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Designing multi-target salesforce incentive contract

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DESIGNING MULTI-TARGET
SALESFORCE INCENTIVE CONTRACT

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DESIGNING MULTI-TARGET SALESFORCE INCENTIVE CONTRACT

by
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ABSTRACT

Designing Multi-Target Salesforce Incentive Contract

by

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Multi-target incentive contracts are widely observed in practice to stimulate salesforce effort. However, little is known about their effectiveness and the issues involved in designing them. In this thesis, we investigate the incentive contracting problem between a manufacturer and an agent when the realized sales of a product are affected by both the agent's selling effort and the type of the agent. The agent's type is uncertain to the manufacturer, whereas the agent can observe the actual type when exerting her selling effort. Again, this is unobservable by the manufacturer. For contract design problem, we develop a principal-agent model with both moral hazard and adverse selection. We examine the manufacturer's optimal contract parameter decisions employing a single multi-target contract for the agent who can be of different types. Because menu contracts are commonly studied in literature for the adverse selection problem, we also study a menu of single-target contracts; and examine the manufacturer's optimal contract parameter decisions. We then compare the performance between the two types of contract. We arrive at a number of managerial insights regarding the design and the performance of multi-target contract and menu contract.

Keywords: Principal-agent model, moral hazard, adverse selection, multi-target contract, menu contract

DECLARATION

I declare that this is an original work based primarily on my own research, and I warrant that all citations of previous research, published or unpublished, have been duly acknowledged.

黄闻欣

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Date 24 Sep 2015

CERTIFICATE OF APPROVAL OF THESIS

DESIGNING MULTI-TARGET SALESFORCE INCENTIVE CONTRACTS

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1 Introduction

Manufacturers usually rely on their salesforce to sell products, resulting in a principal-agent relationship in which the manufacturer is the principal and the salesforce is the agent. The agent can exert an effort to improve the sales level, but because the effort level is generally not observable to the manufacturer, a salesforce incentive scheme is needed to motivate the agent's effort. Contracting problems with hidden actions are regarded as moral hazard: the principal can observe the outcome of the agent's effort, but cannot infer the effort level of the agent. Very often, as the agent is closer to the market than the manufacturer, she possesses better information about the market for the product. Such information will influence the sales and the compensation to the agent. The selling capability of the agent can also affect the realized sales. It's easier for an agent with a higher selling capability to achieve a high sales volume, and it requires a higher sales effort level for the low capability agent to reach an identical sales volume. The agent's capability is also often not observable to the manufacturer. The asymmetric information advantage of the agent over the manufacturer is denoted as the adverse selection problem in principal-agent theory. For convenience, we say that the agent is of a high type if the market condition or her selling capability is high, and of a low type otherwise. A large/small sales volume can result from a high/low effort level of the agent or a high/low type or both; for example, the iPhone sells well in the market even if the agent exerts little effort.

An effective salesforce compensation plan should consider both the effort level and the agent's type, and provide an appropriate incentive to the agent to sell a product. As quota-based contracts are widely employed in practice for salesforce compensation, we investigate the design of such contracts in the presence of both moral hazard and adverse selection problems.

In the literature, a menu of contracts is the most common solution to an adverse selection problem. It is designed such that the agent chooses one of contracts in the menu based on her private information about the product. The use of the menu contract is to reveal an agent's private information, which can benefit the manufacturer by measuring the agent's selling effort more accurately. Menus of quota-based contracts have been extensively studied for salesforce incentives. The contract parameters should be carefully chosen to make sure that it's in the agent's interest to truthfully reveal her private information and to exert an effort at the optimal level. However, it is never easy to design such a contract that exploits the agent's private information and induces her effort level simultaneously, and it's critical to set reasonable sales quotas that reflect a product's sales prospect and to measure the agent's performance.

Although menus of quota-based contracts have been extensively studied in literature, their use in practice is limited. In contrast, multi-target contracts are widely applied by practitioners in a variety of industries, for example, the office equipment industry (Xerox's partnership program 2015), automobile industry (Automotive News 2012 and Crain's Detroit Business 2013), pharmaceutical industry (Sinha, Prabhakant and Zoltners 2001), and so on.

In this thesis, we investigate a menu of single-target contracts and a dual-target contract and ask the following two questions: How is the performance of the widely applied multi-target contract compared with the well-studied menu contract? And when can a multi-target contract outperform a menu contract? To address the above questions and fill the gap between practice and literature, we consider a principal-agent problem in which a risk-neutral agent with limited liability sells a product for a manufacturer. The realized sales are affected by the agent's selling effort and the agent type, both of which are the agent's private information that are unobservable to the manufacturer. The agent type is uncertain and can be either high or low. Although the manufacturer knows the probability for each type to occur, the agent knows precisely the type value when she exerts an effort to sell the product.

After describing the model setting in detail, we proceed to introduce two contractual forms: a menu of single-target contracts and a dual-target contract. We formulate the manufacturer's problem and the agent's optimal response under each contract. Due to the complexity of the manufacturer's problem, it's intractable to obtain a closed-form solution for the manufacturer's problem. Consequently, we conduct a numerical analysis for each contract and find out the manufacturer's optimal contract parameters and expected profit as well as the optimal effort level and the maximum expected profit of each type of agent. Furthermore, we compare the manufacturer's expected profits, the total expected sales and the agent's expected profits under the two types of contracts. Our comparison indicates that the manufacturer's choice of contract is affected by the probability of the high type and the difference between the values of the low and high types.

The rest of the thesis is organized as follows. Section 2 reviews the literature, and in Section 3 we present the models of the menu contract and the dual-target contract, respectively. In Section 4, we conduct a numerical analysis for the menu contract and the dual-target contract, and draw some managerial insights about the relative performance of the two contracts, the manufacturer's optimal choice of contract type, and the optimal parameter values for each type of contract. In Section 5 we discuss some possible future work. Then we conclude in Section 6.

2 Literature Review

This thesis is related to three steams of literature, including marketing, economics and operations management.

2.1 Marketing Literature

The salesforce incentive compensation problem has been extensively studied in the marketing literature. Coughlan (1993) provided a comprehensive review, including model assumptions such as a random demand function, i.e., the realized sales generated by a selling effort involves a random noise term.

Two widely applied simple contractual forms have been examined, the commission contract and the bonus contract. Commission is a compensation method in which agents are paid based on a percentage of the realized sales. Compensation schemes in which commission payment is involved are believed to be the most efficient and the most effective contractual forms because the sales volume is financially rewarded with accuracy.

The bonus contract is a compensation method in which the agent will receive a bonus when the realized sales meet or exceed a pre-specified quota. This compensation scheme is the most appealing to security oriented (risk-averse) agent rather than achievement oriented (risk-neutral or even risk-prone) ones, and is easy to manage. The major limitation of this type of compensation scheme is that it provides no further incentive for the agent when the sales volume reaches the quota (Tosdal 1953).

As reported by Peck (1982), about 18% of salesforces were compensated by salary only, 9% were merely compensated by commissions and 73% were paid through a combination of base salary and performance-based payment. A compensation scheme combining base salary and incentive pay is superior to base salary or incentive payment only and prevents most of the limitations of either one of the two schemes.

Basu et al. (1985) proposed a BLSS model for a moral-hazard problem in which the firm is not capable of inferring the effort exerted by the agent by merely observing her performance (i.e., sales volume). The agent is risk averse, which is represented by a power-utility function. Under such a model setting, Basu et al. (1985) proved the optimality of nonlinear shapes of the optimal compensation scheme. The BLSS model has been widely used to derive general recommendations with respect to the form of compensation plan.

Sinha, Prabhakant and Zoltners (2001) summarized the managerial insights that are found in literature and showed that compensation schemes combining a base salary and some incentives (such as commissions or bonuses) are a common method to compensate the agent.

A number of publications studied both moral hazard and adverse selection problems. Lal and Staelin (1986) considered the design of compensation schemes for heterogeneous and risk-averse salesforce and demonstrated the rationality of the menu contracts. Despite how appealing the nonlinearity of the optimal contracts in the BLSS plan is, in practice, it's intricate to implement, since the manufacturer has to specify a structure of commission rates and sales quotas. Under general assumptions on the agent's utility function, the probability density function of sales, the agent's effort disutility function, linearity of the sales with respect to the agent's effort, Lal and Staelin (1986) relaxed the assumption of homogeneous salesforce and showed that contracts proposed in the BLSS model are not always optimal and provided plausible explanations for multi–target contracts. Lal and Staelin (1986) found that applying an optimal compensation scheme would require the manufacturer to exclude his low skill agents. In comparison with the piecewise-linear-threshold contracts, among all the contractual forms, only linear contracts need to be considered under circumstances in which cumulative sales outcome is only updated periodically given that the agent's utility function is exponential. On the other hand, if the agent has a power utility function, the piecewise-linear-threshold contract significantly dominates the best linear contract.

The subsequent study by Lal and Srinivasan (1993) revealed that simple commission and bonus contracts still outperform other contractual forms even in a multi-product and multi-period setting. Raju and Srinivasan (1996) extended the study of this specific model setting to the multi-territory salesforce situation. It's striking that both the non-optimality incurred by contract shape and salesforce heterogeneity is very small with the total non-optimality within the range of 1%. Raju and Srinivasan (1996) showed that merely varying the quota (without changing the base salary and/or commission rate) captures most of the heterogeneity across salespersons/territories.

Even when the non-linear compensations are proved to be optimal, there are reasons for adopting simpler plans. Plans with a fixed salary and a constant commission rate are simple to administer, easy to understand by agents, and very often not far from optimality. Even better are plans with a salary and a bonus for exceeding a certain sales quota, since these plans preserve the non-linear shape of the BLSS model. Basu and Kalyanaram (1990) compared the performance of linear compensation scheme with the performance of the BLSS model when the salesperson's utility function defines her to be risk averse. Basu and Kalyanaram (1990) proved that when the agent's risk aversion is high, the linear compensation plan is almost as profitable as the BLSS plan, and theoretically justified the widespread use of the linear compensation scheme in practice. On the other hand, when the agent is less risk-averse, a non-linear compensation plan contingent on performance significantly outperforms the linear ones.

Bako and Kalecz (2013) showed that when randomness exists in the realized sales, how the quotas are set affects the behavior of the agent. This quota setting consequently increases the manufacturer's revenue in comparison to single linear (commission) contracts. Their work further indicates that the quotas should be set to target either all or the less-efficient (low-type) agents. When the agent's type values are significantly diversified or the realized sales are highly influenced by randomness, it's in the agent's interest to place higher bonus. Besides, Bako and Kalecz (2013) also proved that the higher the variability between the values of the two agent types, the lower the optimal quotas should be set. Bako and Kalecz (2013) further demonstrated the importance to preserve the general shape of the BLSS plan. The quota-based commission plan preserves the convexity of the BLSS plan and leads to near-optimal results, while the linear plan does not.

Rao (1990) proved the optimality of a menu of single-target contracts for heterogeneous but risk-neutral agent. In the proposed compensation scheme, the agent is offered a menu of contracts to choose from, and her choice truthfully reveals her information. The proposed scheme is continuous, non-linear, and it provides the agent an immediate payment (bonus) for meeting the quota plus a constant commission rate.

2.2 Economics Literature

In the economics literature, the basic moral hazard problem was studied by Holstrom (1987) and (1991). Principal-agent models have been used to investigate the quota-based incentive contracts. This stream of literature focuses on a model setting with a risk-neutral agent whose liability is limited. If both the principal and the agent are risk-neutral, a bunch of contracts, including quota-based bonus contract, are optimal. But when the agent's liability is limited, Kim (1997), Park (1995) and Oyer (2000) proved that the quota-based bonus contract is uniquely optimal. Kim (1997) demonstrated the optimality of a bonus contract in which the revenue from the product sales is shared by the manufacturer and the agent, and the agent takes a lump-sum bonus only when the realized sales exceed a predetermined sales quota. Park (1995) proved the effectiveness of a performance-based bonus contract in achieving the first-best outcome when limited-liability constraint restrains alternative simple contracts from being applicable. In Oyer (2000), an additive form of effort-demand relationship was adopted. When the hazard rate in the demand distribution is monotonically increasing and the agent's participation is only constrained by the limited liability constraint, a quota-based bonus contract is more preferable than other alternative compensation schemes.

2.3 Operations Management Literature

The operations management literature often incorporates inventory consideration in salesforce incentive design. Chu and Lai (2013) found that under demand censorship and quota-based bonus contract, although the first-best solution cannot be achieved, the effectiveness of quota-based bonus contract was proved to approach the first-best solution. In the case of additive demand–effort relationship, it's optimal for a manufacturer to induce a sales effort level and to build an inventory level both greater than the corresponding levels in the first-best solution. For the manufacturer in the case of multiplicative demand–effort relationship, it is optimal to induce an effort level no higher than the first-best effort level, but maintain an inventory level higher than the first-best solution. Combining the moral hazard problem, the adverse selection problem, and the inventory consideration, Chen (2005) showed that a menu of linear contracts dominates Gonik (1978)'s forecast-based compensation scheme since Gonik's solution was not capable of motivating different levels of effort by separating different agent types apart.

Chen and Miller (2009) proved that the optimal compensation scheme is a non-decreasing piecewise linear function which is neither convex nor concave. Their work indicates that the piecewise-linear-threshold contract is significantly superior to the best linear contract when the agent has a power utility function. Dai and Jerath (2013) found that quota plans can be optimal when inventory constraint is considered, although Dai and Jerath (2013) ignored the phenomenon that the manufacturer is biased to build a higher inventory level than that in the first best solution. Chen (2000) showed that the agent's contractual form makes a critical impact on how she is going to put selling effort, which subsequently determines the manufacturer's sales outcome, and eventually how the manufacturer operates his inventory. Furthermore, Chen (2000) proved that it benefits the manufacturer to motivate the agent's selling effort in a way that the realized sales just meet the inventory capacity.

