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DECREASING COST AND PROFIT  
MAXIMIZATION IN COURNOT  
DUOPOLY MODELS

by

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Hong Kong  
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## DECREASING COST AND PROFIT MAXIMIZATION IN COURNOT DUOPOLY MODELS

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## ABSTRACT

This paper raises a case against a maxim in microeconomics which says that to achieve highest profits means to maximize the profit function, and vice versa. The truth is rather that for the Cournot firms in a duopoly market which share the advantage of decreasing cost in production, the maximization of an objective function which takes the revenue-cost ratio rather than profit proper as the maximand may sometimes lead to higher profit than otherwise. This new finding also opens a way to the amendment of a related conventional wisdom in industrial economics which says that although unexploited profit exists in duopoly market, it can not be realized, by whole or by part, if Cournot behaviour is assumed. Actually, depending on the structure of market demand and costs, part of the unexploited profit can sometimes be realized. All the Cournot firms needs to do in such cases is to change their operation strategy from the maximization of a profit function to the maximization of the revenue-cost ratio. The new strategy, as it turns out, will enable these Cournot firms to enjoy higher profits than when they aim at maximizing profits in the first place.

To sum up, this paper endeavours to send a new message to the duopoly world: sometimes it needs to take a round-about strategy to achieve highest profits.

## I. INTRODUCTION

The hypothesis of profit maximization lies at the core of production economics and managerial economics. We have long been taught that to find out an output level corresponding to maximum profit, we simply need to maximize an objective function which takes profit as the maximand and quantity as the decision variable. Given that certain second order conditions are also satisfied, the first order conditions of optimization will then lead us to the optimal solution. This standard and so far unchallenged procedure does work perfectly well in almost all cases discussed in the economics literature. Yet, as this paper endeavours to show, it may break down in the case of duopoly with decreasing cost in production.

The truth is that for the firms in a duopoly market, where both act according to the Cournot behaviour and share the advantage of decreasing cost in production, there may exist some other objective functions which will lead to even higher profits. One of such possibilities is explored in this paper. It is found that in some cases, depending on the structure of demand and costs, the maximization of an objective function which takes the revenue-cost ratio rather than profit proper as the maximand may actually lead to higher profits than otherwise. To maximize a profit function is thus no guarantee to achieve highest profits when the firms are interdependent in the Cournot way.

This new finding also calls our attention to reexamine the role played by decreasing cost in duopoly and oligopoly markets. Traditional oligopoly models have concentrated on industries with either constant or increasing average costs. To the extent that industries with U-shaped average cost curves are also covered in the study, the foci are usually on the investigation of the necessary and/or sufficient conditions for existence, uniqueness, and stability, etc. (Novshek, 1980, 1985; Seade, 1980; Gaudet and Salant, 1991). No one single previous work has raised any doubt about the appropriateness of taking the profit function as the objective function for profit maximization even when decreasing cost is also at the firms' command. Some important results, as a consequence, are missing in the oligopoly literature.

It is true that research works associated with the revenue maximization hypothesis (Baumol,

1967; Amihud and Kamin, 1979) and other managerial theories of the firm (Thompson and Formby, 1993) also challenge the taking of the profit function as the objective function to be optimized. But their targets are totally different from ours. In this paper, the object being challenged is not the hypothesis of profit maximization per se. It is rather the pre-occupied conviction that profit maximization always means maximizing a profit function.

The rest of this paper consists of four sections. The next section examines the necessary and sufficient conditions for optimality when the revenue-cost ratio is maximized. It is followed by a section which makes use of a simple short run decreasing cost model to investigate the differences in profitability under the two alternative objective functions -- the profit function and the revenue-cost ratio function. The general case with a long run U-shaped average cost curve is discussed subsequently. The last section contains the concluding remarks.

