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Competitive market research and product design

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COMPETITIVE MARKET RESEARCH AND PRODUCT DESIGN

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COMPETITIVE MARKET RESEARCH AND PRODUCT DESIGN

by
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ABSTRACT

Competitive Market Research and Product Design

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To learn the uncertainty of customer preference on the attribute of new product, usually a firm needs to do market research. Developing a product on an attribute which is less preferred by customer may lead to a failure. In addition, a firm used to take efforts to design the product. In recent years, we observed a new business model in which the firm does not take effort to design new product, nor does she do market research by herself. She provides rewards to attract outside designers to design new product. Some designers may take effort and design products based on their private information of customer preference. The firm receives designs with different quality and attribute, she chooses one to produce. By solving this game model, we get the equilibrium quality of the design offered by each designer based on their private cost parameter. And we obtain the following insights: When the market size is too small, the firm gives nothing to designers; when the market size is sufficiently big, the firm only gives reward to the designer whose design is produced; otherwise the firm gives both rewards to participated designers and the designer whose design is produced. We find that when the market size is big enough or the disutility is high enough, the new business model dominates the benchmark business model. When both the disutility and market size are small enough, the firm prefers the benchmark business model. And the relative attractiveness of new business model versus benchmark model keeps the same when the market size is small enough. The impact of extra reward on relative attractiveness of new business model versus benchmark model increases with extra reward. When the extra reward is high enough, the firm always prefers the new business model.

Key words: Customer preference; market research; information acquisition; product design; quality.

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

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CERTIFICATE OF APPROVAL OF THESIS

COMPETITIVE MARKET RESEARCH AND PRODUCT DESIGN

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1 Introduction

Recent studies find that failure rate is very high for new products development. Newly launched products have shown notoriously high failure rates over the years, often reaching fifty percent or more (Piller 2010). Most of these failures can be attributed to mismatches between customer preferences and the available products. Compared to technical problems, the poor business prospect leads to unsuccessful new product development projects. In many industries, customer tastes are inherently uncertain. Many firms were faced with hard decisions when developing new products. In the mid-1980s, firms in the disk industry were not sure whether to develop smaller drives or to increase memory capacity. Seagate and Conner Peripherals were faced with this dilemma and contemplating where to dedicate R&D effort (Christensen 1997). In the mid-1990s, a problem emerged in the market of laptops on whether to reduce weight or to improve the performance of the computer. These two attributes cannot be satisfied simultaneously, so Compaq and Toshiba were not sure whether to pursue sense of beauty by allowing only light and interchangeable peripherals or to offer greater convenience by attempting to internally integrate all the peripherals including high-performance components (Bell and Leamon 1999). In the early 2000s, firms in the market of mobile phone were not sure whether to design candy bar or clamshell styles (The Economist 2004). Nokia and Motorola did not know customers' tastes and had to make a choice between these two attributes. The industry of pharmaceuticals also faced the problem of different development directions. For example, Eli Lilly, Pfizer, and Forest were not sure whether to develop antidepressants which can treat acute depression or co-treat physical pain symptoms associated with mild depression. Customer preference is always a major factor considered by firms when they develop new products.

The mismatch with customer preference may have serious consequences. A challenge for firms in selling is how to match supply with uncertain customer preference. This leads to a growing demand for uncertainty-resolving information acquisition. Prior research has found that timely and reliable information on customer preferences and requirements played an important role in successful product development. Conventionally, heavy investments in market research are considered as the main measure to access this information. Manufacturers in fashion and seasonal goods industries (e.g., L. L. Bean and Timberland) have invested in sophisticated information acquisition (Fisher et al. 1994). In the examples mentioned earlier, Compaq conducts market research before development. In Compaq's market research, customers prefer the convenience of all add-ons integrated internally and better performance instead of the concept of interchangeability to reduce laptop weight. Presario notebook line was successfully designed using this information, which garnered dominant market share shortly after being introduced in 1996. Among cell phone firms, Nokia introduced new candy bar handsets without conducting extensive upfront market research. It bet on developing the wrong handset style and lost a significant share in 2004 to Motorola, which produced clamshell phones. In the antidepressant case, Lilly did market research and it revealed that roughly 25% of mildly depressed patients suffering from physical pain. So it pursued a food and drug administration indication for pain in addition to an indication for depression treatment (Ofek 2006). Before making any decisions, P&G invests billions of funds for market research, which makes P&G's customer trends, psychology and preferences very clearly. For example, customers hope clothes can become more translucent after using washing powder. Investigation departments should convert towards "transparency" through certain technical indicators, like whether to add bleach, enzyme and so on. The huge customer data helps P&G make decisions. As we see, changes in customer preference are faster than ever because of the information explosion nowadays. Market research is very useful to predict customer preference which helps firms to design new products preferred by the target market.

After obtaining information about customer preference, firms need to work on new product design. The traditional way of product design is thorough the firm's research and development (R&D) department. And that incurs a R&D cost. Many companies design products by themselves. For instance, P&G designs many products by itself. But some other companies use outsourcing to design products.

In recent years we observed a new business model, which can be illustrated by the practice of Threadless. Threadless is an online community of artists and an E-commerce website. Products of Threadless are created by an online community with 2 million artists. Each week, about 1,000 different designs of clothing and other products are submitted online. These designs may have different attributes, for clothing, some designs are of slim style but some are casual style; some designs are colorful while some are colorless; some designs may follow the latest fashion trend while some other designs may stick to classic style. The staff review and select the designs each week, print them on clothing and other products, and sell them worldwide through the online store and their only retail store in Chicago. Designers whose design is printed receive a reward from Threadless. Under this business model, the company's founders sell about 160-170 thousand T-shirts per month for between \$18 and \$24 a piece with a 30% profit margin on sales. Since 2006, annual growth continued at more than 150%, with similar margins. Sales in 2009 hit almost \$30 million with profits of roughly \$9 million. Threadless has 2 million followers on Twitter and more than 500,000 fans on Facebook. The company's website has 2.5 million logged visitors in August 2010, a 50% increase over the same month last year. Threadless follows an innovative business model that allows it to create products without risk and make heavy investments in market research to acquire customer preferences before production starts.

This business model has one key difference from traditional firms. The firm does not do market research or take efforts to design the product by itself. Outside designers design new product for it. It just chooses one design and puts it into production. At the same time, it acquires the information of customer preference through designs from participating designers. Each design contains the private information of customer preference. These designers have a better knowledge of customer preference. They are professional on the industry and always follow the latest fashion information. Their rich experience could be used in the design. They are loyal fans of the firm. They have experienced the process many times and know the target customers' preference. They are closer to customers and have a good network of potential customers. So the designers understand customer's preference better than the firm.

Some other corporations are utilizing this business model. Jovoto is a company that acts as a intermediary and executes hundreds of innovation contests every year for its clients. It connects those who have ideas with those who need them, provides the necessary tools to make the process entertaining and fair for the creative community and delivers high quality ideas and design to organizations. It enables large brands to brainstorm at scale and to solve product design and innovation challenges with more than 50,000 creative professionals globally.

There are several issues related to this new business model which is worth studying. When should the firm choose this kind of new business model? The traditional business model involves market research, focus groups, testing, reworking, and retesting. But the new business model has become more popular these years. Firms obtain several benefits from this new business model: the competition induced among designers; risk of failures is shifted to designers; potential wage-rate arbitrage or cost savings. These potential benefits attract more and more firms to try this new business model. Greenpeace, Coca-Cola, Unilever, Victorinox, Henkel, Nestle, Unicef and many others have become frequent clients of Jovoto. The global Head of Sales & Marketing SAK at Victorinox AG said: "The 2012 Limited Edition version of the small Swiss pocket knife is the most successful and bestselling collection to date". These global leaders decide to solve their challenges with the talent base of Jovoto, which can give track record consisting of accelerated innovation, sales growth or generated media buzz. Some high-tech industries are not suitable for this business model because of high professional skills. And sometimes companies are not satisfied with the designs submitted but they cannot control the designers. So we plan to investigate when the firm should choose this kind of new business model.

For a given type of innovation problem, what kind of reward structure should the firm provide? Specifically, should the firm provide rewards to every designer or only the designer whose design is picked? The incentive of designers' participation is directly affected by rewards. Four primary motivations for designers' participation are: the opportunity to make money, the opportunity to develop one's creative skills, the potential to take up freelance work, and the love of community (Daren 2010). "Jovoto offers me the freedom to choose if I want to invest time in a contest and how much time

I want to invest. Jovoto also provides the opportunity to work with a great community and to gather valuable contacts and experiences as well," Creative Mind DaHu said. However, the reward structure could differ under different scenarios. Prior research has advocated the use of fixed reward, or a performance contingent reward can lead to better solutions, higher seeker-profits and system efficiency. The reward structure of Betahaus (which cooperated with InnoCentive for co-working environment in 2013) is: reward some designers. While a firm only gives reward to the designer whose design is picked by them in many innovation contests in Zhubajie (a famous Witkey website in China like Innocentive). The reward structure is an interesting issue to study.

Each designer determines the quality independently without knowing others' decision. We want to investigate how designers determine the quality of the design. Do they decide the quality according to their own conditions such as ability and time? Or they consider the ability of other competitors? Sometimes having many designers work on an innovation problem simultaneously may lead to a lower equilibrium effort for each designer, which is undesirable from the perspective of the firm. While an additional benefit of having a large pool of designers is: the firm can benefit from a larger designer population because it obtains a more diverse set of solutions, which may mitigate and sometimes outweigh the effect of underinvestment from each designer. P&G received many ideas each year from the Connect+Develop platform. Despite the fact that it uses some designs from them, many ideas were useless because the poor quality of some designers' submission. But Victorinox gets the bestselling design with a high quality from the innovation contest. Designers' quality decision affects the result significantly.

Under this new business model, the firm receives designs with different qualities and information about customer preference. But how should the firm pick the design? Some firms focus on the quality of the products, such as UNIQLO. The core competency of UNIQLO compared to other brands with a comparably price is the good quality. While some other firms such as FOREVER21 focus on customer preference of the products. The clothes diversification aims at attracting more young people with different tastes. Betahaus has chosen the design with the highest quality in the example mentioned before. While in some other examples, the firm always chooses the design with highest voting or supporting from people, i.e. the firm chooses the one that matches customer preference most closely. But which is more important: quality or customer preference? It is interesting to investigate the way the firm chooses the design.

To answer these questions, we set up the game model and analyze the decisions of customer, designers and the firm. We also build the benchmark model to study the traditional business model. By solving and comparing these game models, we will answer the above questions. Under the new business model, designers do market research and design the product for the firm. We start from the simple case where only two designers compete with each other. After receiving a private signal of customer preference, each of them independently decides whether to participate. And they decide on the attribute and quality if they plan to submit a design. Designers participate when their profit is greater than their reservation utility. They compete with each other to get the reward. Higher quality increases the probability to be chosen by the firm but leads to a higher cost. We get the Pareto-Optimal equilibrium quality. Each participating designer determines a quality based on their private cost parameter. And the quality decreases with the cost parameter of each designer. Designers do not participate if the cost parameter is high enough. We find that the firm can charge a higher price and sell more products with higher quality. So it always chooses the design with higher quality. When the disutility caused by mismatches with customer preference increases, the firm charges less and sells fewer products. When the market size is small enough, the firm gives nothing to designers; when the market size is sufficiently high, the firm only gives reward to the designer whose design is produced; otherwise the firm gives rewards to both participated designers and the designer whose design is produced. When the benchmark quality is sufficiently high, the reward given to the designer whose design is produced decreases with the benchmark quality, fewer designers participate but the overall quality increases. While the benchmark quality is sufficiently low, the reward given to the designer whose design is produced increases with benchmark quality, and more designers participate but the overall quality decreases. However the profit of the firm always decreases with the benchmark quality.

Then we consider the general case in which there are more than two designers compete with each other. Compared with the scenario with two designers, the competition among designers is fiercer. There are more designs with different qualities submitted. The probability that the firm gets a higher quality increased. We find that the firm receives more signals about customer preference. She chooses the design with the attribute that matches customer preference with a higher probability. The expected disutility decreases and the demand becomes greater. Then the company can charge a higher price and get a higher profit at this case. And we find that the accuracy of the signals have positive effects on the profit of the firm, too.

Under the traditional business model, the firm does market research and designs the product by herself. We find that the accuracy of the information and the profit of the firm increases with the market size, and the profit increases faster when the market size is greater. When the disutility of mismatches with customer preference becomes greater, the firm buys information with a higher accuracy but gets a lower profit. By comparing these two models, we find that when the firm has a high R&D cost, the firm benefits from the combination of information and outside design. That means the firm has a lower profit if it gets the market information by itself while designing the product by outsourcing. It can have a higher profit by this new business model in which outside designers design the product and avoid doing extra market research. When the market size is large enough or the disutility is high enough, the new business model dominates the benchmark business model. When both the disutility and market size are small enough, the firm prefers the benchmark business model. And the relative attractiveness of new business model versus benchmark model keeps the same when the market size is small enough. The impact of extra reward on the relative attractiveness of the new business model versus benchmark model increases with extra reward. When the extra reward is high enough, the firm always prefers the new business model.

This thesis is organized as follows: Section 2 presents the literature review. In section 3, we present a game model to study how designers do market research and design product for the firm. In section 4, we consider the simple scenario where there are two designers. In section 5, we consider a more general case where there are more than two designers. In section 6, we analyze the effects of major operational parameters. In section 7, we study the traditional way of market research and product design. We build the model and solve the game. In section 8, we compare the two models and get some insights. We summarize the research findings in section 9.

2 Literature review

This thesis is relevant to four streams of literature. There is a literature on market research. Usually firms need to acquire the market information of customer preference before R&D. With vertical preference, firms are priori uncertain about which attribute all customers will value more. With horizontal preference, there are two distinct segments and each values innovation on only one of the attributes. Lauga and Ofek (2009) model a duopoly in which ex ante identical firms must decide where to direct their innovation efforts. They find that the value of market information to a firm depends on whether the rival is expected to obtain this information in equilibrium. To learn customer preference, there is a process of information acquisition. Some literature about information acquisition is also relevant to this thesis too. Guo and Iyer (2010) examine the interaction between a manufacturer's optimal information acquisition and sharing strategies in a vertical relationship, capturing the impacts of both the flexibility on sequentially control information collection and the flexibility in ex post voluntary sharing. Guo (2009) also investigates the effects of downstream information acquisition in a strategic channel setting. Some other works about information acquisition are based on committees such as the trial group. Gerling et al. (2005) study information acquisition and decision making in committees. Lauga and Ofek (2009) focus on whether to do market research based on competition of rivals or not. We focus on the way of market research of a firm. Many papers related with market research study the uncertainty of market demand, while we study the uncertainty of customer preference. We consider vertical preference instead of horizontal preference. As to the information acquisition, some papers consider sequential information acquisition. Our thesis considers simultaneous information acquisition. And there is no retailer who may provide information about customer preference in our model.

Other research on new product development is also relevant to our thesis. Some papers focus on pricing decision. Xiao and Xu (2012) study the impact of royalty revision on incentives and profits in a two-stage alliance with a marketer and an innovator. Crama et al. (2008) study how innovators can optimally design licensing contracts when there is incomplete information on the licensee's valuation of the innovation, and limited control over the licensee's development efforts. Some papers focus on cost reduction during the product development. Levin and Reiss (1988) analyze R&D policies when the returns to cost reduction are imperfectly appropriable and market structure is endogenous. Chan et al. (2007) develop a model using dynamic-programming techniques which explains why firms vary in their R&D project-management policies. Firms take efforts on managing and balancing their R&D pipelines. Papers related to new product development (e.g. Levin and Reiss (1988)) focus on pricing and cost reduction. Xiao and Xu (2012) study the interaction between the firm and retailer. But our thesis focuses on new product development by outside designers. The firm does not take efforts to design new product. Outside designers compete with each other and the firm gives rewards to them. Then the firm chooses the best one to produce.

Another stream of literature is about outsourcing and crowdsourcing. Hippel et al. (1999) investigate the company 3M which innovates with many customers to develop new products used in surgery. Piller (2006) finds some companies integrate customers into the innovation process, for example, by soliciting new product concepts from them and pursuing the most popular of those ideas. This kind of business model differs with crowdsourcing because designers also provide information about customer preference at the same time. Terwiesch and Xu (2008) find in an innovation contest, a firm (the seeker) facing an innovation-related problem posts this problem to a population of independent agents (the solvers) and then provides an award to the agent chosen to provide the best solution. They analyze the interaction between a seeker and a set of solvers. Hippel et al. (1999) mainly focus on the phenomenon of outsourcing or crowdsourcing from the perspective of management. While our thesis focuses on competitive market research and product design based on both a mathematical model and specific business cases, we consider the business model of Threadless where many designers may participate. This way is related to outsourcing and crowdsourcing. They share the similarity that outside designers design the product for the firm. But the difference is that the firm acquires information about customer preference from designers at the same time instead of doing market research.

The last stream of literature is about game theory. Harsanyi and Selten (1988) achieve a remarkable degree of theoretical unification for game theory as a whole and provides a deeper insight into the nature of game-theoretic rationality by providing solutions for all classes of games, both cooperative and noncooperative, and both those with complete and with incomplete information. Straffin (1993) provides an introduction to both axiomatic mathematical thinking and the fundamental process of mathematical modeling. He gives an insight into both the nature of pure mathematics and the way in which mathematics can be applied to real problems. This thesis focuses on a game problem. Methods to solve game problem are useful to get the equilibrium. We learned from literature about game theory and applied it in our specific cases.

Our results contribute to the literature in several ways. Firstly, to the best of our knowledge, our thesis is the first to study competitive market research. Most of the previous papers about market research such as Lauga and Ofek (2009) focus on the traditional investigation way. But our thesis focuses on the new way of market research. The firm gives reward to attract outside designers but it does not pay extra payment for the information it acquires. The target people of market research are different from the traditional way too. Firms usually investigate potential customers to know their preference but in our thesis designers are not only familiar with customer preference but also professional about the industry. They provide extra professional information for the firm. Through this way of market research, the firm acquires independent signals from each designer. It acquires more valuable information. The uncertainty of market demand is a key research question while we study the uncertainty of customer preference. We consider vertical preference (Lauga and Ofek 2009). Designers always design on the attribute that is the same as the signal they received. Through this way designers can get a higher probability to get the reward.