Incentive compensation has been studied with the involvement of innovative contract forms in a dynamic setting. A typical incentive structure in a multi-period setting is the stair-step incentive compensation scheme widely applied in the automobile industry. Under such an incentive scheme, the agent is paid on different per-unit commissions when the realized sales exceed different thresholds; a base salary might be offered as well (Sohoni et al. 2006). Sohoni et al. (2010) proved that if the manufacturer associates a positive cost with the sales variance, a threshold-based incentive with a positive bonus is superior to other schemes without a bonus offering. Sohoni et al. (2010) showed that an exclusive dealership with a threshold contract boosts sales quantity and decreases sales variance at the same time. But non-exclusive dealership with a threshold-based compensation increases sales variance and decreases sales. In such a dynamic setting, the stair-step salesforce incentive provides an intrinsic incentive to the dealer to exert a large effort at the last period in order to boost the sales and meet the threshold. For this phenomenon, Sohoni et al. (2011) compared an additional marginal pay (commission) with a threshold-based bonus contract and showed that in the case of an exclusive dealership, the bonus contract not only boosts the expected sales but also decreases the sales variance. In the case of a non-exclusive dealership, a commission contract substantially increases the sales variance. Besides, the bonus contract continues to outperform the commission contract in this case. Moreover, Sohoni et al. (2011) proved that the parameters of the incentive compensation scheme and the demand uncertainty affect the agent's optimal effort level decision. Setting up appropriate threshold parameters reduces the sales variance.

Unlike the above publications, in this thesis, we investigate a multi-target contract for the salesforce incentive problem with both moral hazard and adverse selection. The realized sales are random in the agent's effort level. Both the manufacturer and the agent are risk neutral, and the agent has limited liability. In addition, we consider a static setting without inventory decision.

For an incentive problem with both moral hazard and adverse selection, in the literature, menu contracts are a common solution to reveal an agent's private information and induce her selling effort. For the problem with only moral hazard, Basu et al. (1985) proved that the performance of the single-target contract is very close to the optimal nonlinear contract, and the efficiency loss of the single-target contract is within 1%; as shown by Kim (1997), Park (1995) and Oyer (2000), a single-target contract with a bonus will suffice in motivating the agent to exert selling effort. As we discussed in Section 1, the multi-target contract is widely applied in practice for salesforce incentive. If the multi-target contract is designed only to solve the moral hazard problem, then as the above publications have found, even a single-target contract would be sufficient. Therefore it's important to investigate whether the multi-target contract plays dual roles, one is to motivate the agent to exert selling effort, and the other is to reveal her private information.

3 Model

A risk-neutral manufacturer (he) sells a product through a risk-neutral agent (she) into a market. We focus on a single selling season. The realized demand, X, is jointly determined by the agent's selling effort a, a base demand d, the agent's type θ, and a random market noise ε, as shown in the following equation:

$$X = \theta (a + d) + \varepsilon.$$

The multiplicative demand–effort relationship follows the typical principal-agent model in the literature (see, e.g., Rao 1990, Petruzzi and Dada 1999, Agrawal and Seshadri 2000, Chen 2000, Chen 2005, Chen and Xiao 2009, and Chu and Lai 2013).

The agent type can take the value of θ_H or θ_L with the probability $\Pr(\theta = \theta_H) = \rho$ and $\Pr(\theta = \theta_L) = 1 - \rho$, where $0 < \rho < 1$ and $\theta_H > \theta_L > 0$. We denote that the agent is of the high type when her type value is θ_H, and of the low type if her type value is θ_L. The manufacturer knows only the distribution of the agent type, while the agent knows the exact value of her type. Before the manufacturer determines the compensation contract parameters, the distribution of X is known to both parties. The manufacturer offers the salesforce compensation contract to the agent, who then decides how much selling effort to exert given her information about the type.

The random market noise follows a normal distribution $\varepsilon \sim N(0, \sigma^2)$, and the realized demand given the agent's effort level a and her type value θ follows a normal distribution with the cumulative distribution function $F(X \mid a, \theta)$ and the probability density function $f(X \mid a, \theta)$. We assume $\theta d > 3\sigma$ to exclude the situation in which the expected sales go to negative.

The sequence of events is as follows: The manufacturer offers an incentive contract $T(\cdot)$ to the agent; the agent decides whether or not to accept the contract based on her information of θ; upon the selling season begins, the agent makes the effort level decision a; both the manufacturer and the agent observe the realized sales X; and the agent is compensated based on X according to the incentive contract.

Applying principal-agent theory, we analyze the model setting under which the problem faced by the manufacturer is a combination of adverse selection (preliminary private information of type value) and moral hazard (effort decision alongside with the revelation of market demand information). Typically, the manufacturer develops an incentive contract prior to the actual beginning of selling season. Consider the agent's decision when offered an incentive contract. Initially she would consider the expected profit if she participates in the contract. Assume that $T(\cdot)$ is the contract offered, therefore $T(x)$ is the compensation paid to the agent if the realized sales are x. The optimal effort level not only depends on the offered contract, but also on the agent's type value. The asymmetric information advantage permits the agent to make her effort level decision in accordance with her private information. Suppose $a(T(\cdot), \theta)$ is the optimal effort level given the compensation contract $T(\cdot)$ and the actual agent type θ. Assume that given the contract $T(\cdot)$ and the agent's type θ, $U(T(\cdot), \theta)$ is the maximum expected profit achievable for the agent. If $U(T(\cdot), \theta)$ is less than U_0, the agent's reservation profit which represents the expected profit that the agent would achieve from her external opportunities, then optimization of the effort level is meaningless towards the agent. Without further specification, we normalize the reservation profit U_0 to zero.

Let m be the profit margin of the product. Combining moral-hazard problem with adverse selection problem, we follow the typical principal-agent model, which serves as the basic model for understanding the contract designing issues that are involved in a salesforce incentive context. Thereby, the margin m is fixed and does not alter with the volume of sales or selling effort.

We assume that the agent is risk-neutral and has only limited wealth, thereby, she can only receive non-negative payment for any possible sales outcome and that the base salary β should be non-negative. Note this constraint follows the typical principal-agent model as the limited-liability constraint in extant publications (e.g. Sappington 1983, Park 1995, Kim 1997, Oyer 2000, and Poblete 2012).

As it's costly for the agent to exert selling effort, the effort cost is assumed to be quadratic: $V(a) = a^2/2$, an increasing and convex function (e.g. Baker 1992, Bester and Guth 1998, Schaefer 1998, Chen 2005, Chen and Xiao 2009, and Chu and Lai 2013). Other effort cost functions could be applied without fundamentally changing the analytical outcome (Chen 2005). The agent's expected profit is the payment received from the manufacturer minus the effort cost. In order to maximize the expected profit achievable under the compensation scheme $T(\cdot)$, the agent determines the optimal effort level by solving:

$$\max_a E\left(T(X) - V(a) \mid \theta\right).$$

Notice that the agent assesses the contract based on her information of the type value. Thus, the agent's expected profit is based on her type value θ with respect to the random market noise ϵ. The optimal effort level given the contract $T(X)$ and the agent's type is thereby $a(T(X), \theta)$.

Turning to the manufacturer's problem, the manufacturer's expected profit is the revenue of the product sales minus the payment to the agent. In order to maximize the expected profit by optimizing the compensation scheme, the manufacturer's problem could be formulated as:

$$\begin{aligned}
\max\nolimits_{T(x)} \quad & E\left(mX - T(X) \mid \theta\right) \\
s.t. \quad & \\
(LL): \quad & T(X) \geq 0, \\
(IC): \quad & a = a\left(T(X), \theta\right), \\
(IR): \quad & U\left(T(X), \theta\right) \geq 0.
\end{aligned}$$

The first constraint is the limited liability (LL) constraint, ensuring that the agent receives only non-negative payment for any sales realization, which denotes that the agent is restricted from taking risks (without penalty). The second constraint is the incentive compatibility (IC) constraint, indicating that the effort level for each agent type is indeed optimal for the agent (thus compatible with the agent's objective). The last constraint is the individual rationality (IR) constraint, indicating that the agent, regardless of her type, is better off participating compared to her outside opportunities.

3.1 Menu of Single-target Contracts

The agent privately and truly knows the type value while the manufacturer only knows the distribution of the agent type. In the presence of asymmetric information about the agent's type, the agent might not truthfully reveal her type to gain an advantage. According to principal-agent theory, the manufacturer can reveal the agent's type by applying a menu of contracts.

Hence, the manufacturer designs a menu of two single-target contracts for two different types of agents, taking the agent's incentive into consideration. Under the menu of single-target contracts, the incentive payment is contingent on the realized sales. We mainly consider the scenario in which the manufacturer provides two single-target contracts to the agent, who chooses one given her information of the type value. By observing the agent's choice, the manufacturer reveals the agent's type. Specifically, the agent would receive commission when and only when the realized sales reach or surpass the sales quota q. The contract form is as follows:

$$T_i(x) = \begin{cases} \beta_i, & x_i \leq q_i; \\ \beta_i + \alpha_i (x_i - q_i), & x_i > q_i, \end{cases}$$

where $i = H \ or \ L$, and $T_H(x)$ being the contract intended for the high-type agent, and $T_L(x)$ for the low-type, with $\alpha_i \geq 0$.

Note that achieving system efficiency does not mean that there's no information rent. Although the low-type agent's individual rationality (IR) constraint would be kept binding (which would be verified below in Section 4), the high-type agent would obtain a premium as an information rent.

Assuming the agent is of type i, and we try to find her optimal effort level decision under the single-target contract. Notice that the agent's expected profit is:

$$\begin{aligned}
\Pi_S^i(a) &= E\left(T_i(x) \mid a, \theta_i\right) - \frac{1}{2}a^2 \\
&= \beta_i + \alpha_i\left[(a+d)\theta_i - q_i\right]\bar{F}\left(q_i - (a+d)\theta_i\right) \\
&\quad + \alpha_i \sigma f\left(q_i - (a+d)\theta_i\right) - \frac{1}{2}a^2. \qquad (1)
\end{aligned}$$

Maximizing the agent's expected profit with respect to the effort level a, we can obtain the first-order condition,

$$\alpha_i \theta_i \bar{F}\left(q_i - (a+d)\theta_i\right) - a = 0.$$

From the above, the agent's optimal effort level is the solution to

$$a = \alpha_i \theta_i \bar{F}\left(q_i - (a+d)\theta_i\right). \qquad (2)$$

We take the second-order derivative of the agent's expected profit:

$$\frac{\partial^2}{\partial a^2}\Pi_S^i(a) = \alpha_i \theta_i^2 f\left(q_i - (a+d)\theta_i\right) - 1.$$

The agent's optimal effort level is unique when

$$\alpha_i \theta_i^2 f\left(q_i - (a+d)\theta_i\right) - 1 < 0.$$

The manufacturer's problem is formulated as:

$$
\begin{aligned}
\max_{T_i(x)} \Pi_M &= \rho E\left(mx - T_H(x) \mid a_H, \theta_H\right) + (1-\rho) E\left(mx - T_L(x) \mid a_L, \theta_L\right) \\
& \quad s.t. \\
(IR-i) &: \quad \Pi_S^i \geq 0, \\
(LL-i) &: \quad \beta_i \geq 0, \\
(IC-i) &: \quad a_i = \alpha_i \theta_i \bar{F}\left(q_i - (a_i + d)\theta_i\right), \\
(IC-HL) &: \quad \Pi_S^H \geq \beta_L + \alpha_L \left[(a_{HL} + d)\theta_H - q_L\right] \bar{F}\left(q_L - (a_{HL} + d)\theta_H\right) \\
& \quad +\alpha_L \sigma f\left(q_L - (a_{HL} + d)\theta_H\right) - \frac{1}{2} a_{HL}^2, \\
(IC-LH) &: \quad \Pi_S^L \geq \beta_H + \alpha_H \left[(a_{LH} + d)\theta_L - q_H\right] \bar{F}\left(q_H - (a_{LH} + d)\theta_L\right) \\
& \quad +\alpha_H \sigma f\left(q_H - (a_{LH} + d)\theta_L\right) - \frac{1}{2} a_{LH}^2, \\
(FC-i) &: \quad \alpha_i \leq m,
\end{aligned}
\tag{3}
$$

where $i = H$ or L.

The last constraint is the feasibility (FC) constraint which restrains that the commission rate paid to the agent cannot be larger than the manufacturer's profit margin. As demonstrated by Poblete et al. (2012), the feasibility constraint limits the commission rate of the contract.

We first show that given any contract, the high-type agent's expected profit is greater than that of the low-type agent. Differentiating Π_S in (1) with respect to θ and using the first-order condition (2), we can find that

$$\begin{aligned}
\frac{\partial \Pi_S}{\partial \theta} &= \alpha\left(\frac{\partial a}{\partial \theta}\theta + a + d\right)\bar{F}\left(q-(a+d)\theta\right) \\
&\quad +\alpha\left[(a+d)\theta - q\right]\left(\frac{\partial a}{\partial \theta}\theta + a + d\right)f\left(q-(a+d)\theta\right) \\
&\quad +\alpha\left[q-(a+d)\theta\right]\left(\frac{\partial a}{\partial \theta}\theta + a + d\right)f\left(q-(a+d)\theta\right) - a\frac{\partial a}{\partial \theta} \\
&= \alpha\left(\frac{\partial a}{\partial \theta}\theta + a + d\right)\bar{F}\left(q-(a+d)\theta\right) - a\frac{\partial a}{\partial \theta} \\
&= \alpha\left(a+d\right)\bar{F}\left(q-(a+d)\theta\right) > 0.
\end{aligned}$$