## II. MAXIMIZATION OF THE REVENUE - COST RATIO (MRC)

For a firm looking for the maximization of the revenue-cost ratio with respect to output in a monopoly or duopoly market, its objective function simply takes the form of

$$\frac{R(Q)}{C(Q)} \quad (2 - 1)$$

where Q is output, R is revenue, C is cost, and  $\pi$  is profit. The first order condition implies that

$$\frac{MR}{MC} = \frac{R}{C} \left( = \frac{P}{AC} \right) \quad (2 - 2)$$

In contrast to the first order condition under the maximization of a profit function (MPF), (2-2) says that the ratio of marginal revenue (MR) to marginal cost (MC) should not be 1, but instead should be equal to the ratio of price (P) to average cost (AC).

Several implications follow immediately. First, as MR is always smaller than P, MRC requires that MC be smaller than AC. In other words, the MRC solution can only appear in the stage



of decreasing cost<sup>1</sup>. Second, for solutions with positive profits, P must exceed AC which in turn requires that MR be greater than MC. As a consequence, optimal outputs under MRC can never be larger than optimal outputs under MPF. Third, when the demand curve is tangent to the average cost curve, both MRC and MPF have the same output solution which corresponds to zero profit. Fourth, in the classical Cournot duopoly model of mineral water, which incurs no cost other than fixed costs, MRC is also equivalent to MPF. But in that case profits are of course positive for each firm. Fifth, optimal output under MRC is not independent of AC, in contrast to the case under MPF. In this regard, the MRC hypothesis shares a common feature with the hypothesis of revenue maximization (RM) in the theories of the firm.

Next, consider the sufficient condition for optimality under MRC. Maximization is assured when the second derivative of the revenue-cost ratio with respect to Q is negative. That is to say, the sufficient condition is

$$\frac{1}{C^3} [C^2 \times MR' - 2C \times MR \times MC - R \times C \times MC' + 2R \times MC^2] < 0, \quad (2 - 3)$$

where MR' and MC' are the first derivatives of MR and MC respectively. After substitutions with (2-2), (2-3) can be simplified first as

$$[C \times MR' - R \times MC'] < 0, \quad (2 - 4)$$

and further as

$$\left( \frac{MR'}{MR} \right) < \left( \frac{MC'}{MC} \right) \quad (2 - 5)$$

The meaning of (2-5) is quite interesting. It means that instead of having  $MR' < MC'$  as in the case of MPF, it now has the ratio of MR' to MR smaller than the ratio of MC' to MC. Or, if expressing in terms of elasticities, (2-5) implies that the elasticity of MR should be smaller than the elasticity of MC for the sufficiency of optimum, i.e.,

---

1 Decreasing cost can happen in both short run and long run. In short run models with fixed costs, decreasing cost appears when marginal cost is constant. In long run models, decreasing cost could be associated with increasing return to scale.

$$\left(\frac{MR'}{MR}\right)Q < \left(\frac{MC'}{MC}\right)Q \quad (2 - 6)$$

As explained earlier, the optimal output under MRC can never be greater than optimal output under MPF. Nevertheless, it does not follow that profits under MRC will never be larger than profits under MPF unless the firm is a monopoly firm. If the firm is a duopoly firm instead, profits under MRC could be greater than profits under MPF. This is the main point we endeavour to establish in the following pages.

### III. SHORT RUN MODELS WITH DECREASING AC CURVES

#### (A) Cournot - Nash Solutions under the Two Alternative Objective Functions

Consider two firms in a duopoly market which share the same technology and face the same short run cost function as the following:

$$C_i = b + eQ_i, \quad i = 1, 2 ; b > 0, e > 0. \quad (3 - 1)$$

Market demand is given by the inverse demand function

$$P = a - (Q_1 + Q_2), \quad a > 0 \quad (3 - 2)$$

When the profit function is maximized, the Cournot-Nash solution for both firms, denoted by  $Q_b$ , is

$$Q_b = \frac{a - e}{3} \quad (3 - 3)$$

which is independent of fixed cost and therefore of average cost. In contrast, the Cournot-Nash solution for both firms when the revenue-cost ratio is maximized, denoted by  $Q_a$  below, is dependent on fixed cost:

$$Q_a = \frac{-3b + \sqrt{9b^2 + 4abe}}{2e} \quad (3 - 4)$$

This fixed cost factor has a direct role to play in the determination of profit difference under the two hypotheses, as we will see.

