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Information leakage and Stackelberg leadership in Cournot competition

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INFORMATION LEAKAGE AND STACKELBERG LEADERSHIP IN COURNOT COMPETITION

by
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ABSTRACT

Information Leakage and Stackelberg Leadership in Cournot Competition

by

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In duopoly Cournot competition with sequential moves, it is well known that each player prefers Stackelberg leadership without demand uncertainty. We study the same game when the demand is uncertain, and firms possess some private information about the uncertain demand. There are two effects of private information in this game. First, when the Stackelberg leader moves first, its private information is leaked to, or inferred by the Stackelberg follower via the output quantity. Hence, the Stackelberg follower makes decision based on more accurate information than the leader. Second, the leader incurs a cost to signal its information to the follower, which hurts the leader. Both effects hurt the Stackelberg leader, then the follower may earn more ex ante profit than the leader. When the demand is continuous, Gal-or (1987) assumes that firms follow linear decision rules and reports that the follower always sets a higher output quantity than the leader and earns more profit than the leader. However, our study finds that it is true if and only if the demand is unboundedly distributed. Otherwise, the Stackelberg leader's Pareto-optimal output quantity is not linear in its private information unless it observes the highest signal, and the follower does not always earn more ex ante profit than the leader. When the demand is discretely distributed, we study how the number of demand states influences the effect of cost of signaling. With more demand states, the effect of cost of signaling on the leader becomes more significant, and the follower may earn more ex ante profit than the leader.

Keywords: Information leakage, The First-mover advantage, Cournot competition, Signaling

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

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Date 2015.09.11

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1 INTRODUCTION

The importance of information to a firm is apparent in competitive economic environment and information society, especially for firms which sell products in market with uncertain demand. More accurate information can help decision makers better informed of the uncertain demand, hence firms can make decisions more wisely. The information can be sales data, sales forecasts, and so on. In reality, a firm's information may be leaked to other firms directly or indirectly, hence the firm may suffer the information leakage. Singer (1999) gives an example of information leakage. The sales information of music retailer Newbury Comics was leaked to other retailers like Wal-Mart via the SoundScan, an information and sales tracking system. Wal-Mart can use the information for inventory planning and replenishment. Hence, Wal-Mart benefited from the leaked information, and Newbury Comics lost the control of its valuable information.

We consider a Stackelberg competition model, in which two identical firms (a leader and a follower) sell homogeneous product to customers and decide their respective output quantity sequentially. If firms have complete information about the market, the leader earns more profit than the follower and both of them prefer Stackelberg leadership. This is true because as a leader, the firm can preempt its follower by investing a larger capacity, which guarantees a higher profit than the follower.

However, in an incomplete information environment where each firm possesses some private information about the uncertain demand, firms' preference for Stackelberg leadership is not apparent. They will update their beliefs about the uncertain demand based on information available and, then make decisions accordingly. As a rational player, the follower knows that the leader decides its output quantity based on its private information, and will try to infer the leader's private information from its output quantity. The leader's private information is leaked to, or inferred by the follower via the leader's output quantity. The leader only knows its own information, while the follower knows its own information and the inferred information from the leader's output quantity, hence makes decision based on more accurate information than the leader. The follower may benefit from the effect of more accurate information.

The leader knows the follower's rational inference and hence makes decision by taking the follower's inference into consideration. That is, there is a signaling game between the leader and the follower. Different from the case without asymmetric information, the leader's output quantity is an instrument to signal its information. As we know, signaling is costly, the leader incurs a cost of signaling. The leader is worse off with the effect of cost of signaling. On the contrary, the follower is better off with it.

In sum, there are two effects in the game, the effect of the follower's more accurate information and the effect of cost of signaling. With these two effects, the follower position may be improved, and the follower may make more profit than the leader. In this thesis, we study how the leader's information is leaked to the follower, and how firms' private information affects firms' decisions and profits. We also explore whether firms still prefer the Stackelberg leadership when they possess some private information.

We assume that the demand is linear and uncertain. Firms observe private information about the uncertain demand and choose their respective output quantity sequentially. Our model setup is similar as Gal-or's (1987), in which the demand is continuously distributed. Continuous demand distribution is widely used in the literature, such as Li (2002), Zhang (2002), Mishra et al. (2007), Li and Zhang (2008), Ha et al. (2011), Shang et al. (2015). She assumes that firms follow linear decision rules and finds that the follower always earns more profit than the leader. However, we find that Gal-or's results does not hold all the time. Firms' decisions and ex ante profit depend on the highest demand. If the demand is continuously distributed in an unbounded interval, firms' output quantities are linear in the signals and the leader incurs the highest cost to signal its information to the follower and earns less ex ante profit than the follower. Both firms prefer to be the Stackelberg follower. This is consistent with Gal-or's results. However, if the uncertain demand is distributed in a bounded interval, firms does not follow linear decision rules and the leader may earn more ex ante profit than the follower. We know that there are two effects in the game. We are interested in how each effect influences firms' decisions and their preference for Stackelberg leadership. So we build two benchmark models to compare the two effects.

We are also interested in that if the demand is discretely distributed, how the number of demand states affects the leader's cost of signaling and both firms' decisions. The discrete demand information structure is also widely used in the literature, Anand and Goyal (2009), Kong, et al. (2013), Ha and Tong (2008) and economics literature. If the demand distribution is discrete, when the leader observes high demand, it will take an ex post efficient action. However, when the leader observes a lower state, to separate from the high state, the leader will set a lower output quantity to prevent high type leader from mimicing it, thus the leader incurs the cost of signaling. If the number of demand states is two, only the low type may incur the cost of signaling. If the number of demand states is three, both the middle type and the low type leader may incur cost to signal its information, and then incur a higher cost.

Our goal in this work is to explore firms' preference for the Stackelberg leadership in Cournot competition with private information. Thus, the paper bridges and makes a contribution to two streams of research: (1) first-mover advantage and (2) information leakage between competing firms. There have been numerous papers on first-mover advantage. There have been fewer works on information leakage between firms competing in output quantity. How the information leakage affects firms' preference for the Stackelberg leadership has not been explored.

The rest of this thesis is organized as follows. Section 2 reviews the related literature. Section 3 is the model setup. Section 4 and section 5 discuss the impact of demand support and number of demand states on the Stackelberg leadership. Section 6 concludes the main results in this thesis.

2 LITERATURE REVIEW

Our work is related to two streams of research. the first stream studies the information leakage and the second strand of research is about the first-mover advantage.

2.1 Information Leakage

Information leakage is a hot topic in recent literature. Lee and Whang (2000) give examples of information leakage in supply chain, in which a supplier supplies a critical part to two manufacturers competing in the final product market. Either the manufacturer would not share information with the supplies unless it guaranteed that the information is not leaked to the other manufacturer. However, the shared information may be leaked via the observable action that the supplier reacts to the information. Li (2002) considers a supply chain with one manufacturer and multiple retailers. The retailers compete on output quantity and are endowed with private information about the uncertain demand and their cost. He finds that the leakage effect discourages the retailers from sharing their demand information with the manufacturer while encouraging them to share their cost information. Zhang (2002) considers a supply chain with one manufacturer and two retailers competing on either the quantity or the price. The retailers sell differentiated products and possess some private demand information. He finds that complete information sharing can be achieved through side payment if their information is statistically less accurate or they benefit more from the effect of information leakage. Li and Zhang (2008) consider a decentralized supply chain in which one manufacturer supplies to many retailers competing in price. They study the impact of different types of confidentiality agreements on retailers' incentive to share their information with manufacturer. With confidentiality, though retailers cannot observe the shared information directly, they can infer it from the manufacturer's wholesale price. Anand and Goyal (2009) explore how information leakage affects the material flows and information flows of a supply chain with horizontal competition. The informed firm's private information may be leaked to the uninformed one via their common upstream supplier. As a result of information leakage, the informed firm may choose to conceal its information to prevent leakage. And this will leads to operational losses through material flow distortion. Kong et al. (2013) consider a similar model to Anand and Goyal (2009) and study the potential of revenue-sharing contracts to facilitate information sharing in a supply chain and mitigate the negative effects of information leakage.

This line of literature concentrates on information leakage in supply chain, and studies how the downstream firm(s)' information is leaked through their upstream firm and how information leakage affects supply chain participants' decisions and profits. None of these papers in the literature considers the impact of information leakage on firms' preference for Stackelberg leadership in a duopoly market.

2.2 The First-mover Advantage

First-mover advantage is that a firm can be better off than its competitor as a result of being first to market. The first mover and the second mover are symmetric and the difference between them is the leader commits its strategy and moves first. The first-mover advantage is also known as Stackelberg leadership (Turocy 2001). Beginning with the model introduced by von Stackelberg (1934), which demonstrates that the leading firm can obtain higher profits than that of the follower's by committing to a high production quantity, a lot of work has been devoted to find out under which circumstances a first-mover advantage can be achieved. Gal-Or (1985) shows that the first mover (the leader) gains higher profits only if the actions of the leader and the follower are strategic substitutes. Amir and Stepanova (2006) consider a differentiated-product Bertrand duopoly game with general demand and asymmetric linear costs and find that a firm with sufficiently large cost lead over its rival has a first-mover advantage. Rasmusen and Yoon (2012) show that if one player's information about the profitability of new markets is only modestly superior, the possibility of foreclosing the market can lead to first-mover advantage. Bagwell (1995) and Vardy (2004) show that the first-mover advantage is completely lost if the first mover's choice is imperfectly observed or if there are observation costs.

Gal-Or (1987) studies how firms' private information affects the first-mover advantage by considering a leader-follower game with output quantity as strategies. She assumes the demand is linear and stochastic. Firms observe private information about the state of demand. She also assumes that prior and posterior distributions that generate posterior expected values are linear in the observable signals. She concludes that based on these assumptions, firms follow linear decision rules. At a pure strategy equilibrium, the follower is always able to infer the correct private signal of the leader, unless this signal is infinitely noisy. She finds that the follower always produces higher expected output than the leader regardless of the actual accuracies of the private signals, and earns more expected profit than the leader for any signals they observe.

However, we find that linear demand and linear prior and posterior distributions do not mean that firms follow linear decision policy. Thereby, linear decision rules may not hold under all conditions. If so, when do the linear decision rules hold? Does the follower still always earn more profit than the leader? As we know, these questions haven't been discussed in previous literature. We plan to answer these questions in my thesis.

3 MODEL SETUP

We consider a duopoly Cournot game with sequential moves, in which two firms, firm 1 (the leader) and firm 2 (the follower), sell homogeneous products to customers. Firm 1 makes decision first as the Stackelberg leader and firm 2 is the Stackelberg follower. The two firms are profit maximizers. The inverse demand function is given by

$$p = A - (q_1 + q_2),$$

where p is the retail price, q_1 and q_2 are the output quantities set by firm 1 and firm 2, respectively. The intercept A is a random variable and represents uncertain demand. Before making decisions, firm 1 and firm 2 observe private information Y_1 and Y_2 about the uncertain demand, respectively. They will update their beliefs about the demand based on information available. Each player only knows its own information. Firms incur constant retail cost, which we normalize to zero without loss of generality.

This is a two-stage game. Before making decisions, firms observe Y_1 and Y_2, respectively. In the first stage, The leader moves first and decides its output quantity q_1 based on Y_1. In the second stage, observing the leader's output quantity q_1, the follower will try to infer the leader's private information from q_1 and choose q_2 based on its own information and the inferred information. The firms decide their respective output quantity sequentially to maximize their expected profit

$$\begin{aligned} E[\pi_1 | Y_1] &= E[q_1 (A - (q_1 + q_2)) | Y_1], \\ E[\pi_2 | Y_2, q_1] &= E[q_2 (A - (q_1 + q_2)) | Y_2, q_1]. \end{aligned}$$

The sequence of event and decision is as follows:

1. The firm 1 decides output quantity q_1 based on its own information Y_1.

2. Upon receiving q_1, firm 2 chooses an output quantity q_2 based on its own information Y_2 and information inferred from q_1.

3. Demand is realized, and production is completed to meet the demand.

4 THE IMPACT OF DEMAND SUPPORT FOR CONTINUOUS DEMAND

In this section, we assume that the uncertain demand is continuously distributed in a positive interval and the demand can be bounded or unbounded. That is, the highest demand can be a positive finite value or positive infinity. We will analyze how the demand support affects firms decisions and their expected profits.

4.1 Information Structure

The uncertain demand A consists of two parts, a deterministic part a and a random part θ representing demand uncertainty, with $E[\theta] = 0$ and $Var[\theta] = \sigma^2 > 0$, i.e., $A = a + \theta$. The joint probability distribution of (θ, Y_1, Y_2) satisfies the following conditions:

(1) $E[Y_i|\theta] = \theta$ for $i = 1, 2$,

(2) $E[\theta|Y_1, Y_2] = \alpha_0 + \alpha_1 Y_1 + a_2 Y_2$, where α_0, a_1 and α_2 are constants,

(3) Y_1, Y_2 are independent and identically distributed conditional on θ.

Condition (1) says that Y_i is an unbiased estimator of θ and condition (2) states that the conditional expectations are linear in the signals. The two conditions are generally enough to include many conjugate pairs such as normal-normal, gamma-Poisson, beta-binomial. Condition (3) says that the two firms' signals are symmetric in probability distribution. The expected conditional precision of the signal is $\frac{1}{E[Var[Y_i|\theta]]}$. We focus on imperfect demand signal, i.e., $E[Var[Y_i|\theta]] > 0$. The two firms have same signal precision about the uncertain demand. By Lemma 1 of Li (1985), the three conditions alone imply that

$$E[\theta|Y_i] = E[Y_i|Y_j] = \frac{1}{1+s} Y_i,$$

for $i \in \{1,2\}$ and $i \neq j$,

$$E\left[\theta | Y_1, Y_2\right] = \frac{1}{2+s}\left(Y_1 + Y_2\right),$$

where $s \equiv \frac{E[Var[Y_i|\theta]]}{Var[\theta]}$ for $i \in \{1,2\}$, and

$$E[(Y_i)^2] = (1+s)\,\sigma^2,\; E[Y_1 Y_2] = \sigma^2.$$

There is an issue about the nonnegativity of the uncertain demand, i.e., $a + \theta$. According to Li and Zhang (2008), for gamma-Poisson, beta-binomial, we can easily find a condition for the equilibrium solutions to be interior points. For normal-normal case, if σ is small relative to a, the equilibrium outcome will be an interior-point solution for most realizations of demand uncertainty and signals.

4.2 Model Analysis

In the first stage, the leader decides its output quantity q_1 based on Y_1. Though the follower cannot observe Y_1 directly, it will try to infer it from the leader's observable action q_1. How the follower infers information from q_1 depends on the its belief following a functional form $P\left(Y_1\right)$. We restrict the search for equilibria to the subspace where $P\left(Y_1\right)$ is a strictly increasing function of $E\left[\theta|Y_1\right]$, in other words, q_1 is related to Y_1 only through a monotone relationship with $E\left[\theta|Y_1\right]$. We assume that the leader's decision policy takes the form of

$$q_1 = f\left(E\left[\theta|Y_1\right]\right), \text{ this is, } E\left[\theta|Y_1\right] = f^{-1}\left(q_1\right)$$

for some strictly increasing and differentiable function $f(\cdot)$. Observing q_1, the follower believes the leader's belief about the demand is $f^{-1}\left(q_1\right)$. According to Mas-Colell et al. (1995), a separating equilibrium should satisfies the following conditions,

$(C1)$: The leader's strategy is optimal given the follower's strategy,

$(C2)$: The belief function $f^{-1}(q_1)$ is derived from the leader's strategy,

$(C3)$: The follower's strategy is optimal given the belief function and the leader's strategy.

Since the leader's decision policy takes the form of $q_1 = f(E[\theta|Y_1])$, observing a q_1, the follower believes $E[\theta|Y_1]$ is $f^{-1}(q_1)$ condition $C2$ is satisfied.

In the second stage, observing the leader's output quantity, the follower tries to infer the leader's private information according to the belief function $f^{-1}(q_1)$ and updates its belief about the uncertain demand $E[\theta|Y_2, q_1] = \frac{(1+s)f^{-1}(q_1)+Y_2}{2+s}$. The follower chooses q_2 to maximize

$$E[\pi_2|Y_2, q_1] = q_2 (a + E[\theta|Y_2, q_1] - (q_1 + q_2)).$$

From the first-order condition (FOC), we get

$$q_2^*(q_1, Y_2) = \frac{1}{2}\left(a - q_1 + \frac{(1+s) f^{-1}(q_1) + Y_2}{2+s}\right).$$

Given the leader's output quantity and the belief function, the follower's optimal strategy is $q_2^*(q_1, Y_2)$. Condition $C3$ is satisfied.

In the first stage, the leader maximizes its profit,

$$\begin{aligned} E[\pi_1|Y_1] &= E[q_1 (a + \theta - (q_1 + q_2^*(q_1, Y_2)))|Y_1] \\ &= q_1 \left(\frac{1}{2}a + E[\theta|Y_1] - \frac{1}{2}q_1 - \frac{1}{2}\frac{1+s}{2+s} f^{-1}(q_1) - \frac{1}{2}\frac{1}{2+s} E[Y_2|Y_1]\right). \end{aligned}$$

To maximize $E[\pi_1|Y_1]$ over q_1, we set the first derivative to zero, i.e.,

$$\frac{d}{dq_1} E[\pi_1|Y_1] = 0,$$

which can be written as

$$\frac{d}{dq_1}\left\{q_1\left(-\frac{1}{2}q_1-\frac{1}{2}\frac{1+s}{2+s}f^{-1}\left(q_1\right)\right)\right\}+\frac{1}{2}a+E\left[\theta|Y_1\right]-\frac{1}{2}\frac{1}{2+s}E\left[Y_2|Y_1\right]=0.$$

Since $E\left[Y_2|Y_1\right]=E\left[\theta|Y_1\right]$, we replace $E\left[Y_2|Y_1\right]$ by $E\left[\theta|Y_1\right]$ and obtain

$$\frac{d}{dq_1}\left\{q_1\left(-\frac{1}{2}q_1-\frac{1}{2}\frac{1+s}{2+s}f^{-1}\left(q_1\right)\right)\right\}+\frac{1}{2}a+\frac{1}{2}\frac{3+2s}{2+s}E\left[\theta|Y_1\right]=0.$$

For $q_1=f\left(E\left[\theta|Y_1\right]\right)$ to be an equilibrium, the above equality must hold if we replace $E\left[\theta|Y_1\right]$ by $f^{-1}\left(q_1\right)$, i.e.,

$$a-2q_1+f^{-1}\left(q_1\right)-\frac{1+s}{2+s}q_1\frac{d}{dq_1}f^{-1}\left(q_1\right)=0.$$

Solving this equation, we get the solution:

$$f^{-1}(q_1)=\frac{2\lambda}{\lambda-1}\cdot q_1+c\cdot q_1^{\lambda}-a, \tag{1}$$

where $\lambda=\frac{2+s}{1+s}$ and c is an arbitrary constant. For any demand signal Y_1, given a c, there exists an equilibrium for the game as long as $f\left(\cdot\right)$ is increasing with $E\left[\theta|Y_1\right]$. So there are multiple equilibria for the game.

When there are multiple equilibria, the leader will choose a Pareto-optimal equilibrium for its own interest if there is any. In fact, the leader's equilibrium profit is $\frac{1}{2}q_1\left(a+\frac{1}{2}E\left[\theta|Y_1\right]-q_1\right)$, which is increasing in q_1 for any $q_1\leq\frac{1}{2}\left(a+E\left[\theta|Y_1\right]\right)$. Note that (1) implies that, for any given signal Y_1, q_1 is decreasing in c. Therefore, the leader's maximal expected profit is achieved at the lowest constant c. Recall that to fulfill the follower's conjecture, we must keep $f(\cdot)$ strictly increasing. Taking the first-order derivative with respect to q_1 on the right hand side of equation (1), we find that $\frac{df^{-1}(q_1)}{q_1}$ is positive if and only if $c\geq-\frac{2^{\lambda}}{\lambda-1}\cdot\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}$, which holds for any q_1, where $\tilde{Y}$ is the highest demand. It is easy to verify that when $c = -\frac{2^\lambda}{\lambda-1} \cdot \frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}$, the leader achieves its maximal expected profit and $f(\cdot)$ is strictly increasing. Thus, condition $C1$ is satisfied. We summarize the above analysis in the following proposition.