Secondly, we study market research and product design together. The literature studies these two research problems separately. Some papers such as Lauga and Ofek (2009) only study market research. Some papers such as Piller (2011) only study people who are not employees but who design product for the firm. For example, some study open innovation, customer innovation, crowdsourcing and outsourcing.

Market research and product design are independent in some scenarios but in our thesis they are not. In our study, these two things happen in the same process. Actually designers finish these two works together while the traditional market research way is independent from product design. In the traditional way, the firm needs to pay for these two efforts separately, while in our study the firm does not need to pay extra reward for the information it acquired. We find that this kind of business model may help the firm save costs. When the cost of R&D is too high for the firm, this business model is very helpful.

3 The model and preliminary analysis

In this section, we consider the new business model where outside designers do market research and design product for the firm. We present the game model first.

When customers purchase products, they have preferences on characteristics of the product. Taking Threadless for example, customers may prefer slim shirt rather than casual style; customers may prefer colorful shirt rather than colorless style; customers may like following the latest fashion trend rather than classic style. New product may have the wrong attributes which is less preferred by customers (mismatch to customer preference). For example, clothes are casual style but customers actually prefer slim style. To study customer preference about the product, we assume that there are two possible states of attributes: a and b. Let S represent the attribute that is actually preferred by customer. When the new product mismatches the customer preference there will be a disutility of customer's utility. The constant d represents the disutility caused by mismatching of customer preference.

In addition to different preference, customers have different sensitivity to the quality of new product. Some customers are more sensitive about the quality but some others are less sensitive. To capture customers' heterogeneity with the quality of the product, we let the variable θ which accords with uniform distribution on interval $[0, 1]$ represent customers' sensitivity to the quality of the product. When θ is small it means the customer is more sensitive to the quality, otherwise it means the customer is less sensitive to the quality. The customer decides whether to buy the new product. Hence, the utility of a customer is

$$U = \theta Q - \delta d - p. \qquad (1)$$

The parameter Q is the quality of the product. δ is the indicator of the mismatch to customer preference, $\delta = 1$ if the attribute of the product mismatches customer preference, otherwise $\delta = 0$. And p is the price of the product. We assume the reservation utility of customers is zero: A customer will buy the product if and only if the utility is greater than zero. Let λ denote the market size and D denote the demand of the product, then $D = \lambda \Pr(U \geq 0)$.

Let M denote the total number of designers. Each designer needs to decide whether to design. Some designers may have incentive to design but some others may not. We use m $(0 \leq m \leq M)$ to denote the number of designers who submit a design to the firm. These m designers need to decide which attribute they will take effort on. There is common prior belief about customers' preference. We assume that customers prefer a design on attribute a with probability $\alpha \in (0,1)$, and customers prefer design on attribute b with probability $1-\alpha$, i.e.,

$$\Pr(S = a) = \alpha, \qquad \Pr(S = b) = 1 - \alpha.$$

Every designer will receive a signal about customer preference. Every designer has private information of customer preference before he decides whether to design. The community of Threadless is large and active. Some designers with rich experience are professional on apparel industry and always follow the latest fashion information, which could be used in the design for Threadless. They have rich experience in designing the products. Some designers are loyal fans of Threadless, they experience the process many times and know its target customers preference. They pay attention to past designs and learn from them. Some others are amateur designers, they may not have professional knowledge or experience on design. But they are closer to customers and have a good network of potential customers. They understand customer preference much better. Some designers may acquire information about the fashion trend online and hence know better of customer preference. No matter where designers gather the information of customer preference, they will use the information to design a product for a higher probability to get the reward. Let Y_i represent each signal acquired by designer i. Each signal Y_i is independently generated from the true state S with probability $\gamma \in [0.5, 1]$, i.e.,

$$\Pr(Y_i = S) = \gamma.$$

Each designer i will update his belief of customer preference after observing Y_i. Using Bayesian rule, the posterior probability after observing the private signal of designer

i that customers prefer attribute a is

$$\begin{aligned}
\Pr(S = a|Y_i = a) &= \frac{\Pr(S = a)\Pr(Y_i = a|S = a)}{\Pr(S = a)\Pr(Y_i = a|S = a) + \Pr(S = b)\Pr(Y_i = a|S = b)} \\
&= \frac{\alpha\gamma}{\alpha\gamma + (1-\alpha)(1-\gamma)}.
\end{aligned}$$

Similarly, the posterior probability after observing the private signal that customers prefer attribute b is

$$\Pr(S = b|Y_i = b) = \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma + \alpha(1-\gamma)}.$$

When $\gamma \to 0.5$, $\Pr(S = a|Y_i = a) = \alpha$, $\Pr(S = b|Y_i = b) = 1 - \alpha$. It means signal Y_i provides no additional information beyond the prior belief. Then designer i determines the attribute based on common prior belief. When $\gamma \to 1$, $\Pr(S = a|Y_i = a) = \Pr(S = b|Y_i = b) = 1$, it means the signal reveals the true customer preference perfectly. Then designer i determines the attribute based on the signal which is absolutely right.

The firm will browse all of the m designs. There are n_a designs with attribute a and n_b designs with attribute b. With

$$m = n_a + n_b, \qquad n = n_a - n_b.$$

After collecting designs from designers, the firm updates her belief on customer preference. Using Bayesian rule, the posterior probability after receiving m signals that customers prefer attribute a is

$$\begin{aligned}
\Pr(S = a|n_a, n_b) &= \frac{\Pr(S = a)\Pr(n_a, n_b|S = a)}{\Pr(S = a)\Pr(n_a, n_b|S = a) + \Pr(S = b)\Pr(n_a, n_b|S = b)} \\
&= \frac{\alpha\gamma^{n_a}(1-\gamma)^{n_b}}{\alpha\gamma^{n_a}(1-\gamma)^{n_b} + (1-\alpha)(1-\gamma)^{n_a}\gamma^{n_b}} \\
&= \frac{\alpha\gamma^{n}}{\alpha\gamma^{n} + (1-\alpha)(1-\gamma)^{n}}.
\end{aligned}$$

Similarly, the posterior probability after receiving m signals that customers prefer b is

$$\Pr(S=b|n_a,n_b) = 1-\Pr(S=a|n_a,n_b) = \frac{(1-\alpha)(1-\gamma)^n}{(1-\alpha)(1-\gamma)^n+\alpha\gamma^n}.$$

From the probabilities above, we obtain that when $\alpha = 0.5$, the probabilities that customers prefer attributes a and b are respectively

$$\begin{aligned}
\Pr(S=a|n_a,n_b) &= \frac{\gamma^n}{\gamma^n+(1-\gamma)^n}, \\
\Pr(S=b|n_a,n_b) &= \frac{(1-\gamma)^n}{\gamma^n+(1-\gamma)^n}.
\end{aligned}$$

In the rest of this thesis we assume $\alpha = 0.5$ for ease of exposition. When $\alpha \neq 0.5$ we can analyze the game model in the same approach and get similar results. We observe that the probability of customer preference is related on the accuracy of the information γ and the number of designs with each attribute.

Let Q_i represent the quality of the design of designer i. Assume $Q_i \geqslant q_0$ when designer i decides to participate, where q_0 is the benchmark quality. It means the quality of each design should be greater than or equal to a benchmark quality. This assumption is to consider the interior solution. Furthermore, we assume $q_0 > 2d$ throughout the paper. If the quality is too low, firms are not allowed to sell the product.

All designers who submit a design will receive a reward r_a offered by the firm, which is to reward their efforts on the design and information about the customer preference. If the designer's design is picked by the firm, he will receive another reward r_w given by the firm. The designer whose design is picked by the firm benefits from not only the monetary reward but also honor or experience. Threadless posts the design and information of the designer on her website. Everyone can browse this information online. There are 2 million members on the community of Threadless, so this will help the designer gain reputation and increase exposure in the professional field. Designers may get this kind of achievement and satisfaction from the participation. Let r_0 represent this kind of additional reward. Each designer also has a cost to design. We use a quadratic cost, which has commonly been used in the literature (e.g. Ge et al 2013). The random variable $K_i \in [0, 1]$ captures the cost of designer i to achieve a quality. u_r is the reservation utility of the designer. Then the profit of designer i is

$$\pi_i = (r_w + r_0) \times \Pr(\text{his design is picked to produce}) - \frac{1}{2} K_i Q_i^2 + r_a. \qquad (2)$$

The firm receives m designs. Based on the information, the firm determines to pick one design to produce. We assume that the firm must pick up one design to produce. Because we find that at Threadless, she always picks a design to produce when it starts this activity. Threadless is a famous company in the professional field. When this activity is disclosed to designers, if she does not pick a design and give the reward to designers she will lose her reputation. Then fewer designers will participate when she starts again. It is not valuable for her to lose her reputation and fans. So she always chooses one design to produce and rewards the corresponding designer. The profit of the firm will be zero when she does not receive any designs. Otherwise she picks a design and determines the price. Then the profit of the firm is

$$\pi_f = p \times D - r_w - m r_a. \qquad (3)$$

The sequence of events is as follows:

1. The firm determines the rewards r_w and r_a.

2. Each of the M designers observes a private signal Y_i. His random (private) quality cost parameter K_i is realized. The designer determines whether to participate. If he participates he needs to decide the attribute t_i, and the quality Q_i. Then they submit their designs (t_i, Q_i) simultaneously. Every participating designer receives a reward r_a.

3. The firm picks a design with attribute s when she receives designs. And she gives the designer whose design is picked the reward r_w, and then sets the price p.

4. Actual customer preference with attribute is realized. Individual customer chooses whether or not to buy the product. The firm produces to meet the demand.

4 Analysis with two designers

In this section we consider a market with two designers, i.e., $M = 2$. We use backward induction to solve the game model.

4.1 Stage 4: Customer's purchase decision

At stage 4, actual customer preference with attribute is realized. Individual customer buys the product if and only if the utility is non-negative, i.e.,

$$U = \theta Q - \delta d - p \geq 0.$$

Then

$$\theta \geq \frac{\delta d + p}{Q}.$$

It means customer with $\theta \in [\frac{\delta d+p}{Q}, 1]$ buys the product. So the demand volume equals

$$D = \max\left\{\lambda\left(1 - \frac{\delta d + p}{Q}\right), 0\right\}.$$

When $\frac{\delta d+p}{Q} > 1$ there is no sales. While we can show the sales is always positive under the assumption $q_0 > 2d$.

4.2 Stage 3: Firm's design choice

At stage 3, the firm receives m designs and needs to pick one from them. Then she determines the price p. There are three different scenarios which depend on the number of participating designers. We analyze these scenarios one by one.

4.2.1 Both designers participate

In this scenario $m = 2$. The probability with different (n_a, n_b) and the posterior probabilities of customer preference about both attributes are calculated according to the expression in previous section. The distribution of probabilities is shown in table 1.

(n_a, n_b)	Probability	$\Pr(S = a \mid n_a, n_b)$	$\Pr(S = b \mid n_a, n_b)$
$(0,2)$	$C_2^0(0.5)^2 = 0.25$	$\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2}$	$\frac{\gamma^2}{\gamma^2+(1-\gamma)^2}$
$(1,1)$	$C_2^1(0.5)(0.5) = 0.5$	$\frac{1}{2}$	$\frac{1}{2}$
$(2,0)$	$C_2^2(0.5)^2 = 0.25$	$\frac{\gamma^2}{\gamma^2+(1-\gamma)^2}$	$\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2}$

Table 1 Posterior probabilities when $m = 2$

The profit of the firm and the price depend on n_a, n_b and s. So let $\pi_f(n_a, n_b, s)$ and $p(n_a, n_b, s)$ denote the profit of the firm and the price under different values of n_a, n_b and s. When the firm receives two designs which are both on attribute b. Then the firm needs to choose one from them. The difference between these two designs is the quality. So she compares the profits with different quality, and then chooses the one with higher profit. The profit of the firm is

$$\begin{aligned}
& p\lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - 2r_a \\
= \;& p\lambda \left(1 - \frac{(\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2} \times 1 + \frac{\gamma^2}{\gamma^2+(1-\gamma)^2} \times 0)d + p}{Q}\right) - r_w - 2r_a \\
= \;& -\frac{1}{Q}\lambda p^2 + \frac{Q(\gamma^2 + (1-\gamma)^2) - (1-\gamma)^2 d}{Q(\gamma^2 + (1-\gamma)^2)}\lambda p - r_w - 2r_a.
\end{aligned}$$

The equilibrium price and the corresponding profit are

$$\begin{aligned}
p(0,2,b) &= \frac{(\gamma^2 + (1-\gamma)^2)Q - (1-\gamma)^2 d}{2(\gamma^2 + (1-\gamma)^2)}, \\
\pi_f(0,2,b) &= \frac{\lambda(Q(\gamma^2 + (1-\gamma)^2) - (1-\gamma)^2 d)^2}{4Q(\gamma^2 + (1-\gamma)^2)^2} - r_w - 2r_a.
\end{aligned}$$

From the equilibrium we observe that the price increases with the quality, so does the profit. So the firm chooses the design with higher quality. Similarly, we get the same equilibrium result when the firm receives two designs which are both on attribute a. And the firm also chooses the design with higher quality.

When the firm receives one design with attribute a and another design with attribute b, she has to consider both quality and attribute. If she chooses the design with attribute a, her profit is

$$
\begin{aligned}
& p\lambda\left(1-\frac{E(\delta)d+p}{Q}\right) \ -r_w-2r_a \\
= & -\frac{1}{Q}\lambda p^2+\left(\frac{2Q-d}{2Q}\right)\lambda p \ -r_w-2r_a.
\end{aligned}
$$

The equilibrium price and the corresponding profit are

$$
\begin{aligned}
p(1,1,a) &= \frac{2Q-d}{4}, \\
\pi_f(1,1,a) &= \frac{\lambda(2Q-d)^2}{16Q}-r_w-2r_a.
\end{aligned}
$$

We observe that the price and profit increase with the quality. Similarly, when the firm chooses the design with attribute b we get the same equilibrium result. So when the firm receives one design with attribute a and another design with attribute b, she always chooses the design with highest quality no matter which attribute is.

4.2.2 One designer participates

In this scenario $m=1$. There are two cases which depend on the value of n_a and n_b. The distribution of posterior probabilities is shown in table 2.

(n_a,n_b)	Probability	$\Pr(S=a\|n_a,n_b)$	$\Pr(S=b\|n_a,n_b)$
$(0,1)$	0.5	$1-\gamma$	γ
$(1,0)$	0.5	γ	$1-\gamma$

Table 2 Posterior probabilities when $m=1$

For both cases, the profit of the firm is

$$
\begin{aligned}
& p\lambda\left(1-\frac{E(\delta)d+p}{Q}\right) \ -r_w-r_a \\
= & -\frac{1}{Q}\lambda p^2+\frac{Q-(1-\gamma)d}{Q}\lambda p-r_w-r_a.
\end{aligned}
$$

The equilibrium price and corresponding profit are

$$\begin{aligned} p(1,0,a) &= p(1,0,b) = \frac{Q-(1-\gamma)d}{2}, \\ \pi_f(1,0,a) &= \pi_f(1,0,b) = \frac{\lambda(Q-(1-\gamma)d)^2}{4Q} - r_w - r_a. \end{aligned}$$

We observe that the price and profit increase with the quality. For these two cases, the firm receives only one design so she has only one choice.

4.2.3 Neither designer participates

In this scenario $m = 0$. There is no production and the firm collects zero profit.

4.3 Stage 2: Designers' participation and quality decision

At stage 2, each of M designers observes a private signal Y_i. Designer i participates when his profit is greater than the reservation utility. After receiving the private signal, every designer decides whether to participate. If he decides to participate, he determines design on one attribute. There are five pure strategies of designer i. He may always designs on attribute a or b no matter what the signal is; he may design on attribute Y_i; he may design on attribute $\bar{Y}_i$ which is opposite with Y_i; and he may do not participate. In conclusion, the possible pure strategies of designer i are $t_i \in \{a, b, Y_i, \bar{Y}_i, \varnothing\}$. We can easily verify that each designer always designs on the attribute as the same with his private signal, i.e., $t_i^* = Y_i$.

Each designer decides his quality to maximize his expected profit. And each of them has a cost for this quality. But designers do not know others' cost parameter, each of them can only observe his own random (private) quality cost parameter K_i. So each of them makes the decision of the quality based on their private cost parameter. So we assume the strategy of designer j is as follows:

1. When the cost parameter of designer j is high, he submits a smaller quality to reduce his cost. When the cost parameter is low, he submits a higher quality to get a higher probability to get the reward. So we assume Q_j monotonically decreases with K_j when $0 < K_j \leq \bar{k}$, $\bar{k} \in [0, 1]$ is a constant. Let $g(K_j)$ represent this quality

expression.

2. When K_j is high enough i.e. $\bar{k} < K_j < \hat{k}$ ($\hat{k} \in [\bar{k}, 1]$ is a constant), Q_j will be smaller than the benchmark quality q_0 if designer j still submits quality according to $g(K_j)$. This conflicts with our assumption $Q \geqslant q_0$ when designer participates. So we assume $Q_j = q_0$ when $\bar{k} < K_j < \hat{k}$.

3. When the cost parameter is too high i.e $\hat{k} \leqslant K_j < 1$, designer j does not participate because the cost is too high and he cannot get a positive profit. So we assume that designer j does not participate when $\hat{k} \leqslant K_j < 1$.

We have the conclusion that the firm always picks the design with highest quality at stage 3. So it means the probability when the design of designer i is produced equals the probability when his quality is the highest, then the profit of designer i is

$$\begin{aligned}\pi_i &= (r_w + r_0)\Pr(Q_i > Q_j(k_j)) - \frac{1}{2}k_i Q_i^2 + r_a \\ &= (r_w + r_0)(1 - Q_j^{-1}(Q_i)) - \frac{1}{2}k_i Q_i^2 + r_a.\end{aligned}$$

We have the theorem about the equilibrium quality below.