The solution  $Q_a$  in (3-4) is derived as follows. From the optimality condition for MRC

solutions given in last section, the reaction curve for firm  $i$ ,  $RC_i$ , can be obtained implicitly by the following quadratic equation

$$eQ_i^2 + 2bQ_i + (bQ_j - ab) = 0, \quad i \neq j, \quad i, j = 1, 2. \quad (3-5)$$

Then, by imposing  $Q_1 = Q_2 = Q_a > 0$  as an equilibrium condition on (3-5), we will have the Cournot-Nash solution  $Q_a$  just as given in (3-4).

(B) Conditions for Higher Profits for Each Firm under MRC

Given that  $Q_a$  is always smaller than  $Q_b$  except in the cases of zero profit and zero marginal cost, is it possible to have  $\pi_a$  (firm's profits under MRC) greater than  $\pi_b$  (firm's profits under MPF)?

The answer is positive and is provided by the following proposition:

Proposition 1:  $\pi_a > \pi_b$  if and only if  $2Q_a > Q_b$

Proof: By definition,  $\pi_a = P_a Q_a - b - eQ_a$

$$= (a - e - 2Q_a) Q_a - b.$$

Similarly,  $\pi_b = (a - e - 2Q_b) Q_b - b$ . It follows that

$$\begin{aligned} (\pi_a - \pi_b) &= (a - e - 2Q_a) Q_a - (a - e - 2Q_b) Q_b \\ &= 2(Q_b^2 - Q_a^2) - (a - e)(Q_b - Q_a) \\ &= (Q_b - Q_a) [2(Q_b + Q_a) - (a - e)]. \end{aligned}$$

But  $(a - e) = 3Q_b$  from (3 - 3). Therefore,

$$(\pi_a - \pi_b) = (Q_b - Q_a) (2Q_a - Q_b).$$

Since  $Q_b > Q_a$ , the proposition is proved.

Proposition 1 can also be expressed in terms of the three parameters in the model. The condition of  $2Q_a > Q_b$ , after substitutions from (3 - 3) and (3 - 4), is equivalent to the condition of

$$\sqrt{3 [ 9b^2 + 4abe ]} > 9b + e(a - e). \quad (3 - 6)$$

After taking squares on both sides of (3-6) and making rearrangement, it becomes

$$18b(a + e) > e(a - e)^2 \quad (3 - 7)$$

We have, therefore,  $\pi_a > \pi_b$  if and only if (3-7) holds. For example, when  $a = 400$ ,  $b = 250$ , and  $e = 10$ ,  $\pi_a (= 17172)$  is greater than  $\pi_b (= 16650)$ .

Inequality (3-7) also provides a convenient base for simulation studies. The impact of changing  $b$ , the fixed cost factor, on the differences in profitability is the most obvious. Other things being equal, the larger the  $b$  is, the higher is the possibility of having  $\pi_a > \pi_b$ . Impacts from changing  $a$  and  $e$  can also be traced. But we will not perform this sort of simulation experiments here.

#### IV. LONG RUN MODELS WITH U - SHAPED AC CURVES

The duopoly model is now generalized to the following one:

$$\text{demand function: } P = a - b(Q_1 + Q_2), \quad a > 0, b > 0, \quad (4 - 1)$$

$$\begin{aligned} \text{cost functions: } C_i &= dQ_i - eQ_i^2 + fQ_i^3, & i &= 1, 2 \\ & & d, e, f &> 0. \end{aligned} \quad (4 - 2)$$

The cost function in this case corresponds to a cubic cost function with no fixed cost. It represents therefore a long run situation and implies a U-shaped AC curve. One additional parametric restriction,  $3df > e^2$ , is required for this function in order to assure that the marginal cost is always positive.