Note that either $(LL-L)$ or $(IR-L)$ must hold as an equality, otherwise the manufacturer can always decrease the β_H without violating the incentive compatibility constraint. When

$$\begin{aligned}
&-\frac{1}{2}\alpha_L^2\theta_L^2\bar{F}^2\left(q_L-(a_L+d)\theta_L\right)+\alpha_L\left(q_L-\theta_L d\right)\bar{F}\left(q_L-(a_L+d)\theta_L\right) \\
&-\alpha_L\sigma f\left(q_L-(a_L+d)\theta_L\right)\leq 0,
\end{aligned}$$

the $(LL-L)$ constraint should be binding; when

$$\begin{aligned}
&-\frac{1}{2}\alpha_L^2\theta_L^2\bar{F}^2\left(q_L-(a_L+d)\theta_L\right)+\alpha_L\left(q_L-\theta_L d\right)\bar{F}\left(q_L-(a_L+d)\theta_L\right) \\
&-\alpha_L\sigma f\left(q_L-(a_L+d)\theta_L\right)> 0,
\end{aligned}$$

the $(IR-L)$ constraint should be binding. We can show that constraints $(IR-L)$ and $(IC-HL)$ should bind at the optimum for the relaxed problem (without the $IC-LH$ constraint) because otherwise, we can reduce β_L and β_H while making both constraints hold. We can thus solve for β_L and β_H from the binding constraints $(IR-L)$ and $(IC-HL)$ as

$$
\begin{aligned}
& \beta_L \\
= \; & -\frac{1}{2}\alpha_L^2\theta_L^2\bar{F}^2\left(q_L-\left(a_L+d\right)\theta_L\right)+\alpha_L\left(q_L-\theta_L d\right)\bar{F}\left(q_L-\left(a_L+d\right)\theta_L\right) \\
& -\alpha_L\sigma f\left(q_L-\left(a_L+d\right)\theta_L\right),
\end{aligned}
$$

and

$$
\begin{aligned}
& \beta_H \\
= \; & \beta_L+\alpha_L\left[\left(a_{HL}+d\right)\theta_H-q_L\right]\bar{F}\left(q_L-\left(a_{HL}+d\right)\theta_H\right) \\
& +\alpha_L\sigma f\left(q_L-\left(a_{HL}+d\right)\theta_H\right)-\frac{1}{2}a_{HL}^2 \\
& -\alpha_H\left[\left(a_H+d\right)\theta_H-q_H\right]\bar{F}\left(q_H-\left(a_H+d\right)\theta_H\right) \\
& -\alpha_H\sigma f\left(q_H-\left(a_H+d\right)\theta_H\right)+\frac{1}{2}a_H^2 \\
= \; & \alpha_L\left[\left(a_{HL}+d\right)\theta_H-q_L\right]\bar{F}\left(q_L-\left(a_{HL}+d\right)\theta_H\right) \\
& -\frac{1}{2}\alpha_L^2\theta_L^2\bar{F}^2\left(q_L-\left(a_L+d\right)\theta_L\right) \\
& +\alpha_L\sigma f\left(q_L-\left(a_{HL}+d\right)\theta_H\right)-\alpha_L\sigma f\left(q_L-\left(a_L+d\right)\theta_L\right) \\
& -\alpha_H\sigma f\left(q_H-\left(a_H+d\right)\theta_H\right)+\frac{1}{2}a_H^2-\frac{1}{2}a_{HL}^2 \\
& -\alpha_H\left[\left(a_H+d\right)\theta_H-q_H\right]\bar{F}\left(q_H-\left(a_H+d\right)\theta_H\right) \\
& +\alpha_L\left(q_L-\theta_L d\right)\bar{F}\left(q_L-\left(a_L+d\right)\theta_L\right).
\end{aligned}
$$

Substituting the above into the manufacturer's profit function

$$
\begin{aligned}
\Pi_M &= \rho\left\{m\left(a_H+d\right)\theta_H-\beta_H\right. \\
&\quad -\alpha_H\left[\left(a_H+d\right)\theta_H-q_H\right]\bar{F}\left(q_H-\left(a_H+d\right)\theta_H\right) \\
&\quad \left.-\alpha_H\sigma f\left(q_H-\left(a_H+d\right)\theta_H\right)\right\} \\
&\quad +\left(1-\rho\right)\left\{m\left(a_L+d\right)\theta_L-\beta_L\right. \\
&\quad -\alpha_L\left[\left(a_L+d\right)\theta_L-q_L\right]\bar{F}\left(q_L-\left(a_L+d\right)\theta_L\right) \\
&\quad \left.-\alpha_L\sigma f\left(q_L-\left(a_L+d\right)\theta_L\right)\right\},
\end{aligned}
$$

we obtain that

$$
\begin{aligned}
&\Pi_M \\
&= \rho\left\{m\left(a_H+d\right)\theta_H-\beta_L\right. \\
&\quad -\alpha_L\left[\left(a_{HL}+d\right)\theta_H-q_L\right]\bar{F}\left(q_L-\left(a_{HL}+d\right)\theta_H\right) \\
&\quad -\alpha_L\sigma f\left(q_L-\left(a_{HL}+d\right)\theta_H\right)+\frac{1}{2}a_{HL}^2 \\
&\quad +\alpha_H\left[\left(a_H+d\right)\theta_H-q_H\right]\bar{F}\left(q_H-\left(a_H+d\right)\theta_H\right) \\
&\quad +\alpha_H\sigma f\left(q_H-\left(a_H+d\right)\theta_H\right)-\frac{1}{2}a_H^2 \\
&\quad -\alpha_H\left[\left(a_H+d\right)\theta_H-q_H\right]\bar{F}\left(q_H-\left(a_H+d\right)\theta_H\right) \\
&\quad \left.-\alpha_H\sigma f\left(q_H-\left(a_H+d\right)\theta_H\right)\right\} \\
&\quad +\left(1-\rho\right)\left\{m\left(a_L+d\right)\theta_L-\beta_L\right. \\
&\quad -\alpha_L\left[\left(a_L+d\right)\theta_L-q_L\right]\bar{F}\left(q_L-\left(a_L+d\right)\theta_L\right) \\
&\quad \left.-\alpha_L\sigma f\left(q_L-\left(a_L+d\right)\theta_L\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= \rho \left[m\left(a_H+d\right)\theta_H - \beta_L \right. \\
&\quad -\alpha_L\left[\left(a_{HL}+d\right)\theta_H - q_L\right]\bar{F}\left(q_L-\left(a_{HL}+d\right)\theta_H\right) \\
&\quad \left. -\alpha_L\sigma f\left(q_L-\left(a_{HL}+d\right)\theta_H\right)+\frac{1}{2}a_{HL}^2-\frac{1}{2}a_H^2\right] \\
&\quad +\left(1-\rho\right)\left\{m\left(a_L+d\right)\theta_L-\beta_L\right. \\
&\quad -\alpha_L\left[\left(a_L+d\right)\theta_L-q_L\right]\bar{F}\left(q_L-\left(a_L+d\right)\theta_L\right) \\
&\quad \left. -\alpha_L\sigma f\left(q_L-\left(a_L+d\right)\theta_L\right)\right\}.
\end{aligned}
$$

3.2 Dual-target Contract

The dual-target contract is defined as follows:

$$
T\left(x\right)=\begin{cases}
\beta, & x\leq q_1; \\
\beta+\alpha_1\left(x-q_1\right), & q_1<x\leq q_2; \\
\beta+\alpha_2\left(x-q_2\right)+\alpha_1\left(q_2-q_1\right), & x>q_2.
\end{cases}
$$

Notice that the agent's expected profit is:

$$
\begin{aligned}
\Pi_S^i\left(a\right) &= E\left(T\left(x\right)\mid a,\theta_i\right)-\frac{1}{2}a^2 \\
&= \beta-\frac{1}{2}a^2 \\
&\quad +\alpha_1\left[\left(a+d\right)\theta_i-q_1\right]\left(F\left(q_2-\left(a+d\right)\theta_i\right)-F\left(q_1-\left(a+d\right)\theta_i\right)\right) \\
&\quad -\alpha_1\sigma\left(f\left(q_2-\left(a+d\right)\theta_i\right)-f\left(q_1-\left(a+d\right)\theta_i\right)\right) \\
&\quad +\left\{\alpha_2\left[\left(a+d\right)\theta_i-q_2\right]+\alpha_1\left(q_2-q_1\right)\right\}\left(1-F\left(q_2-\left(a+d\right)\theta_i\right)\right) \\
&\quad +\alpha_2\sigma f\left(q_2-\left(a+d\right)\theta_i\right),
\end{aligned}
$$

where $i=H$ or L.

Maximizing the agent's expected profit with respect to the effort level a, we can obtain the first-order condition,

$$
\alpha_1\theta_i\left(F\left(q_2-\left(a+d\right)\theta_i\right)-F\left(q_1-\left(a+d\right)\theta_i\right)\right)+\alpha_2\theta_i\bar{F}\left(q_2-\left(a+d\right)\theta_i\right)-a=0.
$$

From the above, the agent's optimal effort level is the solution to

$$a = \alpha_1 \theta_i \left(F \left(q_2 - (a + d) \theta_i \right) - F \left(q_1 - (a + d) \theta_i \right) \right) + \alpha_2 \theta_i \bar{F} \left(q_2 - (a + d) \theta_i \right).$$

We take the second-order derivative of the agent's expected profit with respect to the agent's effort level when the first-order derivative equals to 0:

$$\begin{aligned} \frac{\partial^2}{\partial a^2} \Pi_S^i (a) &= \alpha_1 \theta_i^2 \left(f \left(q_2 - (a + d) \theta_i \right) - f \left(q_2 - (a + d) \theta_i \right) \right) \\ &\quad + \alpha_2 \theta_i^2 f \left(q_2 - (a + d) \theta_i \right) - 1. \end{aligned}$$

A sufficient condition for the agent's optimal effort level to be unique is:

$$\begin{gathered} \alpha_1 \theta_i^2 \left(f \left(q_2 - (a + d) \theta_i \right) - f \left(q_2 - (a + d) \theta_i \right) \right) \\ + \alpha_2 \theta_i^2 f \left(q_2 - (a + d) \theta_i \right) - 1 < 0. \end{gathered}$$

For convenience, we define:

$$\begin{aligned} H1 &\equiv q_1 - (a_H + d) \theta_H, \quad H2 \equiv q_2 - (a_H + d) \theta_H, \\ L1 &\equiv q_1 - (a_L + d) \theta_L, \quad\ \ L2 \equiv q_2 - (a_L + d) \theta_L. \end{aligned}$$

Thereby, the type-i agent's expected profit is as follows:

$$\begin{aligned} & \Pi_S^i \\ = \ & \beta - \frac{1}{2} a_i^2 \\ & + \alpha_1 \left[(a_i + d) \theta_i - q_1 \right] \left(F (i2) - F (i1) \right) - \alpha_1 \sigma \left(f (i2) - f (i1) \right) \\ & + \left\{ \alpha_2 \left[(a_i + d) \theta_i - q_2 \right] + \alpha_1 (q_2 - q_1) \right\} \bar{F} (i2) + \alpha_2 \sigma f (i2), \end{aligned}$$

For the high-type agent,

$$
\begin{aligned}
& \Pi_{S-H} \\
= \; & \beta - \frac{1}{2}a_H^2 \\
& +\alpha_1 \left[(a_H + d)\,\theta_H - q_1 \right] \left(F\left(H2\right) - F\left(H1\right) \right) - \alpha_1 \sigma \left(f\left(H2\right) - f\left(H1\right) \right) \\
& + \left\{ \alpha_2 \left[(a_H + d)\,\theta_H - q_2 \right] + \alpha_1 \left(q_2 - q_1 \right) \right\} \bar{F}\left(H2\right) + \alpha_2 \sigma f\left(H2\right);
\end{aligned}
$$

For the low-type agent,

$$
\begin{aligned}
& \Pi_{S-L} \\
= \; & \beta - \frac{1}{2}a_L^2 \\
& +\alpha_1 \left[(a_L + d)\,\theta_L - q_1 \right] \left(F\left(L2\right) - F\left(L1\right) \right) - \alpha_1 \sigma \left(f\left(L2\right) - f\left(L1\right) \right) \\
& + \left\{ \alpha_2 \left[(a_L + d)\,\theta_L - q_2 \right] + \alpha_1 \left(q_2 - q_1 \right) \right\} \bar{F}\left(L2\right) + \alpha_2 \sigma f\left(L2\right).
\end{aligned}
$$

The manufacturer's problem is formulated as:

$$
\begin{aligned}
\max_{T(x)} \Pi_M \; &= \; \rho E\left(mx - T(x) \mid a_H, \theta_H\right) + (1-\rho)\, E\left(mx - T(x) \mid a_L, \theta_L\right) \\
& \quad s.t. \\
(IR - i) \; &: \; \Pi_S^i \geq 0, \\
(LL - i) \; &: \; \beta_i \geq 0, \\
(IC - i) \; &: \; a_i = \alpha_i \theta_i \bar{F}\left(q_i - (a_i + d)\,\theta_i\right), \\
(FC - i) \; &: \; \alpha_i \leq m,
\end{aligned}
$$

where $i = H$ or L.

Substituting the agent's expected profit into the manufacturer's expected profit,

$$\begin{aligned}
\Pi_M &= \rho \{ m\theta_H (a_H + d) - \beta \\
&\quad + \alpha_1 H1 (F(H2) - F(H1)) + \alpha_1 \sigma (f(H2) - f(H1)) \\
&\quad - \{-\alpha_2 H2 + \alpha_1 (q_2 - q_1)\} \bar{F}(H2) - \alpha_2 \sigma f(H2) \Big\} \\
&\quad + (1-\rho) \{ m\theta_L (a_L + d) - \beta \\
&\quad + \alpha_1 L1 (F(L2) - F(L1)) + \alpha_1 \sigma (f(L2) - f(L1)) \\
&\quad - \{-\alpha_2 L2 + \alpha_1 (q_2 - q_1)\} \bar{F}(L2) - \alpha_2 \sigma f(L2) \Big\} .
\end{aligned}$$

4 Numerical Analysis

In this section, we conduct a numerical analysis to compare the menu of single-target contracts with the dual-target contract. We fix some parameter values, vary the rest and come out with 98 sets of data: the probability of the high type ρ takes the value from 0.2 to 0.8 in increments of 0.1; the value of the high type is $\theta_H = 3.0$, the value of the low type θ_L takes the value from 1.6 to 2.8 in increments of 0.2; the basic demand is $d = 2$; the manufacturer's profit margin is $m = 2$. For the random market demand noise ε, the mean is $\mu = 0$ and the standard deviation is $\sigma = 1$. For each set of data, we optimize the agent's effort level, and compute the manufacturer's expected profits under the menu of single-target contracts and the dual-target contract, respectively.