Proposition 1 *The Pareto-optimal Bayesian equilibrium for the Stackelberg game is $(q_1^*, q_2^*, f^{-1}(q_1^*))$, where q_1^* satisfies*

$$\frac{2\lambda}{\lambda-1}q_1^* - \frac{2^\lambda}{\lambda-1}\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}(q_1^*)^\lambda = a + E\left[\theta|Y_1\right], \tag{2}$$

and q_2^* satisfies

$$q_2^* = \frac{1}{2}\left(a - q_1^* + \left(\left(\frac{2}{\lambda-1}q_1^* - \frac{1}{\lambda}\frac{2^\lambda}{\lambda-1}\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}(q_1^*)^\lambda\right) + \frac{1}{2+s}Y_2\right)\right),$$

with belief function

$$f^{-1}(q_1^*) = \frac{2\lambda}{\lambda-1}q_1^* - \frac{2^\lambda}{\lambda-1}\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}} \cdot (q_1^*)^\lambda - a.$$

Firms' expected profits are given by

$$\begin{aligned} E\left[\pi_1|Y_1\right] &= \frac{1}{2}q_1^*\left(a + E\left[\theta|Y_1\right] - q_1^*\right), \\ E\left[\pi_2|Y_2, q_1\right] &= q_2^*\left(a + \frac{Y_1+Y_2}{2+s} - (q_1^* + q_2^*)\right). \end{aligned}$$

If the leader observes the highest signal, its optimal output quantity is

$$q_1^*\left(E\left[\theta|\tilde{Y}\right]\right) = \frac{1}{2}\left(a + E\left[\theta|\tilde{Y}\right]\right),$$

which is exactly the highest possible output quantity (recall that any output quantity should satisfy $q_1 \leq \frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right)$). And the leader has no incentive to mimic the highest signal if it observes a signal other than the highest signal.

This coincides with the previous literature on signaling game that the highest type signal sender takes an ex post efficient action.

Corollary 1 *(1) When the leader observes the highest signal $\tilde{Y}$, its output quantity is ex post efficient, i.e., $q_1^*\left(E\left[\theta|\tilde{Y}\right]\right) = \frac{1}{2}\left(a + E\left[\theta|\tilde{Y}\right]\right)$.*

(2) When the leader observes a signal Y_1 lower than $\tilde{Y}$, its output quantity is always less than $\frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right)$.

Proof. Please see the appendix. ■

When the leader observes a signal Y_1, it has an incentive to set an output quantity lower than $\frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right)$ to induce the follower to believe that the leader observes a signal lower than Y_1. Then the follower sets a low output quantity. Thus the leader benefits from it.

4.3 Unbounded Demand

From equation (2), we know that the highest signal plays an important role in deciding the leader's output quantity. If the demand is unbounded, the highest signal, $\tilde{Y}$, goes to positive infinity, hence, the constant c in equation (1) is zero. We find that the leader's output quantity is linear in its private signal.

Lemma 1 *If the demand is unboundedly distributed, the Pareto-optimal Bayesian equilibrium for the Stackelberg game is $(q_1^*, q_2^*, f^{-1}(q_1^*))$, where*

$$\begin{aligned} q_1^* &= \frac{1}{2}\frac{1}{2+s}\left(a + \frac{1}{1+s}Y_1\right), \\ q_2^* &= \frac{1}{2}\frac{1}{2+s}\left(\frac{3+2s}{2}a + \frac{1+2s}{2(1+s)}Y_1 + Y_2\right). \end{aligned}$$

The follower believes the leader's information is

$$f^{-1}(q_1^*) = \frac{2\lambda}{\lambda - 1} \cdot q_1^* - a.$$

Firms' profits are given by

$$
\begin{aligned}
E\left[\pi_1|Y_1\right] &= \frac{1}{8}\frac{3+2s}{(2+s)^2}\left(a+\frac{1}{1+s}Y_1\right)^2, \\
E\left[\pi_2|Y_2,q_1\right] &= \frac{1}{4}\left(\frac{1}{2}\frac{3+2s}{2+s}a+\frac{1+2s}{2(2+s)(1+s)}Y_1+\frac{1}{2+s}Y_2\right)^2,
\end{aligned}
$$

and their ex ante profits are given by

$$
\begin{aligned}
\Pi_1 &= \frac{1}{8}\frac{3+2s}{(2+s)^2}a^2+\frac{1}{8}\frac{3+2s}{(s+1)(2+s)^2}\sigma^2, \\
\Pi_2 &= \frac{1}{16}\frac{(3+2s)^2}{(2+s)^2}a^2+\frac{1}{16}\frac{8s^2+20s+9}{(s+1)(s+2)^2}\sigma^2.
\end{aligned}
$$

When the demand is continuously distributed in an unbounded interval, the leader's maximal equilibrium output quantity is achieved if and only if $c = 0$. Then the leader's output quantity is linear in Y_1 and the follower's output quantity is linear in Y_1 and Y_2. So if the demand is unboundedly distributed, our finding is consistent with Gal-or's (1987) study. Since σ is small relative to a, we compare firms' ex ante profits and find that the follower always earns more ex ante profit than the leader. This is also consistent with findings in Gal-or's (1987) paper.

Corollary 2 *If the demand is unbounded,*

(1) firms follow linear decision rules, that is, firms' output quantities are linear in the observed signals,

(2) the follower always earns more ex ante profit than the leader.

When the demand is unbounded, the Stackelberg leader's profit is lower than that of the Stackelberg follower.Thus firms prefer to be the Stackelberg follower rather than the Stackelberg leader.

4.4 Bounded Demand

If the demand is boundedly distributed, the highest signal $\tilde{Y}$ is finite. Then $c = -\frac{2\lambda}{\lambda-1} \cdot \frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}$ is always smaller than 0. From equation (2), we find that the leader does not follow a linear decision rule.

Corollary 3 *If the demand is boundedly distributed, the leader's output quantity is not linear in its signal unless it observes the highest signal.*

Lemma 2 *Given the signals firms observe,*

(1) q_1^* is decreasing in $\tilde{Y}$, while q_2^* is increasing in $\tilde{Y}$,

(2) $E\left[\pi_1|Y_1\right]$ is decreasing in $\tilde{Y}$ and $E\left[\pi_2|Y_2, q_1^*\right]$ is increasing in $\tilde{Y}$.

Proof. Please see the appendix. ■

Given the signal the leader observes, the higher the $\tilde{Y}$ is, the lower its output quantity and expected profit are. On the contrary, the follower is better off with a higher $\tilde{Y}$. If $\tilde{Y}$ is small enough, the leader sets an output quantity which is always larger than that of the follower's. Thus, the leader can always earn more profit than the follower. However, if $\tilde{Y}$ is large enough, the follower may set a higher output quantity than the leader and earn a high profit. Since it's difficult to compare the two firms' ex ante profits directly, we run numerical experiments to make comparison. We consider beta-binomial distribution, specifically, assume that the demand $A \sim Beta\left(\alpha, \beta\right)$ and the joint probability distribution of (A, Y_1, Y_2) follows a beta-binomial distribution. The probability density function (pdf) of A is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} \left(1-x\right)^{\beta-1},$$

with mean $\frac{\alpha}{\alpha+\beta}$ and variance $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. The signal Y_i, $i \in \{1,2\}$, follows $Binomial(1, A)$. The related probabilities and expectations are given by

$$
\begin{aligned}
\Pr\left(Y_i = k|A\right) &= A^k \left(1-A\right)^{1-k}, \; k = 0, 1. \\
E\left[Y_i|A\right] &= A, \\
Var\left(Y_i|A = x\right) &= A\left(1-A\right), \\
E\left[Y_i\right] &= \frac{\alpha}{\alpha+\beta}, \\
E\left[Var(Y_i|A)\right] &= \frac{\alpha\beta}{\left(\alpha+\beta\right)\left(\alpha+\beta+1\right)}, \\
s &= \frac{E\left[Var(Y_i|A)\right]}{Var\left(A\right)} = \alpha+\beta, \\
E\left[A|Y_i\right] &= \frac{1}{1+s}Y_i + \frac{s}{1+s}E\left[A\right], \\
\Pr\left(Y = k\right) &= \frac{\Gamma\left(\alpha+k\right)\Gamma\left(\beta-k+1\right)}{\Gamma\left(\alpha+\beta+1\right)} \frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}.
\end{aligned}
$$

We assume that the mean of demand is 0.5, i.e., $\frac{\alpha}{\alpha+\beta} = 0.5$. The highest signal is fixed and equal to 1. The firms' ex ante profits are shown in Figure 1

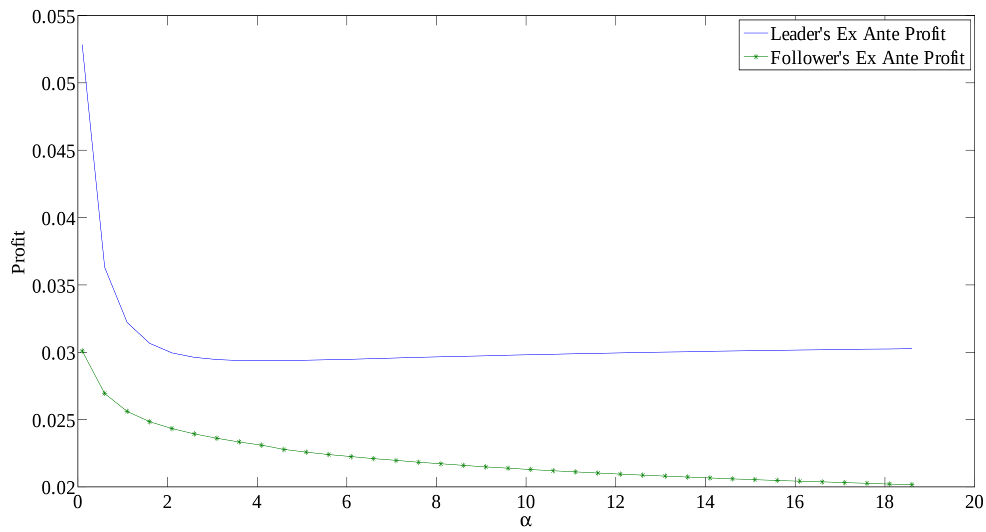

Figure 1: Firms' Ex Ante Profit ($E\left[A\right] = 0.5$)

If the mean of demand is 0.1, i.e., $\frac{\alpha}{\alpha+\beta} = 0.1$. $\tilde{Y} = 1$. The firms' ex ante profits are shown in Figure 2.

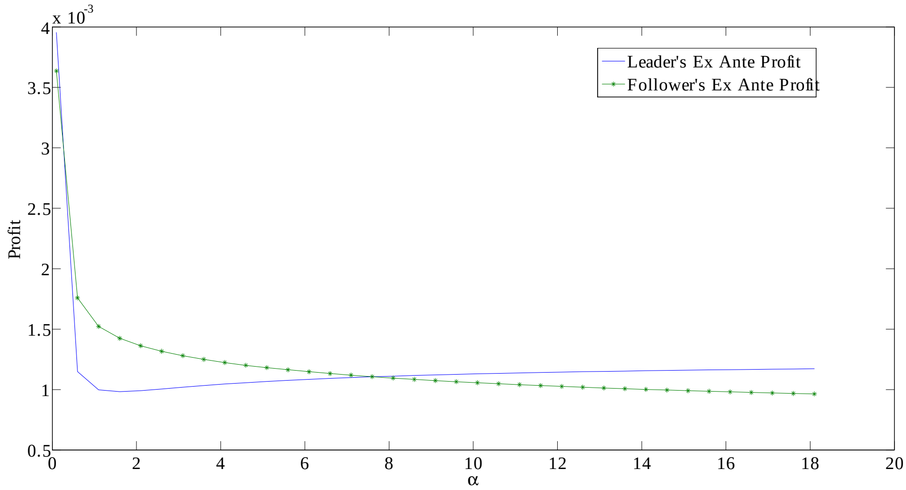

Figure 2: Firms' Ex Ante Profit ($E[A] = 0.1$)

Observation 1 If the demand is boundedly distributed, the follower may earn a higher ex ante profit than the leader.

From Figure 1, we can find that the leader's profit is always higher than that of the follower's. However, when the mean of A is 0.1, from Figure 2, we find that when α is large enough, the leader's profit is always higher than that of the follower's. If α is smaller than 8, the follower may earn a higher profit than the leader. So if the demand is distributed in a bounded interval, our finding is different from Gal-or's (1987).

Knowing the follower will infer its private information from its output quantity, to separate from other signal states, especially from the higher signal states, the leader will set an output quantity which is lower than $\frac{1}{2}(a + E[\theta|Y_1])$. Hence, the leader is worse off by incurring the cost of signaling. On the contrary, the follower is better off. When the cost is large enough, the leader loses its first-mover advantage and the follower earns more ex ante profit than the leader.

We know that the leader is ex post efficient if it observes the highest signal. If the demand is distributed in a bounded interval, the highest signal can be achieved and the leader is ex post efficient when it observes the highest signal. However, if the demand is unbounded, the leader will never observe the highest signal, and it will never be ex post efficient. Recall that given a signal the leader observes, its output quantity is decreasing with the highest signal. Given Y_1, the leader's equilibrium output quantity is decreasing in the $\tilde{Y}$ (recall Lemma 2). If the demand is unboundedly distributed, the highest signal is infinite, thus the leader incurs the largest cost of signaling compared with any other bounded demand distribution. On the contrary, the follower's output quantity is increasing with the highest signal. Given signals firms observe, the follower sets the highest output quantity if the demand distribution is unbound, hence it makes the maximal profit. Thereby the leader's output quantity yields an equilibrium where the follower earns more ex ante profit than the leader.

4.5 The Two Effects

We know that there are two effects in the game, one is the effect of the follower's more accurate information, and the other one is the effect of cost of signaling. To study these two effects, we build two benchmark models. In model 1, we assume that the two firms observe the same signal Y_1 only. And in model 2, we assume that firm 1 observes Y_1 only, and firm 2 observes Y_1 and Y_2. The sequence of events in these two benchmark models is same as the main model. Neither effect plays a role in model 1. The effect of the follower's more accurate information exists in benchmark model 2, but nor does the effect of cost of signaling. Comparing the two benchmark models, we can find the effect of the follower's more accurate information. Comparing the benchmark model 2 with the main model, we can study the effect of cost of signaling.

4.5.1 Benchmark Model 1

Model 1 is similar to that of the base model except that at the beginning both firms observe same signal Y_1 only. Solving the game, we get the following Lemma.

Lemma 3 *If two firms observe same signal Y_1 at the beginning, the unique equilibrium is $\left(q_1^{B*}, q_2^{B*}\right)$, where*

$$
\begin{aligned}
q_1^{B*} &= \frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right), \\
q_2^{B*} &= \frac{1}{4}\left(a + E\left[\theta|Y_1\right]\right).
\end{aligned}
$$

Firms expected profits are

$$
\begin{aligned}
E\left[\pi_1^B|Y_1\right] &= \frac{1}{8}\left(a + E\left[\theta|Y_1\right]\right)^2, \\
E\left[\pi_2^B|Y_1\right] &= \frac{1}{16}\left(a + E\left[\theta|Y_1\right]\right)^2,
\end{aligned}
$$

and their ex ante profits are

$$
\begin{aligned}
\Pi_1^B &= \frac{1}{8}\left(a^2 + \frac{1}{s+1}\sigma^2\right), \\
\Pi_2^B &= \frac{1}{16}\left(a^2 + \frac{1}{s+1}\sigma^2\right).
\end{aligned}
$$

We can find that if firms observe the same signal, the leader's equilibrium output quantity is greater than that of the follower's and the first-mover advantage still works.

4.5.2 Benchmark Model 2

In model 2, the leader observes signal Y_1 only, and the follower observes both Y_1 and Y_2. In this model, the follower possesses more accurate information than the leader. Since the follower knows the leader's information, the leader does not incur the cost to signal its information. So only the effect of the follower's more accurate information exists in this model. Solving the game, we get the following Lemma.

Lemma 4 *If the leader's information Y_1 is public and the follower possesses some private information Y_2 about the uncertain demand, the Subgame Perfect equilibrium for the game is $\left(\hat{q}_1^{B*}, \hat{q}_2^{B*}\right)$, where*

$$
\begin{aligned}
\hat{q}_1^{B*} &= \frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right), \\
\hat{q}_2^{B*} &= \frac{1}{2}\left(\frac{1}{2}a + E\left[\theta|Y_1, Y_2\right] - \frac{1}{2}E\left[\theta|Y_1\right]\right).
\end{aligned}
$$

Firms' expected profits are

$$
\begin{aligned}
E\left[\hat{\pi}_1^B|Y_1\right] &= \frac{1}{8}\left(a + E\left[\theta|Y_1\right]\right)^2, \\
E\left[\hat{\pi}_2^B|Y_1, Y_2\right] &= \frac{1}{16}\left(a + 2E\left[\theta|Y_1, Y_2\right] - E\left[\theta|Y_1\right]\right)^2,
\end{aligned}
$$

and their ex ante profits are

$$
\begin{aligned}
\hat{\Pi}_1^B &= \frac{1}{8}a^2 + \frac{1}{8}\frac{1}{s+1}\sigma^2, \\
\hat{\Pi}_2^B &= \frac{1}{16}a^2 + \frac{1}{16}\frac{1}{s+1}\sigma^2 + \frac{1}{4}\frac{s}{s^2+3s+2}\sigma^2.
\end{aligned}
$$

Since σ is small relative to a, comparing firms' ex ante profits, we find that the leader's ex ante profit is larger than that of the follower regardless of the demand distribution. Although the follower possesses more accurate information than the leader, the leader still earns more ex ante profit than the follower's. With more precise information, the follower's knowledge about the demand is improved and it can make decision wisely. However, the effect of the follower's more accurate information doesn't make the follower better than the leader. Firms still prefer the Stackelberg leadership.

From Corollary 1 and Lemma 4, we find that the leader's equilibrium output quantity q_1^* is no larger than $\hat{q}_1^{B*}$, this is because of the effect of cost of signaling. We define the effect of the follower's more accurate information on the follower is $\left(\hat{\Pi}_2^B - \Pi_2^B\right)$, the effect of cost of signaling on the leader is $\left(\hat{\Pi}_1^B - \Pi_1\right)$, and the effect of cost of signaling on the follower is $\left(\Pi_2 - \hat{\Pi}_2^B\right)$.

If the demand is unboundedly distributed, the two effects are shown in table 1.

Table 1 The two effects

	The follower's more accurate information	The leader's cost of signaling
The leader	0	$\frac{1}{8}\left(\frac{s+1}{s+2}\right)^2 a^2 + \frac{1}{8}\frac{s+1}{(s+2)^2}\sigma^2$
The follower	$\frac{1}{4}\frac{s}{s^2+3s+2}\sigma^2$	$\frac{1}{16}\left(\frac{3s^2+8s+5}{(s+2)^2}a^2 + \frac{3s+5}{(s+2)^2}\sigma^2\right)$

Proposition 2 *If the demand is unboundedly distributed, the effect of cost of signaling on firms is larger than the effect of follower's more accurate information.*

Comparing the two effects shown in table 1, we can find that the effect of cost of signaling is greater than that of the effect of the follower's more accurate information.

If the demand is distributed in a bounded interval, we run a numerical simulation to study the two effects. We assume that the joint probability distribution of (A, Y_1, Y_2) follows a beta-binomial distribution as in section 4.4 and the mean of demand is 0.05. Figure 3 shows firms' ex ante profits and the two effects.

From Figure 3, we find that under some condition, the effect of the follower's more accurate information may be greater than that of cost of signaling. This is different from the unbounded case.

When demand is unbounded, the leader incurs the largest cost to signal its information to the follower, the follower benefits most from the cost of signaling. Hence, the effect of cost of signaling is higher than that if the demand is bounded and is also larger than that of follower's more accurate information. As we know

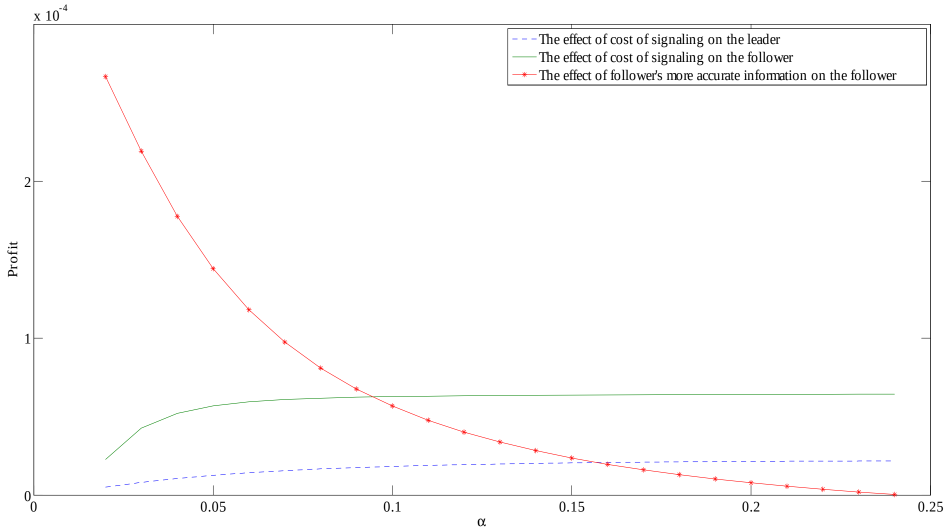

Figure 3: The Two Effects

the cost of signaling depends on the highest signal. If the highest signal is small enough, the effect of cost of signaling may be smaller than that of follower's more accurate information. However, if the highest signal is large enough, the effect of cost of signaling would be larger than that of the follower's more accurate information.

