between reward from static quotation and reward from joint dynamic quotation. Also, DLQ_1 should perform better than DLQ_2. But we do not know whether performances of DLQ_1 and DLQ_2 are closer to performance of joint dynamic quotation or performance of static quotation. We take this as our third research questions.

Literature on lead time quotation and/or lead time and price quotation is quite extensive. As far as we are concerned, existing papers on this topic usually formulate their models based on only one quotation mode, dynamic or static, and consider the optimal policy from many different aspects like multiple stage game. Yet we have Zhao, Stecke and Prasad (2012) compare two modes of static lead time and price quotation, i.e., uniform or differentiated, thus provide guidance for firms to choose between these two quotation modes. Though we have Palaka, Erlebacher, and Kropp (1998), Chatterjee, Slotnick and Sobel (2002), etc., discuss static quotation mode, Duenyas and Hopp (1995), Duenyas (1995), Feng, Liu, Liu (2011), etc., study dynamic quotation mode, we have not found paper compare static quotation mode and dynamic quotation mode.

Before searching for the threshold demand rate where firm can choose between the two types of quotations (dynamic and static), we want to make sure there are firms that both static and dynamic lead time quotations may fit. Take a customized furniture manufacturer, a make-to-order firm, for example. It has certain capacity and faces a market with price and lead time sensitive customers. In this case, the firm may make either a static or dynamic quotation to customers. In fact, those papers we mentioned above may consider different quotation modes by modeling on a same kind of firms, say make-to-order firms. For example, Duenyas and Hopp (1995), Feng, Liu, Liu (2011), etc. considered dynamic lead time policy for manufacturing firms. And yet Chatterjee, Slotnick and Sobel (2002), Rao, Swaminathan and Zhang (2005), etc. also put their static lead time policy studies on manufacturing/make-to-order firms.

Since few papers consider performance comparison of dynamic and static lead time and price quotations while firms can choose both static and dynamic lead time quotation in practice, we consider our work meaningful in helping managers making quotation decisions.

The main contributions of our work are as follows:

1. Without considering information searching cost, like most existing papers on lead time and price quotation do, we compare performance of static and dynamic models, and analyze sensitiveness of some major market factors.

2. As information searching cost are becoming more and more important in customers' purchase process, we add it in demand function and thus find the threshold potential demand rate, based on which managers should choose from dynamic quotation and static quotation.

3. We also consider case of two kinds of dynamic lead time quotation. We compare reward rate of DLQ_1 and DLQ_2 with joint dynamic lead time and price quotation and thus provide insights for managers to decide their quotation mode.

The reminder of this paper is organized as follow. In Chapter 2 we review papers mostly related to lead time (and price) quotation, both on static quotation mode and dynamic quotation mode. In Chapter 3, we build our demand function in an $M/M/1$ queuing model for a make-to-order (MTO) supply system. In Chapter 4, we provide optimal policy for static lead time and price quotation. In Chapter 5, we solve optimal policy for dynamic lead time and price quotation under the average award criterion. In Chapter 6, we use numerical analysis to compare performance of static and dynamic lead time and price quotation and in the case with information cost, we find the existence of potential demand rate. In Chapter 7, we consider dynamic lead time quotation. Lastly in Chapter 8, we conclude the thesis and highlight some management insights.

2. Literature Review

Early research on lead time mainly focuses on its competitive advantages. Stalk (1988) brings out the advantages of time-based competition by analyzing the rise of Japanese companies after the World War II. More specifically, Handfield and Pannesi (1995) analyze time-based competition in make-to-order manufacturing firms. By the time we start our research, there are numerous papers considering lead time-related problems in multiple directions. For example, Handfield and Pannesi (1995) consider cycle-time reduction for four supply-chain management strategies. Other papers study leadtime/cycle-time reduction are Suri (1998) and Hopp, Spearman and Woodruff (1990). The latter works on the cause of excessive lead time and brings out inexpensive and practical method to reduce it by a detailed study of six manufacturing facilities. Another direction in lead time study is predicting manufacturing lead times, e.g., Dongen, Croop and Aalst (2008).

Among all the lead time related directions, lead time quotation in MTO/service /online sales environment is the closest to our research. We first examine the development of lead time quotation problem in literature and then divide them into two parts, static quotation and dynamic quotation.

Although some papers working on lead time quotation problems use "lead time setting / due date setting" to describe their work (Palaka, Erlebacher and Kropp 1998, Duenyas 1992), early papers on lead time/due date setting usually assume an exogenously set lead time/due date and focus on sequencing problems, e.g. Sen and Gupta (1984), Gupta and Kyparisis (1987). Then more papers consider cases where due date are set endogenously, usually for make-to-order firms. Lead time becomes a more important competitive factor in reality, research then add its impact to demand function, some together with price, and instead of cost minimization profit maximization objective are used, e.g. Duenyas and Hopp (1995).

A number of early papers focus on stochastic processing time for lead time quotation and sequencing decisions. In these models processing processes are often facing servicer/machine breakdown or rush jobs. Decisions should be made under careful consideration of the stochastic processing time. Matsuura and Tsubone (1993) present a method for setting lead times and order sequencing under dynamic manufacturing environments with stochastic order processing time. Matsuura, Tsubone, and Kanezashi (1996) extend it to multi-operation jobs. Lawrence (1995) presents a methodology for setting lead times in complex production systems with multiple servers and machine breakdowns. More than one lead time performance objectives are considered, including cost minimization, attainment of service level targets, and minimization of mean absolute lateness and mean squared lateness. Elwany and Baddan (1998) provide a simulation method to calculate the sensitivity of lead time to processing time changes in a single server problem. None of the above papers choose reward/profit as objective.

Given the fact customers and MTO firms are in a two-way selection relationship in practice, a number of works allow customers to leave or firms to reject orders. Firms rejecting orders usually appears together with scheduling problems. Allowing customers to leave leads to a more realistic demand function where demand is affected by firms quotations. Papers with these settings usually choose profit maximization as their objectives. Duenyas and Hopp (1995) consider lead time quotation with a lead time-related demand function that a customer will accept a lead time with a probability which is decreasing in the quoted lead time. They find that optimal lead time is related to the patience of the customers and future arrivals should be taken into consideration when setting lead time. Duenyas (1995) extend this to multiple customer classes. Keskinocak, Ravi, and Tayur (2001) consider both homogeneous customers and two classes of customers for jointly scheduling and lead time quotation. In both models, there is a maximum lead time above which the customer does not place an order, and the quoted lead time are 100% reliable.

Since lead time and price share a similar role in affecting demand in practice, these two are usually studied together as companies decision factors. Besides, capacity is also considered one of the decision parameters sometimes. In Palaka, Erlebacher and Kropp (1998), these three factors are jointly considered in searching for profit maximization. In this model, demand is a function of price and lead time; penalty exists to make up for late order completion; inventory costs exist for early completion. With an $M/M/1$ queue, they find that it is not always better to have higher capacity utilization. Boyaci and Ray (2003) study pricing and lead time decisions for two substitutable services with dedicated capacity. Ray and Jewkes (2004) also consider price and lead time together, where price itself is determined by the length of lead time. A service level constrain is defined. Their results imply that whether customers are price or lead time sensitive should be taken into consideration before setting quotation strategies. Some other factors are also considered together as decision parameters in several papers. Wu et.

al. (2012) add ordering semi-product as a decision factor as well as price and lead time. They study a newsvendor problem to determine optimal price, lead time and ordering quantity simultaneously. Rao, Swaminathan and Zhang (2005) consider situation where late completion risk can be eliminated by outsourcing. Firms' decision factors include price, lead time and production. They show that the optimal lead time has a closed-form solution with a newsvendor-like structure.

Some studies consider coordination between operational department and sales department when deciding lead time policy, leading to models with more than one stage. Erkoc and Wu (2000) build a Nash game model of the coordination between marketing and manufacturing to quote lead times. Pekgn, Griffin and Keskinocak (2008) consider a firm facing both price and lead time sensitive customers, and price and lead time decisions are made by marketing and manufacturing department respectively. They find the inefficiencies created by the decentralization are significant and show that coordination can be achieved using a transfer price contract with bonus payments. Similarly, Liu, Parlar and Zhu (2007) build a Stackelberg game to analyze price and lead time decisions in a decentralized supply chain. Both lead time strategy for supplier and retailer are given and they suggest that before consider coordination with retailers or the marketing department, the supplier should first improve its own internal operation. Chatterjee, Slotnick and Sobel (2002) also consider a two-stage process where sales department making lead time quotation with incomplete information about operation department. They specify conditions for an optimal log-liner decision rule and provide exact expressions for its effect on arrival rate, mean processing time and mean cycle time.

Most papers above have the same fundamental setting: lead time and other decision factors (if any) are predetermined before selling. That is, firms are using static quotation in their model. Now we consider lead time quotation on static model and dynamic model separately.

Customers are favoring uniform lead times guarantees for its low information searching cost in reality. So and Song (1998) model a price and lead time sensitive market and a MTO firm offer uniform lead time with service level constrains (no delay penalty). Capacity is also a decision factor in this paper. So (2000) extends this study to a competitive setting. Other papers discuss uniform lead time quotation including Palaka, Erlebacher, and Kropp (1998), Chatterjee, Slotnick and Sobel (2002), Rao, Swaminathan and Zhang (2005), Ray and Jewkes (2004), etc.. Papers we mentioned in earlier sections mostly set uniform lead time.

In fact, static quotation includes both uniform quotation and differentiates quotation. Differentiate quotation is to dealing with heterogeneous customers. Although lead time and price (if any) are predetermined before selling as in the uniform quotation mode do, differentiate quotation requires information searching cost like in the dynamic quotation mode. Boyaci and Ray (2003) study two substitutable services with only price and lead time differences as we mentioned above. Their model integrates pricing and lead time decisions with capacity requirements and costs. Scenarios where the firm is constrained in capacity for none, one, or both product(s) are considered respectively. They also illustrate their results in a numerical study. Mendelson and Whang (1990) model an $M/M/1$ queuing system that dealing with multiple classes of customers who differ in time sensitiveness. They develop a pricing mechanism which is incentive-compatible that the arrival rates and processing priorities jointly maximize the expected value of the system. Maglaras (2005) studies a system with two non-substitutable services and heterogeneous users. One service uses lead time guarantees and the other uses the "best-effort". They first solve a deterministic relaxation of the original objective to obtain a "fluid-optimal" solution that is subsequently evaluated and refined to account for stochastic fluctuations. Zhao, Stecke and Prasad (2012) extend the work of Boyaci and Ray (2003) by considering more substitutable services different in price and lead time. They compare the performance of the uniform quotation and differentiated quotation and point out that which quotation mode is better depends on multiple factors.