Theorem 1 The equilibrium strategy of designer i is to participate with the equilibrium quality

$$Q_i^* = \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_i}}.$$

if $0 < k_i \leq \bar{k}$ (where $\bar{k} \in [0, 1]$), otherwise he does not participate.

Proof. To find the equilibrium quality, we calculate the first-order derivative of the profit with respect to Q_i

$$\frac{d\pi_i}{dQ_i} = -(r_w + r_0) \times \frac{dQ_j^{-1}(Q_i)}{dQ_i} - k_i Q_i.$$

Set the first-order derivative of the profit with respect to Q_i equal to zero, and we get

$$\frac{dg^{-1}(Q_i)}{dQ_i} = -\frac{k_i Q_i}{(r_w + r_0)}.$$

Then we get

$$Q_i' Q_i = -\frac{(r_w + r_0)}{k_i}.$$

Solve the differential equation and we get the equilibrium of Q_i

$$Q_i^* = \sqrt{C - 2(r_w + r_0)\ln k_i}.$$

C is a constant. The boundary solution is the exact solution cover the interval of $k_i \in [0, 1]$. We consider the scenario when $K_i = \bar{k}$. Under the symmetric situation, the quality when $k_i = \bar{k}$ is

$$Q_i^* = \sqrt{C - 2(r_w + r_0)\ln \bar{k}} = q_0.$$

We get

$$C = q_0^2 + 2(r_w + r_0)\ln \bar{k}.$$

Substitute C into the above equilibrium and we get $g(k_i)$.

Then we check the equilibrium based on our assumption. Given the strategy of designer j

$$\begin{aligned} Q_j &= \sqrt{q_0^2 + 2(r_w + r_0)\ln \frac{\bar{k}}{k_j}} && (\text{when } 0 < k_j \leq \bar{k}), \\ Q_j &= q_0 && (\text{when } \bar{k} < k_j \leq \hat{k}). \end{aligned}$$

Then we plan to find the strategy of designer i. When $0 < k_i \leq \bar{k}$, the profit of designer i is

$$\pi_i = (r_w + r_0)\left(1 - e^{\frac{q_0^2 + 2(r_w + r_0)\ln \bar{k} - Q_i^2}{2(r_w + r_0)}}\right) - \frac{1}{2}k_i Q_i^2 + r_a.$$

The first-order derivative of the profit with respect to Q_i is

$$\frac{d\pi_i}{dQ_i} = Q_i e^{\frac{q_0^2 + 2(r_w + r_0)\ln \bar{k} - Q_i^2}{2(r_w + r_0)}} - k_i Q_i.$$

When the first-order derivative of the profit with respect to Q_i equals to zero, the solution of the equation is the optimal solution. We get the same equilibrium

$$Q_i = \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_i}}.$$

So the strategy of both designers at $0 < k_i \leq \bar{k}$ is the only equilibrium.

When $\bar{k} < k_i \leq \hat{k}$, if designer i participates with q_0, the profit of designer i is

$$\pi_i(q_0) = \frac{1}{2}(2 - \bar{k} - \hat{k})(r_w + r_0) - \frac{1}{2}k_i q_0^2 + r_a.$$

If designer i participates with a higher quality than q_0, i.e. $q_0 + \varepsilon$ (ε is positive and close to zero), then the profit of designer i is

$$\pi_i(q_0 + \varepsilon) = \left(1 - e^{\frac{q_0^2 + 2(r_w + r_0)\ln\bar{k} - (q_0+\varepsilon)^2}{2(r_w+r_0)}}\right)(r_w + r_0) - \frac{1}{2}k_i(q_0 + \varepsilon)^2 + r_a.$$

When ε is close to zero, the limitation of the profit is

$$\lim_{\varepsilon\to 0}\pi_i(q_0 + \varepsilon) = (1 - \bar{k})(r_w + r_0) - \frac{1}{2}k_i q_0^2 + r_a.$$

Compare the profit $\pi_i(q_0)$ and $\lim_{\varepsilon\to 0}\pi_i(q_0+\varepsilon)$. When $\bar{k} < \hat{k}$

$$\lim_{\varepsilon\to 0}\pi_i(q_0 + \varepsilon) > \pi_i(q_0).$$

So there is no equilibrium if $\bar{k} < \hat{k}$, because designers always choose a quality which is little greater than q_0. When $\bar{k} = \hat{k}$

$$\lim_{\varepsilon\to 0}\pi_i(q_0 + \varepsilon) = \pi_i(q_0).$$

This is an equilibrium when $\bar{k} = \hat{k}$. So the optimal solution is the only equilibrium, and it is $Q_i = q_0$ (when $k_i = \bar{k}$). Then the result follows. ∎

There exist multiple equilibria in the reality which equilibrium will be chosen by the designer. We use Pareto-optimal equilibrium as a concept. This equilibrium is

Pareto-optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto-optimal outcome cannot be improved upon without hurting at least one player.

Theorem 2 The Pareto-optimal equilibrium of designer i is to design with quality

$$Q_i = \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{2(r_w + r_0 + r_a)}{(2(r_w + r_0) + q_0^2)k_i}}.$$

if $0 < k_i \leq \frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}$, otherwise he does not participate.

Proof. When $k_i \geq \bar{k}$, the profit of designer i is zero if he does not participate. But when he participates with Q ($Q \geq q_0$), his profit is

$$\pi_i(Q) = \left(1 - \bar{k}e^{\frac{q_0^2 - Q^2}{2(r_w + r_0)}}\right)(r_w + r_0) - \frac{1}{2}k_i Q^2 + r_a.$$

The first derivative of the profit with respect to Q is

$$\frac{d\pi_i(Q)}{dQ} = \bar{k}Qe^{\frac{q_0^2 - Q^2}{2(r_w + r_0)}} - k_i Q.$$

When the first-order derivative of the profit with respect to Q equals to zero, the solution of the equation is the equilibrium solution. So the equilibrium solution is

$$Q = \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_i}}.$$

But we can easily observe that $Q \leq q_0$ because $k_i \geq \bar{k}$, which conflicts with $Q \geq q_0$. So we get $Q = q_0$ and the profit is

$$\pi_i(Q) = (1 - \bar{k})(r_w + r_0) - \frac{1}{2}k_i q_0^2 + r_a.$$

To get an equilibrium, the profit when designer i participates must less than the profit when designer i does not participate, that is

$$\pi_i(Q) \leq 0.$$

Then we get

$$\bar{k} \geq 1 + \frac{r_a}{(r_w + r_0)} - \frac{k_i q_0^2}{2(r_w + r_0)}.$$

Because $\bar{k} \leq k_i < 1$, we get

$$\bar{k} \geq 1 + \frac{r_a}{(r_w + r_0)} - \frac{\bar{k} q_0^2}{2(r_w + r_0)}.$$

Then

$$\bar{k} \geq \frac{2(r_w + r_0 + r_a)}{2(r_w + r_0) + q_0^2}.$$

The expected profit of designer i is

$$\begin{aligned}
\pi_i &= \int_0^{\bar{k}} [(r_w + r_0)(1 - k_i) - \frac{1}{2} k_i (q_0^2 + 2(r_w + r_0) \ln \frac{\bar{k}}{k_i}) + r_a] dk_i \\
&= (r_w + r_0)\bar{k} - \frac{3(r_w + r_0)\bar{k}^2}{4} - \frac{\bar{k}^2 q_0^2}{4} + r_a \bar{k}.
\end{aligned}$$

The first-order derivative of the profit with respect to $\bar{k}$ is

$$\frac{d\pi_i}{d\bar{k}} = (r_w + r_0) - \frac{3(r_w + r_0)\bar{k}}{2} - \frac{1}{2}\bar{k} q_0^2 + r_a.$$

When the first-order derivative of the profit with respect to $\bar{k}$ equals to zero, the solution of the equation is the equilibrium solution of $\bar{k}$. The solution is

$$\bar{k} = \frac{2(r_w + r_0 + r_a)}{3(r_w + r_0) + q_0^2}.$$

But we get the condition $\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2} \leq \bar{k} < 1$ before. And $\frac{d\pi_i}{d\bar{k}} < 0$ among this interval, which means the profit decreases with $\bar{k}$ in the interval $\left[\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}, 1\right)$. So the value of $\bar{k}$ when designer i can get the maximum profit is

$$\bar{k}^* = \frac{2(r_w + r_0 + r_a)}{2(r_w + r_0) + q_0^2}.$$

Then the result follows. ∎

Each designer determines whether to participate based on his cost parameter K_i.

When K_i is less than $\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}$, he participates. And the quality decreases with his cost parameter. But when his cost parameter is greater than $\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}$, the cost is too high and he does not participate. When $k_i < \bar{k}$, we get the probability when the design of designer i is picked by the firm

$$\Pr(\text{his design is picked to produce}) = 1 - k_i.$$

The probability decreases with his cost parameter. Lower cost parameter increases the probability when he gets the rewards.

4.4 Stage 1: Firm's reward decision

At stage 1, the firm determines the rewards. The firm's profit is the expected value at stage 3, we can get probabilities of different value of m as follows

$$\begin{aligned}\Pr(m &= 2) = \bar{k}^2 = \left(\frac{2(r_w + r_0 + r_a)}{2(r_w + r_0) + q_0^2}\right)^2, \\ \Pr(m &= 1) = C_2^1 \bar{k}(1-\bar{k}) = \frac{4(r_w + r_0 + r_a)(q_0^2 - 2r_a)}{(2(r_w + r_0) + q_0^2)^2}.\end{aligned}$$

We got the profits of the firm at stage 3. Then expected profit of the firm is

$$\begin{aligned}\pi_f = \Pr(m=1)\left[E(\tfrac{\lambda(Q-(1-\gamma)d)^2}{4Q}) - r_w - r_a\right] + 0.5\Pr(m=2) \\ \left[E(\tfrac{\lambda(Q(\gamma^2+(1-\gamma)^2)-(1-\gamma)^2 d)^2}{4Q(\gamma^2+(1-\gamma)^2)^2}) + E(\tfrac{\lambda(2Q-d)^2}{16Q}) - 2r_w - 4r_a\right].\end{aligned}$$

Substitute the probabilities into the expression and we get

$$\begin{aligned}\pi_f = \tfrac{\bar{k}(2-\bar{k})\lambda E(Q)}{4} + \lambda \bar{k} d^2 E(\tfrac{1}{Q})\left[\tfrac{\bar{k}(4(1-\gamma)^4+(\gamma^2+(1-\gamma)^2)^2)}{32(\gamma^2+(1-\gamma)^2)^2} + \tfrac{(1-\bar{k})(1-\gamma)^2}{2}\right] \\ - \tfrac{\lambda d \bar{k}^2(3(1-\gamma)^2+\gamma^2)}{8(\gamma^2+(1-\gamma)^2)} - \lambda d\bar{k}(1-\bar{k})(1-\gamma) + \bar{k}(\bar{k}-2)r_w - 2\bar{k}r_a.\end{aligned}$$

Note that $Q = \max(Q_i, Q_j)$. The expected quality $E[Q]$ is

$$\begin{aligned}
E(Q) &= \int_0^{\bar{k}} (\int_0^{K_i} (\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_j}})dk_j \\
&\quad + \int_{K_i}^1 \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_i}} dk_j)dk_i \\
&\quad + \int_{\bar{k}}^1 (\int_0^{\bar{k}} \sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_j}} dk_j dk_i \\
&= 2\int_0^{\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}} (1-k_i)\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{2(r_w + r_0 + r_a)}{(2(r_w + r_0) + q_0^2)k_i}} dk_i.
\end{aligned}$$

The expectation of $E[\frac{1}{Q}]$ is

$$\begin{aligned}
E(\frac{1}{Q}) &= \int_0^{\bar{k}} (\int_0^{K_i} (\frac{1}{\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_j}}})dk_j \\
&\quad + \int_{K_i}^1 \frac{1}{\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_i}}} dk_j)dk_i \\
&\quad + \int_{\bar{k}}^1 (\int_0^{\bar{k}} \frac{1}{\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{\bar{k}}{k_j}}} dk_j dk_i \\
&= 2\int_0^{\frac{2(r_w+r_0+r_a)}{2(r_w+r_0)+q_0^2}} \frac{1-k_i}{\sqrt{q_0^2 + 2(r_w + r_0)\ln\frac{2(r_w+r_0+r_a)}{(2(r_w+r_0)+q_0^2)k_j}}} dk_i.
\end{aligned}$$

We test many sets of parameters and find that the profit function is always jointly concave function in rewards r_w and r_a. When $\gamma = 0.8$, $r_0 = 10$, $q_0 = 10$, $\lambda = 50$, $d = 2$, the profits of the firm with respect to rewards r_w and r_a are

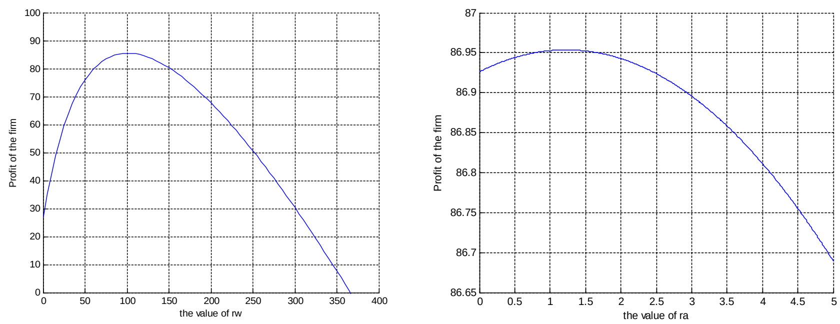

Fig 1 Firm's profit with respect to reward r_w or r_a

We use sequential optimization to show that the profit of the firm is jointly concave in rewards r_w and r_a. We get the first figure by setting a fixed value of $r_a = 10$, then we can find the optimal reward r_w^* from the concave figure. We find that the profit is always concave in r_w for any given r_a. A series value of r_w^* under different value of given r_a can be found, then we show the second figure. So we get a sequential optimal solution of r_w^*, r_a^* from these two figures. There exists optimal rewards $r_w^*, r_a^* > 0$ which make the firm get the maximum profit π_f^*.

5 Analysis with more than two designers

In this section we consider the general scenario when there are more than two designers, i.e. $M > 2$. We use backward induction to solve the game model. The model and sequence of events are the same as section 3. And the analysis of stage 4 is the same as section 4. So we just analyze other three stages in this section.

5.1 Stage 3: Firm's design choice

There are M scenarios of the number of participated designers.

$$m \in \{0, 1, 2, ..., M\}.$$

For $m \in \{0, 1, 2, ..., M\}$, the distribution of posterior probabilities is shown in table 3.

(n_a, n_b)	Probability	$\Pr(a\|n_a, n_b)$	$\Pr(b\|n_a, n_b)$
$(0, m)$	$C_m^0(0.5)^m$	$\frac{\gamma^{-m}}{\gamma^{-m}+(1-\gamma)^{-m}}$	$\frac{(1-\gamma)^{-m}}{\gamma^{-m}+(1-\gamma)^{-m}}$
$(1, m-1)$	$C_m^1(0.5)^m$	$\frac{\gamma^{2-m}}{\gamma^{2-m}+(1-\gamma)^{2-m}}$	$\frac{(1-\gamma)^{2-m}}{\gamma^{2-m}+(1-\gamma)^{2-m}}$
...	...	...	...
$(n_a, m-n_a)$	$C_m^{n_a}(0.5)^m$	$\frac{\gamma^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}$	$\frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}$
...	...	...	...
$(m-1, 1)$	$C_m^{m-1}(0.5)^m$	$\frac{\gamma^{m-2}}{\gamma^{m-2}+(1-\gamma)^{m-2}}$	$\frac{(1-\gamma)^{m-2}}{\gamma^{m-2}+(1-\gamma)^{m-2}}$
$(m, 0)$	$C_m^m(0.5)^m$	$\frac{\gamma^{m}}{\gamma^{m}+(1-\gamma)^{m}}$	$\frac{(1-\gamma)^{m}}{\gamma^{m}+(1-\gamma)^{m}}$

Table 3 Posterior probabilities when $M > 2$

5.1.1 All designers participate

In this scenario $m = M$. When the firm receives M designs on attribute b. The firm's profit is

$$\begin{aligned} & p \times \lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - Mr_a \\ = & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - \frac{\gamma^{-M}}{\gamma^{-M}+(1-\gamma)^{-M}}d}{Q}\right)\lambda p - r_w - Mr_a. \end{aligned}$$

We get the equilibrium price and corresponding profit

$$
\begin{aligned}
p(0, M, b) &= \frac{Q - \frac{(1-\gamma)^M}{\gamma^M + (1-\gamma)^M} d}{2}, \\
\pi_f(0, M, b) &= \frac{\lambda \left(Q - \frac{(1-\gamma)^M}{\gamma^M + (1-\gamma)^M} d \right)^2}{4Q} - r_w - M r_a.
\end{aligned}
$$

We observe that the profit increases with quality. So the firm chooses the design with highest quality from M designs. Similarly, when the firm receives M designs on attribute a, she must choose the design with attribute a. And we get the equilibrium price and corresponding profit

$$
\begin{aligned}
p(M, 0, a) &= \frac{Q - \frac{(1-\gamma)^m}{\gamma^m + (1-\gamma)^m} d}{2}, \\
\pi_f(M, 0, a) &= \frac{\lambda \left(Q - \frac{(1-\gamma)^m}{\gamma^m + (1-\gamma)^m} d \right)^2}{4Q} - r_w - m r_a.
\end{aligned}
$$

She chooses the design with highest quality from M designs too.