5 THE IMPACT OF NUMBER OF DEMAND STATES FOR DISCRETE DEMAND

We are also interested in that how the number of demand states affects firms' decisions and profits if the demand is discrete. Discrete demand distribution is also widely used in the literature, such as Anand and Goyal (2009), Kong et al. (2013). In this section, we discuss how the number of demand states affects the effect of cost of signaling. We discuss two cases, two demand states and three demand states in this section.

5.1 Binary Case

5.1.1 Information Structure

The inverse demand function is given by

$$p = A - (q_1 + q_2),$$

The intercept A is random and can take one of the two values: a *high* value A_H with probability α, where $\alpha \in (0,1)$, and a *low* value A_L ($< A_H$) with probability $(1-\alpha)$. These are common knowledge to all parties. Before making decision, the two firms observe private signal Y_1 and Y_2 about the random intercept A, respectively, where Y_1, $Y_2 \in \{A_H, A_L\}$. Y_1 and Y_2 are independently generated from the true A with probability γ, for $\gamma \in \left(\frac{1}{2}, 1\right)$, i.e., $\Pr(Y_i = Y) = \gamma$. We assume that A is independent of events $\{Y_1 = A\}$ and $\{Y_2 = A\}$, and $\{Y_1 = A\}$ is independent of $\{Y_2 = A\}$. Conditional on private signals, the leader and the follower will update their probabilities of the demand states according to Bayes' rule, specifically, $\Pr(A = A_H | Y_i = A_H) = \frac{\alpha\gamma}{\alpha\gamma + (1-\alpha)(1-\gamma)}$, and $\Pr(A = A_L | Y_i = A_L) = \frac{\gamma(1-\alpha)}{(1-\gamma)\alpha + \gamma(1-\alpha)}$. The parameter γ captures the accuracy of the signals. When $\gamma \to \frac{1}{2}$, a signal provides no additional information since $\Pr(A = A_H | Y_i = A_H) = \alpha$ and

$\Pr(A = A_L|Y_i = A_L) = 1 - \alpha$. When $\gamma \to 1$, the signals becomes perfect because $\Pr(A = A_H|Y_i = A_H) = \Pr(A = A_L|Y_i = A_L) = 1$. The Bayesian updating is shown in appendix and B_1 to B_8 are defined as follows,

$$\begin{aligned}
B_1 &= E\left[A|Y_1 = A_H\right], \\
B_2 &= E\left[A|Y_1 = A_L\right], \\
B_3 &= B_6 \Pr\left(Y_2 = A_H|Y_1 = A_H\right) + B_8 \Pr\left(Y_2 = A_L|Y_1 = A_H\right), \\
B_4 &= B_5 \Pr\left(Y_2 = A_H|Y_1 = A_L\right) + B_7 \Pr\left(Y_2 = A_L|Y_1 = A_L\right), \\
B_5 &= E\left[A|Y_1 = A_H, Y_2 = A_H\right], \\
B_6 &= E\left[A|Y_1 = A_L, Y_2 = A_H\right], \\
B_7 &= E\left[A|Y_1 = A_H, Y_2 = A_L\right], \\
B_8 &= E\left[A|Y_1 = A_L, Y_2 = A_L\right].
\end{aligned}$$

5.1.2 Analysis

Following Anand and Goyal (2009), we assume there exists a q_{1L}^*, if the follower observes $q_1 \leq q_{1L}^*$, it believes the leader observes a low signal. Otherwise, it believes the signal is high. That is, the follower's belief is

$$\Pr_2\left(Y_1 = A_H\right) = \begin{cases} 0 \text{ if } q_1 \leq q_{1L}^*, \\ 1 \text{ otherwise.} \end{cases} \tag{3}$$

In the second stage, the follower observes a demand information Y_2 and the leader's output quantity q_1. The follower's expected profit is given as follow,

$$\begin{aligned}
E\left[\pi_2|Y_2, q_1\right] &= E\left[q_2 p|Y_2, w\right], \\
&= q_2\left(E\left[A|Y_2, q_1\right] - \left(q_1 + q_2\right)\right).
\end{aligned}$$

From the FOC,

$$q_2^*(q_1, Y_2) = \frac{1}{2}\left(E\left[A|q_1, Y_2\right] - q_1\right). \tag{4}$$

That is, if the follower observes a high signal ($Y_2 = A_H$) and the leader sets a high output quantity, then the follower chooses q_{2HH}^* as its output quantity. The first subscript 2 refers to firm 2, the second subscript means the type of the leader in the follower's belief, and the third subscript means that the follower's demand state. To be specific,

$$\begin{aligned}
q_{2HH}^* &= \frac{1}{2}\left(B_5 - q_{1H}\right), \\
q_{2LH}^* &= \frac{1}{2}\left(B_6 - q_{1L}\right), \\
q_{2HL}^* &= \frac{1}{2}\left(B_7 - q_{1H}\right), \\
q_{2LL}^* &= \frac{1}{2}\left(B_8 - q_{1L}\right).
\end{aligned}$$

In the first stage, after the leader observes a signal Y_1, it will decide an output quantity q_1. The leader may choose a q_1 to reveal its type truthfully to the follower. The leader may also choose a q_1 which is different from its true type to mislead the follower if it is profitable to do so.

If the leader decides a q_1 to reveal its information, when it observes a high signal, it will choose q_{1H} as its output quantity. Then the follower's optimal output quantity is $q_2^*(q_{1H}, Y_2)$. The leader believes that the expected demand and the follower's output quantity are $E\left[A|Y_1 = A_H\right]$ and $E\left[q_2^*(q_{1H}, Y_2)|Y_1 = A_H\right]$, respectively. The leader's expected profit is given by

$$E\left[\pi_1\left(q_{1H}|Y_1 = A_H\right)\right] = q_{1H}\left(E\left[A|Y_1 = A_H\right] - \left(q_{1H} + E\left[q_2^*(q_{1H}, Y_2)|Y_1 = A_H\right]\right)\right).$$

Similarly, when the leader observes a low signal, its expected profit is

$$E\left[\pi_1\left(q_{1L}|Y_1 = A_L\right)\right] = q_{1L}\left(E\left[A|Y_1 = A_L\right] - \left(q_{1L} + E\left[q_2^*(q_{1L}, Y_2)|Y_1 = A_L\right]\right)\right).$$

The leader may also choose an output quantity to mislead the follower. When it observes a high signal, it chooses q_{1L} as its output quantity rather than q_{1H} to misguide the follower. The leader believes that the expected demand and the follower's output quantity are $E[A|Y_1 = A_H]$ and $E[q_2^*(q_{1L}, Y_2)|Y_1 = A_H]$, respectively. Hence, the leader maximizes

$$E[\pi_1(q_{1L}|Y_1 = A_H)] = q_{1L}(E[A|Y_1 = A_H] - (q_{1L} + E[q_2^*(q_{1L}, Y_2)|Y_1 = A_H])).$$

Similarly, when the leader observes a low signal, its expected profit is

$$E[\pi_1(q_{1H}|Y_1 = A_L)] = q_{1H}(E[A|Y_1 = A_L] - (q_{1H} + E[q_2^*(q_{1H}, Y_2)|Y_1 = A_L])),$$

where

$$\begin{aligned}
E[q_2^*(q_{1H}, Y_2)|Y_1 = A_H] &= \frac{1}{2}B_1 - \frac{1}{2}q_{1H}, \\
E[q_2^*(q_{1L}, Y_2)|Y_1 = A_L] &= \frac{1}{2}B_2 - \frac{1}{2}q_{1L}, \\
E[q_2^*(q_{1L}, Y_2)|Y_1 = A_H] &= \frac{1}{2}B_3 - \frac{1}{2}q_{1L}, \\
E[q_2^*(q_{1H}, Y_2)|Y_1 = A_L] &= \frac{1}{2}B_4 - \frac{1}{2}q_{1H}.
\end{aligned}$$

So we can simplify the above formulations to

$$\begin{aligned}
E[\pi_1(q_{1H}|Y_1 = A_H)] &= q_{1H}\left(\frac{1}{2}B_1 - \frac{1}{2}q_{1H}\right), \\
E[\pi_1(q_{1L}|Y_1 = A_L)] &= q_{1L}\left(\frac{1}{2}B_2 - \frac{1}{2}q_{1L}\right), \\
E[\pi_1(q_{1L}|Y_1 = A_H)] &= q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right), \\
E[\pi_1(q_{1H}|Y_1 = A_L)] &= q_{1H}\left(B_2 - \frac{1}{2}B_4 - \frac{1}{2}q_{1H}\right).
\end{aligned}$$

For a pure strategy separating equilibrium to exist, q_{1H} and q_{1L} must emerge as a simultaneous solution to the following constrained games: For high type leader, it solves the following problem,

$$\max_{q_{1H}} E\left[\pi_1\left(q_{1H}|Y_1 = A_H\right)\right] = q_{1H}\left(\frac{1}{2}B_1 - \frac{1}{2}q_{1H}\right), \tag{5}$$

subject to

$$\begin{aligned} E\left[\pi_1\left(q_{1H}|Y_1 = A_L\right)\right] &\leq E\left[\pi_1\left(q_{1L}^*|Y_1 = A_L\right)\right] \\ q_{1H}\left(B_2 - \frac{1}{2}B_4 - \frac{1}{2}q_{1H}\right) &\leq q_{1L}^*\left(\frac{1}{2}B_2 - \frac{1}{2}q_{1L}^*\right), \qquad (6) \\ q_{1H} &\geq 0. \qquad (7) \end{aligned}$$

The problem for the low type leader is

$$\max_{q_{1L}} E\left[\pi_1\left(q_{1L}|Y_1 = A_L\right)\right] = q_{1L}\left(\frac{1}{2}B_2 - \frac{1}{2}q_{1L}\right), \tag{8}$$

subject to

$$\begin{aligned} E\left[\pi_1\left(q_{1L}|Y_1 = A_H\right)\right] &\leq E\left[\pi_1\left(q_{1H}^*|Y_1 = A_H\right)\right] \\ q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) &\leq q_{1H}^*\left(\frac{1}{2}B_1 - \frac{1}{2}q_{1H}^*\right), \qquad (9) \\ q_{1L} &\geq 0. \qquad (10) \end{aligned}$$

Inequalities (7) and (10) are the participation constraints that ensure firms earn positive profits. And inequalities (6) and (9) are the incentive compatibility constraints for a separating equilibrium to ensure each type of the leader prefers not to mimic the other. The left hand side (LHS) are off-equilibrium profits and the right hand side (RHS) are the equilibrium profits. The equilibrium profit is no less than the off-equilibrium profit.

The above formulations satisfy: i) the leader's action (the optimal q_1^* derived from (5) and (8)) is the best response to what it knows and its conjecture on the follower's belief, ii) the follower's action (q_2^*) is the best response to what it knows at that point (its own information and the leader's output quantity) and its beliefs on the demand state given by (3), iii) the follower's belief coincides with the leader's conjecture on the follower's belief. Hence, the three conditions in subsection 4.2 are satisfied.

Since only the high type leader has incentives to mimic the low type, so the high type leader sets $q_{1H}^{*} = \frac{1}{2}B_1$ to maximize its profit, and for the low type leader, the game reduces to

$$E\left[\pi_1\left(q_{1L}|Y_1 = A_L\right)\right] = q_{1L}\left(\frac{1}{2}B_2 - \frac{1}{2}q_{1L}\right), \tag{11}$$

subject to

$$q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) \leq \frac{1}{8}B_1^2. \tag{12}$$

The Lagrangian for the above formulation is

$$\pounds_1(q_{1L}, u) = \max_{q_{1L}}\left(q_{1L}\left(\frac{1}{2}B_2 - \frac{1}{2}q_{1L}\right) - u\left(q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) - \frac{1}{8}B_1^2\right)\right),$$

The first-order Karush-Kuhn-Tucker (KKT) conditions for the Lagrangian are:

$$\begin{aligned}
(1a) &: \frac{\partial \pounds_1(q_{1L}, u)}{\partial q_{1L}} \leq 0 \Rightarrow \left(\frac{1}{2}B_2 - q_{1L}\right) - u\left(B_1 - \frac{1}{2}B_3 - q_{1L}\right) + \upsilon_1 = 0,\\
(1b) &: q_{1L}\frac{\partial \pounds_1(q_{1L}, u)}{\partial q_{1L}} = 0 \text{ by complementary slackness},\\
(2a) &: \frac{\partial \pounds_1(q_{1L}, u)}{\partial u} \geq 0 \Rightarrow \left(q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) - \frac{1}{8}B_1^2\right) + \upsilon_2 = 0,\\
(2b) &: u\frac{\partial \pounds_1(q_{1L}, u)}{\partial u} = 0 \text{ by complementary slackness}.
\end{aligned}$$

Solving the above problem we get, for $u = 0$,

$$
\begin{aligned}
q_{1L}^{*} &= \frac{1}{2}B_2, \\
\upsilon_2 &\geq 0 \Rightarrow \left(q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) - \frac{1}{8}B_1^2 \right) \leq 0, \\
\frac{A_H}{A_L} &\in \left(\frac{2b_1 - 2b_3}{b_1^2 + b_2^2 - 4b_1b_2 + 2b_2b_3} + 1, +\infty \right), \text{ if } \frac{1}{8}\left(4b_1b_2 - b_1^2 - b_2^2 - 2b_3b_2\right) < 0,
\end{aligned}
$$

where

$$
\begin{aligned}
b_1 &= \frac{\alpha\gamma}{\gamma\alpha + (1-\gamma)(1-\alpha)}, \\
b_2 &= \frac{\alpha(1-\gamma)}{(1-\gamma)\alpha + \gamma(1-\alpha)}, \\
b_3 &= \frac{\alpha\left(\alpha(1-\alpha)(2\gamma-1)^2 + \gamma(1-\gamma)^2\right)}{(\alpha\gamma + (1-\alpha)(1-\gamma))\left(\alpha(1-\gamma)^2 + (1-\alpha)\gamma^2\right)}.
\end{aligned}
$$

For $u > 0$, we have

$$
q_{1L}^{*} = B_1 - \frac{1}{2}B_3 \pm \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}.
$$

Call the lower root $q_{1L}^{(1)}$ and the upper root $q_{1L}^{(2)}$. We know that the unconstrained maximum of the equation (11) is at $\frac{1}{2}B_2$, while the maximum for the LHS of the constraint (12) is at $\left(B_1 - \frac{1}{2}B_3\right)$, where $\left(B_1 - \frac{1}{2}B_3\right) > \frac{1}{2}B_2$. And we can find that $q_{1L}^{(1)} < \frac{1}{2}B_2 < \left(B_1 - \frac{1}{2}B_3\right) < q_{1L}^{(2)}$. If the constraint (12) binds at q_{1L}^{*}, then any $q_{1L} \in [\frac{1}{2}B_2, \left(B_1 - \frac{1}{2}B_3\right)]$ does not permit a separating equilibrium. Hence, in order to separate out, the low type must decide an output lower than $\frac{1}{2}B_2$ to prevent the high type from mimicing him, so we abandon the upper root, i.e., $q_{1L}^{*} = B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}$.

To guarantee above results are meaningful, we require that firms' output quantity and the realized retail price is non-negative, i.e., the total output quantity is no greater than A_L. In other words (please see appendix), if $\frac{A_H}{A_L} \in \Omega_1$,

$$q_{1L}^* = \frac{1}{2} B_2,$$

if $\frac{A_H}{A_L} \in \Omega_2$,

$$q_{1L}^* = B_1 - \frac{1}{2} B_3 - \frac{1}{2} \sqrt{(B_1 - B_3)(3B_1 - B_3)},$$

where

$$\Omega_1 = \left\{ \frac{A_H}{A_L} : \begin{array}{c} \frac{A_H}{A_L} \in \left[\begin{array}{c} \frac{2b_1 - 2b_3}{b_1^2 + b_2^2 - 4b_1 b_2 + 2b_2 b_3} + 1, \\ 1 + \frac{(\gamma\alpha + (1-\gamma)(1-\alpha))\left(\gamma^2\alpha + (1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha + (1-\gamma^2)(1-\alpha))} \end{array} \right] \\ \text{if } b_1^2 + b_2^2 - 4b_1 b_2 + 2b_2 b_3 > 0 \end{array} \right\},$$

$$\Omega_2 = \left\{ \frac{A_H}{A_L} : \begin{array}{c} \frac{A_H}{A_L} \in \left(1, \min \left(\begin{array}{c} 1 + \frac{(\gamma\alpha + (1-\gamma)(1-\alpha))\left(\gamma^2\alpha + (1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha + (1-\gamma^2)(1-\alpha))}, \\ \frac{2b_1 - 2b_3}{b_1^2 + b_2^2 - 4b_1 b_2 + 2b_2 b_3} + 1 \end{array} \right) \right) \\ \text{if } b_1^2 + b_2^2 - 4b_1 b_2 + 2b_2 b_3 > 0; \\ \frac{A_H}{A_L} \in \left(1, 1 + \frac{(\gamma\alpha + (1-\gamma)(1-\alpha))\left(\gamma^2\alpha + (1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha + (1-\gamma^2)(1-\alpha))} \right], \text{ otherwise.} \end{array} \right\}.$$

We summarize the resultsin the following proposition.

