Optimal Income Tax for China

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I. Introduction

- Per capita annual disposable income of urban households has increased from 343.4yuan in 1978 to 21,809.8 yuan in 2011.
- Personal income distribution has become increasingly unequal among individuals.
- Income tax reform is on the agenda of the government

- Mirrlees (1971) pioneered the optimal income tax theory, followed by Sadka (1976), Seade (1977), Tuomala's (1984), Ebert (1992), Kanbur and Tuomala (1994), Diamond (1998), Dahan and Strawczynski (2000), Fleurbaey and Maniquet (2006), Mankiw, Weinzierl and Yagan(2009), Diamond and Saez (2011), Li, Lin and Zhang (2013), etc.
- However, limited efforts have been made to use the optimal tax theory to design the optimal marginal income tax rates for a specific economy.

- This paper focuses on the optimal personal income tax rates for China.
- We estimate the function of China's skill distribution based on a recent survey and develops a model of optimal marginal and average tax rates for China.
- By assigning plausible values for parameters, we simulate the optimal marginal and average income tax rates for China.
- We find that the actual marginal and average tax rates are significantly different from the optimal marginal and average tax rates.

- Rigorous studies of China’s personal income taxation have emerged recently.
- Zee and Hameed (2006) simulated the effects on different income groups of various reform plans of personal income, with the assumption that the wage in China has the Pareto distribution.
- Piketty and Qian (2009) analyzed income inequality, tax progressivity and the level of tax revenue in China and India.
- So far, there is no study directly addressing the issue of the optimal income tax rates for China.

- The structure of the paper is as follows.
- Section II discusses the evolvement of the personal income tax in China and estimate the skill distribution in China.
- Section III develops a model of the optimal marginal tax rates for China.
- Section IV finds the optimal marginal and average tax rates for China.
- Section V presents some sensitivity analyses.
- Section VI concludes the paper.

II. Personal Income Tax Reforms in China and China's Skill Distribution

- In 1911, the Qing Dynasty government drafted "*Income Tax Prospectus*," which was the first attempt to levy income tax in China.
- In 1914, the government announced "*Income Tax Regulations*," but suspended the regulations in 1916.
- In 1928 and 1929, the government revised "*Income Tax Regulations (Draft)*" and the implementation rules which were on hold for various reasons.
- In 1936, the national government formerly established personal income tax system. The lowest marginal tax rate was 0.5% and the highest 20%.

- In 1950, the government started to collect income taxes. However, the wage income tax was not levied, and the deposit interest income tax was eliminated in 1959.
- On September 1, 1980, the government announced "*Personal Income Tax Law of the People's Republic of China*". The income threshold was set at 800 yuan per month. The lowest marginal tax rate was 5% and the highest 45%.
- Table 1 shows the income tax rates adopted in January 1, 1994. It was a progressive system of nine different marginal tax rates.The lowest marginal tax rate is 5%, and the highest 45%. The tax threshold was 800 yuan per monthly.

Table 1. Personal income tax rates (1994)

Tax brackets for taxable income per month (yuan)		Marginal tax rate (%)
Over	But not over	
0	500	5
500	2,000	10
2,000	5,000	15
5,000	20,000	20
20,000	40,000	25
40,000	60,000	30
60,000	80,000	35
80,000	100,000	40
100,000		45

Source: National People's Congress of China (1994),*Personal Income Tax Law of the People's Republic of China*. http://www.lawxp.com/Statute/s1004363.html.

- From January 1, 2006, the government raised the income threshold to 1600 yuan.
- From March 1, 2008, the income threshold was raised to 2000 yuan.
- In 2011, the income threshold was raised to 3500 yuan and marginal tax rates were reduced from nine to seven levels, with the lowest marginal tax rate being 3% and the highest being 45%.
- Table 2 shows the personal income tax rates based on the 2011 regulation.