When the firm receives n_a designs on attribute a and $M - n_a$ designs on attribute b, the profit if she chooses the design with attribute a is

$$
\begin{aligned}
& p \times \lambda \left(1 - \frac{E(\delta) d + p}{Q} \right) - r_w - M r_a \\
= \; & -\frac{\lambda}{Q} p^2 + \left(\frac{Q - \frac{(1-\gamma)^{2n_a - M}}{\gamma^{2n_a - M} + (1-\gamma)^{2n_a - M}} d}{Q} \right) \lambda p \; - r_w - M r_a.
\end{aligned}
$$

We get the equilibrium price and corresponding profit

$$
\begin{aligned}
p(n_a, m - n_a, a) &= \frac{Q - \frac{(1-\gamma)^{2n_a - M}}{\gamma^{2n_a - M} + (1-\gamma)^{2n_a - M}} d}{2}, \\
\pi_f(n_a, m - n_a, a) &= \frac{\lambda \left(Q - \frac{(1-\gamma)^{2n_a - M}}{\gamma^{2n_a - M} + (1-\gamma)^{2n_a - M}} d \right)^2}{4Q} - r_w - M r_a.
\end{aligned}
$$

We observe that the profit increases with the quality. So when the firm chooses a design on attribute a she chooses the one with the highest quality. Similarly, when she chooses a design on attribute b, the profit is

$$\begin{aligned} & p\times\lambda\left(1-\frac{E(\delta)d+p}{Q}\right) \ -r_w-Mr_a \\ = & \ -\frac{\lambda}{Q}p^2+\left(\frac{Q-\frac{\gamma^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d}{Q}\right)\lambda p \ -r_w-Mr_a. \end{aligned}$$

The equilibrium price and corresponding profit are

$$\begin{aligned} p(n_a,m-n_a,b) &= \frac{Q-\frac{\gamma^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d}{2}, \\ \pi_f(n_a,m-n_a,b) &= \frac{\lambda\left(Q-\frac{\gamma^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2}{4Q}-r_w-Mr_a. \end{aligned}$$

We observe that the profit increases with the quality too. So when the firm chooses a design on attribute b she chooses the one with the highest quality. Hence, when the firm receives n_a $(n_a=1,...,M-1)$ designs on attribute a and $M-n_a$ designs on attribute b, the firm compares the profit of two designs first; one is the design with highest quality on attribute a and the other one is the design with highest quality on attribute b. Then she chooses the one that maximizes the profit. Let Q_a, Q_b denote the highest quality of designs with attribute a and b relatively. Then the difference of the profit when the firm chooses these two designs is

$$\frac{\lambda\left(Q_a-\frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2}{4Q_a}-\frac{\lambda\left(Q_b-\frac{\gamma^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2}{4Q_b}.$$

The necessary and sufficient condition when the difference is greater than zero is

$$\begin{aligned} & Q_aQ_b^2-\left(\left(Q_a-\frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2+\frac{2\gamma^{2n_a-M}dQ_a}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right)Q_b \\ & +\left(\frac{\gamma^{2n_a-M}d}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right)^2Q_a<0. \end{aligned}$$

The solution is

$$
\begin{aligned}
Q_b \quad > \quad & \frac{1}{2Q_a}\left\{\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2 + \frac{2\gamma^{2n_a-M}dQ_a}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right. \\
& -\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)\left[\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2\right. \\
& \left.\left.+\frac{4\gamma^{2n_a-M}dQ_a}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right]^{\frac{1}{2}}\right\},
\end{aligned}
$$

or

$$
\begin{aligned}
Q_b \quad < \quad & \frac{1}{2Q_a}\left\{\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2 + \frac{2\gamma^{2n_a-M}dQ_a}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right. \\
& +\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)\left[\left(Q_a - \frac{(1-\gamma)^{2n_a-M}}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}d\right)^2\right. \\
& \left.\left.+\frac{4\gamma^{2n_a-M}dQ_a}{\gamma^{2n_a-M}+(1-\gamma)^{2n_a-M}}\right]^{\frac{1}{2}}\right\} \qquad (4).
\end{aligned}
$$

Because $Q_b \geq q_0 \geq 2d$, we rule out the first condition. So the condition when the firm chooses the design with attribute a is condition (4). It means when the highest quality of the design with attribute b is less than the value above, the firm chooses the design with highest quality on attribute a.

5.1.2 Some designers participate

In this scenario $2 < m < M$. The firm's profit when she receives m designs with attribute b is

$$
\begin{aligned}
& p \times \lambda\left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - mr_a \\
= \quad & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - \frac{(1-\gamma)^{-m}}{\gamma^{-m}+(1-\gamma)^{-m}}d}{Q}\right)\lambda p - r_w - mr_a.
\end{aligned}
$$

We get the equilibrium price and corresponding profit

$$\begin{aligned}
p(0,m,b) &= \frac{Q-\frac{\gamma^m}{\gamma^m+(1-\gamma)^m}d}{2}, \\
\pi_f(0,m,b) &= \frac{\lambda\left(Q-\frac{\gamma^m}{\gamma^m+(1-\gamma)^m}d\right)^2}{4Q} - r_w - mr_a.
\end{aligned}$$

We observe that the profit increases with the quality. So the firm chooses the design with highest quality from m designs. Similarly, when the firm receives m designs on attribute a, she must choose the design with attribute a. And we get the equilibrium price and corresponding profit

$$\begin{aligned}
p(m,0,a) &= \frac{Q-\frac{(1-\gamma)^m}{\gamma^m+(1-\gamma)^m}d}{2}, \\
\pi_f(m,0,a) &= \frac{\lambda\left(Q-\frac{(1-\gamma)^m}{\gamma^m+(1-\gamma)^m}d\right)^2}{4Q} - r_w - mr_a.
\end{aligned}$$

The firm chooses the design with highest quality from m designs too.

When the firm receives n_a designs on attribute a and $m-n_a$ designs on attribute b, if the firm chooses the design with attribute a, the profit is

$$\begin{aligned}
& p \times \lambda\left(1-\frac{E(\delta)d+p}{Q}\right) - r_w - mr_a \\
= & -\frac{\lambda}{Q}p^2 + \left(\frac{Q-\frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d}{Q}\right)\lambda p - r_w - mr_a.
\end{aligned}$$

We get the equilibrium price and corresponding profit

$$\begin{aligned}
p(n_a,m-n_a,a) &= \frac{Q-\frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d}{2}, \\
\pi_f(n_a,m-n_a,a) &= \frac{\lambda\left(Q-\frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d\right)^2}{4Q} - r_w - mr_a.
\end{aligned}$$

We observe that the profit increases with the quality. So when the firm chooses a design on attribute a, she chooses the one with the highest quality. Similarly, when she chooses a design on attribute b, the profit is

$$
\begin{aligned}
& p \times \lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - mr_a \\
= & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - \frac{\gamma^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d}{Q}\right)\lambda p - r_w - mr_a.
\end{aligned}
$$

The equilibrium price and corresponding profit are

$$
\begin{aligned}
p(n_a, m-n_a, b) &= \frac{Q - \frac{\gamma^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d}{2}, \\
\pi_f(n_a, m-n_a, b) &= \frac{\lambda\left(Q - \frac{\gamma^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d\right)^2}{4Q} - r_w - mr_a.
\end{aligned}
$$

We observe that the profit increases with the quality too. So when the firm chooses a design on attribute b, she chooses the one with the highest quality. In conclusion, when the firm receives n_a ($n_a = 1, ..., m-1$) designs on attribute a and $m-n_a$ designs on attribute b, the firm compares the profit of two designs first; one is the design with highest quality on attribute a and the other one is the design with highest quality on attribute b. Then she chooses the one that maximizes the profit.

Similarly with the last subsection, when

$$
\begin{aligned}
Q_b < & \frac{1}{2Q_a}\left\{\left(Q_a - \frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d\right)^2 + \frac{2\gamma^{2n_a-m}dQ_a}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}\right. \\
& + \left(Q_a - \frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d\right)\left[\left(Q_a - \frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}d\right)^2\right. \\
& \left.\left. + \frac{4\gamma^{2n_a-m}dQ_a}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}}\right]^{\frac{1}{2}}\right\} \qquad (5).
\end{aligned}
$$

It means when the highest quality of the design with attribute b is less than the value above, the firm chooses the design with highest quality on attribute a. Hence, when there are designers participate, the decision of the firm is as follows.

Theorem 3 When all these designers design on the same attribute, the firm chooses the design with highest quality; when some designers design on attribute a but some others design on attribute b, the firm chooses the design with highest quality on attribute a if Q_b is less than the value in expression (5), otherwise she chooses the design with highest quality on attribute b.

Let $f(Q_a, n_a)$ denote the value of equation (5). We test many sets of parameters and find $f(Q_i, n_a)$ follows a same trend. When $M = 100$, $Q_i = 50$, $d = 1$, $\gamma = 0.8$, $f(Q_a, n_a)$ follows the trend below

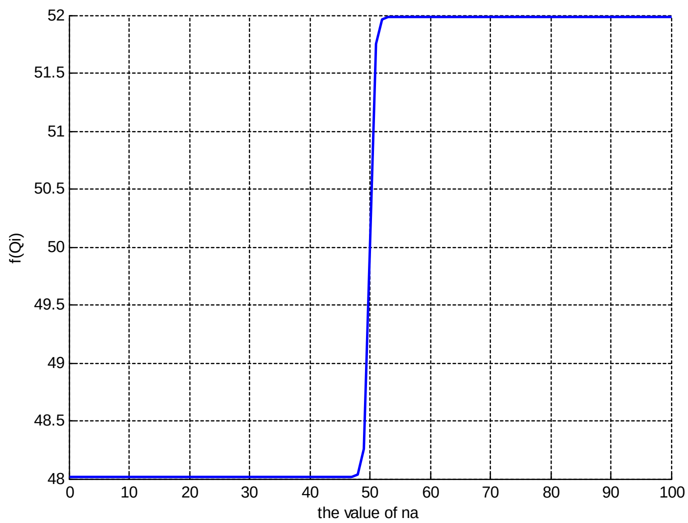

Fig 2 $f(Q_a, n_a)$ with respect to n_a

We observe that when M is even and $n_a < \frac{M}{2}$, $f(Q_a, n_a)$ keeps the same low value, when $n_a > \frac{M}{2}$, $f(Q_a, n_a)$ keeps the same high value. And these two values both increase with Q_a. When $n_a = \frac{M}{2}$, $f(Q_a, n_a) = Q_a$. Combine this observation with the expression of $f(Q_a, n_a)$, we can approximate $f(Q_a, n_a)$ as follows

$$f(Q_a, n_a) = \begin{cases} Q_a - 2d + \frac{d^2}{Q_a} & \text{when } n_a < \frac{M}{2}, \\ Q_a & \text{when } n_a = \frac{M}{2}, \\ \frac{1}{2}(Q_a + 2d + \sqrt{Q_a^2 + 4dQ_a}) & \text{when } n_a > \frac{M}{2}. \end{cases}$$

When Q_a is large enough compared to d, we can observe that the high value and low value are close to Q_a. Study the approximation above and we can get the results as follows

$$f(Q_a, n_a) = \begin{cases} Q_a - 2d & \text{when } n_a < \frac{M}{2}, \\ Q_a & \text{when } n_a = \frac{M}{2}, \\ Q_a + 2d & \text{when } n_a > \frac{M}{2}. \end{cases}$$

When $n_a > \frac{M}{2}$, $Q_a + 2d - \frac{Q_a+2d+\sqrt{Q_a^2+4dQ_a}}{2} = -\frac{\sqrt{Q_a^2+4dQ_a+4d^2}-\sqrt{Q_a^2+4dQ_a}}{2} \approx 0$. So the approximation of $f(Q_a, n_a)$ is suitable. It means when $n_a < \frac{M}{2}$, the firm picks the design with attribute a if the highest quality among designs with attribute b is less than $Q_a - 2d$; when $n_a = \frac{M}{2}$, the firm picks the design with attribute a if the highest quality among designs with attribute b is less than Q_a, i.e. the firm picks the design with highest quality no matter what the attribute is; when $n_a > \frac{M}{2}$, the firm picks the design with attribute a if the highest quality among designs with attribute b is less than $Q_a + 2d$. Otherwise, the firm picks the design with attribute b.

5.1.3 No designers participate

In this scenario $m = 0$. There is no production and the firm collects zero profit.

5.2 Stage 2: Designers' participation and quality decision

Now we analyze the behavior of designers. Designers decide whether to participate and determine the quality if participate. The profit of designer i is the same as expression (2). But the value of the probability when his design is produced is different with subsection 4.3. Refer to the above stage, we analyze the different cases and get the choice of the firm.

When designer i designs on attribute a, there are $M - 1$ cases for other designers. There are $n_a - 1$ designers design on attribute a and $M - n_a$ designers design on attribute b. When designer i designs on attribute b, it is similar because we assumed $\alpha = 0.5$. We get the distribution of the probability when the design of designer i is picked by the firm in table 4.

n_a-1	n_b	Pr(design of designer i is picked)
0	$M-1$	$C^0_{M-1}(0.5)^{M-1}(1-g^{-1}(f(Q_i,n_a)))^{M-1}$
...	...	...
n_a-1	$M-n_a$	$C^{n_a-1}_{M-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{n_a-1}(1-g^{-1}(f(Q_i,n_a)))^{M-n_a}$
...	...	...
$M-1$	0	$C^{M-1}_{M-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{M-1}$

Table 4 Probabilities when the design of designer i is picked

So the probability when the design of designer i is picked to produce is

$$\sum_{n_a-1=0}^{M-1} C^{n_a-1}_{M-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{n_a-1}(1-g^{-1}(f(Q_i,n_a)))^{M-n_a}.$$

The profit of designer i is similar with the scenario when $M=2$. To find the equilibrium quality of designer i, we calculate the first-order derivative of the profit with respect to Q_i

$$\begin{aligned}\frac{d\pi_i}{dQ_i} &= -(r_w+r_0)\sum_{n_a-1=0}^{M-1} C^{n_a-1}_{M-1}(0.5)^{M-1}[(n_a-1)(1-g^{-1}(Q_i))^{n_a-2} \\ &\quad (1-g^{-1}(f(Q_i,n_a)))^{M-n_a}\frac{dg^{-1}(Q_i)}{dQ_i}+(M-n_a)(1-g^{-1}(Q_i))^{n_a-1} \\ &\quad (1-g^{-1}(f(Q_i,n_a)))^{M-n_a-1}\frac{dg^{-1}(f(Q_i,n_a))}{dQ_i}]-k_iQ_i.\end{aligned}$$

We can observe that $g^{-1}(Q_i)$ is corresponding to k_i, $g^{-1}(f(Q_i,n_a))$ is a function on k_i. Set the first-order derivative of the profit with respect to Q_i equal to zero, Then we have

$$\begin{aligned}&(0.5)^{M-1}\textstyle\sum_{n_a-1=0}^{M-1} C^{n_a-1}_{M-1}(1-g^{-1}(Q_i))^{n_a-2}(1-g^{-1}(f(Q_i,n_a)))^{M-n_a-1}[(n_a-1) \\ &(1-g^{-1}(f(Q_i,n_a)))+(M-n_a)(1-g^{-1}(Q_i))g'^{-1}(f(Q_i,n_a))]\tfrac{dg^{-1}(Q_i)}{K_i}=-\tfrac{Q_idQ_i}{(r_w+r_0)}.\end{aligned}$$

Because we assumed that Q_j decreases with k_j, and combined the equation when

$M = 2$ we get the estimated value of $g^{-1}(f(Q_i, n_a))$ is

$$g^{-1}(f(Q_i, n_a)\ ) = \begin{cases} g^{-1}(Q_i) + \Delta & \text{when } n_a < \frac{M}{2}, \\ g^{-1}(Q_i) & \text{when } n_a = \frac{M}{2}, \\ g^{-1}(Q_i) - \Delta & \text{when } n_a > \frac{M}{2}. \end{cases}$$

Δ is a constant. Then we can get the expression below

$$g^{-1}(Q_i + 2d\ ) - g^{-1}(Q_i) = -\Delta.$$

Our approximation is under the assumption when Q_i is far greater than d, so we get

$$g'^{-1}(Q_i) = \frac{g^{-1}(Q_i + 2d\ ) - g^{-1}(Q_i)}{2d} = \frac{-\Delta}{2d}.$$

We know that $g'^{-1}(Q_i) = \frac{1}{g'(k_i)}$. Then we need to show $g'(k_i) = \frac{-2d}{\Delta}$.

While we can get the exact equilibrium quality when $\gamma = 0.5$. Because $f(Q_i, n_a) = Q_i$ for all n_a under this scenario. The exact equilibrium quality is

$$Q_i^* = \sqrt{q_0^2 + 2(r_w + r_0)(M-1)[\sum_{j=0}^{M-3} C_{M-2}^j \frac{(-\bar{k})^{M-2-j} - (-k_i)^{M-2-j}}{M-2-j} + \ln(\frac{\bar{k}}{k_i})]}.$$

Then we get the equation about $\bar{k}$ as follows

$$(r_w + r_0)(1 - \bar{k})^{M-1} - \frac{1}{2}\bar{k}q_0^2 + r_a = 0.$$

We find that $\bar{k}$ keeps the same value when M is sufficiently large. Combining this equation we can ignore the part $(r_w + r_0)(1 - \bar{k})^{M-1}$, then we get

$$\bar{k}^* \approx \frac{2r_a}{q_0^2}$$

The first-order derivative of the $g(k_i)$ is

$$g^{'}(k_i) = \frac{(r_w + r_0)(M-1)[\sum_{j=0}^{M-3} C_{M-2}^{j}(-k_i)^{M-3-j} - \frac{1}{k_i}]}{\sqrt{q_0^2 + 2(r_w + r_0)(M-1)[\sum_{j=0}^{M-3} C_{M-2}^{j} \frac{(-\bar{k})^{M-2-j} - (-k_i)^{M-2-j}}{M-2-j} + \ln \frac{\bar{k}}{k_i}]}}.$$

The part of summation contains $-k_i$, the summation is a small value because the positive parts and negative parts nearly cancelled out. So we get

$$g^{'}(k_i) \approx \frac{-(r_w + r_0)(M-1)}{k_i q_0}.$$

So the value of Δ is

$$\Delta = \frac{2dq_0 k_i}{(r_w + r_0)(M-1)} \approx \frac{dq_0}{(r_w + r_0)(M-1)}.$$

We have the approximation because the mean of k_i is 0.5.