Proposition 3 *A separating equilibrium for the Stackelberg game with two demand states exists and is as follows:*

(1) The leader's equilibrium output quantity is

$$\begin{aligned} q_{1H}^* &= \frac{1}{2} B_1 \textit{ if } Y_1 = A_H, \textit{ and} \\ q_{1L}^* &= \begin{cases} \frac{1}{2} B_2 \textit{ if } Y_1 = A_L \textit{ and } \frac{A_H}{A_L} \in \Omega_1, \\ B_1 - \frac{1}{2} B_3 - \frac{1}{2} \sqrt{(B_1 - B_3)(3B_1 - B_3)} \textit{ if } Y_1 = A_L \textit{ and } \frac{A_H}{A_L} \in \Omega_2. \end{cases} \end{aligned}$$

(2) The follower's equilibrium output quantity is

$$q_{2HH}^{*}=\frac{1}{2}\left(B_5-\frac{1}{2}B_1\right)\ \textit{if}\ Y_2=A_H\ \textit{and}\ \Pr_2\left(Y_1=A_H\right)=1,$$
$$q_{2HL}^{*}=\frac{1}{2}\left(B_7-\frac{1}{2}B_1\right)\ \textit{if}\ Y_2=A_L\ \textit{and}\ \Pr_2\left(Y_1=A_H\right)=1,$$
$$q_{2LH}^{*}=\frac{1}{2}\left(B_6-\frac{1}{2}B_2\right)\ \textit{if}\ Y_2=A_H,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \frac{A_H}{A_L}\in\Omega_1,$$
$$q_{2LL}^{*}=\frac{1}{2}\left(B_8-\frac{1}{2}B_2\right)\ \textit{if}\ Y_2=A_L,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \frac{A_H}{A_L}\in\Omega_1,$$
$$q_{2LH}^{*}=\tfrac{1}{2}\left(B_6-B_1+\tfrac{1}{2}B_3+\tfrac{1}{2}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)$$
$$\textit{if}\ Y_2=A_H,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_2,$$
$$q_{2LL}^{*}=\tfrac{1}{2}\left(B_8-B_1+\tfrac{1}{2}B_3+\tfrac{1}{2}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)$$
$$\textit{if}\ Y_2=A_L,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_2,$$

consistent with its belief that $\Pr_2\left(Y_1=A_H\right)=\begin{cases}0\ \textit{if}\ q_1\leq q_{1L}^{*},\\ 1\ \textit{otherwise.}\end{cases}$

(3) Firms' profits are as follows: the leader earns a expected profit of

$$E\left[\pi_1|Y_1=A_H\right]=\frac{1}{8}B_1^2\ \textit{if}\ Y_1=A_H,\ \textit{and}$$
$$E\left[\pi_1|Y_1=A_L\right]=\begin{cases}\tfrac{1}{8}B_2^2\ \textit{if}\ Y_1=A_L\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_1,\\ \left(B_1-\tfrac{1}{2}B_3-\tfrac{1}{2}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)*\\ \left(\tfrac{1}{2}B_2-\tfrac{1}{2}B_1+\tfrac{1}{4}B_3+\tfrac{1}{4}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)\\ \textit{if}\ Y_1=A_L\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_2.\end{cases}$$

The follower's expected profit is

$$
\begin{aligned}
E\left[\pi_2|Y_2=A_H,q_{1H}\right] &= \tfrac{1}{4}\left(B_5-\tfrac{1}{2}B_1\right)^2 \ \textit{if}\ Y_2=A_H \\
&\quad \textit{and}\ \Pr_2\left(Y_1=A_H\right)=1, \\
E\left[\pi_2|Y_2=A_L,q_{1H}\right] &= \tfrac{1}{4}\left(B_7-\tfrac{1}{2}B_1\right)^2 \ \textit{if}\ Y_2=A_L \\
&\quad \textit{and}\ \Pr_2\left(Y_1=A_H\right)=1, \\
E\left[\pi_2|Y_2=A_H,q_{1L}\right] &= \tfrac{1}{4}\left(B_6-\tfrac{1}{2}B_2\right)^2 \ \textit{if}\ Y_2=A_H, \\
&\quad \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_1, \\
E\left[\pi_2|Y_2=A_L,q_{1L}\right] &= \tfrac{1}{4}\left(B_8-\tfrac{1}{2}B_2\right)^2 \ \textit{if}\ Y_2=A_L, \\
&\quad \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_1, \\
E\left[\pi_2|Y_2=A_H,q_{1L}\right] &= \tfrac{1}{4}\left(B_6-B_1+\tfrac{1}{2}B_3+\tfrac{1}{2}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)^2 \ \textit{if} \\
&\quad Y_2=A_H,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_2, \\
E\left[\pi_2|Y_2=A_L,q_{1L}\right] &= \tfrac{1}{4}\left(B_8-B_1+\tfrac{1}{2}B_3+\tfrac{1}{2}\sqrt{\left(B_1-B_3\right)\left(3B_1-B_3\right)}\right)^2 \ \textit{if} \\
&\quad Y_2=A_L,\ \Pr_2\left(Y_1=A_H\right)=0\ \textit{and}\ \tfrac{A_H}{A_L}\in\Omega_2.
\end{aligned}
$$

Recall that in either demand state (high or low), the leader would like to induce the follower to believe that its demand information is low, using the output quantity as an instrument. When $\frac{A_H}{A_L}$ is high and in the set Ω_1, A_H and A_L are so widely separated that the cost for the high type leader to mimic the low type by setting a low output quantity is exorbitant. So a natural separation exists and both the high type leader and the low type leader convey demand information by setting their output quantities equal to $\frac{1}{2}B_1$ and $\frac{1}{2}B_2$, respectively. However, when $\frac{A_H}{A_L}$ is small and in set Ω_2, if the low type leader sets its output quantity equal to $\frac{1}{2}B_2$, the high type leader would prefer to mimic the low type by setting $q_1 = \frac{1}{2}B_2$ rather than reveal its high type truthfully to the follower by setting $q_1 = \frac{1}{2}B_1$. Hence, if the low type leader wants to send a low signal to the follower, it need to set q_1 smaller than $\frac{1}{2}B_2$ to make it costly for the high type leader to mimic it. The high type leader wouldn't mimic the low type by setting its output quantity lower than $B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}$, since it is not profitable to do so. Hence, the optimal output quantity for the low type leader is $q_{1L}^* = B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}$ and the high type leader sets $q_{1H}^* = \frac{1}{2}B_1$, as stated in Proposition 3. If $\frac{A_H}{A_L} \in \Omega_2$, the low type leader incurs a cost to signal its private information. The cost of signaling makes the leader worse off. However, the follower benefits from the cost of signaling.

Firms' ex ante profits are given as follows. If $\frac{A_H}{A_L} \in \Omega_1$,

$$
\begin{aligned}
\Pi_1 &= E\left[\pi_1 | Y_1 = A_H\right] \Pr\left(Y_1 = A_H\right) + E\left[\pi_1 | Y_1 = A_L\right] \Pr\left(Y_1 = A_L\right) \\
&= \frac{1}{8}B_1^2 \Pr\left(Y_1 = A_H\right) + \frac{1}{8}B_2^2 \Pr\left(Y_1 = A_L\right), \\
\Pi_2 &= E\left[\pi_2 | Y_1 = A_H, Y_2 = A_H\right] \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_H\right] \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_H, Y_2 = A_L\right] \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_L\right] \Pr\left(Y_1 = A_L, Y_2 = A_L\right) \\
&= \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_6 - \frac{1}{2}B_2\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + \frac{1}{4}\left(B_8 - \frac{1}{2}B_2\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_L\right),
\end{aligned}
$$

If $\frac{A_H}{A_L} \in \Omega_2$,

$$\begin{aligned}
\Pi_1 &= \frac{1}{8}B_1^2 \Pr(Y_1 = A_H) + D_1\left(\frac{1}{2}B_2 - \frac{1}{2}D_1\right)\Pr(Y_1 = A_L), \\
\Pi_2 &= \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_H) \\
&\quad + \frac{1}{4}(B_6 - D_1)^2 \Pr(Y_1 = A_L, Y_2 = A_H) \\
&\quad + \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_L) \\
&\quad + \frac{1}{4}(B_8 - D_1)^2 \Pr(Y_1 = A_L, Y_2 = A_L),
\end{aligned}$$

where $D_1 = B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}$.

Proposition 4 *If the number of demand states is two, the leader always earns more ex ante profit than the follower'*

Proof. Please see the appendix. ■

Though the leader incurs the cost of signaling and the follower possesses more accurate information, the leader still earns more ex ante profit than the follower. This seems to be different from the finding in section 4, in which the follower may earn more profit than the leader under some conditions. Recall the Lemma 2 and Figure 1, if the highest signal is small enough, the cost of signaling is not large enough, the leader still benefits from the first-mover advantage, and the Stackelberg leadership is preferred by firms. In the discrete demand model, as mentioned earlier, if $\frac{A_H}{A_L} \in \Omega_1$, the high type leader has no incentive to mimic the low type since it is too costly, a natural separation ensues and the leader does not incur cost of signaling. If $\frac{A_H}{A_L} \in \Omega_2$, the low type leader incurs a cost of signaling, but it is not large enough to make the follower gain more profit than the leader. We conjecture that the leader's cost of signaling depends on the number of demand states. With more demand states, the lower type leader may incur a higher cost of signaling. Hence, the follower benefits more from it and may earn more profit than the leader. The next subsection will study the case when the number of demand states is three.

5.2 Three-state Case

5.2.1 Information Structure

The inverse demand function is given by

$$p = A - (q_1 + q_2).$$

The intercept A is random and can take one of the three values: a *high* value A_H with probability p_H, a *middle* value A_M with probability p_M and a *low* value A_L with probability p_L. We have $0 < A_L < A_M < A_H$, p_H, p_M, $p_L \in (0,1)$ and $p_H + p_M + p_L = 1$. These are common knowledge to all parties. Before Making decisions, the leader and the follower observe private demand information Y_1 and Y_2 about the random intercept A, respectively, and Y_1, $Y_2 \in \{A_H, A_M, A_L\}$. Y_1 and Y_2 are independently generated from the true A with probability γ, and $\gamma \in \left(\frac{1}{2}, 1\right)$, that is, $\Pr(Y_i = A) = \gamma$. We assume that A is independent of event $\{Y_1 = A\}$ and $\{Y_2 = A\}$, and $\{Y_1 = A\}$ is independent of $\{Y_2 = A\}$. Conditional on private signals, the leader and the follower will update their probabilities of the states. The parameter γ captures the informativeness of the signals. When $\gamma \to \frac{1}{2}$, a signal provides no additional information than common knowledge: $\Pr(A = A_H|Y_i = A_H) = p_H$, $\Pr(A = A_M|Y_i = A_M) = p_M$ and $\Pr(A = A_L|Y_i = A_L) = p_L$. When $\gamma \to 1$, the signals reveal the truth with certainty: $\Pr(A = A_H|Y_i = A_H) = \Pr(A = A_M|Y_i = A_M) = \Pr(A = A_L|Y_i = A_L) = 1$.

5.2.2 Analysis

We analyze this game by backward induction. We assume there exists a $\bar{q}_{1H}^*$ and a $\bar{q}_{1L}^{*}$, if the follower observes $\bar{q}_1 \geq \bar{q}_{1H}^{*}$, it believes the demand information state the leader observes is high and if the follower observes $\bar{q}_1 \leq \bar{q}_{1L}^{*}$, it believes that the leader's demand information is low. Otherwise, it believes it is middle, i.e., the follower's belief is

$$\begin{aligned}
\Pr_2 (Y_1 = A_H) &= 1 \text{ if } \bar{q}_1 \geq \bar{q}_{1H}^{*}, \\
\Pr_2 (Y_1 = A_L) &= 1 \text{ if } \bar{q}_1 \leq \bar{q}_{1L}^{*}, \\
\Pr_2 (Y_1 = A_M) &= 1 \text{ otherwise.}
\end{aligned}$$

In the second stage, the follower observes the leader's output quantity $\bar{q}_1$, and it will try to infer the leader's private information from $\bar{q}_1$ based on its belief. The follower's expected profit is,

$$\begin{aligned}
E[\pi_2 | Y_2, \bar{q}_1] &= E[\bar{q}_2 p | Y_2, \bar{q}_1] = E[\bar{q}_2 (A - (\bar{q}_1 + \bar{q}_2)) | Y_2, \bar{q}_1] \\
&= \bar{q}_2 (E[A | Y_2, \bar{q}_1] - (\bar{q}_1 + \bar{q}_2)),
\end{aligned}$$

From the FOC,

$$\bar{q}_2^{*} (\bar{q}_1, Y_2) = \frac{1}{2} (E[A | \bar{q}_1, Y_2] - \bar{q}_1).$$

To be specific,

$$\begin{aligned}
\bar{q}^*_{2HH} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1H}, Y_2 = A_H\right] - \bar{q}_{1H}\right), \\
\bar{q}^*_{2HM} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1H}, Y_2 = A_M\right] - \bar{q}_{1H}\right), \\
\bar{q}^*_{2HL} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1H}, Y_2 = A_L\right] - \bar{q}_{1H}\right), \\
\bar{q}^*_{2MH} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1M}, Y_2 = A_H\right] - \bar{q}_{1M}\right), \\
\bar{q}^*_{2MM} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1M}, Y_2 = A_M\right] - \bar{q}_{1M}\right), \\
\bar{q}^*_{2ML} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1M}, Y_2 = A_L\right] - \bar{q}_{1M}\right), \\
\bar{q}^*_{2LH} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1L}, Y_2 = A_H\right] - \bar{q}_{1L}\right), \\
\bar{q}^*_{2LM} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1L}, Y_2 = A_M\right] - \bar{q}_{1L}\right), \\
\bar{q}^*_{2LL} &= \frac{1}{2}\left(E\left[A|\bar{q}_{1L}, Y_2 = A_L\right] - \bar{q}_{1L}\right).
\end{aligned}$$

The first subscript 2 refers to the firm 2, and the second subscript (H, M or L) means that the leader's output quantity is high, middle or low, and the third subscript means that the follower observes a high, middle or low signal. When the leader observes Y_1, it will decide $\bar{q}_1$. The leader can decides $\bar{q}_1$ to reveal its type truthfully to the follower or to mislead the follower. If the leader decides $\bar{q}_1$ to reveal its type truthfully,

$$\begin{aligned}
E\left[\pi_1\left(\bar{q}_{1H}|Y_1 = A_H\right)\right] &= \bar{q}_{1H}\left(E\left[A|Y_1 = A_H\right] - \left(\bar{q}_{1H} + E\left[\bar{q}^*_2\left(\bar{q}_{1H}\right)|Y_1 = A_H\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_M\right)\right] &= \bar{q}_{1M}\left(E\left[A|Y_1 = A_M\right] - \left(\bar{q}_{1M} + E\left[\bar{q}^*_2\left(\bar{q}_{1M}\right)|Y_1 = A_M\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_L\right)\right] &= \bar{q}_{1L}\left(E\left[A|Y_1 = A_L\right] - \left(\bar{q}_{1L} + E\left[\bar{q}^*_2\left(\bar{q}_{1L}\right)|Y_1 = A_L\right]\right)\right).
\end{aligned}$$

However, if the leader decides $\bar{q}_1$ to mislead the follower,

$$
\begin{aligned}
E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_H\right)\right] &= \bar{q}_{1M}\left(E\left[A|Y_1 = A_H\right] - \left(\bar{q}_{1M} + E\left[\bar{q}_2^*\left(\bar{q}_{1M}\right)|Y_1 = A_H\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_H\right)\right] &= \bar{q}_{1L}\left(E\left[A|Y_1 = A_H\right] - \left(\bar{q}_{1L} + E\left[\bar{q}_2^*\left(\bar{q}_{1L}\right)|Y_1 = A_H\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1H}|Y_1 = A_M\right)\right] &= \bar{q}_{1H}\left(E\left[A|Y_1 = A_M\right] - \left(\bar{q}_{1H} + E\left[\bar{q}_2^*\left(\bar{q}_{1H}\right)|Y_1 = A_M\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_M\right)\right] &= \bar{q}_{1L}\left(E\left[A|Y_1 = A_M\right] - \left(\bar{q}_{1L} + E\left[\bar{q}_2^*\left(\bar{q}_{1L}\right)|Y_1 = A_M\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1H}|Y_1 = A_L\right)\right] &= \bar{q}_{1H}\left(E\left[A|Y_1 = A_L\right] - \left(\bar{q}_{1H} + E\left[\bar{q}_2^*\left(\bar{q}_{1H}\right)|Y_1 = A_L\right]\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_L\right)\right] &= \bar{q}_{1M}\left(E\left[A|Y_1 = A_L\right] - \left(\bar{q}_{1M} + E\left[\bar{q}_2^*\left(\bar{q}_{1M}\right)|Y_1 = A_L\right]\right)\right),
\end{aligned}
$$

where

$$
\begin{aligned}
E\left[\bar{q}_2^*\left(\bar{q}_{1H}\right)|Y_M = A_H\right] &= \frac{1}{2}C_1 - \frac{1}{2}\bar{q}_{1H}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1M}\right)|Y_1 = A_M\right] &= \frac{1}{2}C_2 - \frac{1}{2}\bar{q}_{1M}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1L}\right)|Y_1 = A_L\right] &= \frac{1}{2}C_3 - \frac{1}{2}\bar{q}_{1L}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1M}\right)|Y_1 = A_H\right] &= \frac{1}{2}C_4 - \frac{1}{2}\bar{q}_{1M}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1L}\right)|Y_1 = A_H\right] &= \frac{1}{2}C_5 - \frac{1}{2}\bar{q}_{1L}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1A_H}\right)|Y_1 = A_M\right] &= \frac{1}{2}C_6 - \frac{1}{2}\bar{q}_{1H}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1L}\right)|Y_1 = A_M\right] &= \frac{1}{2}C_7 - \frac{1}{2}\bar{q}_{1L}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1H}\right)|Y_1 = A_L\right] &= \frac{1}{2}C_8 - \frac{1}{2}\bar{q}_{1H}, \\
E\left[\bar{q}_2^*\left(\bar{q}_{1M}\right)|Y_1 = A_L\right] &= \frac{1}{2}C_9 - \frac{1}{2}\bar{q}_{1M}.
\end{aligned}
$$

$$
\begin{aligned}
C_1 &= E\left[A|Y_1 = A_H\right], \\
C_2 &= E\left[A|Y_1 = A_M\right], \\
C_3 &= E\left[A|Y_1 = A_L\right], \\
C_4 &= E\left[A|\bar{q}_{1M}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_H\right) \\
&\quad + E\left[A|\bar{q}_{1M}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_H\right) \\
&\quad + E\left[A|\bar{q}_{1M}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_H\right), \\
C_5 &= E\left[A|\bar{q}_{1L}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_H\right) \\
&\quad + E\left[A|\bar{q}_{1L}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_H\right) \\
&\quad + E\left[A|\bar{q}_{1L}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_H\right), \\
C_6 &= E\left[A|\bar{q}_{1H}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_M\right) \\
&\quad + E\left[A|\bar{q}_{1H}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_M\right) \\
&\quad + E\left[A|\bar{q}_{1H}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_M\right), \\
C_7 &= E\left[A|\bar{q}_{1L}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_M\right) \\
&\quad + E\left[A|\bar{q}_{1L}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_M\right) \\
&\quad + E\left[A|\bar{q}_{1L}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_M\right), \\
C_8 &= E\left[A|\bar{q}_{1H}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_L\right) \\
&\quad + E\left[A|\bar{q}_{1H}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_L\right) \\
&\quad + E\left[A|\bar{q}_{1H}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_L\right), \\
C_9 &= E\left[A|\bar{q}_{1M}, Y_2 = A_H\right] \Pr\left(Y_2 = A_H|Y_1 = A_L\right) \\
&\quad + E\left[A|\bar{q}_{1M}, Y_2 = A_M\right] \Pr\left(Y_2 = A_M|Y_1 = A_L\right) \\
&\quad + E\left[A|\bar{q}_{1M}, Y_2 = A_L\right] \Pr\left(Y_2 = A_L|Y_1 = A_L\right).
\end{aligned}
$$

Hence,

$$\begin{aligned}
E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_H\right)\right] &= \frac{1}{2}\left(C_1-\bar{q}_{1H}\right)\bar{q}_{1H}, \\
E\left[\pi_1\left(\bar{q}_{1M}|Y_1=A_M\right)\right] &= \frac{1}{2}\left(C_2-\bar{q}_{1M}\right)\bar{q}_{1M}, \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1=A_L\right)\right] &= \frac{1}{2}\left(C_3-\bar{q}_{1L}\right)\bar{q}_{1L}, \\
E\left[\pi_1\left(\bar{q}_{1M}|Y_1=A_H\right)\right] &= \bar{q}_{1M}\left(C_1-\left(\frac{1}{2}\bar{q}_{1M}+\frac{1}{2}C_4\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1=A_H\right)\right] &= \bar{q}_{1L}\left(C_1-\left(\frac{1}{2}\bar{q}_{1L}+\frac{1}{2}C_5\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_M\right)\right] &= \bar{q}_{1H}\left(C_2-\left(\frac{1}{2}\bar{q}_{1H}+\frac{1}{2}C_6\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1L}|Y_1=A_M\right)\right] &= \bar{q}_{1L}\left(C_2-\left(\frac{1}{2}\bar{q}_{1L}+\frac{1}{2}C_7\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_L\right)\right] &= \bar{q}_{1H}\left(C_3-\left(\frac{1}{2}\bar{q}_{1H}+\frac{1}{2}C_8\right)\right), \\
E\left[\pi_1\left(\bar{q}_{1M}|Y_1=A_L\right)\right] &= \bar{q}_{1M}\left(C_3-\left(\frac{1}{2}\bar{q}_{1M}+\frac{1}{2}C_9\right)\right).
\end{aligned}$$

For a pure strategy separating equilibrium to exist, $\bar{q}_{1H}$, $\bar{q}_{1M}$ and $\bar{q}_{1L}$ must emerge as a simultaneous solution to the following constrained game: the problem for high type leader is

$$\max_{\bar{q}_{1H}} E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_H\right)\right]=\frac{1}{2}\left(C_1-\bar{q}_{1H}\right)\bar{q}_{1H}, \tag{13}$$

subject to

$$\begin{aligned}
E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_M\right)\right] &\leq E\left[\pi_1\left(\bar{q}^*_{1M}|Y_1=A_M\right)\right], &(14)\\
E\left[\pi_1\left(\bar{q}_{1H}|Y_1=A_L\right)\right] &\leq E\left[\pi_1\left(\bar{q}^*_{1L}|Y_1=A_L\right)\right], &(15)\\
\bar{q}_{1H} &\geq 0. &(16)
\end{aligned}$$

The problem for middle type leader is to maximize

$$\max_{\bar{q}_{1M}} E\left[\pi_1\left(\bar{q}_{1M}|Y_1=A_M\right)\right]=\frac{1}{2}\left(C_2-\bar{q}_{1M}\right)\bar{q}_{1M}, \tag{17}$$

subject to

$$E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_H\right)\right] \leq E\left[\pi_1\left(\bar{q}^*_{1H}|Y_1 = A_H\right)\right], \tag{18}$$

$$E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_L\right)\right] \leq E\left[\pi_1\left(\bar{q}^*_{1L}|Y_1 = A_L\right)\right], \tag{19}$$

$$\bar{q}_{1M} \geq 0. \tag{20}$$

And the problem for low type leader is to maximize

$$\max_{\bar{q}_{1L}} E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_L\right)\right] = \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L}, \tag{21}$$

subject to

$$E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_H\right)\right] \leq E\left[\pi_1\left(\bar{q}^*_{1H}|Y_1 = A_H\right)\right], \tag{22}$$

$$E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_M\right)\right] \leq E\left[\pi_1\left(\bar{q}^*_{1M}|Y_1 = A_M\right)\right], \tag{23}$$

$$\bar{q}_{1L} \geq 0. \tag{24}$$

Inequalities (16), (20) and (24) are participation constraints. Inequalities (14), (15), (18), (19), (22) and (23) are the incentive compatibility constraints for a separating equilibrium. The right hand side of these inequalities are the equilibrium profits which are greater than the LHS off-equilibrium profits. For the high type leader, constraints (14) and (15) do not bind since only the high type leader has an incentive to mimic the lower type. So for the high type leader, the optimal output quantity is $\frac{1}{2}C_1$.