Lastly, we go over papers on dynamic quotation mode. That is, firms make their quotation based on real-time system state. Duenyas and Hopp (1995) and Duenyas (1995) consider their quotation in a dynamic manner as we mentioned earlier. A semi-Markov decision process is built to solve the problem and an average profit per period (i.e. profit per arrival) is defined to describe system performance. They see their SMDP as a discrete time MDP by considering expected return per customer, which are independent of inter-arrival time, thus avoid the complexity of dealing with SMDP. Feng, Liu, Liu (2011) extend Duenyas and Hopp (1995) and derive an optimal policy for joint price and lead time quotation with fixed capacity and homogenous customers. Discounted reward in infinity horizon is used as performance criteria in this paper. Weng (1996) presents a model to study stochastic manufacturing lead time planning problems for make-to-order manufacturing systems facing two types of costumers. One is lead time sensitive and the other is lead time insensitive. He provides a joint lead time and order acceptance rate policy that can reduce cycle time and increase expected profit as well. In his model, although quoted lead time affects processing time, thereby incurring higher production costs for lower leadtimes, it does not affect the arrival rate of demand. Webster (2002) studies pricing, capacity and lead time policies for make-to-order/service companies. They assume time dependent demand and constant processing time. After examining policies for adjusting price and capacity to the dynamically changing in how customers value price and lead time, he suggest that using lead time and price to absorb changes in the market with fixed capacity will be most profitable with stability in throughput. Ata and Olsen (2009) consider dynamic quotation in a similar setting as Feng, Liu, Liu (2011). They provide recommended policies for convex, concave, and convex-concave lead-time cost functions and prove that these policies are asymptotically optimal. Moreover, numerical testes are presented.

Our model assumes homogenous customers [Palaka, Erlebacher, and Kropp (1998), Chatterjee, Slotnick and Sobel (2002), etc.] and set an $M/M/1$ queuing system [Maglaras (2005), Palaka, Erlebacher, and Kropp (1998), etc.]. Demand function is price and lead time related [Ray and Jewkes (2004), So and Song (1998), etc.]. Though joint lead time and price quotation is studied in static and dynamic manners respectively, we find few papers compare performance of these two quotation modes. Also, in order to compare performance of static and dynamic model, we use average reward criterion in our joint dynamic lead time and price quotation model, different from the discounted reward criterion in Feng, Liu, Liu (2011). Thus, we place our thesis in the literature as one of the very few that give management suggestions for managers to decide whether to choose static or dynamic quotation in lead time and price.

3. Demand Function

In this thesis, we consider a scenario where a make-to-order company provides a single product to customers in an infinite horizon. Every customer places one order at a time and they are served in a First-come-first-served (FCFS) manner. We assume a fixed capacity of the firm considering the fact that most make-to-order firms' capacity is settled in the beginning of business and cannot change quickly due to many constrains like cost and site area. Processing time for each product is exponentially distributed with mean μ^{-1} and customers arrive continuous over time according to a Poisson process with rate λ_0. Thus, we use an $M/M/1$ queuing to model the production system. Note that though lead time quotation under an $M/M/1$ setting is not a new studying area, we are the first attempt to compare the performance of static and dynamic lead time quotation. The $M/M/1$ model is chosen because of its traceability.

Every arriving customer will be quoted a price and lead time. We assume customers are homogenous in waiting cost and price sensitivity. Their utility for this product are uniformly distributed. Facing quotation, customer will only make a purchase if his/her cost for getting the product is no more than the utility for the product. Once a customer decides to stay and places an order, the company will get a reward p, the price of the product. However, if fail to meet the quoted lead time, company will suffer a delay penalty at δ per unit time. Here we do not consider firm's manufacturing cost. Also, we assume customers' additional waiting cost from late delivery will be covered exactly by the delay penalty, thus possible delay will not affect customers' purchasing decision.

We build our demand function as $\lambda(l,p)$=$\lambda_0 - \alpha l - \beta p - c$, where l denotes the quoted lead time and p denotes the quoted price, $\alpha(\beta)$ are lead time (price) sensitive factors respectively and c is the information searching cost for customers. Note that this information cost occurs only when price and lead time are vary in dynamic quotation mode while in static quotation mode the information searching cost equals to zero. Without c, this demand function is identical to the demand model in Liu, Parlar and Zhu (2007), Wu, Kazaz, Webster and Yang (2012) and it is consistent with reality where customer will decide whether to place an order or to leave facing a lead time and price quotation. For instance, assume customers' utility is uniformly distributed in [a,b], lead time (price) sensitive factors are $\alpha_*(\beta_*)$ respectively. Then, $\frac{\lambda_0(b-\alpha_* l-\beta_* p)}{b-a}$ is the actual demand, where $\alpha_* l + \beta_* p \geq a$. It has a similar structure with our demand function.

Being ignored in the lead time and price quotation area, customers' information searching cost becomes more significant in this high-speed and fiercely competitive society. Research in Marketing like Punj and Staelin (1983) and Mehta et al. (2003) has already realized the importance of customers' information searching cost and point out that it will affect customers' consideration process. In Mehta et al. (2003), customers are uncertain of prices and/or other information of different brands of product. The information searching process is costly to the customers since it requires investment in time and effort. Consumer rationality implies that these customers will make a trade-off between higher utility from his/her order and the cost of the information searching.

In our paper we do not consider the competition situation however the information searching process is just as costly. In static quotation mode the customer would not face the uncertainty of the information of the product, thus they do not suffer from the information searching cost. In dynamic quotation mode however the customers have to ask to get the information (price, lead time) of the product, this incurs the cost for information searching. The Customers might want to wait for necessary service/ order completion, but they become much less patient for searching information before making order decisions. For example, a customer passing through a shop and is interested by the product there. If the price was already shown on the board, he can directly compare his utility for this product with cost of this product and thus make his decision. However, in the dynamic quotation scenario, he does not know the price unless he get into the shop and ask. This asking procedure is costly (Mehta et al. 2003). Those who has less patience will just leave without even bother to ask for the quotations. This leads to the direct loss of the potential customers. Thus in our demand function we add the information searching cost as a constant number.

Another example is from the on-line hotel reservation. Hotel price is dynamically changing based on demand relations. Also, available status is changing over time. Taobao travel channel, one of the on-line server will only quote a reference price and customers have to ask for available status and price before booking. Customers then suffer from the same information searching cost as in our dynamic quotation model. Last year however Taobao travel channel has developed into Alitrip. In this new platform, real-time quotations are listed on the internet thus customers would not suffer from the information searching cost any more. The new Alitrip has became a much more popular on-line booking agency ever since and the real-time quotation action that helps avoid customers' information searching cost is of great importance.

To be concluded, different from Liu, Parlar and Zhu (2007), we add this information searching cost in our demand function as a direct reduction of potential demand rate. Thus, the optimal policy for our dynamic model with or without information searching cost is the same. In Chapter 5 the proving process for optimal policy can also demonstrate it.

4. Optimal Policy for Static Lead Time and Price Quotation

Before compare performance of dynamic and static lead time and price quotation, we first give optimal policy structure for both quotation modes. We start from the simpler case where lead time and price are quoted to coming customers statically. In our homogeneous customer setting, the joint price and lead time quotation are uniform for all customers. Companies' quotation decisions are determined by previous experience such as customers' coming rate, price and lead time sensitiveness, etc., in advance. Though companies will not be able to adjust quotation to real-time system state, they bare less operational cost and no customers' information searching cost will affect demand. Then the demand function in our static model is $\lambda(l,p)$=$\lambda_0 - \alpha l - \beta p$.

Follow the standard methods of optimization problem in static quotation mode (Liu, Parlar and Zhu 2007,Wu, Kazaz, Webster and Yang 2012), we give the company's optimization problem (COP) as:

$$\max_{l,p} \left\{ \lambda(p - \delta \int_l^{\infty} (x-l) f(x) dx) \right\},$$

where $f(x)$ is the realized lead time density function.

The realized lead time density function in $M/M/1$ model is as follows:

$$f(x) = (\mu - \lambda) e^{-(\mu-\lambda)x}.$$

We then substitute it in the COP, we have

$$\max_{l,p}\left\{\lambda(p-\frac{\delta}{\mu-\lambda}e^{(-(\mu-\lambda)l)})\right\}.$$

Note that the COP is monotonous in l for given λ, we use the following sequential procedure to solve the COP. For a given λ, we can get the optimal l, that is $l^*(\lambda)$. Thus we have $l^*(\lambda)$ as a function of λ. Also, we rewrite p in terms of λ: $p(\lambda)$=$\frac{(\lambda_0-\lambda-\alpha l)}{\beta}$. Then we substitute $l^*(\lambda)$ and $p(\lambda)$ to COP and get a single variable problem.

Lemma 1. *For a fixed λ, the company's best lead time policy is*

$$l^*(\lambda)=R_\lambda^{-1}\frac{\delta-\frac{\alpha}{\beta}}{\delta},$$

where $R_\lambda^{-1}$is the inverse of the realized lead time distribution function R_λ. From s-tandard $M/M/1$ queuing results, $R_\lambda(x)=1-e^{-(\mu-\lambda)t)}$. Thus, we obtain $l^*(\lambda)$=$\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}$.

Proof. Taking the first and second order condition of COP over l, we have $-\frac{\alpha}{\beta}+\delta e^{(-(\mu-\lambda)l}$ and $-(\mu-\lambda)\delta e^{(-(\mu-\lambda)l}$. The second order condition is obvious negtive, and thus root for$-\frac{\alpha}{\beta}+\delta e^{(-(\mu-\lambda)l}=0$ is our optimal l. solving it we have $l^*(\lambda)$=$\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}$. ■

Lemma 1 has a similar structure with the optimal order quantity formula in the standard newsvendor problem and is similar to Liu, Parlar &Zhu (2007), Wu, Kazaz ,Webster & Yang (2012).