Substitute this into the left side of the equation and we get

$$\frac{(0.5)^{M-1}}{K_i}\{\sum_{n_a-1=0}^{\frac{M}{2}-1} C_{M-1}^{n_a-1}[(n_a-1)(1-k_i)^{n_a-2}(1-\Delta-g^{-1}(Q_i))^{M-n_a} + \frac{2dq_0}{(r_w+r_0)(M-1)} (M-n_a)(1-g^{-1}(Q_i))^{n_a-1}(1-\Delta-g^{-1}(Q_i))^{M-n_a-1}] + \sum_{n_a-1=\frac{M}{2}+1}^{M-1} C_{M-1}^{n_a-1}[(n_a-1) (1-g^{-1}(Q_i))^{n_a-2}(1+\Delta-g^{-1}(Q_i))^{M-n_a} + \frac{2dq_0}{(r_w+r_0)(M-1)}(M-n_a)(1-g^{-1}(Q_i))^{n_a-1} (1+\Delta-g^{-1}(Q_i))^{M-n_a-1}] + (M-1)C_{M-1}^{\frac{M}{2}-1}(1-g^{-1}(Q_i))^{M}\}dg^{-1}(Q_i).$$

Get the integral of the left side of the equation and denote it as $h(k_i)$

$$\begin{aligned} h(k_i) = (0.5)^{M-1}\{ &\sum_{n_a-1=0}^{\frac{M}{2}-1} C_{M-1}^{n_a-1} \int \frac{(1-k_i)^{n_a-1}(1-\Delta-k_i)^{M-n_a}}{k_i^2} dk_i \\ &+ \sum_{n_a-1=\frac{M}{2}+1}^{M-1} C_{M-1}^{n_a-1} \int \frac{(1-k_i)^{n_a-1}(1+\Delta-k_i)^{M-n_a}}{k_i^2} dk_i \\ &- (M-1)C_{M-1}^{\frac{M}{2}-1}[\sum_{j=0}^{M-2} C_M^j \frac{(-K_i)^{M-j}}{M-j} + Mk_i - \ln k_i]\}. \end{aligned}$$

So we get the equilibrium quality

$$Q_i = \sqrt{C - 2(r_w + r_0)h(k_i)}.$$

Similarly with the scenario $M = 2$, when $k_i = \bar{k}$

$$Q_i = \sqrt{C - 2(r_w + r_0)h(\bar{k})} = q_0.$$

So we get

$$C = 2(r_w + r_0)h(\bar{k}) + q_0^2.$$

Substitute C into the interior solution. Given the strategy of designer j is

$$Q_j = \begin{cases} \sqrt{q_0^2 + 2(r_w + r_0)(h(\bar{k}) - h(k_j))} & (\text{when } 0 < k_j \leq \bar{k}), \\ q_0 & (\text{when } \bar{k} < k_j \leq \hat{k}). \end{cases}$$

When $0 < k_i \leq \bar{k}$, the profit of designer i is

$$\begin{aligned} \pi_i &= (r_w + r_0) \sum_{n_a - 1 = 0}^{M-1} C_{M-1}^{n_a - 1}(0.5)^{M-1}(1 - g^{-1}(Q_i))^{n_a - 1}(1 - g^{-1}(f(Q_i, n_a)))^{M - n_a} \\ &\quad - \frac{1}{2}k_i Q_i^2 + r_a. \end{aligned}$$

The first-order derivative of the profit with respect to Q_i is

$$\begin{aligned} \frac{d\pi_i}{dQ_i} &= \frac{d[\sum_{n_a-1=0}^{M-1} C_{M-1}^{n_a-1}(0.5)^{M-1}(1 - g^{-1}(Q_i))^{n_a-1}(1 - g^{-1}(f(Q_i, n_a)))^{M-n_a}]}{dQ_i} \times \\ &\quad (r_w + r_0) - k_i Q_i. \end{aligned}$$

When the first-order derivative of the profit with respect to Q_i equals to zero, the solution of the equation is the optimal solution. We get

$$\frac{d[\sum_{n_a-1=0}^{M-1} C_{M-1}^{n_a-1}(0.5)^{M-1}(1 - g^{-1}(Q_i))^{n_a-1}(1 - g^{-1}(f(Q_i, n_a)))^{M-n_a}]}{k_i} = \frac{Q_i dQ_i}{(r_w + r_0)}.$$

It is easily to find that this equation is the same as the process of getting the equilibrium quality. So we get the same equilibrium with the optimal quality.

When $\bar{k} < k_i \leq \hat{k}$, it is similar with the scenario when $M = 2$. So the optimal solution is the only equilibrium, and it is as follows

$$Q_i = q_0 \quad (\text{when } k_i = \bar{k}).$$

When $k_i \geq \bar{k}$, then the profit is zero when designer i does not participate. To get an equilibrium, the profit when designer i participates with q_0 must no more than the profit when designer i does not participate. When the profit equals to zero we can get the equilibrium value of $\bar{k}$ from the equation below

$$(r_w + r_0) \sum_{n_a - 1 = 0}^{M-1} C_{M-1}^{n_a - 1} (0.5)^{M-1} (1 - \bar{k})^{n_a - 1} (1 - g^{-1}(f(q_0)))^{M - n_a} - \frac{1}{2} \bar{k} q_0^2 + r_a = 0.$$

The value of $\bar{k}$ depends on r_w, r_0, r_a, M, q_0 and d. When there are more than two designers, the equilibrium quality of designer i is

$$Q_i = \sqrt{q_0^2 + 2(r_w + r_0)(h(\bar{k}) - h(k_i))} \qquad (\text{When } 0 < k_i < \bar{k}).$$

We test many sets of parameters and find $\bar{k}$ follows the same trend. When $r_a = 10$, $r_0 = 10$, $q_0 = 10$, $r_w = 10$, $d = 1$, we get the figure of $\bar{k}$ with respect to M

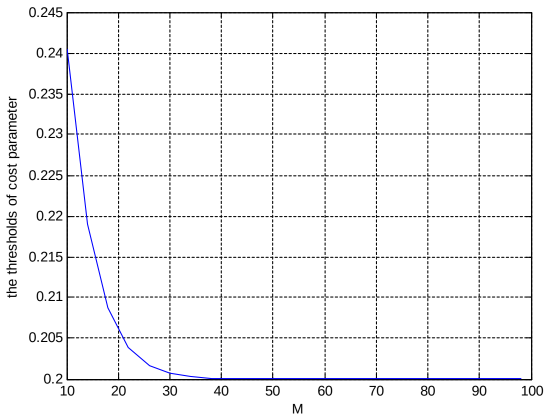

Fig 3 $\bar{k}$ with respect to M

We observe that when there are enough designers participate, the value of $\bar{k}$ keeps the same. Next we test and verify the equilibrium quality under this assumption. We get the figure of $g(k_j)$ when $r_w = 10$, $r_a = 0$, $r_0 = 10$, $q_0 = 10$, $M = 10$, $d = 1$

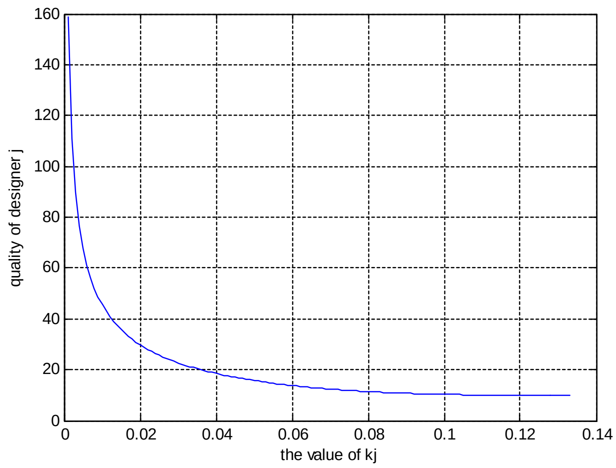

Fig 4 $g(K_j)$ with respect to k_j

Given the strategy of designer j above, then designer i maximizes his profit. We tested many sets of parameters and find our approximation is appropriate. The profit of designer i with respect to his quality when $r_0 = 10$, $q_0 = 10$, $M = 10$, $d = 1$, $\gamma = 0.8$ is

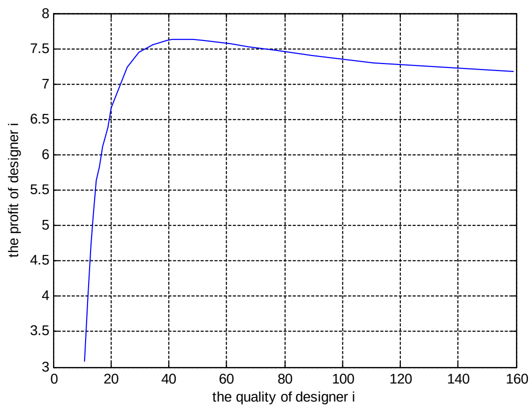

Fig 5 The profit of designer i with respect to his quality

We observe that the optimal quality is 43.22, and we get the corresponding optimal quality from the equilibrium is 43.13. The error is about 0.2%, it is small enough. And the corresponding error of the profit is very small too. We test a series value of quality under different parameters, and get the estimation error between the exact equilibrium quality and the estimated equilibrium quality. The estimation error of the quality is shown in figure 6.

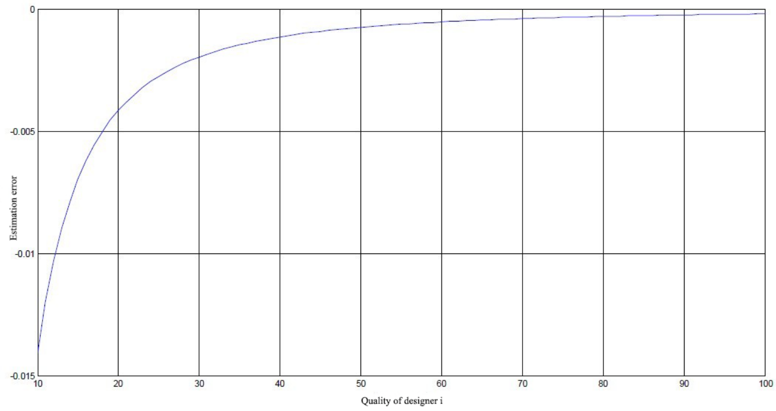

Fig 6 Estimation error of the quality

We observe that the error is small for reasonable quality. All the errors under the set of parameters are small than 1.5%. And the error decreases with the quality. The estimation error is less than 0.5% when the quality is high enough, so our approximation is reasonable.

5.3 Stage 1: Firm's reward decision

In the first stage, the firm determines the rewards. We can get probability of the value of m

$$\Pr(n_a + n_b = m) = C_M^m \bar{k}^m (1 - \bar{k})^{M-m}.$$

So the expected profit of the firm is

$$\begin{aligned} \pi_f \quad = \quad & \Sigma_{m=1}^M C_M^m \bar{k}^m (1-\bar{k})^{M-m} \Sigma_{n_a=0}^m C_m^{n_a} (0.5)^m \times [-r_w - m r_a + \\ & \max E\left(\frac{\lambda(Q_a - \frac{(1-\gamma)^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}} d)^2}{4Q_a}, \frac{\lambda(Q_b - \frac{\gamma^{2n_a-m}}{\gamma^{2n_a-m}+(1-\gamma)^{2n_a-m}} d)^2}{4Q_b} \right) \Bigg] . \end{aligned}$$

The corresponding cost parameter of Q_a, Q_b are K_a, K_b. And Q_i decreases with K_i, so $\Pr(K_a > K_i) = \Pr(K_1 > K_i)\Pr(K_2 > K_i) \cdots \Pr(K_{n_a} > K_i) = (1 - K_i)^{n_a}$. Then $\Pr(K_a < K_i) = 1 - (1 - K_i)^{n_a}$. So the expectation is

$$E(Q_a) = \int_0^{\bar{k}} (1 - (1 - K_i)^{n_a}) \sqrt{q_0^2 + 2(r_w + r_0)(h(\bar{k}) - h(K_i))} dK_i.$$

Similarly,

$$E(Q_b) = \int_0^{\bar{k}} (1-(1-K_i)^{M-n_a})\sqrt{q_0^2 + 2(r_w + r_0)(h(\bar{k}) - h(K_i))}dK_i.$$

Similar with subsection 4.4, we test many sets of parameters that meet the assumptions and find that the profit function is always a concave function in r_w and r_a when the parameters meet our assumptions. We can get a sequential optimal solution of r_w^*, r_a^* from numerical tests. There exists optimal rewards r_w^*, $r_a^* > 0$ which make the firm get the maximum profit π_f^*. To study what kind of rewards the firm gives, we test many sets value of λ and get the corresponding reward r_a^*, r_w^* and the profit π_f^*. And we find the rewards keep a same structure with the market size. When $\gamma = 0.8$, $d = 2$, $r_0 = 10$, $q_0 = 10$, $M = 2$, $u_r = 0$, we get the data shown in table 5.

λ	1	5	10	20	30	50	100	1000	10000
r_a^*	0.00	0.00	2.00	4.00	0.00	0.00	0.00	0.00	0.00
r_w^*	0.00	0.00	0.00	18.00	52.00	120.00	400.00	41000	1×10^6
π_f^*	0.193	0.96	2.01	8.95	26.33	89.51	389.76	40758	3.05×10^6

Table 5 The value of r_a^*, r_w^* and π_f^* under different market size

This rewards structure is robust under different parameters. When the market size is small enough i.e. $\lambda = 1$, 5, the equilibrium rewards are both zero. The demand is too small. If the firm gives reward to designers she may lose money. So no designer receives reward under this scenario. When the market size is more large i.e. $\lambda = 10$, the firm gives reward to all participated designers to thank for their participation but gives no reward to the designer whose design is picked by her. The reward given to all participated designers is small compared to the reward given to the designer whose design is picked. The firm can afford this part of reward under this scenario. Through this way, the firm can attract more designers to participate and acquire more information about customer preference. Then the firm has a higher profit than the scenario when she gives nothing to designers. When the market size reaches higher i.e. $\lambda = 20$, the firm gives both rewards to participated designers and the designer whose design is picked. Under this scenario, the demand is high so the firm has greater ability to provide both rewards. And these rewards improve information of customer and the quality. While the market size is large enough i.e. $\lambda = 30$, 50, 100, 1000, 10000, the firm only gives reward to the designer whose design is picked by her. Because the demand is high enough and the firm is confident about the sales. The effect of the participated designers is less important to the firm. They only provide information about customer preference. And the reward given to the designer whose design is picked already improves the quality. And this reward increases with the market size. Under this scenario, the effect of market size is more important than other parameters.

6 Sensitivity analysis

In this section we analyze the impacts of operational parameters to the decisions (the price and rewards) and profit of the firm. The operational parameters are: number of designers M, benchmark quality q_0, market size λ, accuracy of signal γ, the reward r_0 and the disutility d. We study these impacts one by one.

6.1 The impact of M

We can analyze the impact of the number of designers M by referring the analysis in section 5. First we analyze the impacts to the price. Then we analyze the impacts to the rewards. At last we analyze the impacts to the profit of the firm.

When all designers participate and all designs are designed on attribute b or a, the price increases with M. Because when there are more designers and they all design on the same attribute, the expected disutility decreases. This can be explained by the posterior probability of the customer preference. When there are more designers and they have the same decision on the attribute, the accuracy of the judgment about customer preferences increases. So the product reduces the risk of mismatch with customer preference. Then the firm can charge a higher price.

When some designers design on attribute a but some others design on attribute b, we observe that when $n_a = \frac{M}{2}$ (i.e. each half of the designers design on attribute a and b), the expected disutility of attribute a and b are the same. And the price is not affected by the total number of designers at this scenario. The number of designers does not affect the match of the product to the customer at this scenario. So the firm sets the same price for any number of designers.

When $n_a > \frac{M}{2}$, the expected disutility of attribute a is smaller than the expected disutility of attribute b. If the firm chooses the product with attribute a, the price increases with M. Because at this scenario, the more designers means more designers design on attribute a relatively and this decreases the disutility. So the firm charges a higher price. If the firm chooses the product with attribute b, the price decreases with M. Because at this scenario, the more designers means less designers design on attribute b relatively and this increases the disutility. So the firm charges a lower price.

When $n_a < \frac{M}{2}$, the expected disutility of attribute a is greater than the expected disutility of attribute b. If the firm chooses the product on attribute a, the price decreases with M. Because at this scenario, the more designers means less designers design on attribute a relatively and this increases the disutility. So the firm charges a lower price. If the firm chooses the product with attribute b, the price increases with M. Because at this scenario, the more designers means more designers design on attribute b relatively and this decreases the disutility. So the firm charges a higher price. Similarly, the impacts are the same for the scenario when there are some designers participate. Because the more designers, the greater probability that more designers participate. In conclusion we have corollary below.

Corollary 1 If all designs have the same attribute the firm can charge a higher price with more designers; if there are designs with attribute a and b, then the price increases with the number of designers if the firm chooses the attribute which is chosen by most designers, the price decreases with the number of designers if the firm chooses the attribute which is chosen by least designers.

The number of designers also has effects on the rewards r_w^* and r_a^*. When $\gamma = 0.8$, $d = 2$, $\lambda = 50$, $r_0 = 10$, $q_0 = 10$, rewards r_w^* and r_a^* with respect to M are

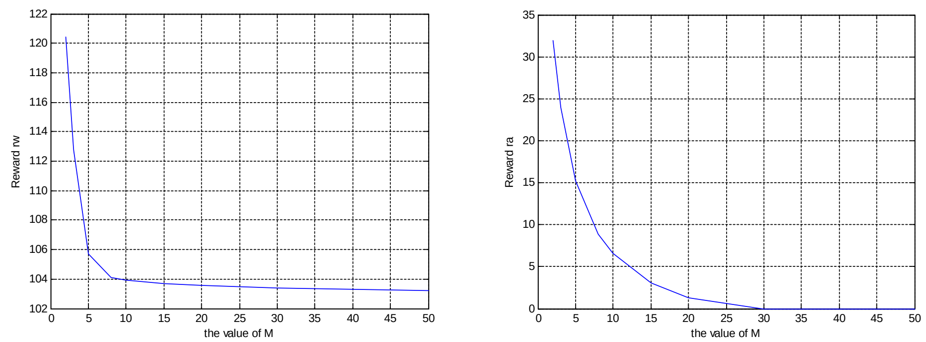

Fig 7 Sensitivity of rewards r_w^* and r_a^* with respect to M

We can find that the optimal rewards r_w^* and r_a^* decrease with M. When there are more designers, the firm can acquire more information about customer preference. The product matches customer preference with higher probability. And the firm has greater probability to receive higher quality, which increases the demand. So there is no need to provide more rewards to attract designers. And when there are enough designers, the firm gives no reward to all participated designers because there is no need to encourage more designers to participate. While the marginal effect to reward r_w^* decreases with the number of designers.