For the middle type leader, constraint (19) does not bind for similar reason as the high type leader. So we need to solve

$$\max_{\bar{q}_{1M}} E\left[\pi_1\left(\bar{q}_{1M}|Y_1 = A_M\right)\right] = \frac{1}{2}\left(C_2 - \bar{q}_{1M}\right)\bar{q}_{1M}, \tag{25}$$

subject to

$$
\begin{aligned}
E\left[\pi_1\left(\bar{q}_{1M} | Y_1 = A_H\right)\right] &\leq E\left[\pi_1\left(\bar{q}_{1H}^{*} | Y_1 = A_H\right)\right], \\
\bar{q}_{1M}\left(C_1 - \left(\frac{1}{2}\bar{q}_{1M} + \frac{1}{2}C_4\right)\right) &\leq \frac{1}{8}C_1^2,
\end{aligned}
\tag{26}
$$

The Lagrangian for the above formulation is

$$
\pounds_2\left(\bar{q}_{1M}, u_1\right) = \frac{1}{2}\left(C_2 - \bar{q}_{1M}\right)\bar{q}_{1M} - u_1\left(\bar{q}_{1M}\left(C_1 - \left(\frac{1}{2}\bar{q}_{1M} + \frac{1}{2}C_4\right)\right) - \frac{1}{8}C_1^2\right),
$$

The first order KKT conditions for the Lagrangian are:

$$
\begin{aligned}
(1a) &: \frac{\partial \pounds_2\left(\bar{q}_{1M}, u_1\right)}{\partial \bar{q}_{1M}} \leq 0 \Rightarrow \left(\begin{array}{c} -u_1\left(\bar{q}_{1M}\left(C_1 - \left(\frac{1}{2}\bar{q}_{1M} + \frac{1}{2}C_4\right)\right) - \frac{1}{8}C_1^2\right) \\ +\frac{1}{2}\left(\left(C_2 - \bar{q}_{1M}\right) - \bar{q}_{1M}\right) + \upsilon_3 \end{array}\right) = 0, \\
(1b) &: \bar{q}_{1M}\frac{\partial \pounds_2\left(\bar{q}_{1M}, u_1\right)}{\partial \bar{q}_{1M}} = 0 \text{ by complementary slackness,} \\
(2a) &: \frac{\partial \pounds_2\left(\bar{q}_{1M}, u_1\right)}{\partial u_1} \geq 0 \Rightarrow \left(\bar{q}_{1M}\left(C_1 - \left(\frac{1}{2}\bar{q}_{1M} + \frac{1}{2}C_4\right)\right) - \frac{1}{8}C_1^2\right) + \upsilon_4 = 0, \\
(2b) &: u_1\frac{\partial \pounds_2\left(\bar{q}_{1M}, u_1\right)}{\partial u_1} = 0 \text{ by complementary slackness.}
\end{aligned}
$$

Solving the above system, we get: for $u_1 = 0$,

$$
\bar{q}_{1M}^{*} = \frac{1}{2}C_2,
$$

$$
\upsilon_2 \geq 0 \Rightarrow \frac{1}{2}C_2\left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4\right)\right) \leq \frac{1}{8}C_1^2.
$$

For $u_1 > 0$, we have

$$
\bar{q}_{1M}^{*} = C_1 - \frac{1}{2}C_4 \pm \frac{1}{2}\sqrt{\left(C_1 - C_4\right)\left(3C_1 - C_4\right)}.
$$

The unconstrained maxima of the equation (25) is at $\frac{1}{2}C_2$, while the maxima for the LHS of the constraint (26) is at $\left(C_1 - \frac{1}{2}C_4\right)$, where $\left(C_1 - \frac{1}{2}C_4\right) > \frac{1}{2}C_2$. Since

$C_1 - \frac{1}{2}C_4 - \frac{1}{2}\sqrt{(C_1 - C_4)(3C_1 - C_4)} < \frac{1}{2}C_2 < C_1 - \frac{1}{2}C_4 + \frac{1}{2}\sqrt{(C_1 - C_4)(3C_1 - C_4)}$. If $\bar{q}_{1M}^* > \frac{1}{2}C_2$ does not permit a separating equilibrium. Hence, in order to separate out, the middle type leader must set an output quantity lower than $\frac{1}{2}C_2$, to prevent the high type leader from mimicing him. So $\bar{q}_{1M}^* = C_1 - \frac{1}{2}C_4 - \frac{1}{2}\sqrt{(C_1 - C_4)(3C_1 - C_4)}$. For the low type leader,

$$\max_{\bar{q}_{1L}} E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_L\right)\right] = \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L}, \tag{27}$$

subject to

$$\begin{aligned}
\bar{q}_{1L}\left(C_1 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_5\right)\right) &\leq \frac{1}{8}C_1^2, \\
\bar{q}_{1L}\left(C_2 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_7\right)\right) &\leq \frac{1}{2}\left(C_2 - \bar{q}_{1M}^*\right)\bar{q}_{1M}^*.
\end{aligned} \tag{28}$$

(1) If $\bar{q}_{1M}^* = \frac{1}{2}C_2$, then constraint (28) does not bind. The game reduces to

$$\max_{\bar{q}_{1L}} E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_L\right)\right] = \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L},$$

subject to

$$\bar{q}_{1L}\left(C_2 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_7\right)\right) \leq \frac{1}{2}\left(C_2 - \bar{q}_{1M}^*\right)\bar{q}_{1M}^*.$$

The Lagrangian for the above formulation is

$$\begin{aligned}
\pounds_3\left(\bar{q}_{1L}, u_2\right) = {} & \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L} \\
& -u_2\left(\bar{q}_{1L}\left(C_2 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_7\right)\right) - \frac{1}{2}\left(C_2 - \bar{q}_{1M}^*\right)\bar{q}_{1M}^*\right),
\end{aligned}$$

Using the first-order KKT conditions, we get if $u_2 = 0$,

$$\begin{aligned}
\bar{q}_{1L}^* &= \frac{1}{2}C_3, \\
\text{If } \frac{1}{2}C_3\left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7\right)\right) &\leq \frac{1}{8}C_2^2.
\end{aligned}$$

if $u_2 > 0$,

$$\bar{q}_{1L}^{*} = C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{(C_2 - C_7)(3C_2 - C_7)}.$$

(2) If $\bar{q}_{1M}^{*} = C_1 - \frac{1}{2}C_4 - \frac{1}{2}\sqrt{(C_1 - C_4)(3C_1 - C_4)}$, the game reduces to

$$\max_{\bar{q}_{1L}} E\left[\pi_1\left(\bar{q}_{1L}|Y_1 = A_L\right)\right] = \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L},$$

subject to

$$\bar{q}_{1L}\left(C_2 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_7\right)\right) \leq \frac{1}{2}\left(C_2 - \bar{q}_{1M}^{*}\right)\bar{q}_{1M}^{*}.$$

The Lagrangian for the above formulation is

$$\begin{aligned} \pounds_4\left(\bar{q}_{1L}, u_3\right) &= \frac{1}{2}\left(C_3 - \bar{q}_{1L}\right)\bar{q}_{1L} \\ &\quad - u_3\left(\bar{q}_{1L}\left(C_2 - \left(\frac{1}{2}\bar{q}_{1L} + \frac{1}{2}C_7\right)\right) - \frac{1}{2}\left(C_2 - \bar{q}_{1M}^{*}\right)\bar{q}_{1M}^{*}\right). \end{aligned}$$

Using the first-order KKT conditions to solve the Lagrangian function, we get if $u_3 = 0$,

$$\begin{aligned} \bar{q}_{1L}^{*} &= \frac{1}{2}C_3, \\ \frac{1}{2}C_3\left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7\right)\right) &\leq \frac{1}{2}\left(C_2 - D_2\right)D_2, \end{aligned}$$

and if $u_3 > 0$,

$$\bar{q}_{1L}^{*} = C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{4D_2^2 - 4D_2C_2 + 4C_2^2 - 4C_2C_7 + C_7^2},$$

where $D_2 = C_1 - \frac{1}{2}C_4 - \frac{1}{2}\sqrt{(C_1 - C_4)(3C_1 - C_4)}$.

Proposition 5 *A separating equilibrium for the Stackelberg game with three demand states exists and is as follows:*

(1) The leader's equilibrium output quantity is

$$
\begin{aligned}
q_{1H}^{*} &= \frac{1}{2}C_1 \textit{ if } Y_1 = A_H, \textit{ and} \\
q_{1M}^{*} &= \begin{cases} \frac{1}{2}C_2 \textit{ if } Y_1 = A_M \textit{ and } (A_H, A_M, A_L) \in \Omega_3, \\ C_1 - \frac{1}{2}C_3 - \frac{1}{2}\sqrt{(C_1 - C_3)(3C_1 - C_3)} \textit{ if } Y_1 = A_M \\ \textit{and } (A_H, A_M, A_L) \in \Omega_4. \end{cases} \\
q_{1L}^{*} &= \begin{cases} \frac{1}{2}C_3 \textit{ if } Y_1 = A_L \textit{ and } (A_H, A_M, A_L) \in \Omega_5 \cup \Omega_7, \\ C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{(C_2 - C_7)(3C_2 - C_7)} \textit{if } Y_1 = A_L \\ \textit{and } (A_H, A_M, A_L) \in \Omega_6, \\ C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{4D_2^2 - 4C_2D_2 + 4C_2^2 - 4C_2C_7 + C_7^2} \\ \textit{if } Y_1 = A_L \textit{ and } (A_H, A_M, A_L) \in \Omega_8. \end{cases}
\end{aligned}
$$

where

$$
\begin{aligned}
\Omega_3 &= \left\{ (A_H, A_M, A_L) : 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) \leq C_1^2 \right\}, \\
\Omega_4 &= \left\{ (A_H, A_M, A_L) : 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) > C_1^2 \right\}, \\
\Omega_5 &= \left\{ (A_H, A_M, A_L) : \begin{array}{c} 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) \leq C_1^2, \\ 4C_3 \left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7 \right) \right) \leq C_2^2 \end{array} \right\}, \\
\Omega_6 &= \left\{ (A_H, A_M, A_L) : \begin{array}{c} 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) \leq C_1^2, \\ 4C_3 \left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7 \right) \right) > C_2^2 \end{array} \right\}, \\
\Omega_7 &= \left\{ (A_H, A_M, A_L) : \begin{array}{c} 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) > C_1^2, \\ C_3 \left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7 \right) \right) \leq (C_2 - D_2) D_2 \end{array} \right\}, \\
\Omega_8 &= \left\{ (A_H, A_M, A_L) : \begin{array}{c} 4C_2 \left(C_1 - \left(\frac{1}{4}C_2 + \frac{1}{2}C_4 \right) \right) > C_1^2, \\ C_3 \left(C_2 - \left(\frac{1}{4}C_3 + \frac{1}{2}C_7 \right) \right) > (C_2 - D_2) D_2 \end{array} \right\}.
\end{aligned}
$$

(2) The follower's equilibrium output quantity is

$$
q_2^{*} = \frac{1}{2} \left(E\left[A | Y_2, q_1^{*} \right] - q_1^{*} \right),
$$

consistent with its belief that

$$
\begin{aligned}
\Pr_2(Y_1 = A_H) &= 1 \textit{ if } \bar{q}_1 \geq \bar{q}^*_{1H}, \\
\Pr_2(Y_1 = A_L) &= 1 \textit{ if } \bar{q}_1 \leq \bar{q}^*_{1L}, \\
\Pr_2(Y_1 = A_M) &= 1 \textit{ otherwise.}
\end{aligned}
$$

(3) The firms' expected profits are given by

$$
\begin{aligned}
E[\pi_1|Y_1] &= \frac{1}{2}q^*_1(E[A|Y_1] - q^*_1), \\
E[\pi_2|Y_2, \bar{q}^*_1] &= \frac{1}{2}(E[A|Y_2, \bar{q}^*_1] - q^*_1)^2.
\end{aligned}
$$

Firms' profits are shown as follows,

If $Y_1 = A_H$,

$$
\begin{aligned}
\bar{q}^*_{1H} &= \frac{1}{2}C_1, \\
E[\pi_1|Y_1 = A_H] &= \frac{1}{8}C_1^2, \\
E[\pi_2|Y_2 = A_H, \bar{q}^*_{1H}] &= \frac{1}{4}\left(E[A|\bar{q}_{1H}, Y_2 = A_H] - \frac{1}{2}C_1\right)^2, \\
E[\pi_2|Y_2 = A_M, \bar{q}^*_{1H}] &= \frac{1}{4}\left(E[A|\bar{q}_{1H}, Y_2 = A_M] - \frac{1}{2}C_2\right)^2, \\
E[\pi_2|Y_2 = A_L, \bar{q}^*_{1H}] &= \frac{1}{4}\left(E[A|\bar{q}_{1H}, Y_2 = A_L] - \frac{1}{2}C_3\right)^2,
\end{aligned}
$$

If $Y_1 = A_M$ and $(A_H, A_M, A_L) \in \Omega_3$,

$$\begin{aligned}
\bar{q}_{1M}^* &= \frac{1}{2}C_2, \\
E\left[\pi_1 | Y_1 = A_M\right] &= \frac{1}{8}C_2^2, \\
E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | \bar{q}_{1M}, Y_2 = A_H\right] - \frac{1}{2}C_2\right)^2, \\
E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_M, \bar{q}_{1M}^*\right] - \frac{1}{2}C_2\right)^2, \\
E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_L, \bar{q}_{1M}^*\right] - \frac{1}{2}C_2\right)^2.
\end{aligned}$$

If $Y_1 = A_M$ and $(A_H, A_M, A_L) \in \Omega_4$,

$$\begin{aligned}
\bar{q}_{1M}^* &= C_1 - \frac{1}{2}C_4 - \frac{1}{2}\sqrt{\left(C_1 - C_4\right)\left(3C_1 - C_4\right)}, \\
E\left[\pi_1 | Y_1 = A_M\right] &= \frac{1}{2}\left(C_2 - D_2\right) D_2, \\
E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | \bar{q}_{1M}, Y_2 = A_H\right] - D_2\right)^2, \\
E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_M, \bar{q}_{1M}^*\right] - D_2\right)^2, \\
E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1M}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_L, \bar{q}_{1M}^*\right] - D_2\right)^2.
\end{aligned}$$

If $Y_1 = A_L$ and $(A_H, A_M, A_L) \in \Omega_5$,

$$\begin{aligned}
\bar{q}_{1L}^* &= \frac{1}{2}C_3, \\
E\left[\pi_1 | Y_1 = A_L\right] &= \frac{1}{8}C_3^2, \\
E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1L}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_H, \bar{q}_{1L}^*\right] - \frac{1}{2}C_3\right)^2, \\
E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1L}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_M, \bar{q}_{1L}^*\right] - \frac{1}{2}C_3\right)^2, \\
E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1L}^*\right] &= \frac{1}{4}\left(E\left[A | Y_2 = A_L, \bar{q}_{1L}^*\right] - \frac{1}{2}C_3\right)^2.
\end{aligned}$$

If $Y_1 = A_L$ and $(A_H, A_M, A_L) \in \Omega_6$,

$$\begin{aligned}
\bar{q}_{1L}^{*} &= \left(C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{(C_2 - C_7)(3C_2 - C_7)}\right), \\
E[\pi_1 | Y_1 = A_L] &= \left(\frac{1}{2}(C_3 - D_3) D_3\right), \\
E[\pi_2 | Y_2 = A_H, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_H, \bar{q}_{1L}^{*}] - D_3\right)^2, \\
E[\pi_2 | Y_2 = A_M, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_M, \bar{q}_{1L}^{*}] - D_3\right)^2, \\
E[\pi_2 | Y_2 = A_L, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_L, \bar{q}_{1L}^{*}] - D_3\right)^2,
\end{aligned}$$

where $D_3 = \left(C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{(C_2 - C_7)(3C_2 - C_7)}\right)$.

If $Y_1 = A_L$ and $(A_H, A_M, A_L) \in \Omega_7$,

$$\begin{aligned}
\bar{q}_{1L}^{*} &= \frac{1}{2}C_3, \\
E[\pi_1 | Y_1 = A_L] &= \frac{1}{8}C_3^2, \\
E[\pi_2 | Y_2 = A_H, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_H, \bar{q}_{1L}^{*}] - \frac{1}{2}C_3\right)^2, \\
E[\pi_2 | Y_2 = A_M, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_M, \bar{q}_{1L}^{*}] - \frac{1}{2}C_3\right)^2, \\
E[\pi_2 | Y_2 = A_L, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_L, \bar{q}_{1L}^{*}] - \frac{1}{2}C_3\right)^2.
\end{aligned}$$

If $Y_1 = A_L$ and $(A_H, A_M, A_L) \in \Omega_8$,

$$\begin{aligned}
\bar{q}_{1L}^{*} &= C_2 - \frac{1}{2}C_7 - \frac{1}{2}\sqrt{4D_2^2 - 4D_2C_2 + 4C_2^2 - 4C_2C_7 + C_7^2}, \\
E[\pi_1(\bar{q}_{1L}^{*} | Y_1 = A_L)] &= \frac{1}{2}(C_3 - \bar{q}_{1L}^{*})\bar{q}_{1L}^{*}, \\
E[\pi_2 | Y_2 = A_H, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_H, \bar{q}_{1L}^{*}] - \bar{q}_{1L}^{*}\right)^2, \\
E[\pi_2 | Y_2 = A_M, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_M, \bar{q}_{1L}^{*}] - \bar{q}_{1L}^{*}\right)^2, \\
E[\pi_2 | Y_2 = A_L, \bar{q}_{1L}^{*}] &= \frac{1}{4}\left(E[A | Y_2 = A_L, \bar{q}_{1L}^{*}] - \bar{q}_{1L}^{*}\right)^2.
\end{aligned}$$

Firms' ex ante profits are

$$\begin{aligned}
\Pi_1 &= E\left[\pi_1 | Y_1 = A_H\right] \Pr\left(Y_1 = A_H\right) + E\left[\pi_1 | Y_1 = A_M\right] \Pr\left(Y_1 = A_M\right) \\
&\quad + E\left[\pi_1 | Y_1 = A_L\right] \Pr\left(Y_1 = A_L\right),
\end{aligned}$$

$$\begin{aligned}
\Pi_2 &= E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1H}^*\right] \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1H}^*\right] \Pr\left(Y_1 = A_H, Y_2 = A_M\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1H}^*\right] \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1M}^*\right] \Pr\left(Y_1 = A_M, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1M}^*\right] \Pr\left(Y_1 = A_M, Y_2 = A_M\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1M}^*\right] \Pr\left(Y_1 = A_M, Y_2 = A_L\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_H, \bar{q}_{1L}^*\right] \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_M, \bar{q}_{1L}^*\right] \Pr\left(Y_1 = A_L, Y_2 = A_M\right) \\
&\quad + E\left[\pi_2 | Y_2 = A_L, \bar{q}_{1L}^*\right] \Pr\left(Y_1 = A_L, Y_2 = A_L\right).
\end{aligned}$$

We analyze firms' ex ante profits numerically below.