Now we substitute $l^*(\lambda)$ and $p(\lambda)$ to COP to solve optimal λ:

$$\begin{aligned}
&\lambda(\frac{\lambda_0-\lambda-\alpha l^*(\lambda)}{\beta}-\frac{\delta}{\mu-\lambda}e^{-(\mu-\lambda)l^*(\lambda)})\\
&=\lambda(\frac{\lambda_0-\lambda-\alpha\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}}{\beta}-\frac{\delta}{\mu-\lambda}e^{-(\mu-\lambda)\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}})\\
&=\lambda(\frac{\lambda_0-\lambda-\alpha\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}}{\beta}-\frac{\delta}{(\mu-\lambda)\beta})
\end{aligned}$$

Denote $\Pi(\lambda)=\lambda(\frac{\lambda_0-\lambda-\alpha\frac{\ln\frac{\alpha}{\delta\beta}}{\lambda-\mu}}{\beta}-\frac{\delta}{(\mu-\lambda)\beta})$. The first and second derivatives with respect to λ of $\Pi(\lambda)$ are

$$\Pi'(\lambda)=\frac{\lambda_0-2\lambda}{\beta}+\frac{\alpha\mu(\ln\frac{\alpha}{\delta\beta}-1)}{\beta(\lambda-\mu)^2} \tag{1}$$

and

$$\Pi''(\lambda)=-2(\frac{1}{\beta}+\frac{\alpha\mu(\ln\frac{\alpha}{\delta\beta}-1)}{\beta(\lambda-\mu)^3}). \tag{2}$$

Since $\delta>\frac{\alpha}{\beta}$,$\alpha\mu(\ln\frac{\alpha}{\delta\beta}-1)<0$. Also we assume $\lambda<\mu$, otherwise the queue length will explode. Thus (2) remains negative, which means that function (1) is decreasing in λ. If (1) is above zero when $\lambda=0$, we can easily get the optimal λ by solving

$$\Pi'(\lambda)=\frac{\lambda_0-2\lambda}{\beta}+\frac{\alpha\mu(\ln\frac{\alpha}{\delta\beta}-1)}{\beta(\lambda-\mu)^2}=0.$$

We remind our readers that purpose of this article is to compare companies' performance under static and dynamic quotation respectively, specified quantitative values will be used in our numerical analysis part. Interested readers can find use of random lead time distribution, which are dependent of λ, lead to a quadratic function of λ in the optimal equation (Wu,Kazaz ,Webster & Yang 2012) and thus find the optimal λ. In fact, we follow the standard method to get optimal policy for static lead time and price quotation just in preparation for comparison of static and dynamic quotation models.

5. Optimal Policy for Dynamic Lead Time and Price Quotation

In dynamic quotation mode, lead time and price are quoted to customers dynamically over time. That is, the company will decide the price and lead time based on real-time system state. In order to compare performance with static model, we derive optimal quotation policy and system performance for this dynamic quotation in a metric of average reward.

Let the number of customers in the system be the system state. Let S denote the set of states, A_s denotes the set of actions (a) in state s, π denotes the policy for every state. Denote $X(t)$ as system state at time t. $X(t)$ will only change either when new customer placing an order or there is a completion of one product. Both events have exponentially distributed interval time, thus the interval time of state transition is exponentially distributed too. Transition rate in each state under any action a is $\gamma(s,a)$=$\lambda(l_s, p_s) + \mu_s$, where p_s and l_s are the quoted price and lead time in state s. μ_s is the order completion rate in state s thus $\mu_0 = 0$, and for $s > 0$, $\mu_s = \mu$. Also, let $r(s,a)$ denote the expected total reward between two decision epochs, and $q(j|s,a)$ denotes the probability that system state transits to j in the next decision epoch when system state is currently i and action a is chosen (as in the embedded Markov chain). Although state transitions due to product completion do not need quotation (no new customer coming), we assume every system state is a decision epoch (firm may not need to decide quotation in state i when no new customer coming, but it may also face situation where new customer coming when system state is i). Thus, we get a continuous-time Markov decision process (CTMDP).

Assume that the process starts at time 0 with state s. For $t \geq 0$, $v_t^\pi(s)$ denote the expected total reward generated by this process under policy π. Let n_t denote the number of decision epoches up to time t, W_t be the random variable representing the state of the natural process at time t, and Y_t be the random variable representing the action at time t. Reward is gained in two parts. First at each decision epoch, a fixed reward $k(X_n, Y_n)$ is gained. Then another reward is accumulated at rate $c(W_u, X_{n_u}, Y_{n_u})$ during the process. Thus, we have

$$v_t^\pi(s) = E_s^\pi \left\{ \int_0^t c(W_u, X_{n_u}, Y_{n_u}) du + \sum_{n=0}^{n_t - 1} k(X_n, Y_n) \right\}.$$

This is an expectation on fixed reward at each decision epoches and the accumulated reward up to time t. We then define the average reward as follow:

$$g^\pi(s) = \liminf_{n \to \infty} \frac{E_s^\pi \left\{ \sum_{i=0}^{n} \left[k(X_i, Y_i) + \int_{\sigma_{i+1}}^{\sigma_i} c(W_t, X_i, Y_i) dt \right] \right\}}{E_s^\pi \left\{ \sum_{i=0}^{n} \tau_i \right\}}.$$

This is the limit inferior of the ratio of the expected total reward up to the nth decision epoch to the expected total time to the nth decision epoch. Our objective is to find the optimal policy that lead to the optimal average reward g.

Different from the static model, our demand function in dynamic model is $\lambda(l_s, p_s)$=$\lambda_0 - \alpha l_s - \beta p_s$-$c$, where c is the information searching cost, l_s is the lead time in state s and p_s is the price in state s and α (β) are lead time (price) sensitive factors respectively. Note that in the following discussion to get the optimal policy, whether c equals to zero or not makes no difference.

Average reward model is more complicated than discounted model. Yet to compare with the reward rate we solve from static quotation model, we need average reward criteria in our dynamic model. For easy analysis, following Puterman (1994), we discretize time and apply the discrete time methods in view of the Markov property to analyze the continuous-time Markov decision process. Note that in the CTMDP we describe above, system state would not be the same after one transition. However, a modified process where system state may be the same after one transition proved to be equal in distribution with the above process (Puterman 1994). Denote all quantities in the transformed model with$\widetilde{}$. Let $\widetilde{S} = S$, $\widetilde{A_s} = A_s$. We assume $\widetilde{A_s} = A$ for all $s \in S$. For all $s \in S$,

$$\widetilde{r}(s, a) = r(s, a)\gamma(s, a)$$

and

$$\widetilde{q}(j|s, a) = \begin{cases} \eta \times q(j|s, a)\gamma(s, a), & j \neq s, \\ 1 + \eta(q(j|s, a) - 1)\gamma(s, a), & j \equiv s. \end{cases}$$

where η is a constant that satisfies

$$0 < \eta < \frac{1}{\gamma(s, a)(1 - q(s|s, a))}$$

for all $a \in A$ and for those $s \in S$ that satisfy $q(s|s, a) < 1$.

The following lemma draws directly from Puterman (1994) and it explains that the transformed discrete-time model has the same optimal time average reward as the orignal CTMDP.

Lemma 2. *Let S be countable. Suppose $(\widetilde{g}, \widetilde{h})$ satisfy the discrete-time optimality equations, $\widetilde{g}^* \equiv g^*$.*

Follow the discrete-time average reward optimality equations, we have

$$h(s) = \max_{a \in A}\{\widetilde{r}(s,a) - \widetilde{g} + \sum_{j \in S} \widetilde{q}(j|s,a)h(j)\},$$

for all $s \in S$,where $\widetilde{g}$ denotes the optimal average reward and $h(s)$ is the unknown value function vector. Note that there are $|S|$ equations and $|S|+1$ unknowns and a max operator in the above equation set, it cannot be solved by using linear algebra techniques such as Gauss elimination.

Now we can compute $r(s,a)$. As defined above, $r(s,a)$ denotes the expected total reward between two decision epochs:

$$r(s,a) = \frac{\lambda(l_s, p_s)}{\gamma(s,a)}(p_s - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t)),$$

where $R_s(t)=\sum_{k=s+1}^{\infty} \frac{(\mu t)^k}{k!} e^{-\mu t}$ denotes the realized lead time distribution for an order coming in state s. Thus, we have

$$\widetilde{r}(s,a) = \lambda(l_s, p_s)(p_s - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t)).$$

Now we consider the transition probabilities. We have

$$q(s+1|s,a) = \frac{\lambda(l_s, p_s)}{\gamma(s,a)},$$

and

$$q(s-1|s,a) = \frac{\mu}{\gamma(s,a)}.$$

Thus,

$$\widetilde{q}(j|s,a) = \begin{cases} \eta\lambda(l_s, p_s), & j = s+1 \\ \eta\mu, & j = s-1 \\ 1 - \eta\gamma(s,a). & j = s \end{cases}$$

This transformation converts rewards to a unit time basis and alter transition structure so that our semi-Markov decision process model agree with the discrete-time model. For easy presentation, we define

$$\overline{h}(s) = \widetilde{r}(s,a) - \widetilde{g} + \sum_{j \in S} \widetilde{q}(j|s,a)h(j).$$

For a given λ, we define the expected reward from one order as $P_{\lambda,s}(l)$. We rewrite price p as a function of λ and l:

$$p_\lambda(l) = \frac{\lambda_0 - \alpha l - \lambda - c}{\beta}.$$

Note that given λ, price p is decreasing in l.

$$P_{\lambda,s}(l) = p_\lambda(l) - \delta \int_l^\infty (t-l)dR_s(t).$$

It is clearly that $P_{\lambda,s}(l)$ is the only component in $\widetilde{r}(s,a)$, thus $\overline{h}(s)$, that relevant to different policies (l_s, p_s). We will use this result in the proof of theorem later.

Now we seek the existence of the lead time l_{*s} that maximize $P_{\lambda,s}(l)$.

Proposition 1. *$l_{*s} = \arg\max_l P_{\lambda,s}(l)$ is the root of $P'_{\lambda,s}(l) = 0$.*

Proof. Consider the first order condition of $P_{\lambda,s}(l)$: $P'_{\lambda,s}(l) = \frac{-\alpha}{\beta} + \delta(1 - R_s(l))$, it is decreasing in l and $P'_{\lambda,s}(0) = \frac{-\alpha}{\beta} + \delta$. Here we assume $\frac{\alpha}{\beta}$ is smaller than δ. If $\frac{\alpha}{\beta}$ is bigger than δ , firm would not hesitate to quote zero lead time to get more profit. Under this assumption, $P'_{\lambda,s}(0) = \frac{-\alpha}{\beta} + \delta > 0$. So, $P_{\lambda,s}(l)$ will increase in l until $P'_{\lambda,s}(l)$ drops below 0. $P_{\lambda,s}(l)$ is maximized when $P'_{\lambda,s}(l) = 0$. ■

Solving equation $P'_{\lambda,s}(0) = \frac{-\alpha}{\beta} + \delta$, we have l_{*s}. Since $R_s(l)$ is obvious decreasing in s for any fixed l , l_{*s} is increasing in s. Now we consider the constraints on lead time l. Assume the largest lead time is l_{max}. $\lambda_0 - \alpha \times l_m ax \equiv 0$, $l \in [0, l_{max}]$. If $l_{max} \leq l_{*s}$, $\frac{-\alpha}{\beta} + \delta(1 - R_s(l_{max})) \geq 0$, $P_{\lambda,s}(l)$ will not get to the mathematical largest point in its codomain.