In Figure 1, we compare the manufacturer's expected profit under the two contractual forms. Figure 2 shows the ratio of the manufacturer's expected profit under the menu contract to that under the dual-target contract. Figures 1 and 2 indicate that, when the probability of the high type is small (i.e., $\rho < 0.4$), the performance of the dual-target contract is always below but nearly as good as that under the menu contract; the difference between the manufacturer's expected profits under these two contractual forms decreases as the low type value gets

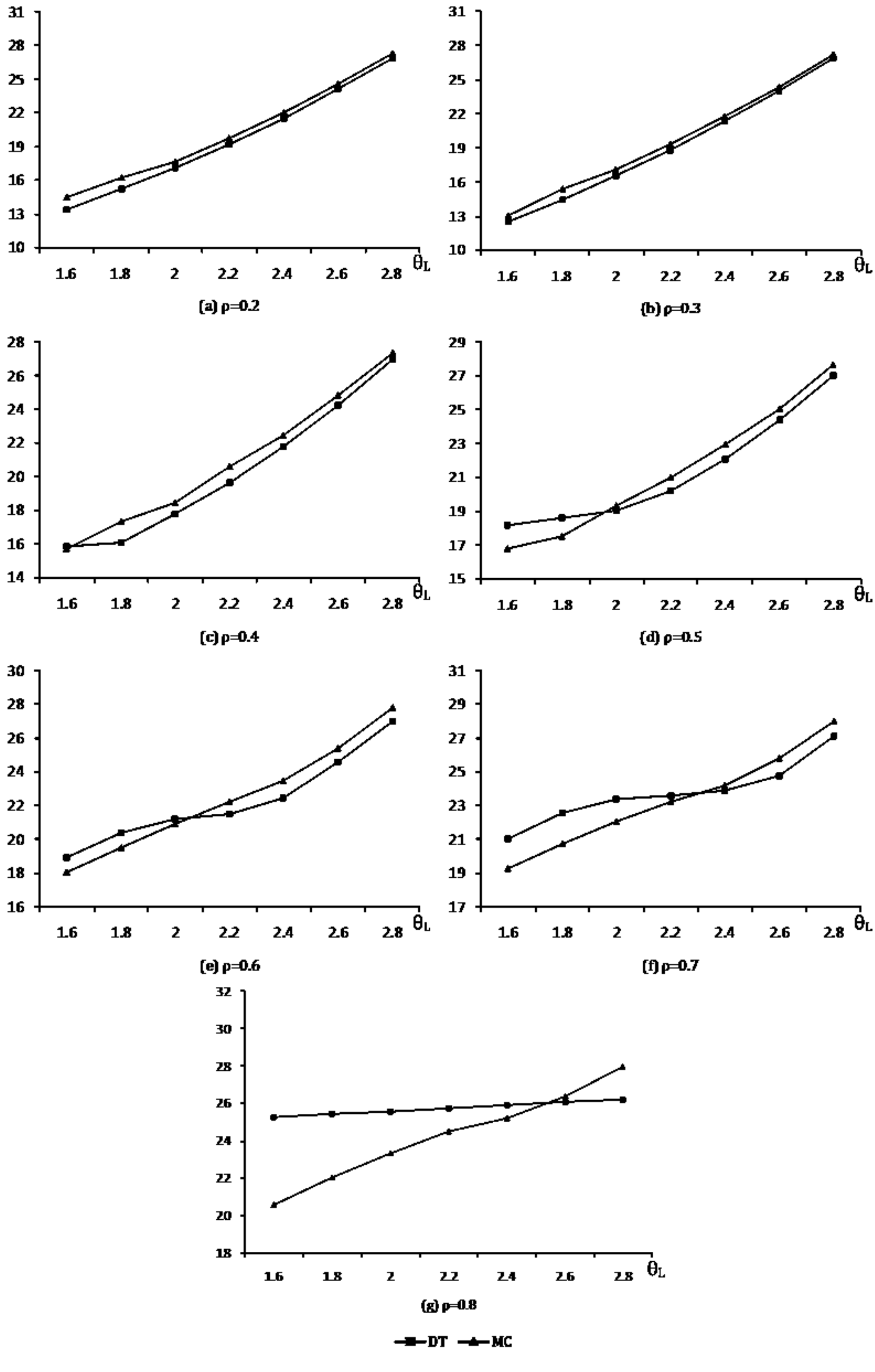

Figure 1: Comparison of the manufacturer's profits under the menu of single-target contracts and the dual-target contract.

closer to the high type value. For medium values of ρ (i.e., $0.4 \leq \rho \leq 0.7$), there exists a threshold of the low type value. When the low type value is below the threshold, the dual-target contract outperforms the menu contract; when the low type value is above the threshold, the dual-target contract underperforms but is nearly as good as the menu contract. Moreover, the difference between the manufacturer's expected profits under these two contractual forms changes with a trend similar to the small ρ scenarios. For large values of ρ (i.e., $\rho \geq 0.8$), again, there exists a threshold of the low type value. When the low type value is below the threshold, the dual-target contract can perform much better than the menu contract in terms of the manufacturer's expected profit, and the profit difference is decreasing as the low type value approaches the high type value. When the low type value is above the threshold, the dual-target contract is worse than the menu contract and the performance difference increases with the low type value.

In general, the dual-target contract performs as good as or even better than the menu contract except for the case when the probability of the high type is very large and the high and low type values are very close, which can be shown in Figure 3. In Figure 3, above the curve, the dual-target contract outperforms the menu of single-target contracts; and below the curve, the menu of single-target contracts outperforms the dual-target contract.

To explore the difference between the two contractual forms and to explain the above phenomena, we investigate each one of the three cases (small/medium/high values of ρ). We plot the manufacturer's expected profit, the expected sales and the high-type agent's expected profit under each case in Figures 4, 5, 6, and 7. The reason why we don't plot the low-type agent's expected profit is that the low-type agent's expected profit is always zero for any ρ value. This result verifies the aforementioned assumption that the low-type agent's (IR) constraint should always be binding.

Recall that in Figure 1, under the cases of $\rho = 0.2$ and $\rho = 0.3$, the menu con-

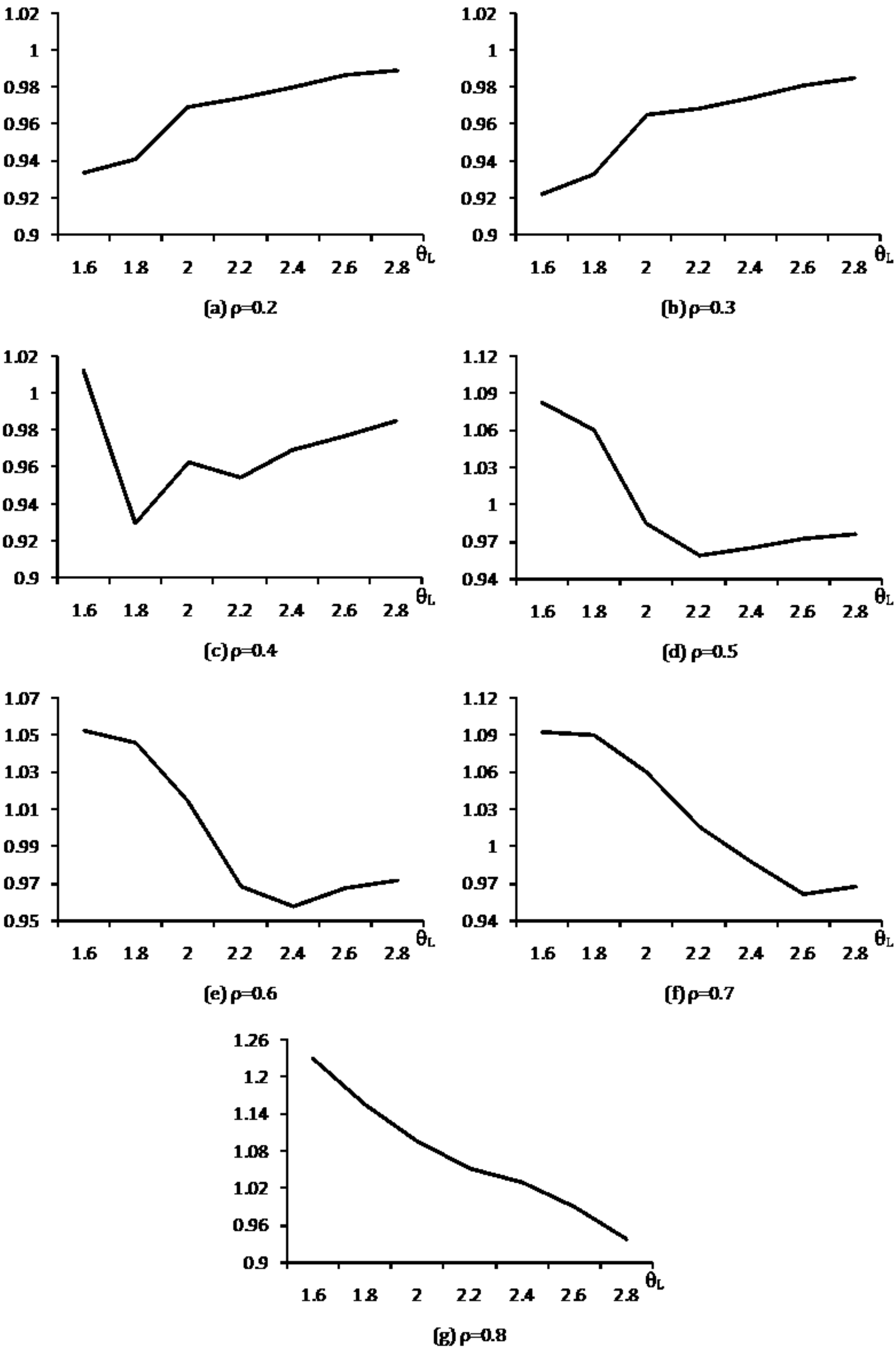

Figure 2: Ratio of the manufacturer's profit under the menu of single-target contracts to that under the dual-target contract.

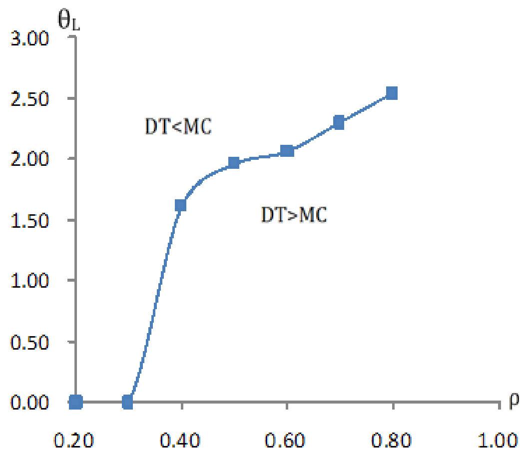

Figure 3: The performance of the dual-target contract versus the menu of single-target contracts.

tract always outperforms the dual-target contract in terms of the manufacturer's expected profit. In Figure 4(b) and Figure 5(b), we find that the expected sales of the dual-target contract are greater than that of the menu contract, except for the case when the low type value θ_L is sufficiently close to the high type value θ_H. Although intuition may indicate that larger sales would result in a larger profit to the manufacturer, Figure 4(c) and Figure 5(c) show that the manufacturer has to pay the high-type agent a larger profit under the dual-target contract than that under the menu contract, and thus gains a lower profit under the dual-target contract.

In Figure 6, under circumstances in which the probability of the high type equals to that of the low type ($\rho = 0.5$), when the low type value θ_L is much lower than the high type value θ_H, it's in the manufacturer's interest to provide no incentive to the low-type agent and a strong incentive to the high-type under the dual-target contract. The contract parameters are chosen to ensure that the high-type agent earns a zero profit, i.e., the high-type agent's (IR) constraint is binding; see Table H in Appendix B. Apparently it's wise for the manufacturer to do so: the performance of the dual-target contract is better than the menu contract in terms of the manufacturer's expected profit when the difference between

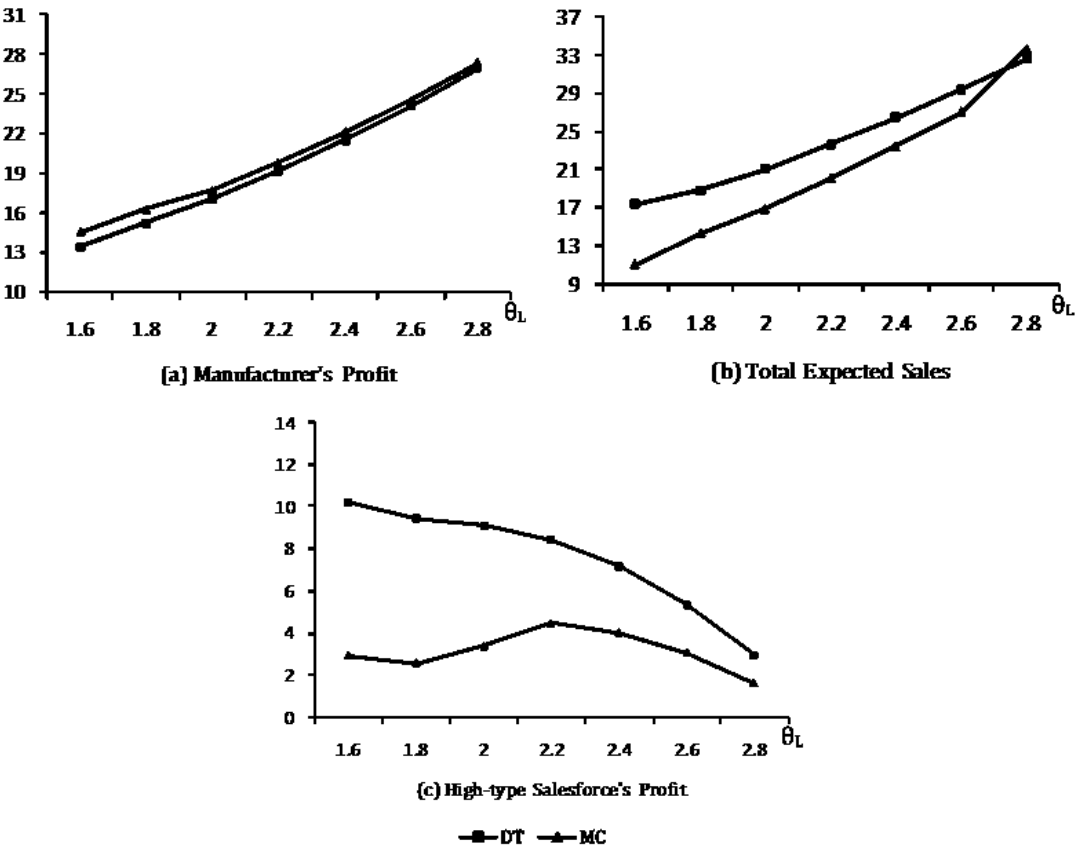

Figure 4: Comparison between the menu of single-target contracts and the dual-target contract for the case of $\rho = 0.2$.