(1) If $A_H = 2$, $A_M = 1.5$, $A_L = 1$, $\bar{q}_H = 0.01$, $\bar{q}_M = 0.001$, $\bar{q}_L = 1 - \bar{q}_H - \bar{q}_M$, and $\gamma = 0.6$, then $\bar{q}_{1H}^* = 0.5100, \bar{q}_{1M}^* = 0.4123$ and $\bar{q}_{1L}^* = 0.3979$, firms' ex ante profits are given by

$$\begin{aligned}
\Pi_1 &= 0.1240, \\
\Pi_2 &= 0.0871.
\end{aligned}$$

The leader earns more profit than the follower.

(2) If $A_H = 2$, $A_M = 1.5$, $A_L = 1$, $\bar{q}_H = 0.1$, $\bar{q}_M = 0.01$, $\bar{q}_L = 1 - \bar{q}_H - \bar{q}_M$,

and $\gamma = 0.6$. Then $\bar{q}_{1H}^{*} = 0.5738$, $\bar{q}_{1M}^{*} = 0.2609$, and $\bar{q}_{1L}^{*} = 0.2453$,

$$\begin{aligned}\Pi_1 &= 0.1038, \\ \Pi_2 &= 0.1251.\end{aligned}$$

Hence, the follower earns more profit than the leader.

Observation 2 If the number of demand states is three, the follower may earn more ex ante profit than the leader.

These results supports our earlier conjecture in section 5.1 that the cost of signaling depends on the number of demand states. In binary case, the leader always earns more profit than the follower. However, If the number of demand states is three, for the middle type leader, it will decides an output quantity to prevent the high type leader from mimicing it, and for the low type leader. It will choose an output quantity to prevent both the high type and the middle type leader from imitating, thus the low type leader incurs a higher cost to signal its information. Therefore, the follower may earn more profit than the leader. With more number of demand states, the leader will set a lower output quantity to separate from higher states, thus the leader incurs a higher cost of signaling when it observes a lower signal. The follower benefits from the cost of signaling. If there are infinite number of demand states in interval $[A_L, A_M]$, we can regards that the demand is continuously distributed in this interval to some extent, which has been discussed in section 4.

5.2.3 The Two Effects

To explore the two effects, we solve the two benchmark models in section 4.5. Solving the two models, we get the following two Lemmas.

Lemma 5 *If both firms observe Y_1 only, the equilibrium for the game is*

$$\begin{aligned} q_1^{B*} &= \frac{1}{2} E\left[A|Y_1\right], \\ q_2^{B*} &= \frac{1}{4} E\left[A|Y_1\right]. \end{aligned}$$

Firms' expected profits are

$$\begin{aligned} E\left[\pi_1|Y_1\right] &= \frac{1}{8}\left(E\left[A|Y_1\right]\right)^2, \\ E\left[\pi_2|Y_1\right] &= \frac{1}{16}\left(E\left[A|Y_1\right]\right)^2. \end{aligned}$$

Lemma 6 *If the leader's information Y_1 is public and the follower possesses some private information Y_2 about the uncertain demand, the Subgame Perfect equilibrium for the game is $\left(\hat{q}_1^{B*}, \hat{q}_2^{B*}\right)$, where*

$$\begin{aligned} \hat{q}_1^{B*} &= \frac{1}{2} E\left[A|Y_1\right], \\ \hat{q}_2^{B*} &= \frac{1}{2}\left(E\left[A|Y_1, Y_2\right] - \frac{1}{2} E\left[A|Y_1\right]\right). \end{aligned}$$

(1) Firms' expected profit are

$$\begin{aligned} E\left[\hat{\pi}_1^B|Y_1\right] &= \frac{1}{8}\left(E\left[A|Y_1\right]\right)^2, \\ E\left[\hat{\pi}_2^B|Y_1, Y_2\right] &= \frac{1}{4}\left(E\left[A|Y_1, Y_2\right] - \frac{1}{2} E\left[A|Y_1\right]\right)^2. \end{aligned}$$

(2) Firms' ex ante profits are

$$\begin{aligned}
\hat{\Pi}_1^B &= E\left[\pi_1|Y_1=A_H\right]\Pr\left(Y_1=A_H\right)+E\left[\pi_1|Y_1=A_M\right]\Pr\left(Y_1=A_M\right) \\
&\quad +E\left[\pi_1|Y_1=A_L\right]\Pr\left(Y_1=A_L\right), \\
\hat{\Pi}_2^B &= E\left[\pi_2|Y_1=A_H,Y_2=A_H\right]\Pr\left(Y_1=A_H,Y_2=A_H\right) \\
&\quad +E\left[\pi_2|Y_1=A_H,Y_2=A_M\right]\Pr\left(Y_1=A_H,Y_2=A_M\right) \\
&\quad +E\left[\pi_2|Y_1=A_H,Y_2=A_L\right]\Pr\left(Y_1=A_H,Y_2=A_L\right) \\
&\quad +E\left[\pi_2|Y_1=A_M,Y_2=A_H\right]\Pr\left(Y_1=A_M,Y_2=A_H\right) \\
&\quad +E\left[\pi_2|Y_1=A_M,Y_2=A_M\right]\Pr\left(Y_1=A_M,Y_2=A_M\right) \\
&\quad +E\left[\pi_2|Y_1=A_M,Y_2=A_L\right]\Pr\left(Y_1=A_M,Y_2=A_L\right) \\
&\quad +E\left[\pi_2|Y_1=A_L,Y_2=A_H\right]\Pr\left(Y_1=A_L,Y_2=A_H\right) \\
&\quad +E\left[\pi_2|Y_1=A_L,Y_2=A_M\right]\Pr\left(Y_1=A_L,Y_2=A_M\right) \\
&\quad +E\left[\pi_2|Y_1=A_L,Y_2=A_L\right]\Pr\left(Y_1=A_L,Y_2=A_L\right).
\end{aligned}$$

Lemma 7 *If the number of demand states is three, though the follower possesses more accurate information, the leader's output quantity is no less than that of the follower's, thus makes more ex ante profit.*

Proof. Please see the appendix. ■

Comparing the two firms' ex ante profits, we find that $\hat{\Pi}_1^B > \hat{\Pi}_2^B$. The result is consistent with that in section 4, the leader always gains more profit than the follower when the leader's information is public. As in section 4.5,the effect of the follower's more accurate information on the follower is $\left(\hat{\Pi}_2^B - \Pi_2^B\right)$, and the effect of cost of signaling on the follower is $\left(\Pi_2 - \hat{\Pi}_2^B\right)$. Based on Lemma 5 and Lemma 6, we can compute the effect of cost of signaling.

We numerically compare the effect of cost of signaling on the leader if the number of demand states is two and three. The mean in two cases is the same.

(1) If the number of demand states is two, $A_H = 1.5$, $A_L = 1$, $\alpha = 0.105$, $\gamma = 0.6$, then

Table 2: The two effects (Two demand states)

	The effect of the follower's more accurate information	The effect of cost of signaling
The leader	0	0.0033
The follower	-0.0027	0.0638

(2) If the number of demand states is three, $A_H = 1.5$, $A_M = 1.25$, $A_L = 1$, $p_H = 0.1$, $p_M = 0.01$, $p_L = 0.89$, $\gamma = 0.6$, then

Table 3: The two effects (Three demand states)

	The effect of follower's more accurate information	The effect of cost of signaling
The leader	0	0.0236
The follower	-0.00012	0.0558

Observation 3 The effect of cost of signaling on leader may increase with more number of demand states.

From table 2 and table 3, we find that the effect of cost of signaling on the leader increases from 0.0033 to 0.0236. We can find same observation if we change the parameters value.

If the demand is discretely distributed, the highest type leader takes ex post efficient action. However, other type leader may incur a cost to signal its information. If the number of demand states is two, only the low type leader may incur the cost to separate from the high type leader. However, if the number of demand states is three, both the middle type leader and the low type leader may incur the cost of signaling. For the middle type leader, it will set an output quantity to prevent the high type leader from mimicing it. For the low type leader, it will set an output quantity to prevent both the middle type and the high type leader from mimicing it, thus may incur a higher cost to separate from them. Hence, the effect of cost of signaling becomes more significant with more demand states.

6 CONCLUSION

In this thesis, we consider a sequential-move game in which two firms are engaged in a Cournot competition and sell homogeneous product to customers. Firms are faced with uncertain demand, and each possesses some private information about the uncertain demand. The leader decides its output quantity first. Upon observing the leader's output quantity, the follower will try to infer leader's private information from the observable action, i.e., its output quantity, and update its belief about the uncertain demand based on the inferred information and its private information, thus the follower makes decision based on more precise information than the leader. Knowing the follower's rational inference, the leader chooses an output quantity by taking the follower's reaction into account. That is, there is a signaling game between the leader and the follower. Signaling is costly, so the leader incurs a cost of signaling, which hurts the leader. There are two effects in this game. One is that the effect caused by the follower's more accurate information. The other one is the effect of the leader's cost of signaling. The follower benefits from the two effects and then may earn more profits than the leader.

This thesis answers the question of how firms' private information affects firms' decisions, profits and their preference for the Stackelberg leadership. We assume the demand is linear and uncertain. The uncertain demand can be continuously distributed or discretely distributed. Both the continuous demand and the discrete demand are widely used in the literature.

When the demand is continuously distributed in an unbounded interval, firms follow linear decision rules and the follower always earns more ex ante profit than the leader. This is consistent with Gal-or's (1987) finding. However, when the demand is continuously distributed in a bounded interval, Gal-or's finding does not hold. Firms' output quantities are not linear in the signals and the leader may earn more ex ante profit than the follower. Without cost of signaling, though the follower has more precise information than the leader, the leader still earns more ex ante profit than the follower regardless of the distribution of demand. Comparing the two effects, we find that the effect of cost of signaling plays an important role in reversing firms' preference for the Stackelberg leadership.

We also study how the number of demand states affects the Stackelberg leader's cost of signaling when the demand follows a discrete distribution. We find that the cost of signaling depends on the number of demand states. With more demand states, the leader will set a lower output quantity to separate from other states and, thus incur a higher cost of signaling.

There are some potential issues that deserve further research. In my thesis, we assume that firms have same precision about the uncertain demand. It would be interesting to extend our framework to address how the signal precision affects firms' decisions, profits and their preference for the Stackelberg leadership.

Appendix

Proof of Corollary 1. When her observes a signal Y_1, taking the first derivative of LHS of (2) over q_1^*, we get

$$\begin{aligned}
\frac{dLHS}{dq_1^*} &= \frac{2\lambda}{\lambda-1} - \frac{2^\lambda}{\lambda-1}\cdot\frac{\lambda}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\left(q_1^*\right)^{\lambda-1} \\
&= \frac{2\lambda}{\lambda-1} - \frac{2\lambda}{\lambda-1}\cdot\left(\frac{q_1^*}{\frac{1}{2}\left(a+\frac{1}{1+s}\tilde{Y}\right)}\right)^{\lambda-1} \\
&= \frac{2\lambda}{\lambda-1}\left(1-\left(\frac{q_1^*}{\frac{1}{2}\left(a+\frac{1}{1+s}\tilde{Y}\right)}\right)^{\lambda-1}\right).
\end{aligned}$$

Since $q_1^* \le \frac{1}{2}\left(a+E\left[\theta|\tilde{Y}\right]\right)$, thus,

$$\frac{dLHS}{dq_1^*} \ge \frac{2\lambda}{\lambda-1}(1-1)=0.$$

So, LHS is increasing with q_1^*. We substitute q_1 with $\frac{1}{2}\left(a+E\left[\theta|Y_1\right]\right)$ into the LHS,

$$\begin{aligned}
LHS &= \frac{1}{2}\frac{2\lambda}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right) - \frac{2^\lambda}{\lambda-1}\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\left(\frac{1}{2}\left(a+E\left[\theta|Y_1\right]\right)\right)^\lambda \\
&= \frac{\lambda}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right) - \frac{1}{\lambda-1}\frac{\left(a+E\left[\theta|Y_1\right]\right)^{\lambda-1}}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\left(a+E\left[\theta|Y_1\right]\right) \\
&= \frac{1}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right)\left(\lambda-\frac{\left(a+E\left[\theta|Y_1\right]\right)^{\lambda-1}}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\right) \\
&= \frac{1}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right)\left(\lambda-\left(\frac{a+E\left[\theta|Y_1\right]}{a+E\left[\theta|\tilde{Y}\right]}\right)^{\lambda-1}\right),
\end{aligned}$$

since $E\left[\theta|Y_1\right] \leq E\left[\theta|\tilde{Y}\right] \Leftrightarrow \left(\frac{a+E[\theta|Y_1]}{a+E\left[\theta|\tilde{Y}\right]}\right) \leq 1$

$$\begin{aligned}
& \frac{1}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right)\left(\lambda-\left(\frac{a+E\left[\theta|Y_1\right]}{a+E\left[\theta|\tilde{Y}\right]}\right)^{\lambda-1}\right) \\
\geq\ & \frac{1}{\lambda-1}\left(a+E\left[\theta|Y_1\right]\right)(\lambda-1) \\
=\ & \left(a+E\left[\theta|Y_1\right]\right) \\
=\ & RHS.
\end{aligned}$$

Since LHS is increasing with q_1^*, so $q_1^* \leq \frac{1}{2}\left(a+E\left[\theta|Y_1\right]\right)$ and only when $Y_1 = \tilde{Y}$, $q_1^* = \frac{1}{2}\left(a+E\left[\theta|\tilde{Y}\right]\right)$.

Proof of Lemma 2. Given the signals firms observe,

(1) q_1^* is decreasing with $\tilde{Y}$, while q_2^* is increasing with $\tilde{Y}$,

(2) $E\left[\pi_1|Y_1\right]$ is decreasing with $\tilde{Y}$ and $E\left[\pi_2|Y_2, q_1^*\right]$ is increasing with $\tilde{Y}$.

(1)

$$\frac{2\lambda}{\lambda-1}\cdot q_1^* - \frac{2^\lambda}{\lambda-1}\cdot\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\cdot\left(q_1^*\right)^\lambda = a+\frac{1}{1+s}Y_1$$

$$\frac{2\lambda}{\lambda-1}\cdot q_1^* - \frac{2^\lambda}{\lambda-1}\cdot\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\cdot\left(q_1^*\right)^\lambda - a - \frac{1}{1+s}Y_1 = 0$$

$$\begin{aligned}
\frac{2^\lambda}{\lambda-1}\cdot\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\cdot\left(q_1^*\right)^\lambda &= \frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1 \\
\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1} &= \frac{1}{\frac{\lambda-1}{2^\lambda\left(q_1^*\right)^\lambda}\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)} \\
\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1} &= \frac{2^\lambda\left(q_1^*\right)^\lambda}{(\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)} \\
\left(a+\frac{1}{1+s}\tilde{Y}\right) &= \left(\frac{2^\lambda\left(q_1^*\right)^\lambda}{(\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)}\right)^{\frac{1}{\lambda-1}}
\end{aligned}$$

$$\tilde{Y} = (1+s)\left(\frac{2^{\lambda}\left(q_1^*\right)^{\lambda}}{(\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)}\right)^{\frac{1}{\lambda-1}} - (1+s)\,a.$$

Taking the first derivative of $\tilde{Y}$ over q_1^*, we get

$$\begin{aligned}
&\frac{d}{dq_1^*}\left((1+s)\left(\frac{2^{\lambda}\left(q_1^*\right)^{\lambda}}{(\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)}\right)^{\frac{1}{\lambda-1}} - (1+s)\,a\right) \\
= \quad & (1+s)\left(\begin{array}{c} \frac{1}{\lambda-1}\left(\frac{2^{\lambda}\left(q_1^*\right)^{\lambda}}{(\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)}\right)^{\frac{2-\lambda}{\lambda-1}} \\ * \left(\frac{2^{\lambda}\lambda(\lambda-1)\left(q_1^*\right)^{\lambda-1}\left(2p_1^* - a - \frac{1}{1+s}Y_1\right)}{\left((\lambda-1)\left(\frac{2\lambda}{\lambda-1}\cdot q_1^* - a - \frac{1}{1+s}Y_1\right)\right)^2}\right) \end{array}\right).
\end{aligned}$$

Since $\lambda \in (1,2)$ and $q_1^* \leq \frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right)$, so

$$\frac{d\tilde{Y}}{dp_1^*} \leq 0, \text{ only when } Y_1 = \tilde{Y},\ \frac{d\tilde{Y}}{dp_1^*} = 0.$$

So $\tilde{Y}$ is decreasing with $q_1 \Leftrightarrow q_1$ is decreasing with $\tilde{Y}$.

$$\begin{aligned}
q_2^* \quad &= \quad \frac{1}{2}\left(a - q_1^* + \left(\frac{2}{\lambda-1}q_1^* - \frac{1}{\lambda}\frac{2^{\lambda}}{\lambda-1}\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\left(q_1^*\right)^{\lambda} + \frac{1}{2+s}Y_2\right)\right) \\
\frac{dq_2^*}{d\tilde{Y}} \quad &= \quad \frac{1}{2}\left\{\begin{array}{c} -\frac{d}{d\tilde{Y}}q_1^* + \frac{2}{\lambda-1}\cdot\frac{d}{d\tilde{Y}}q_1^* \\ -\frac{1}{\lambda}\frac{2^{\lambda}}{\lambda-1}\cdot\left(\left(q_1^*\right)^{\lambda}\frac{d}{d\tilde{Y}}\left(\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\right) + \left(\frac{q_1^*}{a+\frac{1}{1+s}\tilde{Y}}\right)^{\lambda-1}\cdot\lambda\frac{d}{d\tilde{Y}}q_1^*\right) \end{array}\right\} \\
&= \quad \frac{1}{2}\left\{\begin{array}{c} \left(-1 + \frac{2}{\lambda-1} - \frac{2^{\lambda}}{\lambda-1}\cdot\left(\frac{q_1^*}{a+\frac{1}{1+s}\tilde{Y}}\right)^{\lambda-1}\right)\frac{d}{d\tilde{Y}}q_1^* \\ -\frac{1}{\lambda}\frac{2^{\lambda}}{\lambda-1}\cdot\left(q_1^*\right)^{\lambda}\frac{d}{d\tilde{Y}}\left(\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\right) \end{array}\right\}.
\end{aligned}$$

$$\frac{d}{d\tilde{Y}}\left(\frac{1}{\left(a+\frac{1}{1+s}\tilde{Y}\right)^{\lambda-1}}\right) = -(\lambda-1)^2\left(a+\frac{1}{1+s}\tilde{Y}\right)^{-\lambda} < 0\,,$$

$$\text{So}\quad\left(-1+\frac{2}{\lambda-1}-\frac{2^\lambda}{\lambda-1}\cdot\left(\frac{q_1^*}{a+\frac{1}{1+s}\tilde{Y}}\right)^{\lambda-1}\right) < 0\,,$$

$$\text{Hence}\quad\frac{dq_2^*}{d\tilde{Y}} > 0\,.$$

q_2^* in increasing with $\tilde{Y}$.

Proof. (2)

$$\begin{aligned}
E\left[\pi_1|Y_1\right] &= \frac{1}{2}q_1\left(a-q_1^*+\frac{1}{1+s}Y_1\right),\\
\frac{d}{dq_1}\left(E\left[\pi_1|Y_1\right]\right) &= \frac{1}{2}\left(a-q_1^*+\frac{1}{1+s}Y_1\right)-\frac{1}{2}q_1^*\\
&= \frac{1}{2}a-q_1^*+\frac{1}{2}\frac{1}{1+s}Y_1\\
&\geq 0.
\end{aligned}$$

$$\begin{aligned}
E\left[\pi_2|Y_2,q_1^*\right] &= \frac{1}{4}\left(a+\frac{1}{2+s}Y_1+\frac{1}{2+s}Y_2-q_1^*\right)^2,\\
\frac{d}{dp_1}\left(E\left[\pi_2|Y_2,q_1^*\right]\right) &= \frac{1}{2}\left(q_1^*-\left(a+\frac{1}{2+s}Y_1+\frac{1}{2+s}Y_2\right)\right)\\
&= \frac{1}{2}\left(q_1^*-\left(a+E\left[\theta|Y_1,Y_2\right]\right)\right)\\
&\leq \frac{1}{2}\left(\frac{1}{2}\left(a+\frac{1}{1+s}Y_1\right)-\left(a+E\left[\theta|Y_1,Y_2\right]\right)\right)\\
&= \frac{1}{2}\left(-\frac{1}{2}a-\frac{1}{2}\frac{s}{(1+s)(2+s)}Y_1-\frac{1}{2+s}Y_2\right)\\
&\leq \frac{1}{2}\left(-\frac{1}{2}a+\frac{1}{2}\frac{s}{(1+s)(2+s)}a+\frac{1}{2+s}a\right)\\
&= -\frac{1}{4}\frac{s^2}{(1+s)(2+s)}a\\
&< 0.
\end{aligned}$$

To sum up, $E\left[\pi_1|Y_1\right]$ is increasing with q_1^*, and $E\left[\pi_2|Y_2,q_1^*\right]$ is decreasing with q_1^*, while q_1^* is decreasing with $\tilde{Y}$, so $E\left[\pi_1|Y_1\right]$ is decreasing with $\tilde{Y}$ and $E\left[\pi_2|Y_2,q_1^*\right]$

is increasing with $\tilde{Y}$. ■

Proof of Lemma 4. We analyze the benchmark model by backward induction.