Proposition 2. *$\frac{-\alpha}{\beta} + \delta(1 - R_s(l_{max})) \geq 0$ holds if and only if $\delta > \frac{\alpha}{\delta}$ for some finite s.*

Proof. $R_s(l_{max})$ is obvious decreasing in s and $\lim_{s\to\infty} R_s(l_{max}) \equiv 0$. ■

Now we define the admission threshold T.

$$T = \begin{cases} \min_s \frac{-\alpha}{\beta} + \delta(1 - R_s(l_{max})) \geq 0, & s \leq |S| - 1 \\ S. & \text{otherwise} \end{cases}$$

Theorem 1. *When $s \geq T$, quote l_{max} to reject the customer; when $s < T$, because $l_{*s} < l_{max}$, l_{*s} belongs to the codomain, quote l_{*s}.*

Proof. Define p_{min} and p_{max} as follows: $p_{min} = 0$, $\lambda_0 - \beta p_{max} = 0$.

$$\max_{a \in A} \overline{h}(s) \equiv \max_{\lambda} \max_{a:\lambda(a)} \overline{h}(s).$$

As we discussed before, $P_{\lambda,s}(l)$ is the only component dependent of the decision for any given λ. So l_{*s} or l_{max} (when $l_{max} < l_{*s}$), not only maximize $P_{\lambda,s}(l)$, but also $\overline{h}(s)$. First we consider situation when $s \geq T$. Since $R_s(l)$ is obvious decreasing in s for any fixed l , $l_{*s} \geq l_{max}$ under this condition. Then $P_{(\lambda,s)}(l)$ is increasing of l in $[0, l_{max}]$ and is maximized in l_{max}. Note that $\lambda_0 - \alpha \times l_{max} = 0$, so quoting l_{max} to customers equals to rejecting customers. If this customer were accepted and quoted a lead time $l_l < l_{max}$, then, to maximize $P_{(\lambda,s)}(l)$, l_l should increase and p should decrease. Thus the policy is (p_{min}, l_l) . Reward from this customer is worse than reject him/her. What is more, when consider optimal equation in dual liner program, we have

$$\widetilde{g} = \max_{a} \sum_{s} \widetilde{r}(s,a)\pi(s,a),$$

where $\pi(s,a)$ is the fraction of time spent in state i in steady state under policy a. Accepting this customer instead of rejecting him/her will decreasing $\pi(s,a)$ for every s less than T and add a positive probability, $\pi(i,a)$ with negative $\widetilde{r}(i,a)$, (i is bigger than T). Thus, rejecting this customer is optimal in this case.

When $s \leq T$, $l_{*s} < l_{max}$. we divide the decision domain into two regions:

$$E = a : \lambda(p, l) > \lambda(p_{min}, l_{*s}),$$

$$F = a : \lambda(p, l) \leq \lambda(p_{min}, l_{*s}).$$

In region E, we know by definition that $l < l_{*s}$, so $P'_{\lambda,s}(l) > 0$. Thus the optimal l in this region is the possibly largest one. For any λ, we know $\overline{h}(s, a)$ is maximized when $p = p_{min}$. From discussion above this $\widetilde{g}$ will be worse than that from (p, l_{*s}). In region F, we know from above l_{*s} maximize $P_{\lambda,s}(l)$ and also $\overline{h}(s, a)$. Thus, optimal lead time policy is l_{*s}. ■

Given optimal l , we can solve the MDP with only one variable p and thus get the optimal p. The following theorem allows us use dichotomy in our process solving optimal p and thus avoid vast computation in exhaustion algorithm.

Theorem 2. *$\overline{h}(s)$ is unimodal in p.*

Proof.

$$\begin{aligned}
&\frac{\partial \overline{h}(s)}{\partial p} \\
=&\frac{\partial \lambda(p, l)}{\partial p}(p - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t)) + \lambda(p, l) + [h(s+1) - h(s)]\eta \frac{\partial \lambda(p, l)}{\partial p} \\
=&[-\frac{1}{\lambda(p, l)} \frac{\partial \lambda(p, l)}{\partial p}(p - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t) - (h(s+1) - h(s))\eta) - 1](-\lambda(p, l))
\end{aligned}$$

Let $\Phi = [-\frac{1}{\lambda(p,l)} \frac{\partial \lambda(p,l)}{\partial p}(p - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t) - (h(s+1) - h(s))\eta) - 1]$, Ψ=$p - \delta \int_{l_s}^{\infty} (t - l_s) dR_s(t) - (h(s+1) - h(s))\eta$. Since $\lambda(p, l) > 0$, thus $-\lambda(p, l) < 0$, whether $\frac{\partial h(s)}{\partial p}$ is positive is determined by Φ. It is obvious $-\frac{1}{\lambda(p,l)} \frac{\partial \lambda(p,l)}{\partial p} > 0$ and is nondecreasing in p in our setting. Also Ψ is increasing in p. So if $\Psi > 0$, Φ is increasing in p. And if $\Psi < 0$, $\Phi < 0$. Thus, Φ will have at most one zero point, so does $\frac{\partial h(s)}{\partial p}$. Thus, $\overline{h}(s)$ is unimodal in p. ■

Average reward criterion is not used much in dynamic quotation problems due to its complication in continuous time models. However, since we have optimal reward rate in static model, average reward criterion is necessary in our dynamic model in order to compare performance with the static model. Duenyas and Hopp (1995) define an average profit per period (i.e. profit per arrival) thus see their SMDP as a discrete time MDP by considering expected return per customer, which are independent of interarrival time. In addition, they do not have a clear optimal policy. We have done a more general work by considering average reward per unit time to compare optimal performance in static and dynamic quotation mode. Feng, Liu and Liu (2011) draw an explicit optimal policy for dynamic lead time and price quotation in a discounted reward criterion. Our optimal policy in this average reward criterion has a similar structure with theirs. Though the optimal equation for discounted criterion and average criterion is not the same, we manage to define the expected reward from one order and thus solve the optimal lead time and price in a sequential procedure.

6. The Threshold Potential Demand Rate

In order to find the threshold potential demand rate, we need to compare performance of dynamic and static lead time and price quotation based on models we have built on the last subsection. Two MATLAB programs have been developed to compute both optimal policy and reward of these two models. Optimal policy and reward rate of static model can be solved simply following the algorithm in Chapter 4. However, we only solved optimal lead time policy for our dynamic model. Here we state our relative value iteration algorithm used to solve optimal average reward g in dynamic model.

Step 1: choose an arbitrary value for $\overline{h}^0$, choose an arbitrary state $s^* \in S$, specify ε, set $J^0 = \overline{h}^0 - \overline{h}^0(s^*) \times \overrightarrow{e}$. Set iteration times $n = 0$.

Step 2: define minimum price p_{min} and maximum price p_{max}, define p_{mid}.

Step 3: compute $\overline{h}^{n+1} = \max\limits_{p}\{\widetilde{r}(s,a) + \sum\limits_{j \in S} \widetilde{q}(j|s,a)J^n\}$ and $J^{n+1} = \overline{h}^{n+1} - \overline{h}^{n+1}(s^*) \times \overrightarrow{e}$.

Step 4: if$\max\limits_{s \in S}[\overline{h}^{n+1}(s) - \overline{h}^n(s)] - \min\limits_{s \in S}[\overline{h}^{n+1}(s) - \overline{h}^n(s)] < \epsilon$, go to step 5. Otherwise increment n by 1 and return to step 3.

Step 5: choose $a \in \arg\max\limits_{p}[\widetilde{r}(s,a) + \sum\limits_{s \in S} \widetilde{q}(j|s,a)h^n]$. Average reward $\widetilde{g} = \overline{h}^n(s^*)$.

Note that in step 3 we used dichotomy to solve the optimal p. Since we proved in Theorem 2 that $\overline{h}(s)$ is unimodal in p, dichotomy is a more convenient way to solve the optimal p other than exhaustion. This algorithm renormalizes $\overline{h}^n$ at each iteration by subtracting $\overline{h}^n(s^*)$ and outputs average reward g as we needed.

α	β	δ	λ_0	g_d	g_s	$\frac{g_d-g_s}{g_s}$
0.1	1	1	1	0.2458	0.2149	14.38%
0.1	1	1	3	2.1908	2.1128	3.39%
0.1	1	1	5	6.0474	5.9356	1.88%
0.1	1	1	7	11.7962	11.5745	1.92%
0.1	1	1	9	19.3284	18.7222	3.24%
0.1	1	1	10	23.6388	22.6900	4.18%
0.5	1	1	1	0.2418	0.1664	45.31%
0.5	1	1	3	2.1526	1.9145	12.44%
0.5	1	1	5	5.91	5.4956	7.54%
0.5	1	1	7	11.3975	10.7307	6.21%
0.5	1	1	9	18.3843	17.2762	6.41%
0.5	1	1	10	22.3280	20.9013	6.83%
0.9	1	1	1	0.2408	0.1538	56.57%
0.9	1	1	3	2.1409	1.8608	15.05%
0.9	1	1	5	5.8620	5.3786	8.99%
0.9	1	1	7	11.2531	10.5170	7.00%
0.9	1	1	9	18.0570	16.9333	6.64%
0.9	1	1	10	21.8814	20.4879	6.80%

Table 1: Performance of Static and Dynamic Model(a)

6.1. Performance Comparison of Dynamic and Static Models Without Information Searching Cost

First we start to consider the question: how does dynamic lead time and price quotation model without information searching cost perform compared to static model?

In the remainder of the numerical analysis, we assume company's capacity $\mu = 5$ without loss of generality. This company may face markets with potential demand rates vary from 1 to 10.

To examine performance of dynamic and static quotation models under different potential demand rates, λ_0 increases from 1 to 10 in 1 increment. We list average rewards of both static model and dynamic model under different value of factors in Table 1, Table 2, Table 3 and Table 4.