Refer to expression (4), when $\gamma = 0.8$, $d = 2$, $\lambda = 50$, $r_0 = 10$, $r_w = 10$, $r_a = 0$, $q_0 = 10$, the profit with respect to M is

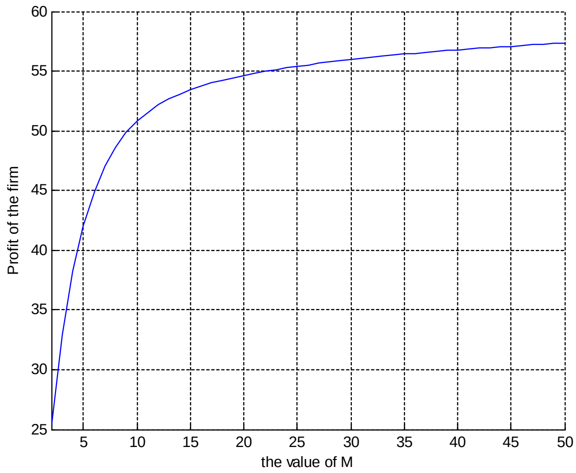

Fig 8 Sensitivity of firm's profit respect to M

We observe that the profit increases with M. More designers increase the opportunity of the firm to produce a product which matches customer preference closer and with a higher quality. In conclusion, we have the observation below.

Observation *When there are more designers participate, the firm gets a higher profit while gives lower rewards to the designers.*

6.2 The impact of λ

According to the expression of the price under different scenarios, we observe that the market size λ has no effects on the price. The price is only affected by the quality and the expected disutility.

As to the effect on the rewards r_w^* and r_a^*, we can observe from table 5 and get the figures

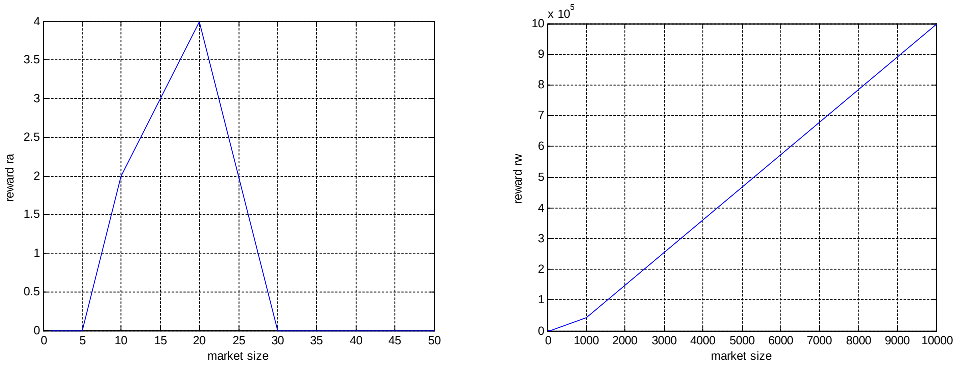

Figure 9 Sensitivity of rewards r_w^* and r_a^* with respect to λ

We observe that the reward r_w^* increases with the market size. When the market size is greater, the firm can make more profits because of higher demand, she has the incentive to give more rewards to designers. And this will let the designers have more motivation to submit a design with higher quality. So the firm can benefit from giving a higher reward r_w^*. While the firm gives higher reward r_a^* to all designers participated to thank for their participation when the market size is low enough. Through this way, designers have more incentive to participate and provide more information. But when the market size is large enough, the firm gives lower reward r_a^* to participated designers. Because the great market size already ensures high sales. The effect of information and quality becomes less important. So she does not need to give reward to every participated designer. The corresponding profit of the firm with respect to λ is

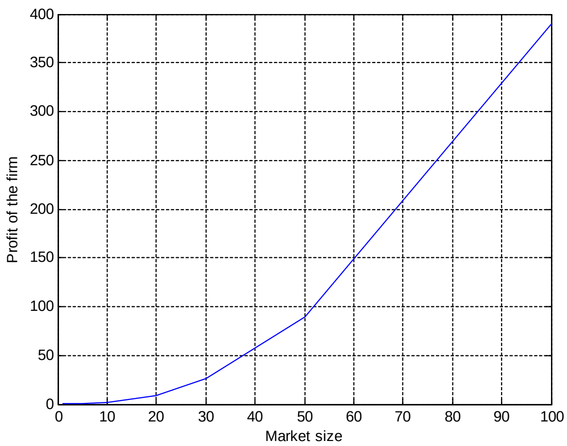

Fig 10 Sensitivity of firm's profit with respect to λ

We observe that the profit increases with the market size. The demand $D = \lambda\left(1 - \frac{\delta d+p}{Q}\right)$ is always increasing with λ. So the expected profit of the firm increases with market size. In conclusion, we have the observation below.

Observation *When the market size becomes greater, the firm gives a higher reward to the designer whose design is picked. The firm gets a higher profit. But the reward given to all designs is concave in market size.*

6.3 The impact of q_0

Higher benchmark quality leads to less designers' participation, then less information could be acquired by the firm. But there is higher probability to receive a higher quality. So the benchmark q_0 has positive effect on the price when it is low enough, when q_0 is higher the quality of all the designs becomes higher, and then the firm can charge a higher price. While the benchmark quality is high enough, less information may lead to mismatch to customer preference. Then the expected disutility is higher and the firm charges a lower price.

When $\gamma = 0.8$, $d = 2$, $\lambda = 50$, $r_0 = 10$, $r_a = 10$, $M = 2$, rewards r_w^* and r_a^* with respect to q_0 are

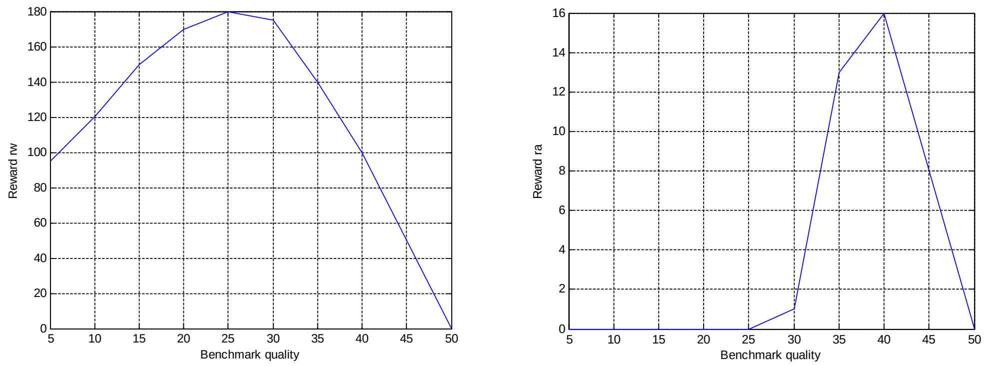

Figure 11 Sensitivity of rewards r_w^* and r_a^* with respect to q_0

When the benchmark quality is sufficiently high, both rewards decrease with the benchmark quality. The benchmark quality stops some designers with high cost parameter to participate. They cannot afford to give a quality greater than the high benchmark quality. Then the firm will lose more information about customer preference. This increases the risk of mismatch to customer preference. So the firm gives lower rewards to designers. While the benchmark quality is sufficiently low, the rewards increases with benchmark quality. This attracts more designers to participate. The firm can benefit from more information about customer preference. The corresponding profit of the firm is

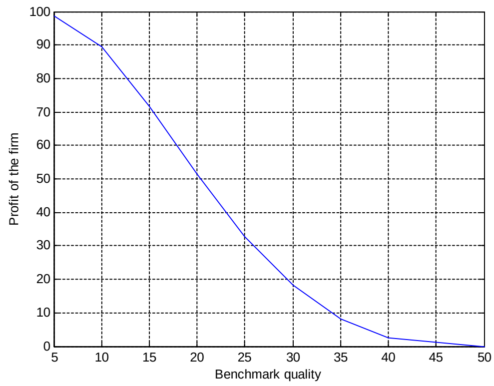

Figure 12 Sensitivity of firm's profit with respect to q_0

While the profit of the firm always decreases with the benchmark quality. Higher benchmark quality leads fewer designers to participate. Less information about customer preference hurts the firm's profit. In conclusion, we have the observation below.

Observation *When the benchmark quality is sufficiently high, the price and rewards given to the designers decrease with the benchmark quality. When the benchmark quality is sufficiently low, the price and rewards increase with benchmark quality. While the profit of the firm always decreases with the benchmark quality.*

6.4 The impact of γ

The accuracy of signals acquired by designers has positive effect on the price. The disutility becomes smaller when the signals are closer to the true customer preference. Then the firm can charge a higher price when the signals are more accurate. We have the corollary below.

Corollary 2 When the accuracy of information becomes greater, the firm charges a higher price.

When $d = 2$, $\lambda = 15$, $r_0 = 10$, $q_0 = 10$, $M = 10$, rewards r_w^* and r_a^* with respect to γ are

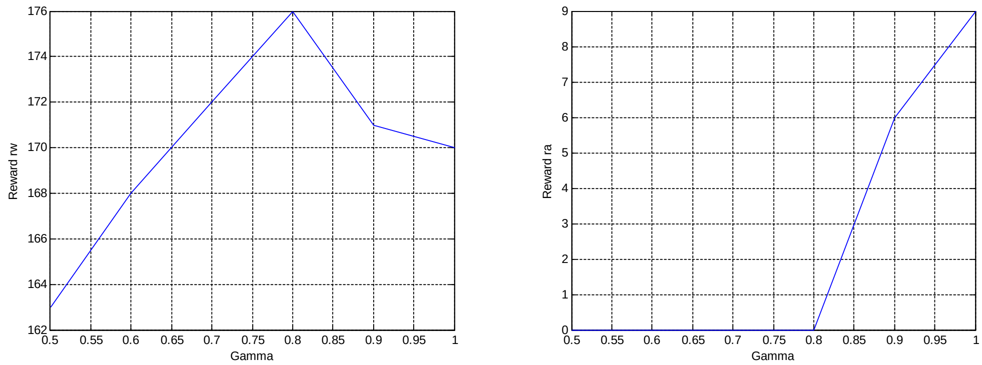

Fig 13 Sensitivity of rewards r_w^* and r_a^* with respect to γ

We observe that the optimal reward r_a^* always increases with the accuracy of the information. Because when the accuracy increased, the value of the designers' information increased. The firm can benefit from a higher matching with customer preference. So the firm should give higher reward to attract designers give this information to her. Otherwise the firm cannot get more valuable information with a smaller reward. While the accuracy of the information is high enough, the firm gives lower reward to the designer whose design is picked. Under this scenario, the firm already benefits enough from the information she acquired and there are enough designers participate. So she does not waste money on the only designer. The corresponding profit with respect to γ is

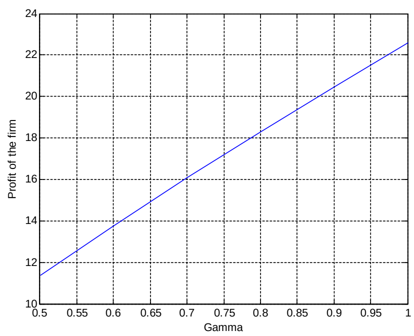

Fig 14 Sensitivity of firm's profit with respect to γ

We observe that π_f increases with γ. Because when the accuracy of the information acquired by designers increases, the probability of mismatch is smaller. So the profit will be higher. In conclusion, we have the observation below.

Observation *When the accuracy of information becomes greater, the firm always gives a higher reward to all participated designers, and the firm always gets a higher profit. When the accuracy of information is low enough, the firm gives higher reward to the designer whose design is picked, while gives lower reward to him when the accuracy is high enough.*

6.5 The impact of d

The disutility caused by mismatch of customer preference has negative effects on the price. When the disutility is high enough, the customer may not buy the product if the price is high. The customer will buy the product if and only if the price is low enough to make the utility of customer greater than zero. When the disutility is low, the firm can charge a high price that ensures the customer still buy the product. So the price decreases with the disutility. We have the corollary below.

Corollary 3 When the disutility caused by mismatch of customer preference becomes greater, the firm charges a lower price.

As to the effect on the rewards r_w^* and r_a^*, when $\gamma = 0.8$, $\lambda = 20$, $r_0 = 10$, $q_0 = 10$, $M = 2$, rewards r_w^* and r_a^* with respect to d are

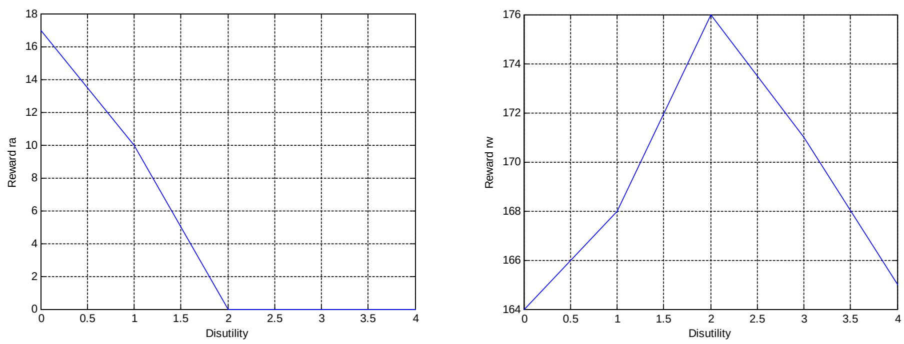

Fig 15 Sensitivity of rewards r_w^* and r_a^* with respect to d

We observe that when the disutility is low enough, the firm gives higher reward to the designer whose design is picked. The firm hopes get more information and higher quality to increase the demand and counteract the negative effect of the disutility. While both rewards r_w^* and r_a^* decrease with the disutility when the disutility is high enough. Under this scenario, the demand is lower, rewards hardly counteract the negative effects and cost the firm too much. So the firm gives lower rewards. The corresponding profit is

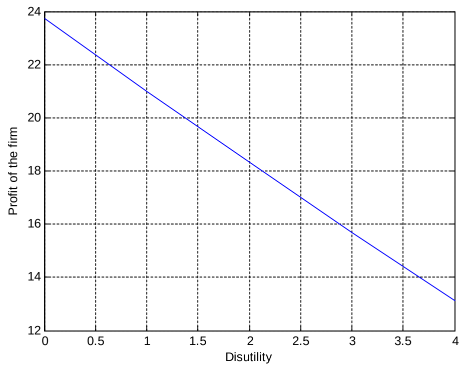

Fig 16 Sensitivity of firm's profit with respect to d

The profit of the firm decreases with the disutility. Higher disutility leads to lower demand and price. In conclusion, we have the observation below.

Observation *When the disutility of mismatch to customer preference becomes greater, the firm gives lower reward to all participated designers and gets lower profit. When the disutility is low enough, the reward given to the designer whose design is picked increases with the disutility. But when the disutility is high enough, the reward given to the designer whose design is picked decreases with the disutility.*

6.6 The impact of r_0

The price increases with the extra reward r_0. When the reward r_0 is higher, designers are willing to submit a higher quality. Then the firm receives a higher quality and she can charge a higher price. We have the corollary below.

Corollary 4 When the extra reward becomes greater, the firm charges a higher price.

When $\gamma = 0.8$, $d = 2$, $\lambda = 30$, $q_0 = 10$, $M = 2$, rewards r_w^* and r_a^* with respect to r_0 are

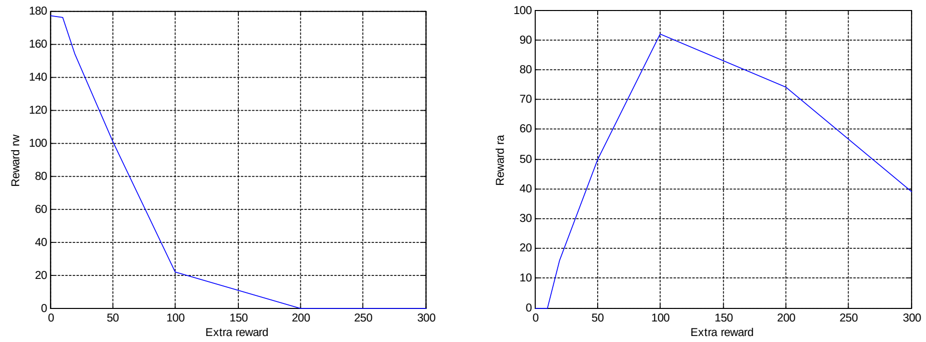

Fig 17 Sensitivity of rewards r_w^* and r_a^* with respect to r_0

We observe that the reward r_w^* decreases with r_0. Because when the designer whose design is picked get a higher extra reward r_0, he can accept a lower reward r_w^* from the firm respectively. The firm has no need to give high rewards to him. Higher extra reward helps the firm get a higher profit. While the reward which is given to all participated designers increases with extra reward when it is low enough. The firm needs to encourage designers to participate when they are not expected to receive high extra reward. But the reward which is given to all participated designers decreases with extra reward when it is high enough. Similarly, the firm has no need to give high rewards because designers may receive high enough reward from the participation. The corresponding profit is

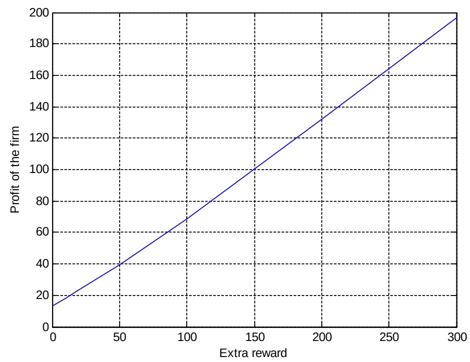

Fig 18 Sensitivity of firm's profit with respect to r_0

And we observe that the profit of the firm increases with r_0. When designers obtain higher reputation or experience from the process of participation, it means they have more incentive to participate and have a higher profit. So they give a higher quality to the firm, the firm can get a higher profit with the same reward given to the designer whose design is picked. The non-pecuniary reward helps designers gain honor in the professional industry. Some new designers cherish this kind of opportunity to exposure to the industry. They can gain more experience and learn from the professional designers. So this kind of benefits helps improve the quality of designs and increase the profit of designers as well as the firm. In conclusion, we have the observation below.