In the second stage, the follower uses information available to it to maximize its expected profit

$$E\left[\pi_2^B|Y_1,Y_2\right] = q_2^B\left(a + E\left[\theta|Y_1,Y_2\right] - \left(q_1^B + q_2^B\right)\right),$$

where the script B means the benchmark model. From the FOC, we get

$$q_2^{B*} = \frac{a + E\left[\theta|Y_1,Y_2\right] - q_1^B}{2}.$$

In the first stage, the leader decides the output quantity

$$\begin{aligned} E\left[\pi_2^B|Y_1\right] &= q_1^B\left(a + E\left[\theta|Y_1\right] - \left(q_1^B + E\left[q_2^{B*}|Y_1\right]\right)\right) \\ &= q_1^B\left(\frac{1}{2}a + \frac{1}{2}E\left[\theta|Y_1\right] - \frac{1}{2}q_1^B\right). \end{aligned}$$

From the FOC, we get

$$q_1^{B*} = \frac{1}{2}\left(a + E\left[\theta|Y_1\right]\right),$$

Thus, the follower's equilibrium output quantity is

$$q_2^{B*} = \frac{1}{2}\left(\frac{1}{2}a + E\left[\theta|Y_1,Y_2\right] - \frac{1}{2}E\left[\theta|Y_1\right]\right).$$

Firms' expected profits are given by

$$\begin{aligned} E\left[\pi_1^B|Y_1\right] &= \frac{1}{8}\left(a + \frac{1}{1+s}Y_1\right)^2, \\ E\left[\pi_2^B|Y_1,Y_2\right] &= \frac{1}{16}\left(a + \frac{2}{2+s}\left(Y_1+Y_2\right) - \frac{1}{1+s}Y_1\right)^2. \end{aligned}$$

Taking expectation of $E\left[\pi_1^B|Y_1\right]$ and $E\left[\pi_2^B|Y_1,Y_2\right]$ with respect to signal, we get firms' ex ante profits,

$$\begin{aligned}\Pi_1^B &= \frac{1}{8}a^2+\frac{1}{8}\frac{\sigma^2}{s+1},\\ \Pi_2^B &= \frac{1}{16}a^2+\frac{1}{16}\frac{\sigma^2}{s+1}+\frac{1}{4}\frac{s}{s^2+3s+2}\sigma^2.\end{aligned}$$

We compare firms' ex ante profits

$$\Pi_1^B-\Pi_2^B=\frac{1}{16}a^2-\frac{1}{16}\frac{3s-2}{s^2+3s+2}\sigma^2.$$

Since σ is small relative to a,

$$\Pi_1^B-\Pi_2^B>\frac{1}{16}a^2\frac{s^2+4}{s^2+3s+2}>0.$$

Therefore, the leader always more profit than that of the follower's.

Bayesian Updating for Binary Case. $\Pr(Y_i=A_H)=\gamma\alpha+(1-\gamma)(1-\alpha)$.

$\Pr(Y_i=A_L)=\alpha(1-\gamma)+(1-\alpha)\gamma$.

$\Pr(Y_1=H,Y_2=H)=\gamma^2\alpha+(1-\gamma)^2(1-\alpha)$.

$\Pr(Y_1=H,Y_2=L)=\gamma(1-\gamma)$.

$\Pr(Y_1=L,Y_2=H)=\gamma(1-\gamma)$.

$\Pr(Y_1=L,Y_2=L)=(1-\gamma)^2\alpha+\gamma^2(1-\alpha)$.

$\Pr(Y=A_H|Y_i=A_H)=\frac{\alpha\gamma}{\alpha\gamma+(1-\alpha)(1-\gamma)}$.

$\Pr(Y=A_L|Y_i=A_H)=\frac{(1-\gamma)(1-\alpha)}{\alpha\gamma+(1-\gamma)(1-\alpha)}$.

$\Pr(Y=A_H|Y_i=A_L)=\frac{\alpha(1-\gamma)}{\alpha(1-\gamma)+(1-\alpha)\gamma}$.

$\Pr(Y=A_L|Y_i=A_L)=\frac{\gamma(1-\alpha)}{\alpha(1-\gamma)+(1-\alpha)\gamma}$.

$\Pr(Y_i=A_H|Y_j=A_H)=\frac{\gamma^2\alpha+(1-\gamma)^2(1-\alpha)}{\gamma\alpha+(1-\gamma)(1-\alpha)}$.

$\Pr(Y_i = A_L|Y_j = A_H) = \frac{\gamma(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)}$.

$\Pr(Y_i = A_L|Y_j = A_L) = \frac{(1-\gamma)^2\alpha+\gamma^2(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)}$.

$\Pr(Y_i = A_H|Y_j = A_L) = \frac{\gamma(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)}$.

$\Pr(A_H|Y_i = A_H, Y_j = A_H) = \frac{\gamma^2\alpha}{\gamma^2\alpha+(1-\gamma)^2(1-\alpha)}$.

$\Pr(A_L|Y_i = A_H, Y_j = A_H) = \frac{(1-\gamma)^2(1-\alpha)}{\gamma^2\alpha+(1-\gamma)^2(1-\alpha)}$.

$\Pr(A_H|Y_i = A_H, Y_j = A_L) = \alpha$.

$\Pr(A_L|Y_i = A_H, Y_j = A_L) = (1-\alpha)$.

$\Pr(A_H|Y_i = A_L, Y_j = A_L) = \frac{(1-\gamma)^2\alpha}{(1-\gamma)^2\alpha+\gamma^2(1-\alpha)}$.

$\Pr(A_L|Y_i = A_L, Y_j = A_L) = \frac{\gamma^2(1-\alpha)}{(1-\gamma)^2\alpha+\gamma^2(1-\alpha)}$.

Requirement for Binary Case. We should require that firms' output quantities and the realized retail price are non-negative, that is

$$
\begin{aligned}
q_1 &\geq 0, \\
q_2 &\geq 0, \\
q_1 + q_2 &\leq A_L.
\end{aligned}
$$

1. If $u = 0$,

$$
\begin{aligned}
&\left(q_{1L}\left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}q_{1L}\right) - \frac{1}{8}B_1^2\right) \leq 0 \\
&\Rightarrow \frac{A_H}{A_L} \in \left(\frac{2b_1 - 2b_3}{b_1^2 + b_2^2 - 4b_1b_2 + 2b_2b_3} + 1, +\infty\right), \\
&\text{if } \left(\frac{1}{2}b_1b_2 - \frac{1}{8}b_1^2 - \frac{1}{8}b_2^2 - \frac{1}{4}b_3b_2\right) < 0\,.
\end{aligned}
$$

It's easy to verify that $q_1^* \geq 0$. We know that $B_5 > B_1 > B_6 = B_7 > B_2 >$

B_8. We should guarantee that

$$\begin{aligned} q^*_{2HH} &= \frac{1}{2}\left(B_5 - \frac{1}{2}B_1\right) \geq 0, \\ q^*_{2LH} &= \frac{1}{2}\left(B_6 - \frac{1}{2}B_2\right) \geq 0, \\ q^*_{2HL} &= \frac{1}{2}\left(B_7 - \frac{1}{2}B_1\right) \geq 0, \\ q^*_{2LL} &= \frac{1}{2}\left(B_8 - \frac{1}{2}B_2\right) \geq 0. \end{aligned}$$

q^*_{2HH} and q^*_{2LH} is obviously larger than 0.

$$\begin{aligned} &\left\{\frac{1}{2}\left(B_7 - \frac{1}{2}B_1\right) \geq 0\right\} \\ \Rightarrow &\left\{\begin{array}{c} \frac{A_H}{A_L} > 1 \text{ if } \left(2\gamma\alpha + 2\left(1-\gamma\right)\left(1-\alpha\right) - \gamma\right) \geq 0; \\ \frac{A_H}{A_L} \leq \frac{(\alpha-1)(2\gamma\alpha+(1-2\alpha)(1-\gamma))}{\alpha(2\gamma\alpha+2(1-\gamma)(1-\alpha)-\gamma)} \\ \text{if } 2\gamma\alpha + 2\left(1-\gamma\right)\left(1-\alpha\right) - \gamma < 0 \end{array}\right\}. \\ &\left\{\frac{1}{2}\left(B_8 - \frac{1}{2}B_2\right) \geq 0\right\} \\ \Rightarrow &\left\{\begin{array}{c} \frac{A_H}{A_L} > 1 \text{ if } \alpha + 2\gamma - 3\gamma^2 - 4\alpha\gamma + 4\alpha\gamma^2 \geq 0; \\ \frac{A_H}{A_L} \leq \frac{\gamma\left(\alpha-\gamma^2-4\alpha\gamma+4\alpha\gamma^2\right)}{(1-\gamma)(\alpha+2\gamma-3\gamma^2-4\alpha\gamma+4\alpha\gamma^2)} \\ \text{if } \alpha + 2\gamma - 3\gamma^2 - 4\alpha\gamma + 4\alpha\gamma^2 < 0 \end{array}\right\} \end{aligned}$$

$$\begin{aligned} q^*_{1H} + q^*_{2HH} &= \frac{1}{4}B_1 + \frac{1}{2}B_5 \leq A_L, \\ q^*_{1H} + q^*_{2HL} &= \frac{1}{4}B_1 + \frac{1}{2}B_7 \leq A_L, \\ q^*_{1L} + q^*_{2LH} &= \frac{1}{4}B_2 + \frac{1}{2}B_6 \leq A_L, \\ q^*_{1L} + q^*_{2LL} &= \frac{1}{4}B_2 + \frac{1}{2}B_8 \leq A_L. \end{aligned}$$

Since $B_5 > B_6 = B_7 > B_8$ and , so if $\frac{1}{4}B_1 + \frac{1}{2}B_5 \leq A_L$ and $\frac{1}{4}B_2 + \frac{1}{2}B_6 \leq A_L$, then $\frac{1}{4}B_1+\frac{1}{2}B_7\le A_L$ and $\frac{1}{4}B_2+\frac{1}{2}B_8\le A_L$.

$$\begin{aligned}\frac{1}{4}B_1+\frac{1}{2}B_5 &\le A_L \Rightarrow \frac{A_H}{A_L}<1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma\left(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha)\right)},\\ \frac{1}{4}B_2+\frac{1}{2}B_6 &\le A_L \Rightarrow \frac{A_H}{A_L}<1+\frac{((1-\gamma)\alpha+\gamma(1-\alpha))}{\alpha(1-\gamma)+2\alpha((1-\gamma)\alpha+\gamma(1-\alpha))}.\end{aligned}$$

To sum up,

$$\frac{A_H}{A_L}\in\begin{array}{c}\left[\begin{array}{c}\frac{2b_1-2b_3}{b_1^2+b_2^2-4b_1b_2+2b_2b_3}+1,\\ 1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\end{array}\right]\\ \text{if } b_1^2+b_2^2-4b_1b_2+2b_2b_3>0.\end{array}$$

$$\begin{aligned}\Omega_1 &= \left\{\frac{A_H}{A_L}:\begin{array}{c}\frac{A_H}{A_L}\in\left[\begin{array}{c}\frac{2b_1-2b_3}{b_1^2+b_2^2-4b_1b_2+2b_2b_3}+1,\\ 1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\end{array}\right]\\ \text{if } b_1^2+b_2^2-4b_1b_2+2b_2b_3>0\end{array}\right\},\\ \Omega_2 &= \left\{\frac{A_H}{A_L}:\begin{array}{c}\frac{A_H}{A_L}\in\left(1,\min\left(\begin{array}{c}1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))},\\ \frac{2b_1-2b_3}{b_1^2+b_2^2-4b_1b_2+2b_2b_3}+1\end{array}\right)\right)\\ \text{if } b_1^2+b_2^2-4b_1b_2+2b_2b_3>0;\\ \frac{A_H}{A_L}\in\left(1,1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\right],\text{ otherwise.}\end{array}\right\}.\end{aligned}$$

2. If $u>0$, it's easy to verify that $q_1^*\ge 0$. we should guarantee that

$$\begin{aligned}q_{2HH}^* &= \frac{1}{2}\left(B_5-\frac{1}{2}B_1\right)\ge 0,\\ q_{2HL}^* &= \frac{1}{2}\left(B_7-\frac{1}{2}B_1\right)\ge 0,\\ q_{2LH}^* &= \frac{1}{2}\left(B_6-B_1+\frac{1}{2}B_3+\frac{1}{2}\sqrt{(B_1-B_3)(3B_1-B_3)}\right)\ge 0,\\ q_{2LL}^* &= \frac{1}{2}\left(B_8-B_1+\frac{1}{2}B_3+\frac{1}{2}\sqrt{(B_1-B_3)(3B_1-B_3)}\right)\ge 0.\end{aligned}$$

It's easy to verify that $q_{2HH}^*>0$ and $q_{2LH}^*>0$.

$$\left\{\frac{1}{2}\left(B_7 - \frac{1}{2}B_1\right) \geq 0\right\}$$

$$\Rightarrow \left\{ \begin{array}{c} \frac{A_H}{A_L} > 1 \text{ if } \left(2\gamma\alpha + 2\left(1-\gamma\right)\left(1-\alpha\right) - \gamma\right) \geq 0; \\ \frac{A_H}{A_L} \leq \frac{(\alpha-1)(2\gamma\alpha+(1-2\alpha)(1-\gamma))}{\alpha(2\gamma\alpha+2(1-\gamma)(1-\alpha)-\gamma)} \\ \text{if } 2\gamma\alpha + 2\left(1-\gamma\right)\left(1-\alpha\right) - \gamma < 0 \end{array} . \right\}$$

Since $B_1 + \frac{1}{2}B_3 + \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)} < \frac{1}{2}B_2$, so when

$$\left\{ \begin{array}{c} \frac{A_H}{A_L} > 1 \text{ if } \alpha + 2\gamma - 3\gamma^2 - 4\alpha\gamma + 4\alpha\gamma^2 \geq 0; \\ \frac{A_H}{A_L} \leq \frac{\gamma\left(\alpha-\gamma^2-4\alpha\gamma+4\alpha\gamma^2\right)}{(1-\gamma)(\alpha+2\gamma-3\gamma^2-4\alpha\gamma+4\alpha\gamma^2)} \\ \text{if } \alpha + 2\gamma - 3\gamma^2 - 4\alpha\gamma + 4\alpha\gamma^2 < 0 \end{array} \right\}$$

$$\Rightarrow \frac{1}{2}\left(B_8 - B_1 + \frac{1}{2}B_3 + \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)}\right) > 0.$$

$$\begin{aligned} q_{1H}^* + q_{2HH}^* &= \frac{1}{4}B_1 + \frac{1}{2}B_5 \leq A_L, \\ q_{1H}^* + q_{2HL}^* &= \frac{1}{4}B_1 + \frac{1}{2}B_7 \leq A_L, \\ q_{1L}^* + q_{2LH}^* &= \left\{ \begin{array}{c} \left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)}\right) \\ +\frac{1}{2}\left(B_6 - B_1 + \frac{1}{2}B_3 + \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)}\right) \end{array} \right\} \leq A_L, \\ q_{1L}^* + q_{2LL}^* &= \left\{ \begin{array}{c} \left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)}\right) \\ +\frac{1}{2}\left(B_8 - B_1 + \frac{1}{2}B_3 + \frac{1}{2}\sqrt{\left(B_1 - B_3\right)\left(3B_1 - B_3\right)}\right) \end{array} \right\} \leq A_L. \end{aligned}$$

We only need to $q_{1H}^* + q_{2HH}^* \leq A_L$ and $q_{1L}^* + q_{2LH}^* \leq A_L$.

$$\begin{aligned} q_{1H}^* + q_{2HH}^* &\leq A_L \Rightarrow \frac{A_H}{A_L} \leq 1 + \frac{\left(2\alpha\gamma - \gamma - \alpha + 1\right)\left(\gamma^2\alpha + \left(1-\gamma\right)^2\left(1-\alpha\right)\right)}{\alpha\gamma\left(3\gamma^2\alpha + \left(1-\gamma^2\right)\left(1-\alpha\right)\right)}, \\ q_{1L}^* + q_{2LH}^* &\leq A_L \Rightarrow \frac{A_H}{A_L} \leq 1 + \frac{3b_1 - 2b_3 + 2\alpha + 2\sqrt{\left(b_1 - b_3\right)\left(2b_1 - b_3 + \alpha\right)}}{8\alpha b_1 - 4\alpha b_3 + b_1^2 + 4\alpha^2}. \end{aligned}$$

To sum up,

$$\left\{ \frac{A_H}{A_L}: \begin{array}{c} \frac{A_H}{A_L} \in \left(1, \min\left(\begin{array}{c} 1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}, \\ \frac{2b_1-2b_3}{b_1^2+b_2^2-4b_1b_2+2b_2b_3}+1 \end{array}\right)\right) \\ \text{if } b_1^2+b_2^2-4b_1b_2+2b_2b_3>0; \\ \frac{A_H}{A_L} \in \left(1, 1+\frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\right], \text{ otherwise.} \end{array} \right\}.$$

B_1 to B_8 are shown as follows,

$$B_1 = A_H\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)} + A_L\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)}, \tag{29}$$

$$B_2 = A_H\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)} + A_L\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)},$$

$$B_3 = \left\{ \begin{array}{c} A_H\left(\frac{\alpha\left(\alpha\gamma(1-\alpha)+\alpha(1-\alpha)(1-\gamma)(1-4\gamma)+(1-\gamma)^2\gamma\right)}{(\alpha\gamma+(1-\alpha)(1-\gamma))\left(\alpha(1-\gamma)^2+(1-\alpha)\gamma^2\right)}\right) \\ +A_L\left(\frac{(1-\alpha)\left(\alpha(1-\alpha)(1-\gamma)+\left(4\alpha\gamma^2(1-\alpha)-3\alpha\gamma(1-\alpha)+\gamma^2(1-\gamma)\right)\right)}{\left((1-\gamma)^2\alpha+\gamma^2(1-\alpha)\right)(\gamma\alpha+(1-\gamma)(1-\alpha))}\right) \end{array} \right\},$$

$$B_4 = \left\{ \begin{array}{c} A_H\left(\frac{\alpha\left(\alpha(1-\alpha)(1-\gamma)+\left(4\alpha\gamma^2(1-\alpha)-3\alpha\gamma(1-\alpha)+\gamma^2(1-\gamma)\right)\right)}{\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)((1-\gamma)\alpha+\gamma(1-\alpha))}\right) \\ +A_L\left(\frac{(1-\alpha)(\alpha\gamma(1-\alpha)+(\gamma(1-\gamma)-4\alpha\gamma(1-\alpha)+\alpha(1-\alpha))(1-\gamma))}{\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)((1-\gamma)\alpha+\gamma(1-\alpha))}\right) \end{array} \right\},$$

$$B_5 = A_H\frac{\gamma^2\alpha}{\gamma^2\alpha+(1-\gamma)^2(1-\alpha)} + A_L\frac{(1-\gamma)^2(1-\alpha)}{\gamma^2\alpha+(1-\gamma)^2(1-\alpha)}, \tag{30}$$

$$B_6 = A_H\alpha + A_L(1-\alpha),$$

$$B_7 = A_H\alpha + A_L(1-\alpha), \tag{31}$$

$$B_8 = A_H\frac{(1-\gamma)^2\alpha}{(1-\gamma)^2\alpha+\gamma^2(1-\alpha)} + A_L\frac{\gamma^2(1-\alpha)}{(1-\gamma)^2\alpha+\gamma^2(1-\alpha)}.$$

Proof of Proposition 4. The firms ex ante payoffs are as follows, If $\frac{A_H}{A_L} \in \Omega_1$,

$$\begin{aligned}
\Pi_1 &= E\left[\pi_1 | Y_1 = A_H\right] \Pr\left(Y_1 = A_H\right) + E\left[\pi_1 | Y_1 = A_L\right] \Pr\left(Y_1 = A_L\right) \\
&= \frac{1}{8} B_1^2 \Pr\left(Y_1 = A_H\right) + \frac{1}{8} B_2^2 \Pr\left(Y_1 = A_L\right), \\
\Pi_2 &= E\left[\pi_2 | Y_1 = A_H, Y_2 = A_H\right] \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_H\right] \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_H, Y_2 = A_L\right] \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_L\right] \Pr\left(Y_1 = A_L, Y_2 = A_L\right) \\
&= \frac{1}{4}\left(B_5 - \frac{1}{2} B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_6 - \frac{1}{2} B_2\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_7 - \frac{1}{2} B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + \frac{1}{4}\left(B_8 - \frac{1}{2} B_2\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_L\right).
\end{aligned}$$

If $\frac{A_H}{A_L} \in \Omega_2$,

$$\begin{aligned}
\Pi_1 &= E\left[\pi_1 | Y_1 = A_H\right] \Pr\left(Y_1 = A_H\right) + E\left[\pi_1 | Y_1 = A_L\right] \Pr\left(Y_1 = A_L\right) \\
&= \frac{1}{8} B_1^2 \Pr\left(Y_1 = A_H\right) + D_1\left(\frac{1}{2} B_2 - \frac{1}{2} D_1\right) \Pr\left(Y_1 = A_L\right), \\
\Pi_2 &= E\left[\pi_2 | Y_1 = A_H, Y_2 = A_H\right] \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_H\right] \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_H, Y_2 = A_L\right] \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + E\left[\pi_2 | Y_1 = A_L, Y_2 = A_L\right] \Pr\left(Y_1 = A_L, Y_2 = A_L\right) \\
&= \frac{1}{4}\left(B_5 - \frac{1}{2} B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_6 - D_1\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\
&\quad + \frac{1}{4}\left(B_7 - \frac{1}{2} B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \\
&\quad + \frac{1}{4}\left(B_8 - D_1\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_L\right),
\end{aligned}$$

where $D_1 = \left(B_1 - \frac{1}{2}B_3 - \frac{1}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)}\right)$.