It is clearly that average reward in dynamic model is better than that in

α	β	δ	λ_0	g_d	g_s	$\frac{g_d - g_s}{g_s}$
0.1	1	2	1	0.2446	0.2080	17.60%
0.1	1	2	3	2.1588	2.0851	3.53%
0.1	1	2	5	5.9303	5.8732	0.97%
0.1	1	2	7	11.5742	11.4497	1.09%
0.1	1	2	9	19.0606	18.4947	3.06%
0.1	1	2	10	23.3794	22.4017	4.36%
1.0	1	2	1	0.2360	0.1020	131.37%
1.0	1	2	3	2.0761	1.6254	27.73%
1.0	1	2	5	5.6532	4.8719	16.04%
1.0	1	2	7	10.8216	9.6224	12.45%
1.0	1	2	9	17.3567	15.5500	11.62%
1.0	1	2	10	21.0445	18.8430	11.68%
1.9	1	2	1	0.2345	0.0831	182.19%
1.9	1	2	3	2.0577	1.5310	34.40%
1.9	1	2	5	5.5857	4.6706	19.59%
1.9	1	2	7	10.6392	9.2767	14.69%
1.9	1	2	9	16.9705	15.0311	12.90%
1.9	1	2	10	20.5265	18.2328	12.58%

Table 2: Performance of Static and Dynamic Model(b)

α	β	δ	λ_0	g_d	g_s	$\frac{g_d - g_s}{g_s}$
0.1	1	1.5	1	0.2450	0.2109	16.17%
0.1	1	1.5	3	2.1736	2.0966	3.67%
0.1	1	1.5	5	5.9862	5.8990	1.48%
0.1	1	1.5	7	11.6811	11.5009	1.57%
0.1	1	1.5	9	19.1879	18.5873	3.23%
0.1	1	1.5	10	23.5008	22.5186	4.36%
0.5	1	1.5	1	0.2401	0.1493	60.82%
0.5	1	1.5	3	2.1269	1.8414	15.50%
0.5	1	1.5	5	5.8255	5.3365	9.16%
0.5	1	1.5	7	11.2283	10.4410	7.54%
0.5	1	1.5	9	18.1274	16.8127	7.82%
0.5	1	1.5	10	22.0330	20.3431	8.31%
1.3	1	1.5	1	0.2374	0.1161	104.48%
1.3	1	1.5	3	2.0974	1.6923	23.94%
1.3	1	1.5	5	5.7177	5.0152	14.01%
1.3	1	1.5	7	10.9316	9.8714	10.74%
1.3	1	1.5	9	17.4868	15.9283	9.78%
1.3	1	1.5	10	21.1706	19.2899	9.75%

Table 3: Performance of Static and Dynamic Model(c)

α	β	δ	λ_0	g_d	g_s	$\frac{g_d - g_s}{g_s}$
0.6	0.5	2	1	0.4828	0.3224	49.75%
0.6	0.5	2	3	4.2961	3.7852	13.50%
0.6	0.5	2	5	11.7847	10.8957	8.16%
0.6	0.5	2	7	22.6861	21.2862	6.58%
0.6	0.5	2	9	36.4990	34.2697	6.51%
0.6	0.5	2	10	44.2709	40.8144	8.47%
0.6	1.5	2	1	0.1578	0.0737	114.11%
0.6	1.5	2	3	1.3777	1.1109	24.02%
0.6	1.5	2	5	3.7409	3.3063	13.14%
0.6	1.5	2	7	7.1829	6.5162	10.23%
0.6	1.5	2	9	11.5996	10.5201	10.26%
0.6	1.5	2	10	14.1171	12.7431	10.78%
0.2	0.3	2	1	0.8182	0.6866	19.17%
0.2	0.3	2	3	7.3193	6.9237	5.71%
0.2	0.3	2	5	20.2036	19.5176	3.51%
0.2	0.3	2	7	39.2433	38.0475	3.14%
0.2	0.3	2	9	63.7998	61.4375	3.85%
0.2	0.3	2	10	77.7244	74.4063	4.46%

Table 4: Performance of Static and Dynamic Model(d)

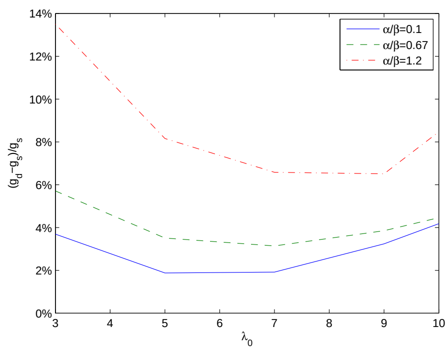

Figure 1: Example of Change Pattern of The Difference Ratio

static model. All the numerical results satisfy this property. As demand rate grows, both reward rates increase and the difference becomes larger. To see this difference more clearly, we consider the difference ratio, that is, $\frac{g_d - g_s}{g_s}$ (g_d: optimal average reward in dynamic model; g_s: optimal average reward in static model). We find that this ratio is the largest when $\lambda_0 = 1$ and then decreases as λ_0 increases. For most cases, this decreases would end as λ_0 increases to some point and then start to increase. However, for cases with α bigger than 1, this increasing pattern disappears. First we try to analyze the 'first decreasing then increasing' pattern. (We leave the analysis of the 'only decreasing' pattern at the end of Section 6.2). We draw picture of difference reward ratios facing three different kinds of customers: $\alpha = 0.1$, $\beta = 1$, ($\alpha/\beta = 0.1$); $\alpha = 0.2$, $\beta = 0.3$, ($\alpha/\beta = 0.67$); $\alpha = 0.6$, $\beta = 0.5$, ($\alpha/\beta = 1.2$) in Figure 1. Note that the ratio under $\lambda_0 = 1$ is much bigger then those under other potential demand rate, we delete this $\lambda_0 = 1$ point to draw the change pattern more clearly. Take the case $\alpha/\beta = 0.6$ for example, we further compare lead time, price and realized demand rate under these three potential demand rates($\lambda_0 = 1, \lambda_0 = 7, \lambda_0 = 10$). In table 5, we list expectation and variance of quoted price, lead time and realized demand rate, which are respectively marked as E[p], E[l], E[λ], Var[p], Var[l], Var[λ], in dynamic models and also the quoted price (p_{static}), lead time (l_{static}) and realized demand rate (λ_{static}) in static models. One possible explanation is that when potential demand rate is lower or higher, dynamic quotation model can provide more flexible lead time and price quotation than that when potential demand rate is in an average level. However, we can see from this table that all Var[p], Var[l], Var[λ] are not changing in this first increasing and then decreasing pattern, they keep increasing as λ_0 increases instead. It seems that the bigger the market is ,the more flexible of lead time and price quotations are. This fits our intuition because lead time and price quotation are made under constraints that $\lambda_0 - \alpha l - \beta p > 0$. When market is bigger, or λ_0 is bigger, lead time and price has more possible values and thus the variances are bigger.

Refer to the fact that the static quotation model is just a special case of dynamic model, we try to compare expectations of price and lead time, thus realized demand rate, in dynamic model with those in static models. See in Table 6. We use $(E[\lambda] - \lambda_{static})/\lambda_{static}$ to describe realized demand rate differences of dynamic models over static models. $(E[\lambda] - \lambda_{static})/\lambda_{static}$ is the largest, that is 9.50% , when $\lambda_0 = 1$ and it becomes the least, 3.70%, when λ_0=7. Realized demand rate is decided by price and lead time decisions. Higher price can get more revenue from one order but cause decrease in demand rate. Higher lead time can avoid some delay penalty but also cause decrease in demand rate.

Since realized demand rate is not the only component that has a direct impact on average reward, we also consider differences of price and lead time. $(E[l] - l_{static})/l_{static}$ are all below zero under three potential demand rate , which means optimal lead time quotation policy in dynamic models can lead to less expected quoted lead time to gain a higher realized demand rate. First we consider the small market. When $\lambda_0 = 1$, $(E[l] - l_{static})/l_{static} = -90\%$. This big difference in lead time allow the expected price quotation bigger than the price policy in static model and yet reach a higher demand rate. We consider delay penalty, $\delta \int_{l_s}^{\infty} (t - l_s) dR_s(t)$. When the market is very small, say $\lambda_0 = 1$, system state s, or queue length tends to be very small. Thus, expected completion time of an order becomes much shorter in this few accumulated workload circumstance. Hence, we believe the delay penalty at least will not be severe enough to eliminate the extra revenue drew from higher price and higher realized demand rate. This explains the observation that advantages of dynamic model over static model is more significant in small market. We conclude that dynamic quotation can quote shorter lead time hence increase realized demand rate without suffering severe delay penalty in small market. Note that queue tends to be longer as market increases, flexible lead time's role then becomes less significant.

Now we consider the big market, for example, when $\lambda_0 = 10$. In this case, $(E[p] - p_{static})/p_{static}$ is -4.90%, smaller than -2.10% for $\lambda_0 = 7$ and $(E[l] - l_{static})/l_{static}$ is -12.04%, bigger than -34% for $\lambda_0 = 7$. Opposite to small market, big market are more likely to have a relatively longer queue. Recall that we get our optimal lead time in Theorem 1. It is independent of realized demand rate and concerns only on reward from one order while optimal price is obtained from global optimization considering future queue length. Hence, dynamic price quotation plays an important role to control queue length here. However, due to longer expected queue, deviation of lead time is the smallest among under these three potential demand rates. Note that for $\lambda_0 = 7$ and $\lambda_0 = 10$, expectations of price are above 10 and expectations of lead time are below 1. Thus, deviation of -4.9% on price plays an more important role on reward than deviation of -34% on lead time.

In summary, dynamic quotation model's advantage over static model is that it can make full use of capacity by quoting flexible lead time and price regarding to different demand rates and system states. Flexible lead time policy contributes

λ_0	E[p]	E[l]	E[λ]	Var[p]	Var[l]	Var[λ]	p_{static}	l_{static}	λ_{static}
1	1.6795	0.0238	0.4921	$2.24e-005$	0.0058	0.0002	1.6746	0.2414	0.4493
7	12.0458	0.3980	3.3002	0.0277	0.2127	0.0198	12.3224	0.6044	3.1824
10	18.3849	0.9509	4.2973	0.4948	0.6563	0.1358	19.3332	1.0811	3.9838

Table 5: Static and Dynamic Policy Comparison Under Three Potential Demand Rates

λ_0	$(E[\lambda]-\lambda_{static})/\lambda_{static}$	$(E[p]-p_{static})/p_{static}$	$(E[l]-l_{static})/l_{static}$
1	9.50%	0.29%	-90.00%
7	3.70%	-2.10%	-34.00%
10	7.87%	-4.9%	-12.04%

Table 6: Expectation Comparison

more to better capacity utilization for dynamic model when market is small. Its impact becomes less obvious as potential demand rate increases. On the contrary, flexible price policy's role is more significant when the market is big. When the market is small, there are fewer backlog thus price's role on adjust workload becomes less important.