Observation *When the extra reward* r_0 is higher*, the firm gives lower reward to the designer whose design is picked. While the reward given to all participated designers increases with the extra reward first but then decreases. When the extra reward* r_0 is higher*, the firm gets a higher profit.*

7 Benchmark

In this section, we consider a game problem that the firm takes effort to do market research and design the product. We build the model of this game.

7.1 The model

There are only the firm and customer under this business model. Customer decides whether to buy the new product. The utility of a customer is the same with the new business model. In the benchmark model the firm takes effort to do market research, the cost parameter is k_a. She determines to buy the information about customer preference as a whole with accuracy $\gamma > 0.5$. Higher accuracy helps the firm match customer preference more accurately but costs more, while lower accuracy helps save cost but increases the risk of mismatch with customer preference. And she also takes effort to design the product with a cost parameter k_f. We use quadratic cost to model these two costs. Then the profit of the firm is

$$\pi_f = pD - \frac{1}{2}k_f Q^2 - \frac{1}{2}k_a(\gamma - 0.5)^2.$$

The sequence of events is as follows:

1. The firm determines to acquire information with accuracy γ.

2. The firm decides the attribute s, quality Q and the price p.

3. Customers' actual preference with the attribute realizes. Individual customer chooses whether or not to buy the product. The firm produces to meet the demand.

7.2 Analysis

We use backward induction to analyze the above multistage game. Stage 4 is the same as section 4.1, so we start from stage 3.

7.2.1 Stage 2: Firm's quality, attribute and price decision

At stage 2, the firm sets the price to maximize her profit. The profit is

$$p\lambda(1-\frac{E(\delta)d+p}{Q})\ -\frac{1}{2}k_fQ^2-\frac{1}{2}k_a(\gamma-0.5)^2.$$

The equilibrium price and corresponding profit are

$$\begin{aligned} p &= \frac{Q-E(\delta)d}{2}, \\ \pi_f &= \frac{\lambda(Q-E(\delta)d)^2}{4Q}-\frac{1}{2}k_fQ^2-\frac{1}{2}k_a(\gamma-0.5)^2. \end{aligned}$$

The firm chooses the attribute s and decides the quality Q to maximize her profit. The profit is

$$\frac{\lambda(Q-E(\delta)d)^2}{4Q}-\frac{1}{2}k_fQ^2-\frac{1}{2}k_a(\gamma-0.5)^2\ .$$

After acquiring information about customer preference, the firm decides to design on which attribute. Let Y represent the attribute acquired from the information. $\bar{Y}$ is the opposite attribute with the signal. Similar with the choice of outside designers in the new business model, she has five possible choices $s \in \{a,\ b,\ Y,\ \bar{Y},\ \varnothing\}$. We can easily observe that the profit of $s = Y$ is the largest one. So the firm always chooses the attribute which exactly the same with the signal. The profit of the firm is

$$\frac{\lambda(Q^2-2dQ(1-\gamma)+(1-\gamma)d^2)}{4Q}-\frac{1}{2}k_fQ^2-\frac{1}{2}k_a(\gamma-0.5)^2.$$

Then the firm determines the quality to maximize her profit. We have the theorem below.

Theorem 4 The equilibrium quality is

$$Q^* = \frac{1}{12k_f}\left\{\frac{\lambda^2}{\sqrt[3]{\lambda^3 + 12k_f\lambda d\sqrt{3}\sqrt{108(1-\gamma)^2d^2k_f^2 - \lambda^2(1-\gamma)} - 216\lambda(1-\gamma)d^2k_f^2}}\right.$$
$$\left. + \sqrt[3]{\lambda^3 + 12k_f\lambda d\sqrt{3}\sqrt{108(1-\gamma)^2d^2k_f^2 - \lambda^2(1-\gamma)} - 216\lambda(1-\gamma)d^2k_f^2} + \lambda\right\}.$$

Proof. The firm determines the quality to maximize the profit. Get the first-order derivative of the profit with respect to quality

$$\frac{d\pi_f}{dQ} = \frac{\lambda(Q^2 - (1-\gamma)d^2)}{4Q^2} - k_f Q.$$

When the first-order derivative equals to zero, we can get the equilibrium quality. Then we get the equation

$$4k_f Q^3 - \lambda Q^2 + \lambda(1-\gamma)d^2 = 0.$$

Solve the equation and we get the theorem above. ■

7.2.2 Stage 1: Firm's information accuracy decision

At stage 1, the firm determines to buy the information with accuracy γ. The expected profit of the firm is

$$\frac{\lambda(Q^* - (1-\gamma)d)^2}{4Q^*} - \frac{1}{2}k_f Q^{*2} - \frac{1}{2}k_a(\gamma - 0.5)^2.$$

We get the first-order derivative of the profit with respect to γ and set it to zero. Solve the equation and we can get the equilibrium γ. We test many sets of parameters and take a set parameter for example: $k_f = 0.5$, $k_a = 1000$, $\lambda = 100$, $d = 2$, the profit of the firm with respect to γ is

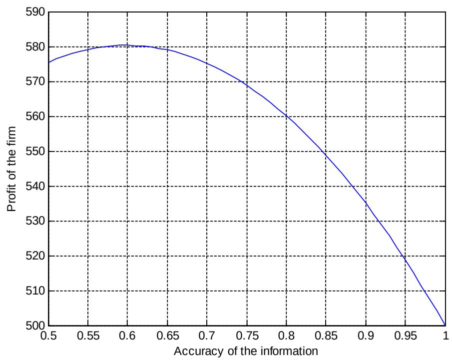

Fig 19 Firm's profit with respect to γ

We observe that firm's profit increases with the accuracy of the information first. More accurate information benefits the firm by matching customer preference perfectly. It costs too much to improve the accuracy when the information is accurate enough. So the profit of the firm decreases with the accuracy of the information when the accuracy is high enough. And she determines an optimal cost and gets the information about customer preference with corresponding accuracy.

7.3 Sensitivity analysis

7.3.1 The impact of λ

According to the expression of the quality, it increases with the market size. When the market size is smaller, the firm will provide a smaller quality. She bears a higher effort cost respectively under this scenario. But she prefers to provide a higher quality when the market size is greater because of large sales. Then she can get a higher profit. We observe that the market size has the same effects on the price. When the market size is small, she cannot charge a high price because no one wants to pay too much to buy a product with poor quality. But she can charge a higher price when the market size is greater. She has product with good quality to attract customers. Then we have the observation below.

Observation *When the market size becomes greater, the firm provides a higher quality and charges a higher price.*

The market size has great effects on the accuracy of information and profit of the firm. When $k_f = 0.5$, $k_a = 1000$, $d = 2$, the accuracy of the information and firm's profit with respect to λ are

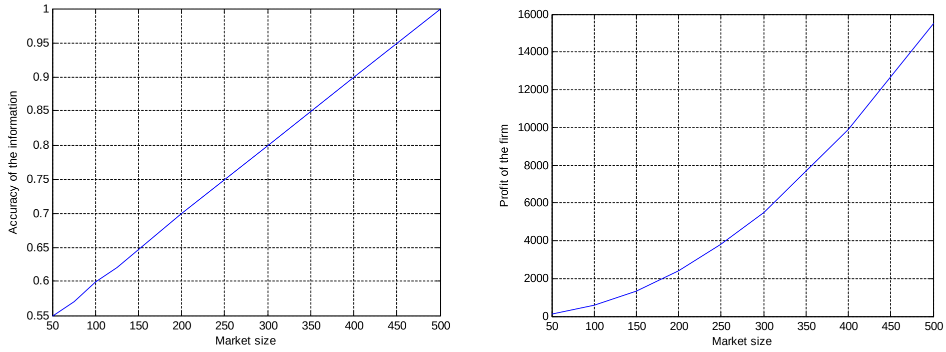

Figure 20 Optimal accuracy of information and firm's profit with respect to λ

We observe that when the market size is greater, the optimal accuracy of information and profit are higher. The effect to the optimal accuracy of information keeps the same. But when the market size is small, it has little effects on the profit because its limited effect to the demand. But when the market size is greater, the profit of the firm increases faster with it. It has greater positive effect to the profit with the same sales compared to small market size. The profit will be improved more effectively. So we have the observation below.

Observation *The accuracy of the information and the profit of the firm increase with the market size, and the profit increases faster when the market size is greater.*

7.3.2 The impact of d

When $k_f = 0.5$, $k_a = 1000$, $\lambda = 100$, optimal accuracy of information and the profit with respect to d are

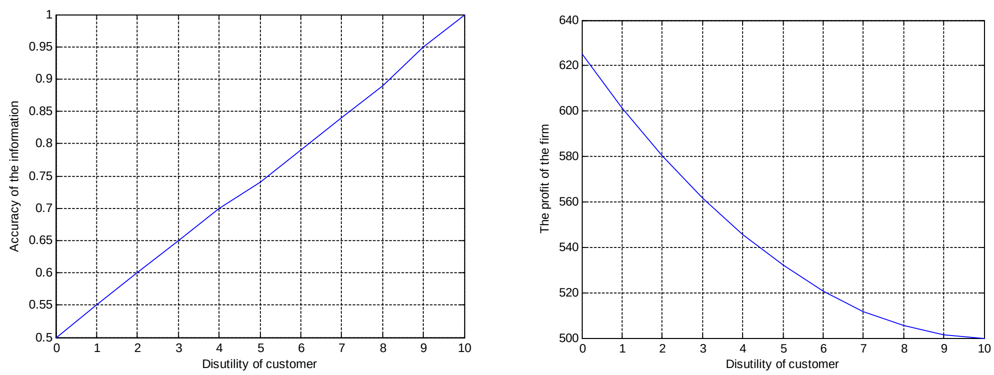

Figure 21 Optimal accuracy of information and the profit with respect to d

Optimal accuracy of information increases with the disutility. When disutility caused by mismatch to customer preference is higher, the firm needs to buy information with higher accuracy to avoid the high disutility. She has to reduce the risk of mismatch. Otherwise, the sale is expected too small with high probability. While the profit decreases with the disutility. When the disutility becomes greater the firm bears more risk to mismatch the customer preference, the demand is smaller and she collects fewer profits. In conclusion, we have the observation below.

Observation *When the disutility of mismatch to customer preference becomes greater, the firm buys information with higher accuracy but gets a lower profit.*

8 Comparing the two game models

In this section, we mainly compared different models and different ways the firm chooses the design.

8.1 Comparing different models

Our research focuses on the way of market research and product design. The firm or outside designers complete these two processes so there are four different scenarios of combination. We considered the scenario when designers complete the two processes and the scenario when the firm designs the product and does market research. When $\gamma = 0.5$ in our main model, the acquired information is useless for the firm. It means designers design the product only and the firm has no information about customer preference. In the benchmark model, the firm has information of customer preference and designs the product herself. We want to study the performance of combination of information and whether designers design the product. The profit of the firm when outside designers design the product but the firm has no information is

$$\pi_f^O = 0.5\bar{k}^2\{\tfrac{\lambda}{2}E(Q) + \tfrac{\lambda d^2}{8}E(\tfrac{1}{Q}) - \tfrac{\lambda d}{2}\} + C_2^1\bar{k}(1-\bar{k})$$
$$[\tfrac{\lambda}{4}E(Q) + \tfrac{\lambda d^2}{16}E(\tfrac{1}{Q}) - \tfrac{\lambda d}{4}] + (\bar{k}^2 - 2\bar{k})r_w - 2\bar{k}r_a.$$

The profit of the firm when she has information and designs the product is

$$\pi_f^I = \frac{\lambda(Q-(1-\gamma)d)^2}{4Q} - \frac{1}{2}k_f Q^2 - \frac{1}{2}k_a(\gamma - 0.5)^2.$$

The profit of the firm when outside designers design the product and the firm has information is

$$\pi_f^{IO} = 0.5\bar{k}^2\{\tfrac{\lambda}{2}E(Q) + \tfrac{\lambda(1-\gamma)^4 d^2}{4(\gamma^2+(1-\gamma)^2)^2}E(\tfrac{1}{Q}) + \tfrac{\lambda d^2}{16}E(\tfrac{1}{Q}) - \tfrac{\lambda(1-\gamma)^2 d}{2(\gamma^2+(1-\gamma)^2)} - \tfrac{\lambda d}{4}\}$$
$$+ C_2^1\bar{k}(1-\bar{k})[\tfrac{\lambda}{4}E(Q) + \tfrac{\lambda(1-\gamma)^2 d^2}{4}E(\tfrac{1}{Q}) - \tfrac{\lambda(1-\gamma)d}{2}] + (\bar{k}^2 - 2\bar{k})r_w - 2\bar{k}r_a.$$

Compare $\pi_f^O + \pi_f^I$ and π_f^{IO}, when the cost parameter k_f is small enough, $\pi_f^O + \pi_f^I > \pi_f^{IO}$; when the cost parameter k_f is high enough, $\pi_f^O + \pi_f^I < \pi_f^{IO}$. The profit of the second model decreases with the cost factor k_f. And the profit of the first model is stationary. When k_f is too large the traditional firm gets little profit, then she should choose the new way to avoid the large innovate cost and get more profits from the new way.

As shown in figure 22 ($q_0 = 20$, $M = 2$, $r_0 = 10$, $k_f = 0.5$, $k_a = 10$), we have the observation below.

Observation *When the market size is large enough or the disutility is high enough, the new business model dominates the benchmark business model. When both the disutility and market size are small enough, the firm prefers the benchmark business model. And the relative attractiveness of new business model versus benchmark model keeps the same when the market size is small enough.*

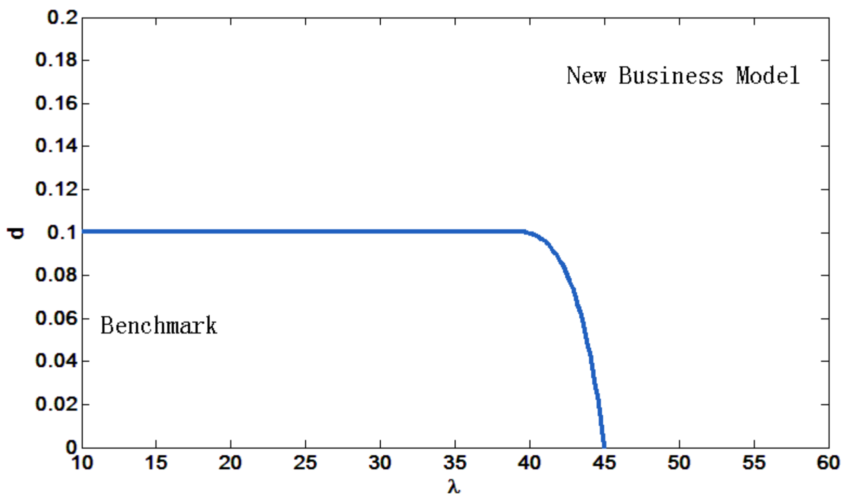

Fig 22 Two business models (effects of disutility and market size)

We observe that both demands are large enough when the market size is large enough. In the benchmark model, the firm takes more effort to provide higher quality with higher cost. However, in the new business model, the quality is provided by outside designers. They determine the quality according to their private cost parameter and this quality is not affected by the market size. So when the market size is large enough, the firm costs more in the benchmark. We also observe that the new business model dominates the benchmark business model when the disutility is high enough. Match customer preference is more important when the disutility is high enough. In the benchmark model, the firm needs to provide a higher quality and buy the information with higher accuracy to counteract the negative effect of high disutility. While in the new business model, both quality and accuracy of the information are provided by outside designers and they are not affected by the disutility. So the new business model has advantages when the market size is high enough or the disutility of customer is high enough. When both the disutility and market size are small enough, the firm prefers the benchmark business model. The relative attractiveness of new business model versus benchmark model keeps the same when the market size is small enough. It means the firm should not change the business model she used before. The benefit of outside designers' competitive product design is limited, so does the firm. So both business models cannot be better because of the limitation of the market size.

As shown in figure 21 ($q_0 = 15$, $M = 2$, $k_a = 100$, $d = 2$, $\lambda = 20$), we have the observation below.

Observation *The impact of extra reward on relative attractiveness of new business model versus benchmark model increases with extra reward. When the extra reward is high enough, the firm always prefers the new business model.*

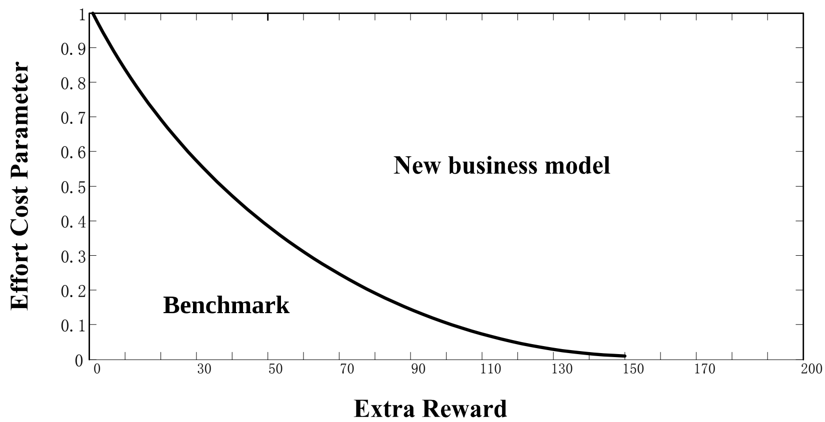

Fig 23 Two business models (effects of extra reward and effort cost)

When the extra reward is low enough, the designer whose design is picked by the firm cannot gain enough benefit from the participation. Then he hopes to receive higher reward form the firm. When extra reward is smaller, the firm needs to pay more to designers. And fewer designers participate because of the expected small extra reward. Then the firm receives less information about customer preference, expected disutility is higher and the demand is lower. This would hurt the attractiveness of new business model versus benchmark model. When the extra reward is high enough, outside designers may receive enough reward from the participation. The firm gives no rewards to them and still can attract enough designers to participate. That means the firm has no cost for the information about customer preference and the quality of the design. Even the cost of the firm in the benchmark model is zero, the attractiveness of new business model versus benchmark model is high enough. So the new business model has an advantage when the extra reward is high enough.