We will prove following inequality, when $\frac{A_H}{A_L} \in \Omega_1$,

$$1a) \quad : \quad \frac{1}{8}B_1^2 \Pr(Y_1 = A_H) \geq \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_H) \\ +\frac{1}{4}\left(B_6 - \frac{1}{2}B_2\right)^2 \Pr(Y_1 = A_H, Y_2 = A_L) \end{array} \right\},$$

$$1b) \quad : \quad \frac{1}{8}B_2^2 \Pr(Y_1 = A_L) \geq \left\{ \begin{array}{c} \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_L, Y_2 = A_H) \\ +\frac{1}{4}\left(B_8 - \frac{1}{2}B_2\right)^2 \Pr(Y_1 = A_L, Y_2 = A_L) \end{array} \right\},$$

When $\frac{A_H}{A_L} \in \Omega_2$,

$$2a) \quad : \quad \frac{1}{8}B_1^2 \Pr(Y_1 = A_H) \geq \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_H) \\ +\frac{1}{4}(B_6 - D_1)^2 \Pr(Y_1 = A_H, Y_2 = A_L) \end{array} \right\},$$

$$\begin{aligned} 2b) \quad : \quad D_1\left(\frac{1}{2}B_2 - \frac{1}{2}D_1\right)\Pr(Y_1 = A_L) \geq & \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \Pr(Y_1{=}A_L, Y_2{=}A_H) \\ & + \frac{1}{4}(B_8 - D_1)^2 \Pr(Y_1{=}A_L, Y_2{=}A_L), \end{aligned}$$

1a)

$$\begin{aligned} & \frac{1}{8}B_1^2 \Pr(Y_1 = A_H) \\ \geq \; & \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_H) \\ +\frac{1}{4}\left(B_6 - \frac{1}{2}B_2\right)^2 \Pr(Y_1 = A_H, Y_2 = A_L) \end{array} \right\}. \end{aligned}$$

From 2a), we know that

$$\begin{aligned} & \frac{1}{8}B_1^2 \Pr(Y_1 = A_H) \\ > \; & \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr(Y_1 = A_H, Y_2 = A_H) \\ +\frac{1}{4}(B_6 - D_1)^2 \Pr(Y_1 = A_H, Y_2 = A_L) \end{array} \right\}, \end{aligned}$$

and

$$\begin{aligned} D_1 &< \frac{1}{2}B_2, \\ \frac{1}{4}\left(B_6 - D_1\right)^2 &> \frac{1}{4}\left(B_6 - \frac{1}{2}B_2\right)^2. \end{aligned}$$

So

$$\begin{aligned} & \frac{1}{8}B_1^2 \Pr\left(Y_1 = A_H\right) \\ > & \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\ +\frac{1}{4}\left(B_6 - D_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \end{array} \right\} \\ > & \left\{ \begin{array}{c} \frac{1}{4}\left(B_5 - \frac{1}{2}B_1\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_H\right) \\ +\frac{1}{4}\left(B_6 - \frac{1}{2}B_2\right)^2 \Pr\left(Y_1 = A_H, Y_2 = A_L\right) \end{array} \right\}. \end{aligned}$$

1b)

$$\begin{aligned} & \frac{1}{8}B_2^2 \Pr\left(Y_1 = A_L\right) \\ \geq & \left\{ \begin{array}{c} \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_H\right) \\ +\frac{1}{4}\left(B_8 - \frac{1}{2}B_2\right)^2 \Pr\left(Y_1 = A_L, Y_2 = A_L\right) \end{array} \right\}. \end{aligned}$$

This is equivalent to prove

$$\begin{aligned} \frac{1}{8}B_2^2 &\geq \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2, \\ \frac{1}{8}B_2^2 &\geq \frac{1}{4}\left(B_8 - \frac{1}{2}B_2\right)^2. \end{aligned}$$

(1) First, we prove $\frac{1}{8}B_2^2 \geq \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2$.

$$
\begin{aligned}
&\frac{1}{8}B_2^2 \geq \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \\
&\left(\begin{array}{c} A_H\left(\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)} - \sqrt{2}\alpha + \frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)}\right) \\ +A_L\left(\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)} + \frac{\sqrt{2}}{2}\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)} + \sqrt{2}\left(1-\alpha\right)\right) \end{array}\right) \geq 0\,,
\end{aligned}
$$

if $\gamma > -\frac{\left(6\alpha+9\sqrt{2}\alpha-8\sqrt{2}\alpha^2+2\sqrt{-6\alpha+6\sqrt{2}\alpha-5\sqrt{2}\alpha^2+\frac{11}{2}\alpha^2-2\sqrt{2}+2}-2\sqrt{2}-4\right)}{-8\alpha-20\sqrt{2}\alpha+16\sqrt{2}\alpha^2+6\sqrt{2}+4}$ where $\alpha \in (0.5, 1)$,

$$
\left(\frac{\alpha\left(1-\gamma\right)}{\left(1-\gamma\right)\alpha+\gamma\left(1-\alpha\right)} - \sqrt{2}\alpha + \frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+\left(1-\gamma\right)\left(1-\alpha\right)}\right) < 0.
$$

We know that $\frac{A_H}{A_L} \leq \left(1 + \frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\right)$. Taking the first derivative of $\left(1 + \frac{(\gamma\alpha+(1-\gamma)(1-\alpha))\left(\gamma^2\alpha+(1-\gamma)^2(1-\alpha)\right)}{\alpha\gamma(3\gamma^2\alpha+(1-\gamma^2)(1-\alpha))}\right)$ over α and γ, we find that it is decreasing with α and γ, so $\frac{A_H}{A_L} < 1.38$.

$$
\begin{aligned}
&\left(\begin{array}{c} A_H\left(\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)} - \sqrt{2}\alpha + \frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)}\right) \\ +A_L\left(\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)} + \frac{\sqrt{2}}{2}\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)} + \sqrt{2}\left(1-\alpha\right)\right) \end{array}\right) \\
> &\left(\begin{array}{c} 1.38L\left(\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)} - \sqrt{2}\alpha + \frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)}\right) \\ +A_L\left(\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)} + \frac{\sqrt{2}}{2}\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)} + \sqrt{2}\left(1-\alpha\right)\right) \end{array}\right) \\
> &\ 0.
\end{aligned}
$$

Else

$$
\left(\frac{\alpha\left(1-\gamma\right)}{\left(1-\gamma\right)\alpha+\gamma\left(1-\alpha\right)} - \sqrt{2}\alpha + \frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+\left(1-\gamma\right)\left(1-\alpha\right)}\right) > 0.
$$

$$\begin{aligned}
&\left(\begin{array}{c} A_H\left(\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)}-\sqrt{2}\alpha+\frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)}\right) \\ +A_L\left(\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)}+\frac{\sqrt{2}}{2}\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)}+\sqrt{2}\,(1-\alpha)\right)\end{array}\right) \\
> &\left(\begin{array}{c} A_L\left(\frac{\alpha(1-\gamma)}{(1-\gamma)\alpha+\gamma(1-\alpha)}-\sqrt{2}\alpha+\frac{\sqrt{2}}{2}\frac{\alpha\gamma}{\gamma\alpha+(1-\gamma)(1-\alpha)}\right) \\ +A_L\left(\frac{\gamma(1-\alpha)}{(1-\gamma)\alpha+\gamma(1-\alpha)}+\frac{\sqrt{2}}{2}\frac{(1-\alpha)(1-\gamma)}{\gamma\alpha+(1-\gamma)(1-\alpha)}+\sqrt{2}\,(1-\alpha)\right)\end{array}\right) \\
= &\ \frac{1}{2}A_L\left(3\sqrt{2}-4\sqrt{2}\alpha+2\right) \\
> &\ 0.
\end{aligned}$$

(2) Next, we prove $\frac{1}{8}B_2^2 \geq \frac{1}{4}\left(B_8-\frac{1}{2}B_2\right)^2$.

$$\begin{aligned}
\frac{1}{8}B_2^2 &\geq \frac{1}{4}\left(B_8-\frac{1}{2}B_2\right)^2 \\
B_2 &\geq \sqrt{2}\left(B_8-\frac{1}{2}B_2\right) \\
\left(1+\frac{\sqrt{2}}{2}\right)B_2-\sqrt{2}B_8 &\geq 0.
\end{aligned}$$

Since $\left(1+\frac{\sqrt{2}}{2}\right) > \sqrt{2}$ and $B_2 > B_8$, So $\left(1+\frac{\sqrt{2}}{2}\right)B_2-\sqrt{2}B_8 \geq 0$.

2b) We will prove

$$\begin{aligned}
&D_1\left(\frac{1}{2}B_2-\frac{1}{2}D_1\right)\Pr\left(Y_1=A_L\right) \\
> &\left\{\begin{array}{c} \frac{1}{4}\left(B_7-\frac{1}{2}B_1\right)^2\Pr\left(Y_1=A_L,Y_2=A_H\right) \\ +\frac{1}{4}\left(B_8-D_1\right)^2\Pr\left(Y_1=A_L,Y_2=A_L\right)\end{array}\right\}.
\end{aligned}$$

This is equivalent to prove

$$\begin{aligned}
D_1\left(\frac{1}{2}B_2-\frac{1}{2}D_1\right) &> \frac{1}{4}\left(B_7-\frac{1}{2}B_1\right)^2, \\
D_1\left(\frac{1}{2}B_2-\frac{1}{2}D_1\right) &> \frac{1}{4}\left(B_8-D_1\right)^2.
\end{aligned}$$

(1) We prove that $D_1 > \frac{1}{\sqrt{2}}\left(B_7 - \frac{1}{2}B_1\right)$.

$$\begin{aligned}
D_1 &> \frac{1}{\sqrt{2}}\left(B_7 - \frac{1}{2}B_1\right) \\
&\Leftrightarrow \sqrt{2}B_1 - \frac{\sqrt{2}}{2}B_3 - B_7 + \frac{1}{2}B_1 > \frac{\sqrt{2}}{2}\sqrt{(B_1 - B_3)(3B_1 - B_3)} \\
&\Leftrightarrow \left(\sqrt{2}B_1 - \frac{\sqrt{2}}{2}B_3 - B_7 + \frac{1}{2}B_1\right)^2 > \frac{1}{2}(B_1 - B_3)(3B_1 - B_3) \\
&\Leftrightarrow \left(\sqrt{2}B_1 - \frac{\sqrt{2}}{2}B_3 - B_7 + \frac{1}{2}B_1\right)^2 - \frac{1}{2}(B_1 - B_3)(3B_1 - B_3) > 0.
\end{aligned}$$

$\left(\sqrt{2}B_1 - \frac{\sqrt{2}}{2}B_3 - B_7 + \frac{1}{2}B_1\right)^2 - \frac{1}{2}(B_1 - B_3)(3B_1 - B_3)$ is decreasing with B_3, so

$$\begin{aligned}
&\left(\sqrt{2}B_1 - \frac{\sqrt{2}}{2}B_3 - B_7 + \frac{1}{2}B_1\right)^2 - \frac{1}{2}(B_1 - B_3)(3B_1 - B_3) \\
> &\left(\left(\sqrt{2}B_1 - B_7 + \frac{1}{2}B_1\right)^2 - \frac{3}{2}B_1^2\right) \\
> &B_7^2 > 0.
\end{aligned}$$

Hence, $D_1 > \frac{1}{\sqrt{2}}\left(B_7 - \frac{1}{2}B_1\right) > \frac{1}{\sqrt{2}}\left(B_7 - \frac{1}{2}B_1\right)$.

$$\begin{aligned}
D_1 &> \frac{1}{\sqrt{2}}\left(B_7 - \frac{1}{2}B_1\right) \\
\frac{1}{2}D_1^2 &> \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2 \\
\frac{1}{2}D_1^2 &= \frac{1}{2}D_1D_1 < \frac{1}{2}D_1(B_2 - D_1) = D_1\left(\frac{1}{2}B_2 - \frac{1}{2}D_1\right).
\end{aligned}$$

Since $q_{1L}^* < \frac{1}{2}B_2$, so

$$\begin{aligned}
\frac{1}{2}D_1^2 &< \frac{1}{2}D_1(B_2 - D_1) = D_1\left(\frac{1}{2}B_2 - \frac{1}{2}D_1\right) \\
D_1\left(\frac{1}{2}B_2 - \frac{1}{2}D_1\right) &> \frac{1}{4}\left(B_7 - \frac{1}{2}B_1\right)^2.
\end{aligned}$$

(2)

$$D_1\left(\frac{1}{2}B_2-\frac{1}{2}D_1\right)>\frac{1}{4}\left(B_8-D_1\right)^2.$$

Since $B_2>B_8$, so $D_1\left(\frac{1}{2}B_2-\frac{1}{2}D_1\right)>D_1\left(\frac{1}{2}B_8-\frac{1}{2}D_1\right)$.

If $\frac{1}{2}D_1\left(B_8-D_1\right)>\frac{1}{4}\left(B_8-D_1\right)^2$, i.e., $D_1>\frac{B_8}{3}$, $\frac{1}{2}D_1\left(B_2-D_1\right)>\frac{1}{4}\left(B_8-D_1\right)^2$. So we only need to prove $D_1>\frac{B_8}{3}$. We know that D_1 is the lower root of equation $q_{1L}\left(B_1-\frac{1}{2}B_3-\frac{1}{2}q_{1L}\right)=\frac{1}{8}B_1^2$. Taking $q_{1L}=\frac{B_8}{3}$ into $\frac{1}{8}B_1^2-q_{1L}\left(B_1-\frac{1}{2}B_3-\frac{1}{2}q_{1L}\right)$, we get

$$\begin{aligned}
&\frac{1}{8}B_1^2-q_{1L}\left(B_1-\frac{1}{2}B_3-\frac{1}{2}q_{1L}\right)\\
=\ &\frac{1}{8}B_1^2-\frac{1}{3}B_1B_8+\frac{1}{18}B_8^2+\frac{1}{6}B_3B_8\\
>\ &\frac{1}{72}\left(3B_1-4B_8\right)^2\geq 0.
\end{aligned}$$

Hence $D_1>\frac{B_8}{3}$.

2a)

$$\begin{aligned}
&\frac{1}{8}B_1^2\Pr\left(Y_1=A_H\right)\\
>\ &\left\{\begin{array}{c}\frac{1}{4}\left(B_5-\frac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)\\+\frac{1}{4}\left(B_6-D_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{array}\right\}.
\end{aligned}$$

$D_1>\frac{1}{\sqrt{2}}\left(B_7-\frac{1}{2}B_1\right)>\frac{1}{2}\left(B_7-\frac{1}{2}B_1\right)$,

so

$$\begin{aligned}
&\left\{\begin{array}{c}\frac{1}{4}\left(B_5-\frac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)\\+\frac{1}{4}\left(B_6-D_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{array}\right\}\\
<\ &\left\{\begin{array}{c}\frac{1}{4}\left(B_5-\frac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)\\+\frac{1}{4}\left(\frac{1}{2}B_7+\frac{1}{4}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{array}\right\}.
\end{aligned}$$

Take (29), (30) and (31) into

$$\begin{gathered}\tfrac{1}{8}B_1^2\Pr\left(Y_1=A_H\right)-\tfrac{1}{4}\left(B_5-\tfrac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)\\-\tfrac{1}{4}\left(\tfrac{1}{2}B_7+\tfrac{1}{4}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{gathered}$$

and we know that $\alpha\in(0,1)$ and $\gamma\in(0.5,1)$, so

$$\left\{\begin{gathered}\tfrac{1}{8}B_1^2\Pr\left(Y_1=A_H\right)-\tfrac{1}{4}\left(B_5-\tfrac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)\\-\tfrac{1}{4}\left(\tfrac{1}{2}B_7+\tfrac{1}{4}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{gathered}\right\}>0,$$

that is,

$$\begin{aligned}&\frac{1}{8}B_1^2\Pr\left(Y_1=A_H\right)\\>&\left\{\begin{gathered}\tfrac{1}{4}\left(B_5-\tfrac{1}{2}B_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_H\right)+\\\tfrac{1}{4}\left(B_6-D_1\right)^2\Pr\left(Y_1=A_H,Y_2=A_L\right)\end{gathered}\right\}.\end{aligned}$$

In sum, the leader earns more ex ante profit than the follower.

Proof of Lemma 7. From Lemma 6, we know that firms' strategies are given by

$$\begin{aligned}\hat{q}_1^*&=\frac{1}{2}E\left[A|Y_1\right],\\\hat{q}_2^*&=\frac{1}{2}\left(E\left[A|Y_1,Y_2\right]-\frac{1}{2}E\left[A|Y_1\right]\right).\end{aligned}$$

We require the total output quantity less than A_L, i.e.,

$$\begin{cases} \hat{q}_1^* + \hat{q}_2^* \leq A_L \\ \quad 0 \leq \hat{q}_1^* \\ \quad 0 \leq \hat{q}_2^* \end{cases}$$

$$\Rightarrow \begin{cases} \frac{1}{2}E\left[A|Y_1\right] + \frac{1}{2}\left(E\left[A|Y_1, Y_2\right] - \frac{1}{2}E\left[A|Y_1\right]\right) \leq A_L \\ 0 \leq \hat{q}_1^* \leq A_L \\ 0 \leq \hat{q}_2^* \leq A_L \end{cases}$$

$$\Rightarrow \begin{cases} A_L \leq E\left[A|Y_1\right] \leq \frac{4}{3}A_L \\ A_L \leq E\left[A|Y_1, Y_2\right] \leq \frac{3}{2}E\left[A|Y_1\right] \end{cases}.$$

$$\begin{aligned} \hat{q}_1^* - \hat{q}_2^* &= \frac{1}{2}E\left[A|Y_1\right] - \frac{1}{2}\left(E\left[A|Y_1, Y_2\right] - \frac{1}{2}E\left[A|Y_1\right]\right) \\ &= \frac{3}{4}E\left[A|Y_1\right] - \frac{1}{2}E\left[A|Y_1, Y_2\right] \\ &\geq \frac{3}{4}E\left[A|Y_1\right] - \frac{1}{2}\frac{3}{2}E\left[A|Y_1\right] \\ &= 0. \end{aligned}$$

so $\bar{q}_1^* \geq \bar{q}_2^*$ regardless of the demand signals they observe.

Firms' realized profits are given by

$$\begin{aligned} \bar{\pi}_1 &= \bar{q}_1^*\left(A - \bar{q}_1^* - \bar{q}_2^*\right), \\ \bar{\pi}_2 &= \bar{q}_2^*\left(A - \bar{q}_1^* - \bar{q}_2^*\right). \end{aligned}$$

Since $\bar{q}_1^* \geq \bar{q}_2^*$, then $\bar{\pi}_1 \geq \bar{\pi}_2$. So the leader always earns more ex ante profit than that of the follower's.

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