6.2. Sensitivity Analysis

Now we start to analyze sensitivities of factors α, β and δ. First we consider lead time sensitive factor α. See in Figure 2, when keep $\beta = 1$, $\delta = 1$, difference ratio $\frac{g_d - g_s}{g_s}$ increases significantly as α increases from 0.1 to 0.9. More specifically, when $\lambda_0 = 1$, $\frac{g_d - g_s}{g_s}$ increases from 14.38% to 56.57% while when $\lambda_0 = 10$, $\frac{g_d - g_s}{g_s}$ increases from 4.18% to 6.80%. This result fits our discussion in Chapter 6.1: when the market is big, the role of the lead time on the dominant advantage of dynamic model over static model becomes less significant. We then set $\beta = 1$, $\delta = 2$, $\lambda_0 = 5$ and increase α from 0.1 to 1.0 to further discuss the impact of α. As we can see in Table 7, with other factors fixed, increasing in α will cause significant increasing in difference ratio $\frac{g_d - g_s}{g_s}$. Both g_d and g_s drops as α increases and g_s drops quicker than g_d does. When α increases from 0.1 to 1, the average

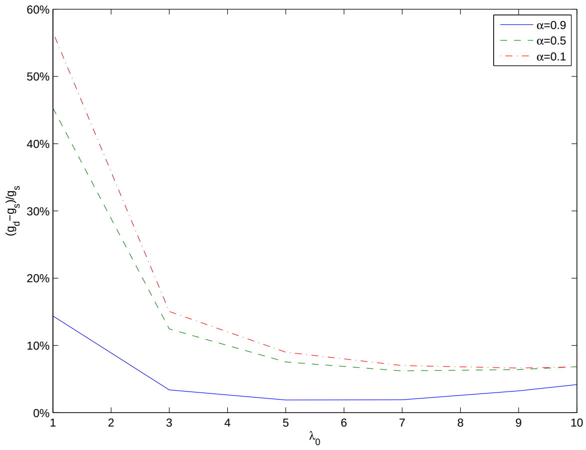

Figure 2: Impact of Lead Time Sensitive Factor

α	0.1	0.2	0.3	0.4	0.5	0.7	0.9	1.0
g_d	5.9303	5.8712	5.8251	5.7876	5.7560	5.7060	5.6684	5.6532
g_s	5.8732	5.6479	5.4775	5.3408	5.2276	5.0515	4.9228	4.8719
$\frac{g_d-g_s}{g_s}$	0.97%	3.95%	6.35%	8.37%	10.11%	12.96%	15.15%	16.04%

Table 7: Impact of Lead Time Sensitive Factor

reward on dynamic model drops by 4.67% while reward rate in static model drops by 17.05%. That is to say, dynamic quotation model performs much better than static model when dealing with the increasing sensitiveness of customers on lead time. Thus, we conclude that as α increases, the advantages of the dynamic model over the static model becomes more significant. This result makes sense because the more sensitive customers on lead time, the more important the role of lead time to balance realized demand rate and firms' operational constraints.

Then we consider the impact of β on performance of dynamic model and static model. Setting $\alpha = 0.6$, $\delta = 2$, we compute average reward of static and dynamic model from $\beta = 0.5$ to $\beta = 1.5$. We denote average reward for dynamic model and static model when $\beta = 0.5$ as g_{d1} and g_{s1}, respectively. Also, average reward for dynamic model and static model when $\beta = 1.5$ are g_{d2} and g_{s2}, respectively.

λ_0	1	3	5	7	9	10
g_{d1}	0.4828	4.2961	11.7847	22.6861	36.4990	44.2709
g_{d2}	0.1578	1.3777	3.7409	7.1829	11.5996	14.1171
g_{s1}	0.3224	3.7852	10.8957	21.2862	34.2697	40.8144
g_{s2}	0.0737	1.1109	3.3063	6.5162	10.5201	12.7431
$\frac{g_{d1}-g_{d2}}{g_{d1}}$	-67.31%	-67.93%	-68.26%	-68.34%	-68.22%	-68.11%
$\frac{g_{s1}-g_{s2}}{g_{s1}}$	-77.14%	-70.65%	-69.66%	-69.39%	-69.30%	-68.78%

Table 8: Impact of Price Sensitive Factor

δ	1	1.2	1.4	1.6	1.8	2.0
$\frac{g_d-g_s}{g_s}$	2.39%	2.62%	2.99%	3.20%	3.32%	3.51%

Table 9: Impact of Delay Penalty

In Table 8, we can see that rewards of both dynamic model and static model are dropping rapidly as price sensitive factor increases. Dynamic model only has a slight advantage over static model when facing the increasing in price sensitive factor. This is a major difference with the case of lead time sensitive factor.

Now we consider impact of delay penalty δ. We set other parameters fixed as follows: $\mu = 5, \alpha = 0.2, \beta = 0.3, \lambda_0 = 5$. Table 9 shows how the difference ratio $\frac{g_d-g_s}{g_s}$ becomes larger as δ increases. In this case, the optimal average rewards under both quotation models do not drop so quickly as in the last case where β increases. And although not much, difference ratio $\frac{g_d-g_s}{g_s}$ becomes larger as δ increases. We also compute dropping ratio of rewards of both dynamic model and static model as δ increases from 1 to 1.5. When $\alpha = 0.1$ and $\lambda_0 = 1$, reward of dynamic model drops by 0.32% while static model drops by 1.89%. Increasing in potential demand rate will decrease the difference. When $\lambda_0 = 10$, reward in dynamic model drops by 0.58% and reward in static model drops by 0.76%. Recall that in our objective functions (both dynamic model's and static model's), delay penalty δ affects the reward only by lead time. Since the effect of the lead time on reward is the least when $\lambda_0 = 10\%$, the fact that decreasing ratios of rewards of static model and dynamic model are the closet at this potential demand rate just makes sense.

We have analyzed impact of α, β and δ. All these three factors' increasing will cause the increase advantage of the dynamic model over the static model and the increase of α brings the most obvious increase in advantage of the dynamic model while the increase of β leads to the least significant increase in advantage of the dynamic model.

In Section 6.1, we find some cases when α is bigger than 1, difference ratio $\frac{g_d - g_s}{g_s}$ does not follow the 'first decreasing then increasing' pattern as potential demand rate increases, it just decreases as λ_0 increases. As we said before, lead time plays a most significant role in the dominant advantage of dynamic model over static model when potential demand rate is 1. And this role becomes less obvious as λ_0 increases. Price however plays a most obvious role when $\lambda_0 = 10$ and its impact decreases as market decreases. Now that we have found increasing in α will cause a much bigger difference in $\frac{g_d - g_s}{g_s}$ than increasing in β does. Then when α is big enough, decreasing impact of lead time will eliminate the increasing impact of price as potential demand rate grows. So for some cases when α is bigger than 1, difference ratio $\frac{g_d - g_s}{g_s}$ just decreases as λ_0 increases.

6.3. Existence of The Threshold Potential Demand Rate

We have seen from above discussion that without information searching cost, dynamic quotation model always has a higher optimal average reward than static quotation model. This result fits our intuition that dynamic quotation model

λ_0	1	2	3	4	5	6	7	8	9	10
g_s	0.69	2.98	6.92	12.44	19.52	28.09	38.05	49.24	61.44	74.41
g_{dc}	0.67	2.95	6.84	12.34	19.41	28.03	38.15	49.68	62.64	76.29

Table 10: Average Reward in Dynamic and Static Models

can provides more flexible lead time and price quotation thus attract more customers and decrease delay penalty. However as we discussed before, nowadays customers' information searching cost becomes more significant. In this suction, we analyze performances of dynamic and static quotations considering customers' information searching cost.

Add an information searching cost $c = 0.1$ to demand function with other parameters' value fixed as time sensitive factor $\alpha = 0.2$, price sensitive factor $\beta = 0.3$, delay penalty per unit time $\delta = 2$ and capacity $\mu = 5$, we begin to investigate the existence of threshold potential demand rate. First we list optimal average rewards of static and dynamic models g_s, g_{dc}in table 10. g_{dc} is less than g_s when λ_0=$1, 2, 3, 4, 5, 6$ and then g_s becomes less than g_{dc} when λ_0=$7, 8, 9, 10$. Both g_s and g_dc are increasing as λ_0 grows and it seems that g_{dc} increases faster than g_s. To be more clearly, we compute ratio $\frac{g_{dc}-g_s}{g_s}$. It is clearly that $\frac{g_{dc}-g_s}{g_s}$ increases as λ_0 grows and it turns from negative to positive somewhere between $\lambda_0 = 6$ and λ_0=7. See in Figure 3, this change pattern differs from that we draw from last case: when potential demand rate is larger or smaller, advantages of dynamic quotation model become more significant. Note that this case only differs with the last one by adding a constant number $c = 0.1$ to the demand function, we keep digging the effect of $c = 0.1$. Consider$\frac{g_d-g_{dc}}{g_{dc}}$. $\frac{g_d-g_{dc}}{g_{dc}}$ becomes smaller when λ_0 increases. When λ_0=1, $\frac{g_d-g_{dc}}{g_{dc}}$=23.41% and when$\lambda_0$=10, $\frac{g_d-g_{dc}}{g_{dc}}$ drops to 1.88%. So it is clearly that effect of information searching cost c on

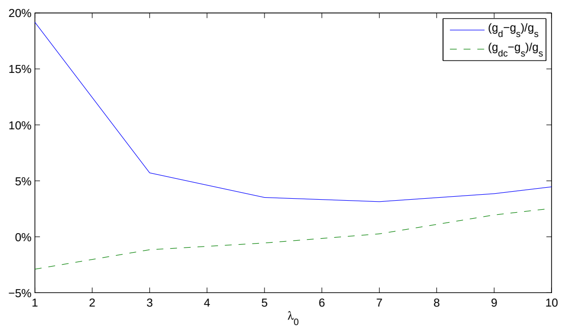

Figure 3: $\frac{g_{dc}-g_s}{g_s}$ and $\frac{g_d-g_s}{g_s}$

average reward becomes smaller when λ increases. This also fit our discussion in Section 6.1. we can see information cost as a direct decrease in potential demand rate. When market is small, queue length is relatively short and any decrease in market will cause more severe decreasing in revenue. However when market is relatively bigger, queue length becomes longer and company tend to use price policy to adjust workload. In this case, some small decrease in market will not cause lost of revenue as severe as when market is small. Thus, we explain the existence of threshold potential demand rate. When potential demand rate is smaller than this threshold, static quotation model can provide bigger reward and when potential demand rate is bigger than this threshold, dynamic quotation model is more profitable.