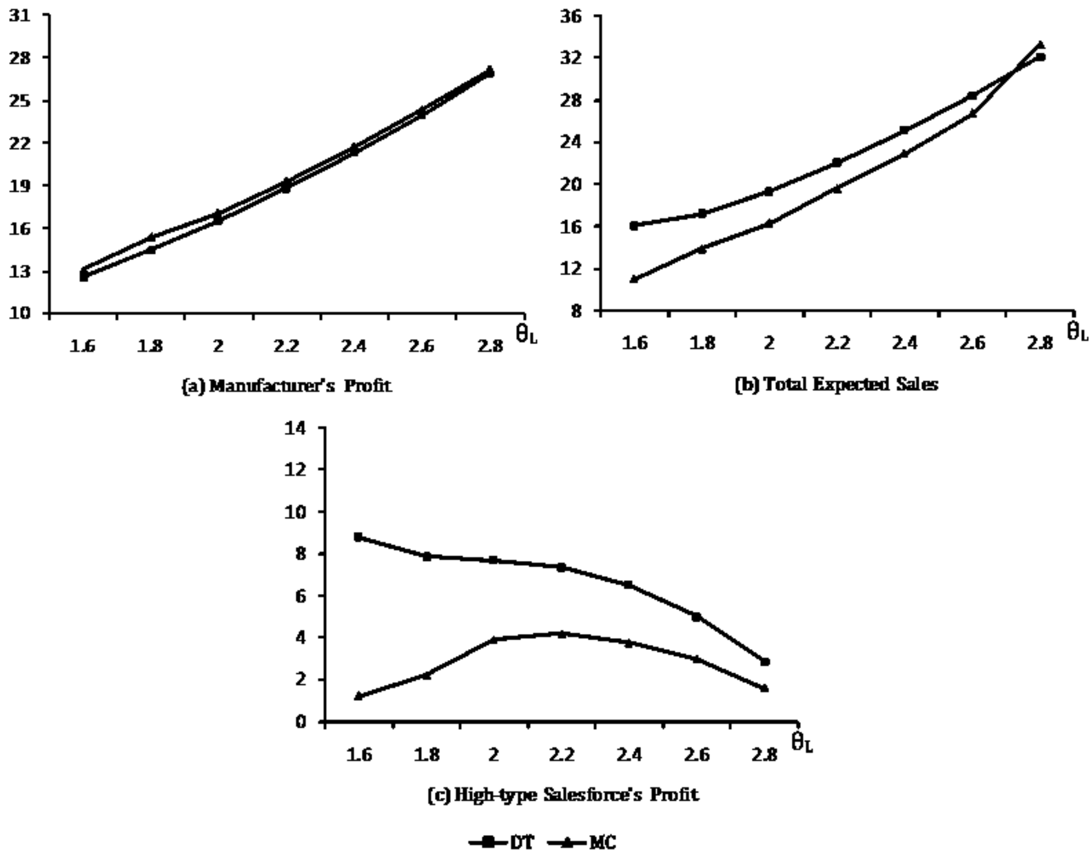

Figure 5: Comparison between the menu of single-target contracts and the dual-target contract for the case of $\rho = 0.3$.

the values of the two types is large, although this difference decreases with the low type value. Similar to the case of low values of ρ, a threshold for the low type value θ_L exists: the relative performance of the two contractual forms reverses after the low type value θ_L exceeds the threshold, and the exceeding amount of the manufacturer's expected profit under the menu contract over that under the dual-target contract is decreasing when the low type value θ_L gets closer to the high type one.

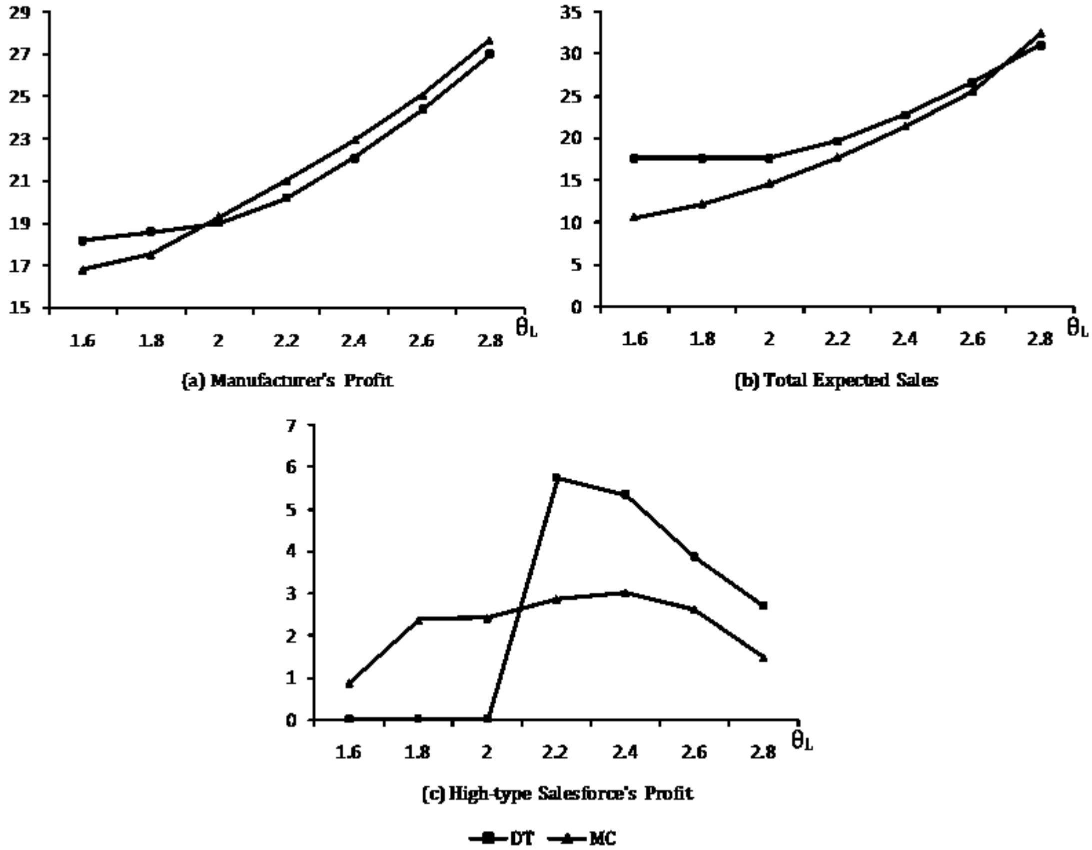

Figure 6: Comparison between the menu of single-target contracts and the dual-target contract for the case of $\rho = 0.5$.

In Figure 7, under circumstances in which the probability of the high type is much larger than that of the low type ($\rho = 0.8$). Again, there exists a threshold: in terms of the manufacturer's expected profit, the dual-target contract performs much better when the low type value is much lower than the high type one but the performance difference of the two types of contracts decreases as the low type value gets closer to the threshold; and after the low type value exceeds the threshold, the menu contract outperforms the dual-target contract and the performance difference increases with the low type value. Under the dual-target contract, no incentive is provided for the low-type agent and the optimal dual-target contract is solely designed for the high-type agent. According to Table K, the dual-target contract's parameters are chosen to make the high-type agent's (IR) constraint binding. As inducing the agent's selling effort is costly, and the expected sales generated by the low-type agent is considerably low relative to her payment; it's better to provide no incentive to the low-type agent when the low type value is much lower than the high type value.

Unlike the dual-target contract in which it is optimal for the manufacturer to provide no incentive to the low-type agent, the menu contract always provides incentive for both types of agent. Furthermore, both the manufacturer's expected profit and the expected sales grow with the decreasing difference between the two type values. As the menu contract outperforms the dual-target contract after the low type value exceeds the threshold, it is no longer appropriate to offer the low-type agent zero incentive.

In Figure 8, we plot the expected sales of each type of agent and the corresponding optimal quotas under the menu contract. The dashed lines represent the quotas set in the contract for each type and the solid lines represent the expected sales. In general, the optimal quota for each type is set below the expected sales of that type, and the difference between the expected sales and the sales quotas increases when the low type value θ_L gets close to the high type value θ_H. When θ_L is sufficiently close to θ_H and the probability of the high type takes small to medium values ($0.2 \leq \rho < 0.7$), the optimal quotas of both types are set below the low-type agent's expected sales. As shown in Tables E and F in Appendix B, both the high-type agent's and the low-type agent's effort levels are increasing in the low type value. This corresponds to a situation that when the difference be-

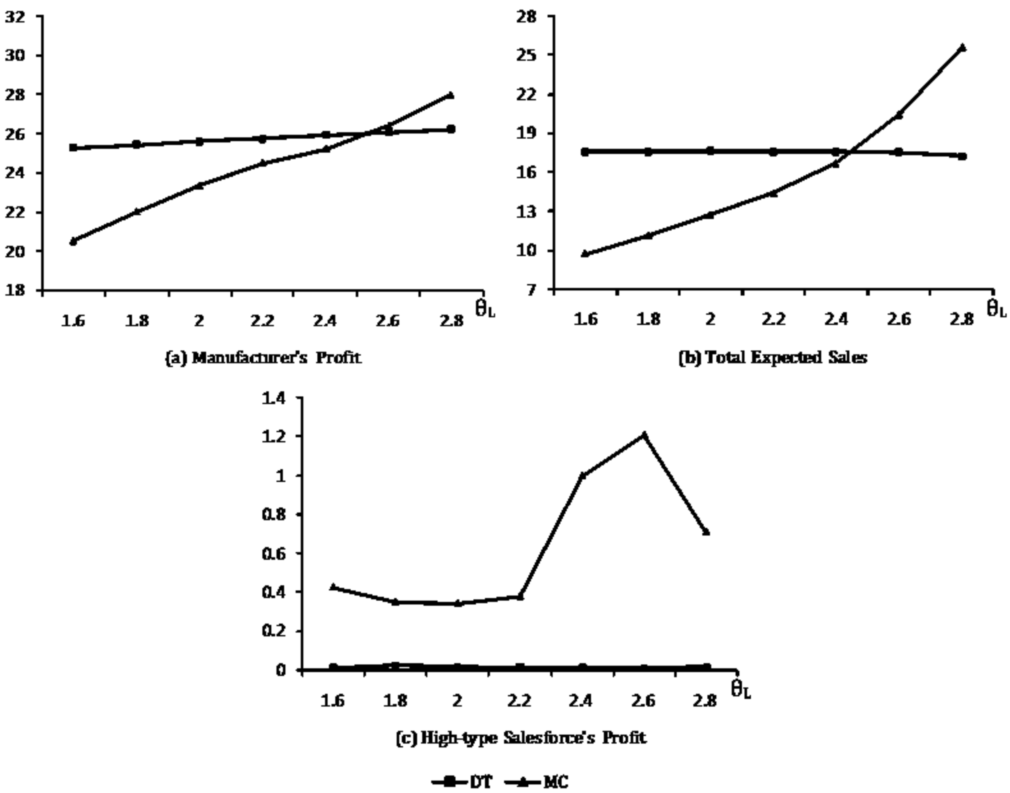

Figure 7: Comparison between the menu of single-target contracts and the dual-target contract for the case of $\rho = 0.8$.

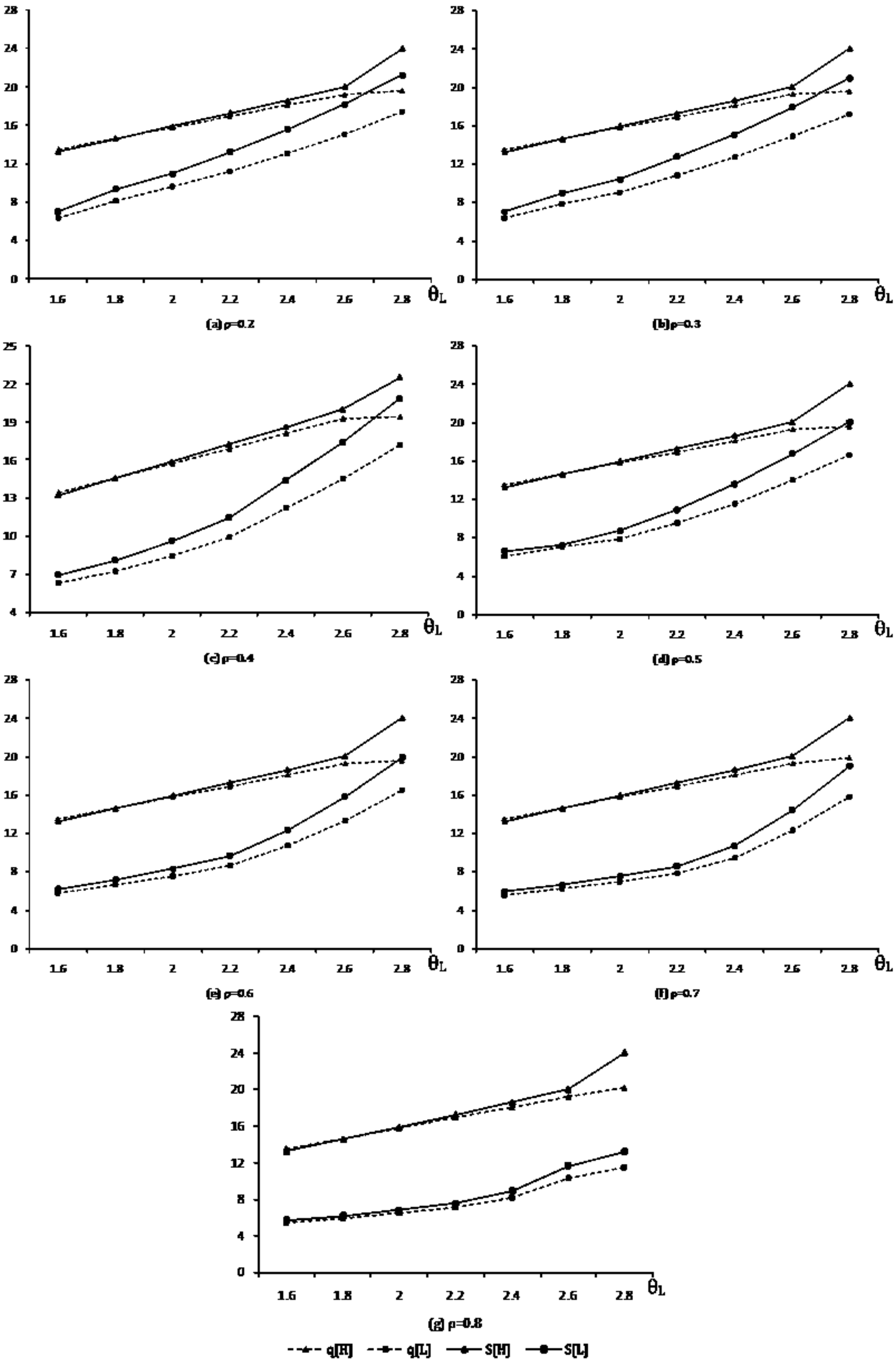

Figure 8: Comparison between the sales quota and the expected sales of each type of agent under the menu of single-target contracts.

tween the two type values is large, the large difference affects the performance of the menu contract in terms of the expected sales and the manufacturer's expected profit.