##### (A) Cournot - Nash Solutions under MPF and MRC

The Cournot - Nash solutions under the MPF hypothesis can be derived in a similar way as is done in last section. First, from the optimality condition  $MR = MC$ , the reaction functions  $RC_i$  are obtained:

$$\begin{aligned} RC_i: \quad 3fQ_i^2 - 2(e - b)Q_i - (a - d - bQ_j) &= 0, & i \neq j, \\ & i, j = 1, 2. \end{aligned} \quad (4 - 3)$$

The equilibrium condition  $Q_1 = Q_2 = Q_b > 0$  is then imposed on either one of the  $RC_i$  in order to solve for an equilibrium output. Since  $RC_i$  is a quadratic equation, there will be two solutions. But only the one associated with a positive squared root is acceptable based on the ground that when there are two outputs corresponding to  $MR = MC$  only the larger one is the true optimum. The final result is

$$Q_b = \frac{(2e - 3b) + \sqrt{(2e - 3b)^2 + 12f(a - d)}}{6f} \quad (4 - 4)$$

Solutions under MRC can be derived by the same procedures. From the optimality condition  $(P/MR) = (AC/MC)$ , reaction functions appear as

$$RC_i: \quad bfQ_i^2 - 2f(a - bQ_j)Q_i + (ae - bd - beQ_j) = 0. \quad i \neq j, \\ i, j = 1, 2 \quad (4 - 5)$$

After imposing  $Q_1 = Q_2 = Q_a$  on (4-5) two solutions are obtained as follows:

$$Q_a = \frac{(2af + be) \pm \sqrt{(2af + be)^2 - 12bf(ae - bd)}}{6bf} \quad (4 - 6)$$

The positive squared root solution, however, is not acceptable in this case. Because, if the positive squared root is taken, the MR for each firm, which equals to  $(a - 3bQ_a)$ , will become negative. The feasible solution, therefore, must be the one associated with the negative squared root.

Two additional points should be mentioned before going further. First, in order to have  $Q_a$  greater than zero,  $(ae - bd)$  must be non-negative. This is just a condition for non-trivial solutions. Second, in order to obtain a real value solution for  $Q_a$ , imaginary root must be ruled out. A sufficient condition for achieving this is simply  $(af \geq 2be)$ .

#### (B) Conditions for Higher Profits for Each Firm under MRC

Consider the case with positive profits under both MRC and MPF. Since then  $Q_a$  is always smaller than  $Q_b$ , total revenues for each firm must always be lower under MRC as price becomes higher and demand is elastic. But cost is also lower under MRC. So it is not necessary that profits under MPF will be higher as we usually think it is. To be sure, if monopoly is what we are considering, the result will definitely be in favour of MPF. But if duopoly is the case we are considering, results may turn out differently. Higher profits from MRC can still be achieved if the reduction in costs exceeds the reduction in revenues following the change from  $Q_b$  to  $Q_a$ . The converse is also true. In other words, we have  $\pi_a > \pi_b$  if and only if  $\Delta C > \Delta R$ , where  $\Delta C = C_b - C_a$ , and  $\Delta R = R_b - R_a$ . The detail is given in the following proposition.