6.4. Equalizing Information Searching Cost

Although we have found the existence of the threshold potential demand rate, real market cannot be infinity. It usually has a certain upper bound. If the information cost is too small, for example $c = 0.0000001$, its decreasing effect on reward might be ignored. In this case, the threshold potential demand rate will

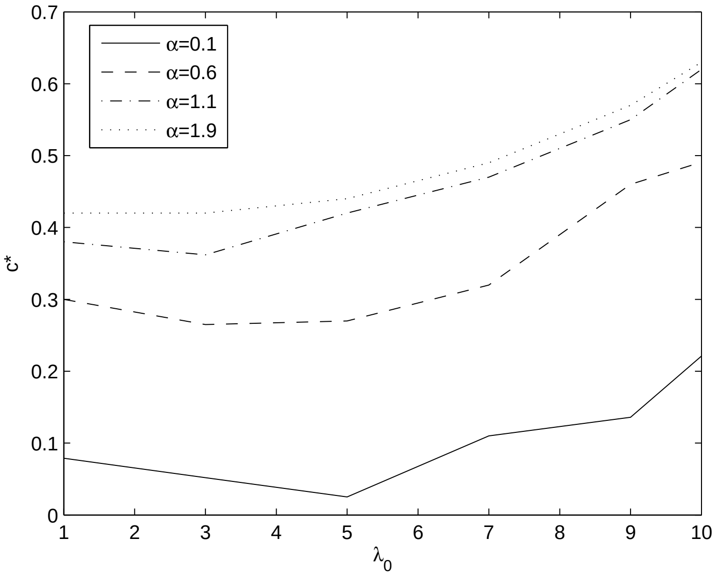

Figure 4: Change Pattern of c*

be very close to zero. Also, if information searching cost is too big, the threshold potential demand rate will be above the upper bound. To give manager more practical guidance in choosing between dynamic lead time and price quotation or static lead time and price quotation, we compare static and dynamic model under different information searching cost c. More specifically, we consider the value of information cost that average reward of dynamic model and static model are equal (marked as c^*). Thus, certain company with fixed capacity in certain market can decide its quotation model based on customer's information searching cost. If the current information cost is less than c^*, dynamic quotation will be better than static model, otherwise, static model is better on the point of reward maximization. In Figure 4 we display the change pattern of $c*$. Since information searching cost causes a direct decreasing in demand, c^* should have a similar change pattern as the gap of static model and dynamic model (without information searching cost). Plus, as we discussed before, the effect of information searching cost c on average reward is the most significant when $\lambda_0 = 1$ and will decrease as λ_0 increases. Hence, though gap between static model and dynamic model are the most, $c*$ in $\lambda_0 = 1$ is less than $c*$ in $\lambda_0 = 10$.

7. Dynamic Lead Time Quotation

Except for joint dynamic lead time and price quotation, in reality there are still lots of firms who quote only lead time dynamically over time. We define the case with optimal price as DLQ_1 and the case with price from static quotation as DLQ_2. In this chapter, we consider the performances of these two 'partial' dynamic quotations. The result comes out immediately that performance of these two 'partial' dynamic quotation should be between static quotation and jointly dynamic lead time and price quotation by intuition (if information searching cost equals zero). We further study whether performances of DLQ_1 and DLQ_2 closer to the performance of static quotation or closer to the performance of joint dynamic lead time and price quotation.

We build a same CTMDP as in Chapter 5 except that price here is fixed. And we also discretize time and turn the CTMDP into a MDP following Puterman (1994).

Optimal equations are the same with that in Chapter 5:

$$h_l(s) = \max_{a \in A}\{\widetilde{r_l}(s,a) - \widetilde{g_l} + \sum_{j \in S} \widetilde{q_l}(j|s,a)h_l(j)\},$$

for all $s \in S$, where $\widetilde{g_l}$ denotes the optimal average reward and $h_l(s)$ is the unknown value function vector.

To solve optimal policy and optimal reward for DLQ_1, we have two decisions to make, lead time and price. Theorem 1 also fits here. Then we use dual linear programming to solve optimal price and average reward g_{l1}. With optimal lead time is drawn from Theorem 1, we can simply solve the dual linear programming

α	β	δ	λ_0	g_s	g_d	g_{l1}	$\frac{g_d - g_{l1}}{g_d}$
0.3	1	2	1	0.1644	0.2411	0.24	0.46%
0.3	1	2	3	1.9062	2.1275	2.1185	0.42%
0.3	1	2	5	5.4775	5.8252	5.8072	0.31%
0.3	1	2	7	10.6975	11.2741	11.2512	0.20%
0.3	1	2	9	17.2225	18.3291	18.3039	0.14%
0.3	1	2	10	20.8364	22.3545	22.3168	0.17%
0.6	1	2	1	0.1280	0.2382	0.2372	0.42%
0.6	1	2	3	1.7469	2.0987	2.0890	0.46%
0.6	1	2	5	5.1324	5.7291	5.7069	0.26%
0.6	1	2	7	10.0771	11.0179	10.9849	0.30%
0.6	1	2	9	16.2443	17.7678	17.7225	0.25%
0.6	1	2	10	19.6648	21.5937	21.5341	0.25%
1	1	2	1	0.1020	0.2360	0.2350	0.42%
1	1	2	3	1.6254	2.0761	2.0653	0.52%
1	1	2	5	4.8719	5.6532	5.6265	0.47%
1	1	2	7	9.6224	10.8216	10.7762	0.42%
1	1	2	9	15.5500	17.3567	17.2848	0.41%
1	1	2	10	18.8430	21.0445	20.9507	0.45%
1.6	1	2	1	0.0855	0.2347	0.2323	1.02%
1.6	1	2	3	1.5433	2.0603	2.0456	0.71%
1.6	1	2	5	4.6967	5.5969	5.5605	0.65%
1.6	1	2	7	9.3214	10.6726	10.6021	0.66%
1.6	1	2	9	15.0978	17.0447	16.9172	0.75%
1.6	1	2	10	18.3111	20.6275	20.4584	0.82%
1.9	1	2	1	0.0831	0.2345	0.2306	1.67%
1.9	1	2	3	1.5310	2.0577	2.0379	0.96%
1.9	1	2	5	4.6706	5.5857	5.5351	0.91%
1.9	1	2	7	9.2767	10.6392	10.5312	1.01%
1.9	1	2	9	15.0311	16.9705	16.7622	1.23%
1.9	1	2	10	18.2328	20.5265	20.2480	1.35%

Table 11: Performance of DLQ_1(a)

as a single variable problem. We list some of our results in the following Table 11 and Table 12.

To solve optimal policy and reward for DLQ_1, we only need to decide the lead time. Price is fixed the same as in the static quotation mode. We simply use relative value iteration to solve it and list results in Table 13.

It is clearly that reward from the DLQ_1 is very close to the reward from joint

α	β	δ	λ_0	g_s	g_d	g_{l1}	$\frac{g_d - g_{l1}}{g_d}$
0.5	1	1	1	0.1664	0.2418	0.2415	0.12%
0.5	1	1	3	1.9145	2.1526	2.1494	0.32%
0.5	1	1	5	5.4956	5.91	5.9021	0.13%
0.5	1	1	7	10.7307	11.3975	11.3819	0.14%
0.5	1	1	9	17.2762	18.3843	18.3440	0.22%
0.5	1	1	10	20.9013	22.3280	22.2583	0.31%
0.5	1	1.5	1	0.1493	0.2401	0.2395	0.25%
0.5	1	1.5	3	1.8414	2.1269	2.1210	0.28%
0.5	1	1.5	5	5.3365	5.8255	5.8119	0.23%
0.5	1	1.5	7	10.4410	11.2283	11.2073	0.19%
0.5	1	1.5	9	16.8127	18.1274	18.0914	0.20%
0.5	1	1.5	10	20.3431	22.0330	21.9769	0.25%
0.6	0.5	2	1	0.3224	0.4828	0.4819	0.19%
0.6	0.5	2	3	3.7852	4.2961	4.2891	0.16%
0.6	0.5	2	5	10.8957	11.7847	11.7656	0.16%
0.6	0.5	2	7	21.2862	22.6861	22.6257	0.27%
0.6	0.5	2	9	34.2697	36.4990	36.2747	0.61%
0.6	0.5	2	10	40.8144	44.2709	43.8923	0.86%
0.6	1.5	2	1	0.0737	0.1578	0.1565	0.82%
0.6	1.5	2	3	1.1109	1.3777	1.3650	0.92%
0.6	1.5	2	5	3.3063	3.7409	3.7132	0.74%
0.6	1.5	2	7	6.5162	7.1829	7.1442	0.53%
0.6	1.5	2	9	10.5201	11.5996	11.5573	0.36%
0.6	1.5	2	10	12.7431	14.1171	14.0723	0.32%

Table 12: Performance of DLQ_1(b)

α	β	δ	λ_0	g_s	g_{l2}	g_d	$\frac{g_d - g_{l2}}{g_d}$
0.6	1	2	1	0.1280	0.1665	0.2382	30.10%
0.6	1	2	3	1.7479	1.9368	2.0987	7.71%
0.6	1	2	5	5.1324	5.5738	5.7291	2.71%
0.6	1	2	7	10.0771	10.9150	11.0179	0.93%
0.6	1	2	9	16.2443	17.6555	17.7678	0.63%
0.6	1	2	10	19.0164	21.4242	21.5937	0.78%
1	1	2	1	0.1020	0.1442	0.2360	38.90%
1	1	2	3	1.6254	1.8460	2.0761	11.08%
1	1	2	5	4.8719	5.3734	5.6532	4.95%
1	1	2	7	9.6224	10.5314	10.8216	2.68%
1	1	2	9	15.5500	17.0039	17.3567	2.03%
1	1	2	10	18.8430	20.6156	21.0445	2.03%
1.6	1	2	1	0.0855	0.1192	0.2347	49.21%
1.6	1	2	3	1.5433	1.7528	2.0603	14.93%
1.6	1	2	5	4.6967	5.1763	5.5969	7.51%
1.6	1	2	7	9.3214	10.1589	10.6726	4.81%
1.6	1	2	9	15.0978	16.3681	17.0447	3.88%
1.6	1	2	10	18.3111	19.8167	20.6275	3.93%
1.9	1	2	1	0.0831	0.1062	0.2345	54.71%
1.9	1	2	3	1.5310	1.7124	2.0577	16.78%
1.9	1	2	5	4.6706	5.0973	5.5857	8.74%
1.9	1	2	7	9.2767	10.0135	10.6392	5.88%
1.9	1	2	9	15.0311	16.1151	16.9705	5.04%
1.9	1	2	10	18.2328	19.4954	20.5265	5.02%

Table 13: Performance of DLQ_2

dynamic lead time and price quotation. In fact, when we consider the difference ratio between g_d and g_{l1}, that is $\frac{g_d - g_{l1}}{g_d}$, they stays below 1.67% for all different set of factors. That is to say, reward from DLQ_1 and joint dynamic lead time and price quotation are really close (difference ratio is not higher than 1.67% in our cases), firm may be satisfied with dynamic lead time quotation when they can decide the fixed lead time as the optimal one.