In Figure 9, we plot the optimal commission rate α_L for the low-type agent under the menu of single-target contracts. Note that in Figure 9, the optimal commission rate of the low-type contract under the menu contract displays a convex pattern that it decreases to a certain extent and increases afterwards, while the high-type commission rate is set at the maximum value restrained by the feasibility constraint (3).

In Figure 9(a), the optimal commission rate of the low type contract is set at the maximum value 2 when $\theta_L = 1.6$, $\rho = 0.2$. Recall that in Figure 8(a) (also see Table L in Appendix C), the expected sales are 0.25 below the low quota in this scenario, which means that it's quite possible ($> 50\%$) that the low-type agent will not receive commission part of the payment. So it's optimal for the manufacturer to combine a high commission and a high quota in designing the low-type contract to provide an incentive strong enough for the low-type agent.

Recall that in Figures 4 and 5, the high-type agent's expected profit displays a concave pattern in which it increases with the low type value until the low type value meets a threshold and decreases afterwards. The reason that the high-type agent's expected profit changes in the opposite way against that of the optimal low-type commission rate could be explained as follows: since the base salary for the low-type agent increases as the low type value increases (see Table A in Appendix A), it's in the manufacturer's interest to lower the commission rate to make the low-type agent's (IR) constraint binding. As the base salary of the low-type contract increases, the base salary of the high-type contract increases correspondingly. The high-type commission rate which is set at the maximum value boosts the high-type agent's expected profit before the low type value reaches the threshold. However, when the low type value exceeds the threshold, it is no

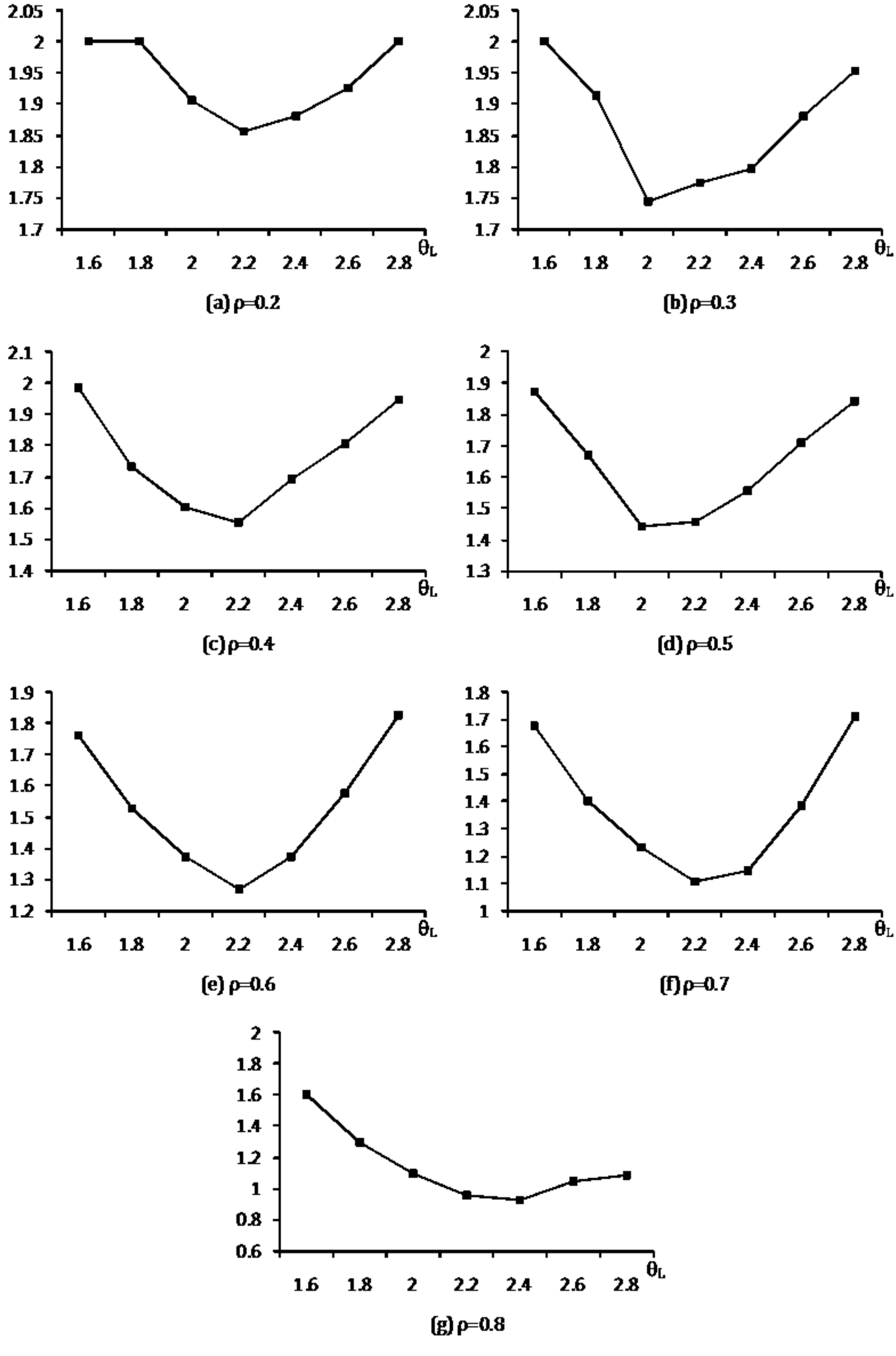

Figure 9: The optimal commission rate α_L for the low-type agent under the menu of single-target contracts.

longer capable for the manufacturer to provide an incentive strong enough for the low-type agent (although the $(IR-L)$ constraint is still binding) by only enlarging the low-type commission rate. Thereby, the manufacturer has to jointly increase the commission rate and the base salary of the low-type contract after the low-type value surpasses the threshold. This consideration of the manufacturer explains the concavity of the high-type agent's expected profit.

The above corresponds to a situation in which the probability of the high type ρ takes low values ($0.2 \leq \rho \leq 0.4$). The optimal base salaries of the high-type contract and the low-type contract are very close when the low type value θ_L is either much lower than or very close to the high type value θ_H (see Table A in Appendix A). However, when the low type value θ_L takes medium values (e.g., $\theta_L = 2.2$), the base salary of the high-type contract is much higher than that of the low-type contract, compared to the cases when the low type takes either very small or very large value. This indicates that when θ_L takes medium values, the high-type agent has a motivation to imitate the low-type agent and choose a low-type contract. In order to prevent this from happening, the manufacturer has to give the high-type agent a higher premium. Thus, the high-type agent can gain a higher profit.

Finally, in Figure 10, we plot the expected sales of each type of agent and the corresponding optimal quotas under the dual-target contract. The dashed lines represent the two quotas set in the contract and the solid lines represent the expected sales. Note that the optimal quotas under the dual-target contract are in the following way: when the probability of the high-type ρ is small (i.e., $\rho < 0.4$), the quotas are set such that the high-type agent's expected sales are between the low quota and the high quota but close to the high one; when ρ takes medium to high values (i.e., $0.5 \leq \rho < 0.8$), there exists a threshold for the low type value. When the difference between the two type values is large, the dual-target contract is solely designed for the high-type agent while providing no incentive to the low-type agent. Thereby, the optimal quota changes in a similar way with that of the high-type agent's expected sales. After the low type value exceeds the threshold, the manufacturer has to provide the low-type agent an incentive. Whenever the low-type agent's selling effort is induced, the low quota should be set below the low-type agent's expected sales within a 3σ range and far below the high quota. Hence, it is important to notice that although the high-type agent's expected sales are below the high quota q_2, the below part is no larger than 1σ. This indicates that there stands a good probability ($> 30\%$) for the high-type agent to reach, or even exceed q_2. When ρ takes high values (i.e., $\rho \geq 0.8$), the optimal dual-target contract solely provides incentive to the high-type agent, the high-type agent's expected sales are always between the low quota and the high quota but within a 3σ range of the high one. The low quota is set far below the high one. In extant publications, contract settings which consider deterministic sales outcome with varying type always come to the conclusion that the sales quota should be set exactly at the expected sales. In contrast, our result shows that due to the randomness in the sales outcome, setting the sales quota above the expected sales is possible. If the sales quota is set much higher than the expected sales, the agent will perceive it as unapproachable. However, it is possible for the agent to meet or exceed the sales quota under the optimal dual-target contract, although the sales quota is set closely above the expected sales.

In Figure 11, we plot the optimal commission rate α_1 for sales between the low and high quotas under the dual-target contract. The commission rate for sales above the high quota α_2 is set at the maximum value, and that for sales between the low and high quotas is smaller than the maximum value.

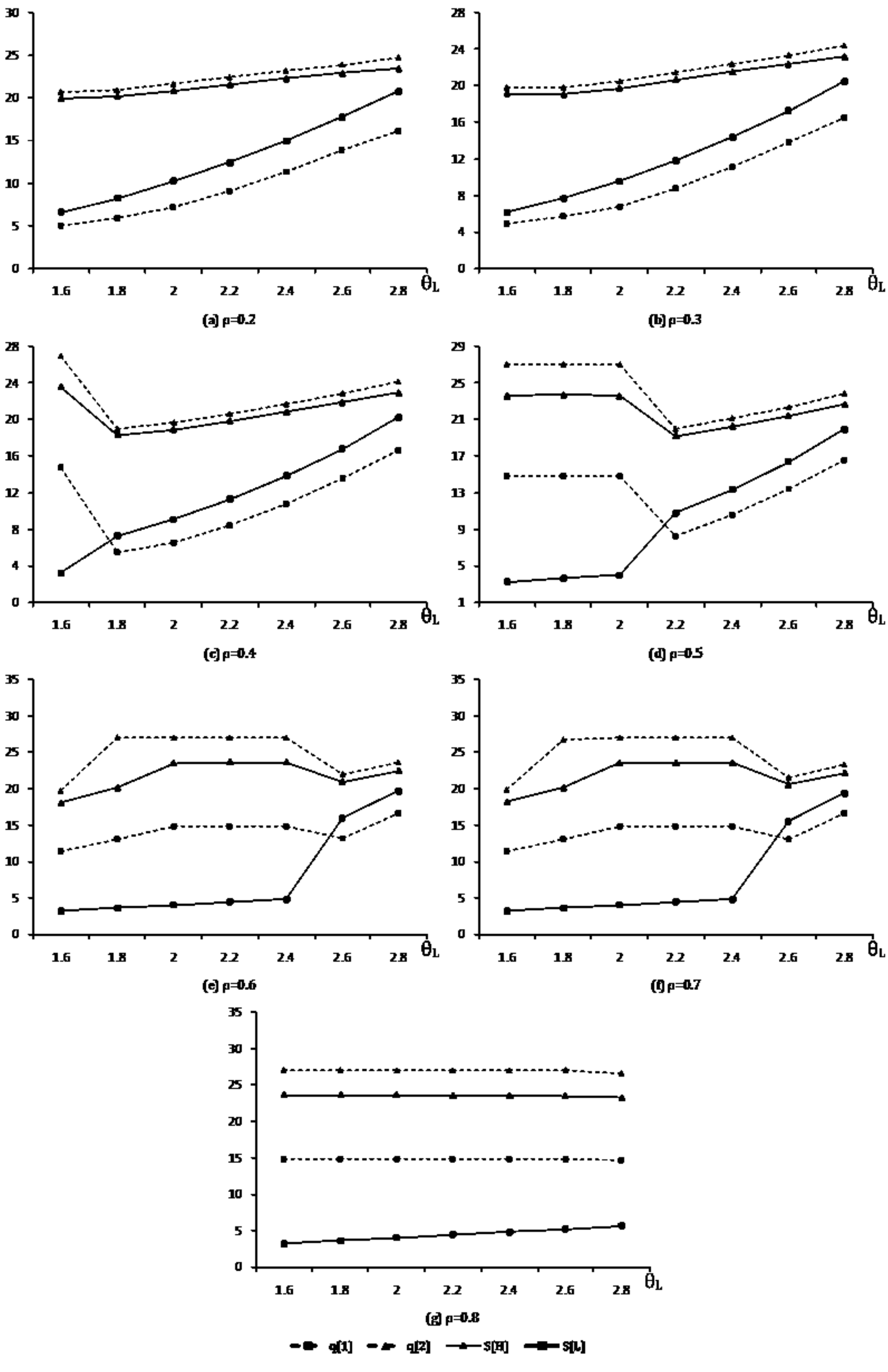

Figure 10: Comparison between the sales quotas and the expected sales of each type of agent under the dual-target contract.

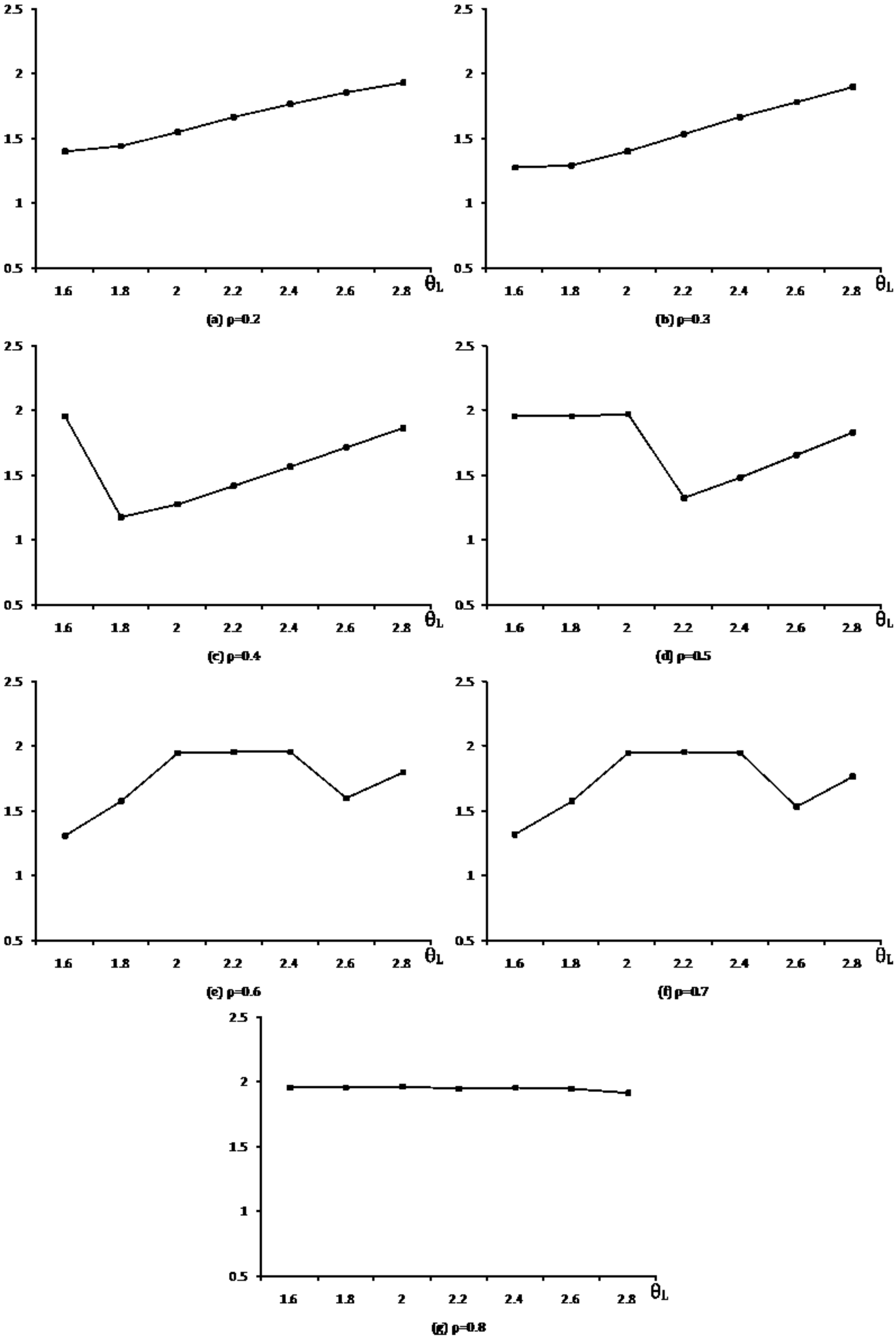

Figure 11: The optimal commission rate α_1 for the low-type agent under the dual-target contract.