Proposition 2: Profits under MRC will be higher than profits under MPF if and only if

$$bQ_a > (Q_b - Q_a) [f(2Q_b + Q_a) + b - e]. \quad (4 - 7)$$

Proof: Since  $\Delta R = P_b Q_b - P_a Q_a$

$$= (a - 2bQ_b)Q_b - (a - 2bQ_a)Q_a$$

$$= (Q_b - Q_a) [a - 2b(Q_b + Q_a)],$$

and  $\Delta C = d(Q_b - Q_a) - e(Q_b^2 - Q_a^2) + f(Q_b^3 - Q_a^3)$

$$= (Q_b - Q_a) [d - e(Q_b + Q_a) + f(Q_b^2 + Q_a Q_b + Q_a^2)],$$

so  $(\pi_a - \pi_b) = \Delta C - \Delta R$

$$= (Q_b - Q_a) [(d - a) + (2b - e)(Q_b + Q_a) + f(Q_b^2 + Q_a Q_b + Q_a^2)]$$

(4 - 8)

Now, let  $k = Q_b - Q_a > 0$ . It follows that:

- i.  $Q_b + Q_a = 2Q_b - k$ ;
- ii.  $Q_a Q_b = Q_b^2 - kQ_b$ ; and
- iii.  $Q_a^2 = Q_b^2 - 2kQ_b + k^2$ .

Substituting these results into (4-8) and rearranging the terms, we have the following:

$$(\pi_a - \pi_b) = k \{ [3fQ_b^2 - (2e - 3b)Q_b - (a - d)] + [(b - 3fk)Q_b + k(fk - 2b + e)] \}.$$

(4 - 9)

But the terms in the first bracket on the right hand side add up to zero due to (4-3). Therefore,

$$(\pi_a - \pi_b) = k [(b - 3fk)Q_b + k(fk - 2b + e)]. \quad (4 - 10)$$

At this point it is clear that  $(\pi_a - \pi_b) > 0$  if and only if

$$bQ_b + fk^2 + ek > 3fkQ_b + 2bk. \quad (4 - 11)$$

Inequality (4-11) can also be expressed as

$$bQ_b > k (3fQ_b - fk + 2b - e). \quad (4 - 12)$$

Putting  $k = (Q_b - Q_a)$  back to (4-12), it becomes

$$bQ_b > (Q_b - Q_a) [f(2Q_b + Q_a) + 2b - e].$$

Or equivalently,

$$bQ_a > (Q_b - Q_a) [f(2Q_b + Q_a) + b - e]$$

This is the necessary and sufficient condition for  $\pi_a > \pi_b$ .

Q.E.D.

Inequality (4-7) can also be expressed in terms of the five parameters (a, b, d, e, and f) in the model. But the resulting expression will be quite complicated. Suffice to say here is that a variety of parametric configuration could satisfy (4-7). It is not impossible, therefore, to have  $\pi_a$  be larger than  $\pi_b$ .

For example, when  $a = 465$ ,  $b = 4$ ,  $d = 400$ ,  $e = 10$ , and  $f = 1/3$ , we have  $Q_a = 9.8$  and  $Q_b = 13$ . Inequality (4-7) is thus satisfied as the value on its left hand side becomes 39.2 and on right hand side is only 19. In this example, profit under MRC ( $\pi_a$ ) is 515, and under MPF ( $\pi_b$ ) is only 451. Profit can therefore be raised by 14.2% as a result of simply changing the firms' operation strategy from maximizing a profit function to maximizing a revenue-cost ratio function. Other examples can also be constructed.

## V. CONCLUDING REMARKS

In the industrial economics literature, it becomes well known that in the Cournot oligopoly model industry profit is not maximized. Each firm could increase profits by reducing output, provided its rivals responded in the same way. But outcomes of this sort cannot be attained if Cournot behaviour is assumed.

But this generally accepted view is only partly true. In this paper we have demonstrated that it is possible to raise the profits for each firm by reducing output even though the Cournot behaviour is still assumed, provided that decreasing cost technology in short run or increasing return to scale technology in long run is also at the firms' command. To achieve that possible result all the firms need to do is to maximize a revenue-cost ratio function instead of a profit function. Under certain circumstances, the maximizing of the revenue-cost ratio will not only lead to the highest revenue-cost ratio for the firms but also lead to higher profits than when the firms aim at maximizing their profits in the first place. Part of the unexploited profits existed before could thus be realized. This is a new finding which has been neglected in the literature all along.

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