However, when we look at performance of the dynamic quotation when price is fixed the same as in the static quotation, difference ratio $\frac{g_d - g_{l2}}{g_d}$ is much bigger. It's performance is closer to static quotation. An interesting finding is that when α increasing, the difference ratio increasing. How does this happen? Recall that in this DLQ_2 quotation mode, price is the same as in the static quotation mode and stay the same during the whole period. So even we can quote lead time dynamically over time, it's range is strictly limited. See in our demand function, $\lambda(l, p) = \lambda_0 - \alpha l - \beta p - c$. With a given potential demand rate λ_0 and a constraint that realized demand rate should be above zero, lead time can not change out of some certain range. When α is increasing, range of lead time is shorten and thus it's control ability decreasing. Hence we can see that when α is increasing, the difference ratio $\frac{g_d - g_{l2}}{g_d}$ is increasing. Also, when potential demand rate is increasing, impact of lead time is decreasing as we discussed in Chapter 5. Thus here in the situation where lead time's control ability is shorten, difference ratio between joint dynamic lead time and price quotation and DLQ_2 is decreasing as potential demand rate increasing. So we concluded that when lead time sensitive factor is small and potential demand rate is high, performance of DLQ_2 will be closer to that of the joint dynamic quotation mode. Firms do not need to struggle to decide a new optimal price.

8. Conclusions

This thesis extends the existing literature about lead time and/or price quotation in three ways. Firstly, it further considers how does dynamic quotation outperforms static quotation. Secondly, it highlights the impact of information searching cost thus point out that static quotation can perform better than static quotation considering impact of information searching cost on demand. Lastly, it compares performance of two kinds of dynamic lead time quotation and joint dynamic lead time and price quotation thus provides guidance for managers to choose their quotation modes.

We formulate policy and reward algorithm in an average time criterion for static lead time and price quotation model, dynamic joint lead time and price quotation model, dynamic lead time model. Four corresponding MATLAB programs have been developed to compute optimal reward and optimal policy with specific factor values.

With numerical results, we first compare performance of static model and joint dynamic lead time and price quotation models. The dominant advantage of dynamic models (without considering information searching cost) over static models are quite obvious, which fits our intuition. Further considering relationship between policy and advantages, we explain how does dynamic quotation model leading to better capacity utilization: when market is small, firms who use dynamic quotation can quote relatively small lead time to attract more customers without suffering from server delay penalty; when market is big, firms more rely on price quotation to adjust queue length, or workload. Analysis on price and lead time sensitiveness showed that when customers are more sensitive to lead time, dynamic quotation's advantage over static quotation is increasing significantly while when customers are more sensitive to price, this advantage is not increasing so obviously.

Then we consider performance of dynamic quotation and static quotation with impact of information searching cost. Since we find that impact on dynamic model's advantage of this additional cost from customers' aspect is decreasing as market is increasing, so the threshold potential demand rate does exist. When potential demand rate is smaller than this threshold rate, static quotation leads to a better reward while when potential demand rate is bigger than this threshold rate, dynamic quotation provides a better reward. Consider the fact that real market can not be infinity and this threshold can be very big or small regrading to different market properties, we consider the value of information searching cost that equals average reward of static and dynamic models. We find that this value has a similar change pattern with the gap of dynamic model (without information searching cost) and static model. Managers can compare real information searching cost with this value to evaluate threshold rate thus choose quotation mode from dynamic and static quotation.

In Chapter 7, we consider performance of two dynamic lead time quotation modes. In reality lots of firms can not change their price dynamically over time, so we consider the case where only lead time are quoted dynamically. After numerical study, we find that when firm can decide a new optimal price, difference between dynamic lead time quotation and joint dynamic lead time and price quotation is very small. Firms may be satisfied with the performance of the dynamic lead time quotation. However, when firms can not change the original price in the market, performance of the dynamic quotation mode will not be so good. We find the performance of this kind of dynamic lead time quotation will be much close to the performance of the joint dynamic quotation mode when lead time sensitive factor is small and the potential demand rate is high.

Limitations of our work is that we modeled our problem in a $M/M/1$ queueing system. This helps us trace the queueing system conveniently however lose generality in a certain degree. Further study can work on a more general model to compare performance of static and dynamic quotation model.

As the first to consider the fact that firms may yield a better reward using static lead time and price quotation, we find our working meaningful in providing management guidance for firms to decide lead time and price quotation mode. Performance comparison of dynamic lead time and price quotation with static quotation not only helps understand better capacity utilization of dynamic model, but also helps decide the threshold potential demand rate when impact of information searching cost can not be ignored. When potential demand rate is lower than the threshold demand rate, firms should choose static quotation and when potential demand rate is higher than the threshold demand rate, firms should choose dynamic lead time and price quotation. For firms who can not change price dynamically, we also point out that only quote lead time dynamically over time is just enough when they can decide their price. Or when they can not decide their price, when customers are not very sensitive to lead time and the market is big, the dynamic lead time quotation would also provide a similar reward with the joint dynamic lead time quotation.

9. Reference

[1] Ata, B., Olsen, T. L., 2009. Near-optimal dynamic lead-time quotation and scheduling under convex-concave customer delay costs. Operations Research 57 (3), 753–768.

[2] Boyaci, T., Ray, S., 2003. Product differentiation and capacity cost interaction in time and price sensitive markets. Manufacturing & Service Operations Management 5 (1), 18–36.

[3] Chatterjee, S., Slotnick, S. A., Sobel, M. J., 2002. Delivery guarantees and the interdependence of marketing and operations. Production and Operations Management 11 (3), 393–410.

[4] Duenyas, I., 1992. Due date setting and sequencing in a time competitive environment. Ann Arbor 1001, 48109.

[5] Duenyas, I., Hopp, W. J., 1995. Quoting customer lead times. Management Science 41 (1), 43–57.

[6] Elwany, M. H., Baddan, E. A., 1998. The sensitivity of jobshop due date lead time to changes in the processing time. Computers & Industrial Engineering 35 (3), 411–414.

[7] Erkoc, M., Wu, S., 2000. Competitive due-date coordination between marketing and manufacturing. In: Manufacturing and Service Operations Management Conference Proceedings, Ann Arbor, MI, June. pp. 29–30.

[8] Feng, J., Liu, L., Liu, X., 2011. An optimal policy for joint dynamic price and lead-time quotation. Operation Research 59, 1523–1527.

[9] Gupta, S. K., Kyparisis, J., 1987. Single machine scheduling research. Omega 15 (3), 207–227.

[10] Handfield, R., Pannesi, R., 1995. Antecedents of leadtime competitiveness in make-to-order manufacturing firms. The International Journal of Production Research 33 (2), 511–537.

[11] Hopp, W. J., Spearman, M. L., Woodruff, D. L., 1990. Practical strategies for lead time reduction. Manufacturing Review 3 (2), 78–84.

[12] Keskinocak, P., Ravi, R., Tayur, S., 2001. Scheduling and reliable lead-time quotation for orders with availability intervals and lead-time sensitive revenues. Management Science 47 (2), 264–279.

[13] Lawrence, S. R., 1995. Estimating flowtimes and setting due-dates in complex production systems. IIE transactions 27 (5), 657–668.

[14] Liu, L., Parlar, M., Zhu, S. X., 2007. Pricing and lead time decisions in decentralized supply chains. Management Science 53 (5), 713–725.

[15] Maglaras, C., Zeevi, A., 2005. Pricing and design of differentiated services: Approximate analysis and structural insights. Operations Research 53 (2), 242–262.

[16] Matsuura, H., Tsubone, H., 1993. Setting planned leadtimes in capacity requirements planning. Journal of the Operational Research Society, 809–816.

[17] Matsuura, H., Tsubone, H., Kanezashi, M., 1996. Setting planned lead times for multi-operation jobs. European Journal of Operational Research 88 (2), 287–303.

[18] Mehta, N., Rajiv, S., Srinivasan, K., 2003. Price uncertainty and consumer search: A structural model of consideration set formation. Marketing Science 22 (1), 58–84.

[19] Mendelson, H., Whang, S., 1990. Optimal incentive-compatible priority pricing for the $M/M/1$ queue. Operations Research 38 (5), 870–883.

[20] Palaka, K., Erlebacher, S., Kropp, D. H., 1998. Lead-time setting, capacity utilization, and pricing decisions under lead-time dependent demand. IIE Transactions 30 (2), 151–163.

[21] Pekgn, P., Griffin, P. M., Keskinocak, P., 2008. Coordination of marketing and production for price and leadtime decisions. IIE transactions 40 (1), 12–30.

[22] Punj, G. N., Staelin, R., 1983. A model of consumer information search behavior for new automobiles. Journal of Consumer Research, 366–380.

[23] Puterman M L. Markov decision processes: Discrete dynamic stochastic programming[J]. New York, NY: John Wiley. doi, 1994, 10: 9780470316887.

[24] Rao, U. S., Swaminathan, J. M., Zhang, J., 2005. Demand and production management with uniform guaranteed lead time. Production and Operations Management 14 (4), 400–412.

[25] Ray, S., Jewkes, E. M., 2004. Customer lead time management when both demand and price are lead time sensitive. European Journal of Operational Research 153 (3), 769–781.

[26] Sen, T., Gupta, S. K., 1984. A state-of-art survey of static scheduling research involving due dates. Omega 12 (1), 63–76.

[27] So, K. C., 2000. Price and time competition for service delivery. Manufacturing & Service Operations Management 2 (4), 392–409.

[28] So, K. C., Song, J.-S., 1998. Price, delivery time guarantees and capacity selection. European Journal of Operational Research 111 (1), 28–49.

[29] Stalk, G., 1988. Time–the next source of competitive advantage. Harvard business review.

[30] Suri, R., 1998. Quick response manufacturing: A company-wide approach to lead time reduction. Productivity Press.

[31] van Dongen, B. F., Crooy, R., van der Aalst, W. M., 2008. Cycle time prediction: when will this case finally be finished? In: On the Move to Meaningful Internet Systems: OTM 2008. Springer, pp. 319–336.

[32] Webster, S., 2002. Dynamic pricing and lead-time policies for make-to-order systems. Decision Sciences 33 (4), 579–600.

[33] Weng, Z. K., 1996. Manufacturing lead times, system utilization rates and lead-time-related demand. European Journal of Operational Research 89 (2), 259–268.

[34] Wu, Z., Kazaz, B., Webster, S., Yang, K.-K., 2012. Ordering, pricing, and lead-time quotation under lead-time and demand uncertainty. Production and Operations Management 21 (3), 576–589.

[35] Zhao, X., Stecke, K. E., Prasad, A., 2012. Lead time and price quotation mode selection: Uniform or differentiated? Production and Operations Management 21 (1), 177–193.