5 Future Work

We can consider the model setting in which the realized sales consists of a deterministic part and a random part. The deterministic part is determined by the agent's type and her effort level in a multiplicative demand–effort relationship. Note that the multiplicative demand–effort relationship has been used by a number of publications in the literature such as Rao 1990, Petruzzi and Dada 1999, Agrawal and Seshadri 2000, Chen 2000, Chen 2005, Chen and Xiao 2009, and Chu and Lai 2013. For our future work, we can investigate the case of additive demand–effort relationship and find out if our major findings and insights based on the multiplicative demand–effort relationship can still hold.

In addition, we assume that the randomness in the realized sales follows a normal distribution. We can test the robustness of our major results by modeling the randomness with other distributions that have been used in the literature, including the gamma, compound Poisson, or uniform distribution.

6 Conclusion

In this thesis, we considered the problem where a manufacturer sells a product through his agent who has private information about her selling effort and the type, i.e., a problem involving both moral hazard and adverse selection. To explore the design of salesforce compensation schemes, we considered two types of contract: the multi-target contracts that are widely observed in practice and the menu of contracts that are commonly considered in the literature to address the adverse selection problem. For the adverse selection problem, we assumed the agent can be one of two types. We formulated the manufacturer's problem for the optimal contract parameters under a menu of single-target contracts and for those under a dual-target contract. Then, for each contract, we conducted an extensive numerical analysis to find out the optimal contract parameters, the optimal effort level and the maximum expected profit of each type of agent, and the manufacturer's optimal expected profit.

We compared the performances of the two contracts in terms of their effectiveness in generating a profit for the manufacturer, and our results generated a number of managerial implications about the manufacturer's optimal choice of a contract type. In general, neither contract strictly dominates over the other, the circumstances at which the conclusion can be drawn need to be specified. To be specific, the relative performance of the menu of single-target contracts and the dual-target contract is dependent on the probability of the high type and the difference between the low and high type values.

When the probability of the high type is low, the menu of single-target contracts outperforms the dual-target contract, but the dual-target contract is nearly as good as the menu contract.

When the probabilities of the high and low types are close, there exists a threshold for the low type value. If the low type value is below the threshold (i.e., the low type value is much lower compared with the high type value), the manufacturer will be better off from adopting the dual-target contract. This situation corresponds to a large uncertainty about the agent type. In this situation, the dual-target contract provides no incentive to the low-type agent. On the other hand, if the low type value is above the threshold, the agent type is less uncertain, the menu contract is preferred, but again the dual-target contract is nearly as good as the menu contract. Moreover, if the manufacturer adopts the dual-target contract, the contract should provide incentives to both types of agent.

When the probability of the high type is significantly larger than that of the low one, there exists a similar threshold for the low type value. The manufacturer will receive a larger revenue by applying the dual-target contract. In this situation, the dual-target contract provides no incentive to the low-type agent, even if the low type value is close to the high one. In the meantime, if the low type value exceeds the threshold, the menu contract outperforms the dual-target contract, and the surpassing amount of the manufacturer's expected profit under the menu contract over that under the dual-target contract is increasing.

Recall the first research question raised at the beginning of this thesis: How is the performance of the widely applied multi-target contract compared with the well-studied menu contract? We came to the conclusion that in general, the dual-target contract performs as good as or even better than the menu contract except for the scenario in which the probability of the high type is very large and the high/low type values are very close.

After going through the question which contract should be adopted, we explore the designing issue of the menu contract and the dual-target contract. The optimal low-type commission rate in the menu of single-target contracts displays a convex pattern. It decreases to a certain extent and increases afterwards with the optimal high-type commission rate set at the maximum value restrained by the feasibility constraint. This pattern indicates the manufacturer's joint consideration to make the low-type agent's individual rationality constraint binding and to share the least profit with the high-type agent. By jointly adjusting the base salary and the commission rate, the manufacturer's able to seize the maximum profit from the realized sales. For the menu of single-target contracts, in general, the quota for each type is set above the expected sales of that type, but the difference increases with the low type value, rendering that each type of agent only gets more than a half probability to reach or even surpass the quota.

For the dual-target contract, the commission rate for sales above the high quota is set at the maximum value, and that for sales between the low and high quotas is smaller than the maximum value. The quotas are set such that the expected sales of the high-type agent are above the low quota and closely below the high quota. When the low-type agent is induced an effort, her expected sales are above and close to the low quota. This indicates that whenever the low-type agent is retained in the game, she has a great probability to receive not only the base salary, but a small commission. In contrast to extant publications which considers deterministic sales outcome with respect to the varying type, our results indicate that setting sales quota above the expected sales is possible.

Appendices

A The Optimal Contract Parameters

A.1 Menu of Single-target Contracts

Table A: The optimal contract parameters of the menu of single-target contracts in the cases of ρ=0.2 to 0.5.

		Low Type			High Type		
ρ	θ_L	β_L	α_L	q_L	β_H	α_H	q_H
0.2	1.6	1.21	2.00	6.35	3.54	2.00	13.48
	1.8	2.55	2.00	8.13	5.88	2.00	14.62
	2.0	3.40	1.91	9.61	7.91	2.00	15.77
	2.2	4.22	1.86	11.17	10.36	2.00	16.93
	2.4	5.31	1.88	13.04	11.42	2.00	18.09
	2.6	6.52	1.93	15.09	12.20	2.00	19.25
	2.8	8.07	2.00	17.48	10.82	2.00	19.60
0.3	1.6	1.21	2.00	6.35	3.54	2.00	13.48
	1.8	2.18	1.91	7.85	5.51	2.00	14.62
	2.0	2.63	1.74	9.01	8.41	2.00	15.77
	2.2	3.81	1.77	10.85	10.03	2.00	16.93
	2.4	4.83	1.80	12.67	11.16	2.00	18.09
	2.6	6.22	1.88	14.87	12.10	2.00	19.25
	2.8	7.71	1.95	17.21	10.78	2.00	19.60
0.4	1.6	1.16	1.99	6.31	3.50	2.00	13.48
	1.8	1.46	1.73	7.25	4.80	2.00	14.62
	2.0	1.99	1.60	8.46	7.71	2.00	15.77
	2.2	2.71	1.55	9.91	8.57	2.00	16.93
	2.4	4.25	1.69	12.19	10.84	2.00	18.09
	2.6	5.75	1.81	14.49	11.95	2.00	19.25
	2.8	7.68	1.95	17.18	12.61	2.00	20.42
0.5	1.6	0.86	1.87	6.03	3.20	2.00	13.48
	1.8	1.05	2.00	7.05	5.70	2.00	14.62
	2.0	1.32	1.44	7.80	6.92	2.00	15.77
	2.2	2.25	1.46	9.49	8.74	2.00	16.93
	2.4	3.53	1.56	11.55	10.42	2.00	18.09
	2.6	5.16	1.71	14.00	11.75	2.00	19.25
	2.8	6.87	1.84	16.55	10.68	2.00	19.60

Table B: The optimal contract parameters of the menu of single-target contracts in the cases of ρ=0.6 to 0.8.

		Low Type			High Type		
ρ	θ_L	β_L	α_L	q_L	β_H	α_H	q_H
0.6	1.6	0.61	1.76	5.77	2.97	2.00	13.48
	1.8	0.80	1.53	6.58	4.14	2.00	14.62
	2.0	1.07	1.37	7.52	5.58	2.00	15.77
	2.2	1.39	1.27	8.57	7.26	2.00	16.93
	2.4	2.60	1.37	10.66	9.85	2.00	18.09
	2.6	4.38	1.58	13.31	11.48	2.00	19.25
	2.8	6.76	1.83	16.46	10.67	2.00	19.60
0.7	1.6	0.44	1.67	5.57	2.83	2.00	13.48
	1.8	0.50	1.40	6.19	3.85	2.00	14.62
	2.0	0.63	1.23	6.95	5.14	2.00	15.77
	2.2	0.79	1.11	7.79	6.66	2.00	16.93
	2.4	1.53	1.15	9.44	9.11	2.00	18.09
	2.6	3.33	1.39	12.29	11.09	2.00	19.25
	2.8	5.95	1.71	15.78	11.01	2.00	19.82
0.8	1.6	0.33	1.60	5.41	2.75	2.00	13.48
	1.8	0.29	1.29	5.88	3.68	2.00	14.62
	2.0	0.32	1.09	6.44	4.85	2.00	15.77
	2.2	0.37	0.96	7.07	6.25	2.00	16.93
	2.4	0.65	0.92	8.16	8.40	2.00	18.09
	2.6	1.60	1.04	10.27	10.34	2.00	19.25
	2.8	1.94	1.01	11.49	11.10	2.00	20.20

A.2 Dual-target Contract

Table C: The optimal contract parameters of the dual-target contract under the case from ρ=0.2 to 0.5.

			Low Type		High Type	
ρ	θ_L	β	α_1	q_1	α_2	q_2
0.2	1.6	0.00	1.40	5.01	2.00	20.62
	1.8	0.00	1.44	5.94	2.00	20.89
	2.0	0.00	1.55	7.10	2.00	21.60
	2.2	1.00	1.66	9.02	2.00	22.40
	2.4	2.60	1.76	11.35	2.00	23.15
	2.6	4.48	1.85	13.88	2.00	23.88
	2.8	5.65	1.93	16.10	2.00	24.72
0.3	1.6	0.00	1.28	4.87	2.00	19.80
	1.8	0.00	1.29	5.70	2.00	19.83
	2.0	0.00	1.40	6.79	2.00	20.53
	2.2	0.92	1.53	8.71	2.00	21.47
	2.4	2.51	1.66	11.09	2.00	22.41
	2.6	4.53	1.78	13.77	2.00	23.36
	2.8	6.68	1.90	16.56	2.00	24.40
0.4	1.6	0.00	1.96	14.80	2.00	27.00
	1.8	0.00	1.18	5.52	2.00	19.04
	2.0	0.00	1.28	6.55	2.00	19.68
	2.2	0.83	1.42	8.43	2.00	20.67
	2.4	2.35	1.57	10.82	2.00	21.74
	2.6	4.43	1.72	13.59	2.00	22.86
	2.8	6.93	1.87	16.63	2.00	24.11
0.5	1.6	0.00	1.96	14.80	2.00	27.00
	1.8	0.00	1.96	14.80	2.00	27.00
	2.0	0.00	1.97	14.86	2.00	27.00
	2.2	0.77	1.32	8.18	2.00	19.99
	2.4	2.20	1.49	10.56	2.00	21.13
	2.6	4.30	1.66	13.39	2.00	22.38
	2.8	7.04	1.83	16.63	2.00	23.83

Table D: The optimal contract parameters of the dual-target contract under the case from ρ=0.6 to 0.8.