Next we investigate the different ways the firm chooses the design. The way when the firm always chooses the design which combined the quality and customer preference has been considered in section 5. We consider the way the firm only considers quality or customer preference now.

8.2 Comparing different ways of choosing design

We consider two other ways the firm chooses the design similar with section 5.

8.2.1 Highest quality

Stage 3: Firm's design choice When the firm receives n_a designs on attribute a and $m - n_a$ designs on attribute b, if the firm always chooses the design with highest quality, the profit is

$$\begin{aligned} & p \times \lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - mr_a \\ = & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - 0.5d}{Q}\right)\lambda p - r_w - mr_a. \end{aligned}$$

We get the equilibrium price and corresponding profit

$$\begin{aligned} p(n_a, m - n_a, a) &= \frac{Q - 0.5d}{2}, \\ \pi_f(n_a, m - n_a, a) &= \frac{\lambda (Q - 0.5d)^2}{4Q} - r_w - mr_a. \end{aligned}$$

Stage 2: Designers' participation and quality decision

$$\frac{d\pi_i}{dQ_i} = -(r_w + r_0)(M-1)(1-g^{-1}(Q_i))^{M-2}\frac{dg^{-1}(Q_i)}{dQ_i} - K_iQ_i.$$

Set the first-order derivative of the profit with respect to Q_i equal to zero, Then we have

$$Q_iQ_i' = -(r_w + r_0)(M-1)\frac{(1-K_i)}{K_i}^{M-2}.$$

The equilibrium quality is

$$Q_i^* = \sqrt{q_0^2 + 2(r_w + r_0)(M-1)\left[\sum_{j=0}^{M-2} C_{M-2}^{j}((-\bar{k})^{j-1} - (-K_i)^{j-1})\right]}.$$

And $\bar{k}$ can be solved from the equation below

$$(r_w + r_0)(1-\bar{k}^*)^{M-1} - 0.5\bar{k}^*q_0^2 + r_a = 0.$$

Stage 1: Firm's reward decision In the first stage, the firm determines the rewards. We can get probability of the value of m

$$\Pr(n_a + n_b = m) = C_M^m\bar{k}^m(1-\bar{k})^{M-m}.$$

So the expected profit of the firm is

$$\pi_f = \Sigma_{m=1}^{M} C_M^m \bar{k}^m (1-\bar{k})^{M-m} \Sigma_{n_a=0}^{m} C_m^{n_a} (0.5)^m E[\frac{\lambda(Q-0.5d)^2}{4Q} - r_w - mr_a].$$

The expectations related with the quality are

$$\begin{aligned} E(Q) &= \int_0^{\bar{k}} (1-(1-K_i)^m)Q_i^* dK_i. \\ E(\frac{1}{Q}) &= \int_0^{\bar{k}} \frac{(1-(1-K_i)^m)}{Q_i^*} dK_i. \end{aligned}$$

We can easily find that this scenario is the special case when $\gamma = 0.5$ in section 5. We conclude that the profit of the firm increases with the accuracy of the information in section 6.4. So the firm always gets a higher profit from the way choosing the design combined quality and customer preference compared with the way choosing the highest quality without consideration of customer preference.

8.2.2 Customer preference

Stage 3: Firm's design choice The firm chooses the design with attribute a if and only if $n_a > 0.5m$, the profit is

$$\begin{aligned} & p \times \lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - mr_a \\ = & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - \frac{(1-\gamma)^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d}{Q}\right)\lambda p - r_w - mr_a. \end{aligned}$$

We get the equilibrium price and corresponding profit

$$\begin{aligned} p(n_a, m - n_a, a) &= \frac{Q - \frac{(1-\gamma)^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d}{2}, \\ \pi_f(n_a, m - n_a, a) &= \frac{\lambda\left(Q - \frac{(1-\gamma)^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d\right)^2}{4Q} - r_w - mr_a. \end{aligned}$$

We observe that the profit increases with the quality. So when the firm chooses a design on attribute a she chooses the one with the highest quality. Similarly, she chooses a design on attribute b if $n_a < 0.5m$, the profit is

$$\begin{aligned} & p \times \lambda \left(1 - \frac{E(\delta)d + p}{Q}\right) - r_w - mr_a \\ = & -\frac{\lambda}{Q}p^2 + \left(\frac{Q - \frac{\gamma^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d}{Q}\right)\lambda p - r_w - mr_a. \end{aligned}$$

The equilibrium price and corresponding profit are

$$\begin{aligned} p(n_a, m - n_a, b) &= \frac{Q - \frac{\gamma^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d}{2}, \\ \pi_f(n_a, m - n_a, b) &= \frac{\lambda\left(Q - \frac{\gamma^{2n_a - m}}{\gamma^{2n_a - m} + (1-\gamma)^{2n_a - m}}d\right)^2}{4Q} - r_w - mr_a. \end{aligned}$$

We observe that the profit increases with the quality too. So when the firm chooses a design on attribute b she chooses the one with the highest quality.

We can conclude the profit of the firm is

$$\pi_f(n_a, m-n_a) = \frac{\lambda\left(Q - \frac{(1-\gamma)^{|2n_a-m|}}{\gamma^{|2n_a-m|}+(1-\gamma)^{|2n_a-m|}}d\right)^2}{4Q} - r_w - mr_a.$$

Note that $Q = Q_a$ (when $n_a > 0.5m$) or Q_b (when $n_a < 0.5m$).

Stage 2: Designers' participation and quality decision When designer i designs on attribute a, there are $M-1$ cases for other designers. There are $n_a - 1$ designers design on attribute a and $M - n_a$ designers design on attribute b.

$n_a - 1$	n_b	Pr(design of designer i is picked)
0	$M-1$	0
...	...	...
$n_a - 1$	$M - n_a$	$C_{M-1}^{n_a-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{n_a-1}$
...	...	...
$M-1$	0	$C_{M-1}^{M-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{M-1}$

Table 6 Probabilities when the design of designer i is picked

So the probability when the design of designer i is picked to produce is

$$\sum_{n_a=0.5M}^{M-1} C_{M-1}^{n_a-1}(0.5)^{M-1}(1-g^{-1}(Q_i))^{n_a-1}.$$

The profit of designer i is similar with the scenario when $M = 2$. To find the equilibrium quality of designer i, we calculate the first-order derivative of the profit with respect to Q_i

$$\frac{d\pi_i}{dQ_i} = -(r_w + r_0)\sum_{n_a=0.5M}^{M-1} C_{M-1}^{n_a-1}(0.5)^{M-1}(n_a-1)(1-g^{-1}(Q_i))^{n_a-2}\frac{dg^{-1}(Q_i)}{dQ_i} - K_iQ_i.$$

Set the first-order derivative of the profit with respect to Q_i equal zero, Then we have

$$Q_i Q_i' = -(r_w + r_0)(0.5)^{M-1} \sum_{n_a=0.5M}^{M-1} C_{M-1}^{n_a-1}(n_a - 1)(\frac{1}{K_i} - 1)^{n_a-2}.$$

The equilibrium quality is

$$Q_i^* = \sqrt{\frac{q_0^2 + 2(r_w + r_0)(0.5)^{M-1} \sum_{n_a=0.5M}^{M-1} C_{M-1}^{n_a-1}(n_a - 1)}{\left[(\bar{k} - \ln \bar{k} + \sum_{j=2}^{n_a-2} C_{n_a-2}^{j} \frac{(-1)^{n_a-3-j}}{(j-1)\bar{k}^{(j-1)}}) - (K_i - \ln K_i + \sum_{j=2}^{n_a-2} C_{n_a-2}^{j} \frac{(-1)^{n_a-3-j}}{(j-1)K_i^{(j-1)}})\right]}}.$$

And $\bar{k}$ can be solved from the equation below

$$(r_w + r_0) \sum_{n_a=0.5M}^{M-1} C_{M-1}^{n_a-1}(0.5)^{M-1}(1 - \bar{k}^*)^{n_a-1} - 0.5\bar{k}^* q_0^2 + r_a = 0.$$

Compare the equilibrium quality with the result in section 5.2, we observe that this equilibrium quality is smaller.

Stage 1: Firm's reward decision In the first stage, the firm determines the rewards. We can get probability of the value of m

$$\Pr(n_a + n_b = m) = C_M^m \bar{k}^m (1 - \bar{k})^{M-m}.$$

So the expected profit of the firm is

$$\begin{aligned}
\pi_f &= \Sigma_{m=1}^{M} C_M^m \bar{k}^m (1 - \bar{k})^{M-m} \Sigma_{n_a=0}^{m} C_m^{n_a} (0.5)^m \\
&\quad E\left[\frac{\lambda\left(Q - \frac{(1-\gamma)^{|2n_a-m|}}{\gamma^{|2n_a-m|} + (1-\gamma)^{|2n_a-m|}} d\right)^2}{4Q} - r_w - m r_a\right].
\end{aligned}$$

The expectations related with the quality are

$$\begin{aligned}
E(Q) &= \int_0^{\bar{k}} (1 - 0.5(1 - K_i)^{n_a} - 0.5(1 - K_i)^{M-n_a}) Q_i^* dK_i. \\
E(\frac{1}{Q}) &= \int_0^{\bar{k}} \frac{(1 - 0.5(1 - K_i)^{n_a} - 0.5(1 - K_i)^{M-n_a})}{Q_i^*} dK_i.
\end{aligned}$$

Similarly, the profit is equal to or smaller than the profit in section 5.3. Next we compare the profit under the way only considering the quality or customer preference. Designers compete with a quality greater than other $M-1$ designers' (the way choosing highest quality), and a quality greater than other $n_a - 1$ (or $n_b - 1$) designers' (the way choosing customer preference). We observe that the equilibrium quality by choosing the highest quality is always greater than the quality by choosing customer preference. The expected disutility when the firm chooses customer preference is smaller. So when the design chosen by customer preference happens to be the one with highest quality, the firm gets a higher profit by choosing customer preference. The quality is much greater than the disutility under our assumption. So when the design chosen by customer preference is not the one with highest quality, the firm gets a higher profit by choosing highest quality. We test many sets of parameters with different values of disutility and show the result when $\gamma = 0.8$ (or 1), $\lambda = 20$, $q_0 = 10$, $M = 2$, $r_0 = 10$

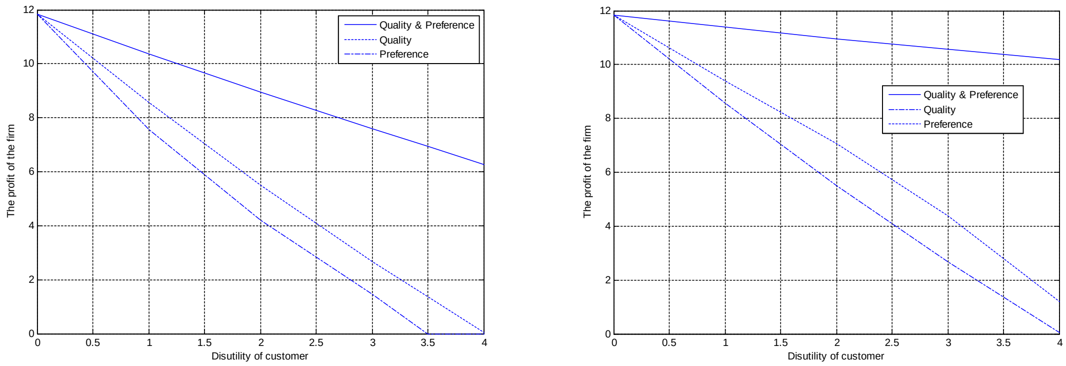

Fig 24 Three ways of choosing design ($\gamma = 0.8$ or 1)

The left figure shows result when $\gamma = 0.8$ and the right one shows result when $\gamma = 1$. We observe that when the disutility is low enough i.e. equals to zero, the firm gets the same profit by choosing the design with three different ways. None disutility means customer preference does not affect the sales. Note that the accuracy of information also has significant effects on profit. When there is disutility, we observe that quality is always more important than customer preference when the information accuracy is low enough. When the accuracy of information is low enough, the value of the information is less important. The firm cannot get an accurate design with customer preference because the information is not reliable. So the firm also prefers choosing highest quality. But when the accuracy of the information or the disutility is high enough, the firm prefers choosing customer preference. Under this scenario, the expected disutility is high. And the relative dominance increases with accuracy of the information or the disutility.

9 Summary

In this thesis, we investigate a new way that the firm does not take efforts to do market research or design the product. Outside competitive designers develop the product for the firm. The firm determines a reward to all participating designers and another reward to the designer whose design is picked to attract designers to propose their designs for the company. There are many designers who may participate and compete with each other for the rewards. The designers determine whether or not to begin a design according the rewards and their private cost. If they decide to design then they should also consider the customer preference and decide to design on which attribute. After designers submit their designs, the firm picks a design for production and determines the price of the product. And only the corresponding designer whose design is picked gets the reward. The customer decides whether or not to buy the product. Another business way is the traditional way in the benchmark. The firm determines to buy the information about customer preference with a suitable accuracy, then decides to design on which attribute. Then the firm starts to produce the product with a suitable quality and sets the price. At last customer's actual preference with the attribute is realized. Individual customer chooses whether or not to buy the product. The firm produces to meet the demand. We plan to answer the following questions: When should a firm choose this kind of new business model? What kind of reward structure the firm provides? How do designers determine the quality of the design? How does a firm choose the design?

By solving the game models, we answered the questions. We find that when the market size is large enough or the disutility is high enough, the new business model dominates the benchmark business model. When both the disutility and market size are small enough, the firm prefers the benchmark business model. And the relative attractiveness of new business model versus benchmark model keeps the same when the market size is small enough. The impact of extra reward on the relative attractiveness of new business model versus benchmark model increases with extra reward. When the extra reward is high enough, the firm always prefers the new business model.

When there are more designers that participate, the firm gives lower rewards to designers. When the market size becomes greater, the firm gives a higher reward to the designer whose design is picked. While the reward given to all designers increases with market size when it is low enough, and decreases when the market size is high enough. When the benchmark quality is sufficiently high, the rewards decrease with the benchmark quality. When the benchmark quality is sufficiently low, the rewards increase with benchmark quality. When the accuracy of information becomes greater, the firm always gives a higher reward to all designers who have participated. When the accuracy of information is low enough, the firm gives a higher reward to the designer whose design is picked, while giving a lower reward to him when the accuracy is high enough. When the disutility of mismatches with customer preference becomes greater, the firm gives a lower reward to all participated designers. When the disutility is low enough, the reward given to the designer whose design is picked increases with the disutility. But when the disutility is high enough, the reward given to the designer whose design is picked decreases with the disutility. When the extra reward is higher, the firm gives lower reward to the designer whose design is picked. However, the reward given to all designers who have participated increases with the extra reward first, but then decreases.

A designer participates if and only if his cost parameter is less than k. When the cost parameter is high enough i.e. greater than k, designers do not participate because they cannot afford a quality which is greater than benchmark quality. Under this scenario, he cannot have a profit greater than the reservation utility. And they can hardly get the only reward which is given to the designer whose design is picked by the firm, so they prefer not waste their effort. If a designer participates, each of them determines the quality according to his private cost parameter without knowing others' cost parameter because they cannot observe them. He submits a lower quality when his cost parameter is higher, while he submits a higher quality with a lower cost parameter. Each designer determines the quality according to the Pareto-optimal equilibrium quality we obtained.

The firm chooses one design by comparing the profits of different choices. It always chooses the one which maximizes its profit. We find that it always chooses the one with the highest quality when there are two designers. If there are more than two, it chooses the one with the highest quality when all designs are with the same attribute. While some designers design with attribute a but some others design with attribute b, the firm chooses the design with the highest quality with attribute a if the highest quality of design with attribute b is low enough. Otherwise it chooses the design with the highest quality with attribute b. That means both the quality and signals of customer preference determine the choice of the firm. When the benefit of high quality is greater than the benefits of the attribute with more signals, the firm chooses the one with higher quality, while it may choose the one with a lower quality but with more signals to match customer preference. When half designers design on each attribute, the firm always chooses the design with the highest quality no matter what the attribute is.

When the disutility is low enough, the firm prefers choosing the design with the highest quality. Low disutility means the effect of customer preference is minimal. Quality is more important with low disutility. When the accuracy of information is low enough, the value of the information is less important. The firm cannot get an accurate design with customer preference because the information is not reliable. So the firm also prefers choosing the highest quality. But when the accuracy of the information or the disutility is high enough, the firm prefers choosing customer preference. Under this scenario, the expected disutility is high. And the relative dominance increases with the accuracy of the information or the disutility.

Operational parameters have significant effects on the decisions and the profits. When there are more designers that participate, if all designs have the same attribute the firm can charge a higher price; if there are designs with attributes a and b, then the price is higher if the firm chooses the attribute chosen by most designers, while the price is lower if the firm chooses the attribute chosen by least designers. The price decreases with the benchmark quality when the benchmark quality is sufficiently high. The price increases with benchmark quality when the benchmark quality is sufficiently low. When the accuracy of information or the extra reward becomes greater, the firm charges a higher price, while when the disutility caused by mismatches with customer preference becomes greater, the firm charges a lower price.

When there are more designers that participate, the firm gets a higher profit. And the margin benefit becomes smaller when there are enough designers. When the market size becomes greater, the firm gets a higher profit. And the benefit becomes greater when the market size is greater enough. However, the profit of the firm always decreases with the benchmark quality and the disutility. When the accuracy of information or extra reward is higher, the firm gets a higher profit.

The analysis and results of this thesis is suitable for vertical preference. Products which can be divided into vertical preference can be applied by this model. For those that can be divided into horizontal preference, we may analyze the model using horizontal preference. This is one possible topic of our future research. Only a business model like Threadless's which combines market research with product design can be applied by this model.

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