			Low Type		High Type	
ρ	θ_L	β	α_1	q_1	α_2	q_2
0.6	1.6	0.00	1.31	11.32	2.00	19.73
	1.8	0.00	1.57	13.08	2.00	27.00
	2.0	0.00	1.95	14.75	2.00	27.00
	2.2	0.00	1.96	14.80	2.00	27.00
	2.4	0.00	1.96	14.79	2.00	27.00
	2.6	4.15	1.59	13.19	2.00	21.93
	2.8	7.12	1.80	16.61	2.00	23.56
0.7	1.6	0.00	1.32	11.32	2.00	19.81
	1.8	0.00	1.57	13.08	2.00	26.77
	2.0	0.00	1.95	14.77	2.00	27.00
	2.2	0.00	1.95	14.77	2.00	27.00
	2.4	0.00	1.95	14.78	2.00	27.00
	2.6	4.01	1.54	13.01	2.00	21.49
	2.8	7.19	1.76	16.59	2.00	23.27
0.8	1.6	0.00	1.95	14.79	2.00	27.00
	1.8	0.00	1.96	14.79	2.00	27.00
	2.0	0.00	1.96	14.82	2.00	27.00
	2.2	0.00	1.95	14.77	2.00	27.00
	2.4	0.00	1.95	14.78	2.00	27.00
	2.6	4.15	1.95	14.75	2.00	27.00
	2.8	7.12	1.91	14.60	2.00	26.56

B Performance Comparison between the Menu of Single-target Contracts and the Dual-target Contract

Table E: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.2.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	2.37	2.41	11.02	0.00	2.90	13.41
	1.8	3.17	2.85	14.27	0.00	2.55	15.37
	2.0	3.49	3.30	16.88	0.00	3.40	17.07
	2.2	3.99	3.75	20.02	0.00	4.49	19.31
	2.4	4.49	4.20	23.39	0.00	4.02	21.73
	2.6	5.00	4.67	27.03	0.00	3.07	24.33
	2.8	5.60	6.00	33.68	0.00	1.61	27.18
Dual-target Contract	1.6	2.11	4.65	17.31	0.00	10.19	12.52
	1.8	2.57	4.72	18.79	0.00	9.46	14.46
	2.0	3.09	4.94	21.01	0.00	9.10	16.54
	2.2	3.65	5.19	23.61	0.00	8.42	18.81
	2.4	4.23	5.42	26.42	0.00	7.17	21.30
	2.6	4.82	5.63	29.42	0.00	5.34	24.00
	2.8	5.41	5.82	32.61	0.00	2.94	26.90

Table F: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.3.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	2.37	2.41	11.02	0.00	1.21	14.53
	1.8	2.96	2.85	13.88	0.00	2.18	16.29
	2.0	3.19	3.30	16.27	0.00	3.90	17.70
	2.2	3.79	3.75	19.58	0.00	4.16	19.80
	2.4	4.28	4.20	22.88	0.00	3.76	22.08
	2.6	4.88	4.67	26.72	0.00	2.97	24.55
	2.8	5.47	6.00	33.31	0.00	1.57	27.34
Dual-target Contract	1.6	1.84	4.37	16.06	0.00	8.78	13.39
	1.8	2.27	4.37	17.18	0.00	7.85	15.20
	2.0	2.79	4.58	19.32	0.00	7.67	17.08
	2.2	3.36	4.88	22.05	0.00	7.34	19.17
	2.4	3.99	5.18	25.10	0.00	6.48	21.51
	2.6	4.64	5.46	28.44	0.00	5.01	24.09
	2.8	5.32	5.73	32.08	0.00	2.86	26.92

Table G: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.4.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	2.34	2.41	11.02	0.00	1.17	15.64
	1.8	2.50	2.85	13.88	0.00	1.56	17.28
	2.0	2.81	3.30	16.27	0.00	3.21	18.43
	2.2	3.21	3.75	19.58	0.00	2.71	20.57
	2.4	4.01	4.20	22.88	0.00	3.43	22.47
	2.6	4.69	4.67	26.72	0.00	2.82	24.79
	2.8	5.46	5.17	33.31	0.00	1.57	27.36
Dual-target Contract	1.6	0.00	5.87	10.97	0.00	0.01	15.83
	1.8	2.03	4.10	13.06	0.00	6.74	16.05
	2.0	2.54	4.30	15.51	0.00	6.59	17.74
	2.2	3.12	4.62	18.31	0.00	6.47	19.62
	2.4	3.76	4.95	22.23	0.00	5.89	21.78
	2.6	4.47	5.30	26.22	0.00	4.70	24.21
	2.8	5.22	5.65	30.78	0.00	2.77	26.95

Table H: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.5.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	2.09	2.41	10.57	0.00	0.87	16.79
	1.8	1.99	2.85	12.15	0.00	2.37	17.53
	2.0	2.35	3.30	14.60	0.00	2.41	19.29
	2.2	2.95	3.75	17.73	0.00	2.87	21.01
	2.4	3.65	4.20	21.37	0.00	3.01	22.92
	2.6	4.43	4.67	25.55	0.00	2.62	25.06
	2.8	5.15	6.00	32.43	0.00	1.47	27.65
Dual-target Contract	1.6	0.00	5.87	17.60	0.00	0.01	18.19
	1.8	0.00	5.87	17.60	0.00	0.02	18.59
	2.0	0.00	5.87	17.60	0.00	0.02	18.99
	2.2	2.90	4.39	19.55	0.00	5.75	20.15
	2.4	3.55	4.75	22.79	0.00	5.37	22.10
	2.6	4.30	5.14	26.60	0.00	3.88	24.37
	2.8	5.13	5.56	31.03	0.00	2.69	26.99

Table I: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.6.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	1.86	2.41	10.20	0.00	0.64	18.00
	1.8	1.98	2.85	12.12	0.00	0.81	19.49
	2.0	2.16	3.30	14.21	0.00	1.07	20.89
	2.2	2.37	3.75	16.47	0.00	1.39	22.21
	2.4	3.14	4.20	20.15	0.00	2.44	23.47
	2.6	4.08	4.67	24.62	0.00	2.35	25.38
	2.8	5.11	6.00	32.31	0.00	1.46	27.83
Dual-target Contract	1.6	0.00	4.02	12.07	0.00	0.75	18.94
	1.8	0.00	4.72	14.16	0.00	0.00	20.39
	2.0	0.00	5.84	17.51	0.00	0.01	21.19
	2.2	0.00	5.87	17.60	0.00	0.01	21.51
	2.4	0.00	5.87	17.60	0.00	0.01	22.48
	2.6	4.13	4.99	25.73	0.00	4.14	24.55
	2.8	5.03	5.47	30.50	0.00	2.60	27.04

Table J: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.7.

	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	1.69	2.41	9.92	0.00	0.50	19.25
	1.8	1.67	2.85	11.57	0.00	0.52	20.72
	2.0	1.76	3.30	13.41	0.00	0.63	22.05
	2.2	1.88	3.75	15.38	0.00	0.79	23.26
	2.4	2.45	4.20	18.48	0.00	1.70	24.19
	2.6	3.55	4.67	23.24	0.00	1.96	25.78
	2.8	4.78	6.00	31.40	0.00	1.36	28.01
Dual-target Contract	1.6	0.00	4.06	12.18	0.00	0.81	21.03
	1.8	0.00	4.72	14.16	0.00	0.41	22.59
	2.0	0.00	5.85	17.56	0.00	0.01	23.39
	2.2	0.00	5.85	17.56	0.00	0.04	23.63
	2.4	0.00	5.85	17.56	0.00	0.01	23.87
	2.6	3.97	4.85	24.87	0.00	3.88	24.77
	2.8	4.93	5.39	29.95	0.00	2.52	27.10

Table K: Comparison of the performance between the menu of single-target contracts and the dual-target contract under the case of ρ=0.8.

D	θ_L	a_L	a_H	Expected Sales	Π_S^L	Π_S^H	Π_M
Menu of Single-target Contracts	1.6	1.55	2.41	9.70	0.00	0.42	20.53
	1.8	1.43	2.85	11.13	0.00	0.35	22.02
	2.0	1.40	3.30	12.69	0.00	0.34	23.33
	2.2	1.42	3.75	14.37	0.00	0.38	24.48
	2.4	1.70	4.20	16.70	0.00	1.00	25.18
	2.6	2.48	4.67	20.47	0.00	1.21	26.37
	2.8	2.71	6.00	25.58	0.00	0.71	27.97
Dual-target Contract	1.6	0.00	5.86	17.59	0.00	0.01	25.26
	1.8	0.00	5.87	17.60	0.00	0.02	25.42
	2.0	0.00	5.88	17.65	0.00	0.01	25.58
	2.2	0.00	5.85	17.55	0.00	0.01	25.74
	2.4	0.00	5.86	17.57	0.00	0.01	25.90
	2.6	0.00	5.84	17.51	0.00	0.01	26.06
	2.8	0.00	5.74	17.22	0.00	0.01	26.20

C The Difference between Expected Sales and Sales Quotas

C.1 Menu of Single-target Contracts

Table L: Comparison between the sales quota and the expected sales of each type of agent under the menu of single-target contracts in the cases of ρ=0.2 to 0.5.

	θ_L	q_H	q_L	$S_H - q_H$	$S_L - q_L$
$\rho = 0.2$	1.6	13.48	6.35	-0.25	0.65
	1.8	14.62	8.13	-0.06	1.18
	2	15.77	9.61	0.12	1.38
	2.2	16.93	11.17	0.32	2.01
	2.4	18.09	13.04	0.53	2.53
	2.6	19.25	15.09	0.77	3.11
	2.8	19.60	17.48	4.40	3.80
$\rho = 0.3$	1.6	13.48	6.35	-0.25	0.65
	1.8	14.62	7.85	-0.06	1.08
	2	15.77	9.01	0.12	1.36
	2.2	16.93	10.85	0.32	1.89
	2.4	18.09	12.67	0.53	2.40
	2.6	19.25	14.87	0.77	3.03
	2.8	19.60	17.21	4.40	3.71
$\rho = 0.4$	1.6	13.48	6.31	-0.25	0.63
	1.8	14.62	7.25	-0.06	0.85
	2	15.77	8.46	0.12	1.16
	2.2	16.93	9.91	0.32	1.55
	2.4	18.09	12.19	0.53	2.23
	2.6	19.25	14.49	0.77	2.90
	2.8	20.42	17.18	1.09	3.70
$\rho = 0.5$	1.6	13.48	6.03	-0.25	0.52
	1.8	14.62	7.05	-0.06	0.14
	2	15.77	7.80	0.12	0.90
	2.2	16.93	9.49	0.32	1.39
	2.4	18.09	11.55	0.53	2.00
	2.6	19.25	14.00	0.77	2.73
	2.8	19.60	16.55	4.40	3.48

Table M: Comparison between the sales quota and the expected sales of each type of agent under the menu of single-target contracts in the cases of ρ=0.6 to 0.8.

	θ_L	q_H	q_L	$S_H - q_H$	$S_L - q_L$
	1.6	13.48	5.77	-0.25	0.41
	1.8	14.62	6.58	-0.06	0.58
	2	15.77	7.52	0.12	0.80
$\rho = 0.6$	2.2	16.93	8.57	0.32	1.05
	2.4	18.09	10.66	0.53	1.68
	2.6	19.25	13.31	0.77	2.49
	2.8	19.60	16.46	4.40	3.45
	1.6	13.48	5.57	-0.25	0.33
	1.8	14.62	6.19	-0.06	0.42
	2	15.77	6.95	0.12	0.57
$\rho = 0.7$	2.2	16.93	7.79	0.32	0.74
	2.4	18.09	9.44	0.53	1.23
	2.6	19.25	12.29	0.77	2.13
	2.8	19.82	15.78	4.18	3.22
	1.6	13.48	5.41	-0.25	0.26
	1.8	14.62	5.88	-0.06	0.29
	2	15.77	6.44	0.12	0.36
$\rho = 0.8$	2.2	16.93	7.07	0.32	0.45
	2.4	18.09	8.16	0.53	0.73
	2.6	19.25	10.27	0.77	1.38
	2.8	20.20	11.49	3.80	1.69

C.2 Dual-target Contract

Table N: Comparison between the sales quotas and the expected sales of each type of agent under the dual-target contract in the cases of ρ=0.2 to 0.5.

	θ_L	q_1	q_2	$S_H - q_1$	$S_H - q_2$	$S_L - q_1$	$S_L - q_2$
$\rho = 0.2$	1.6	5.01	20.62	14.93	-0.68	1.56	-14.05
	1.8	5.94	20.89	14.23	-0.72	2.28	-12.67
	2	7.10	21.60	13.72	-0.78	3.09	-11.41
	2.2	9.02	22.40	12.55	-0.83	3.42	-9.96
	2.4	11.35	23.15	10.91	-0.88	3.60	-8.20
	2.6	13.88	23.88	9.01	-0.99	3.84	-6.15
	2.8	16.10	24.72	7.36	-1.26	4.65	-3.96
$\rho = 0.3$	1.6	4.87	19.80	14.25	-0.68	1.28	-13.66
	1.8	5.70	19.83	13.41	-0.73	1.98	-12.15
	2	6.79	20.53	12.96	-0.78	2.78	-10.96
	2.2	8.71	21.47	11.94	-0.82	3.09	-9.67
	2.4	11.09	22.41	10.44	-0.88	3.27	-8.04
	2.6	13.77	23.36	8.61	-0.97	3.49	-6.09
	2.8	16.56	24.40	6.63	-1.20	3.93	-3.91
$\rho = 0.4$	1.6	14.80	27.00	8.81	-3.40	-11.60	-23.80
	1.8	5.52	19.04	12.79	-0.73	1.73	-11.79
	2	6.55	19.68	12.34	-0.78	2.52	-10.61
	2.2	8.43	20.67	11.43	-0.82	2.83	-9.42
	2.4	10.82	21.74	10.05	-0.87	3.01	-7.91
	2.6	13.59	22.86	8.30	-0.96	3.22	-6.05
	2.8	16.63	24.11	6.31	-1.18	3.60	-3.89
$\rho = 0.5$	1.6	14.80	27.00	8.81	-3.40	-11.60	-23.80
	1.8	14.80	27.00	8.81	-3.40	-11.20	-23.40
	2	14.86	27.00	8.75	-3.40	-10.86	-23.00
	2.2	8.18	19.99	10.98	-0.82	2.59	-9.21
	2.4	10.56	21.13	9.70	-0.87	2.78	-7.80
	2.6	13.39	22.38	8.04	-0.95	2.98	-6.01
	2.8	16.63	23.83	6.05	-1.16	3.33	-3.88

Table O: Comparison between the sales quotas and the expected sales of each type of agent under the dual-target contract in the cases of ρ=0.6 to 0.8.

	θ_L	q_1	q_2	$S_H - q_1$	$S_H - q_2$	$S_L - q_1$	$S_L - q_2$
$\rho = 0.6$	1.6	11.32	19.73	6.75	-1.66	-8.12	-16.53
	1.8	13.08	27.00	7.08	-6.84	-9.48	-23.40
	2	14.75	27.00	8.76	-3.49	-10.75	-23.00
	2.2	14.80	27.00	8.81	-3.40	-10.40	-22.60
	2.4	14.79	27.00	8.81	-3.40	-9.99	-22.20
	2.6	13.19	21.93	7.79	-0.95	2.75	-5.98
	2.8	16.61	23.56	5.81	-1.14	3.08	-3.87
$\rho = 0.7$	1.6	11.32	19.81	6.86	-1.63	-8.12	-16.61
	1.8	13.08	26.77	7.08	-6.61	-9.48	-23.17
	2	14.77	27.00	8.79	-3.44	-10.77	-23.00
	2.2	14.77	27.00	8.79	-3.44	-10.37	-22.60
	2.4	14.78	27.00	8.79	-3.44	-9.98	-22.20
	2.6	13.01	21.49	7.54	-0.94	2.52	-5.97
	2.8	16.59	23.27	5.56	-1.12	2.80	-3.88
$\rho = 0.8$	1.6	14.79	27.00	8.80	-3.41	-11.59	-23.80
	1.8	14.79	27.00	8.80	-3.40	-11.19	-23.40
	2	14.82	27.00	8.83	-3.35	-10.82	-23.00
	2.2	14.77	27.00	8.78	-3.45	-10.37	-22.60
	2.4	14.78	27.00	8.79	-3.43	-9.98	-22.20
	2.6	14.75	27.00	8.76	-3.49	-9.55	-21.80
	2.8	14.60	26.56	8.62	-3.34	-9.00	-20.96

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