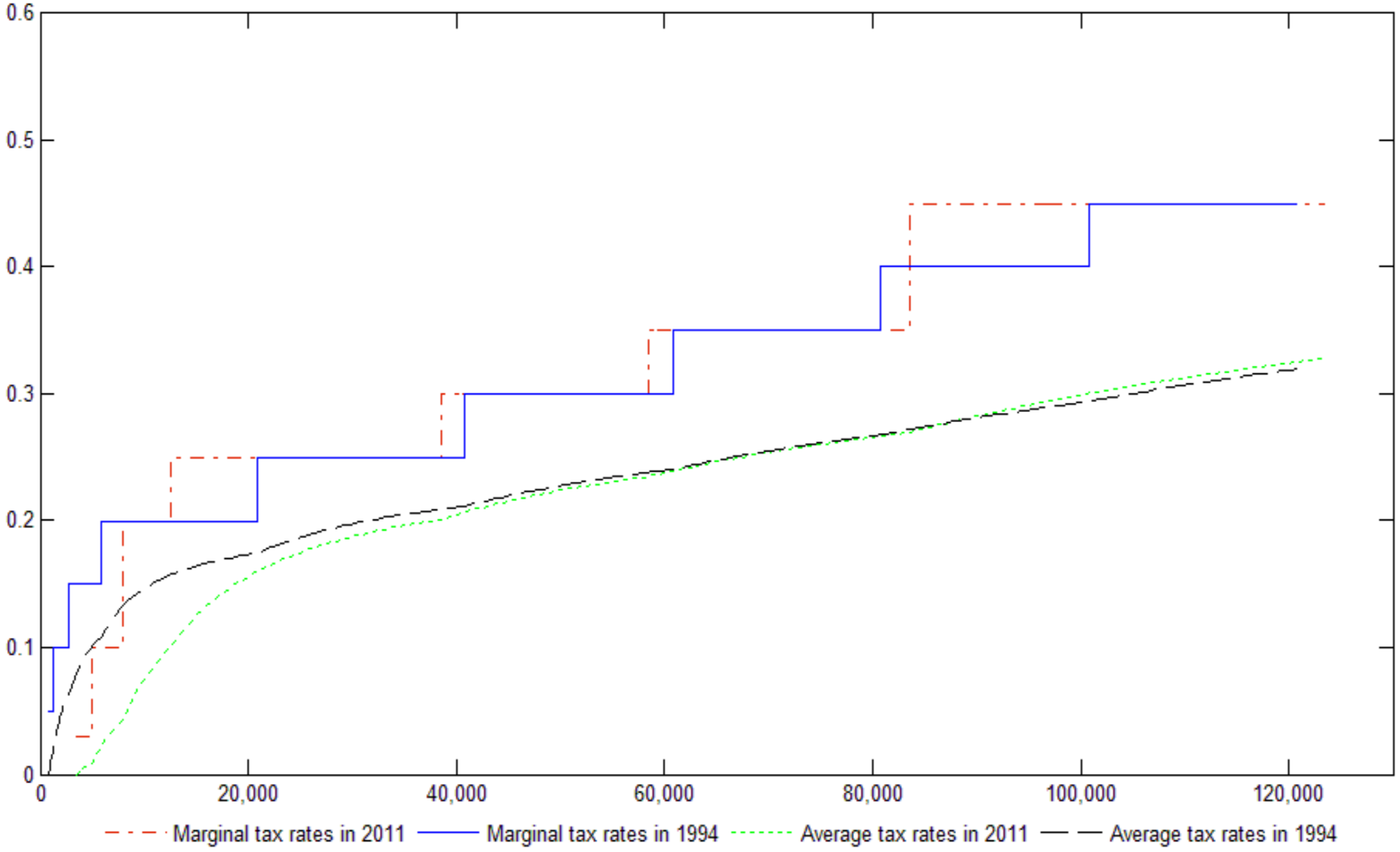
Table 2. Personal income tax rates (2011)

Tax brackets for taxable income per month (yuan)		Marginal tax rate (%)
Over	But not over	
0	1,500	3
1,500	4,500	10
4,500	9,000	20
9,000	35,000	25
35,000	55,000	30
55,000	80,000	35
80,000		45

Sources: National People's Congress of China (2011),Personal Income Tax Law of the People's Republic of China.http://www.lawxp.com/Statute/s1059645.html.

- It can be seen that the reform in 2011 reduced the low-income and middle-income group’s burden.
- Many taxpayers do not need to pay personal income tax again after increasing the threshold from 2000 yuan to 3500 yuan.
- In addition, the reform made the tax system more progressive, with marginal tax rate being 45% tax rate for the group with income higher than 80,000 yuan per month, instead of 100,000 yuan in the 1994 tax regulation.

Figure 1. China's marginal and average tax rates in 1994 and 2011



- To analyze the optimal marginal income tax rate schedule for China, we must first estimate the distribution function of skills in China.
- The data are from China Family Panel Studies (CFPS) 2009, carried out by the Institute of Social Science Survey of Peking University.
- First, we omit the observations for those who have no work and whose hourly wage less than 2 yuan (about US$0.3) because they are too low to be reliable.

- Second, we compute relative hourly wage income by dividing hourly wage income by the average hourly wage income, and use the relative hourly wage income to represent one's skill level *n*.
- Third, we calculate the proportion of the individuals, whose skills are lower than *n*, and the proportion is the cumulative distribution function (CDF), or the distribution function *F* with respect to the skill level *n*, and calculate the probability density function is denoted by *f*. Figure 2a shows the distribution of *F* with respect to *n*.

- Forth, find ln(n) and ln(1-F).
- The approximate distribution function F of n:

$$F(n) = 1 - \left(\frac{b}{n}\right)^{a} = 1 - \left(\frac{0.426}{n}\right)^{1.501} \qquad n \in [0.426, +\infty)$$

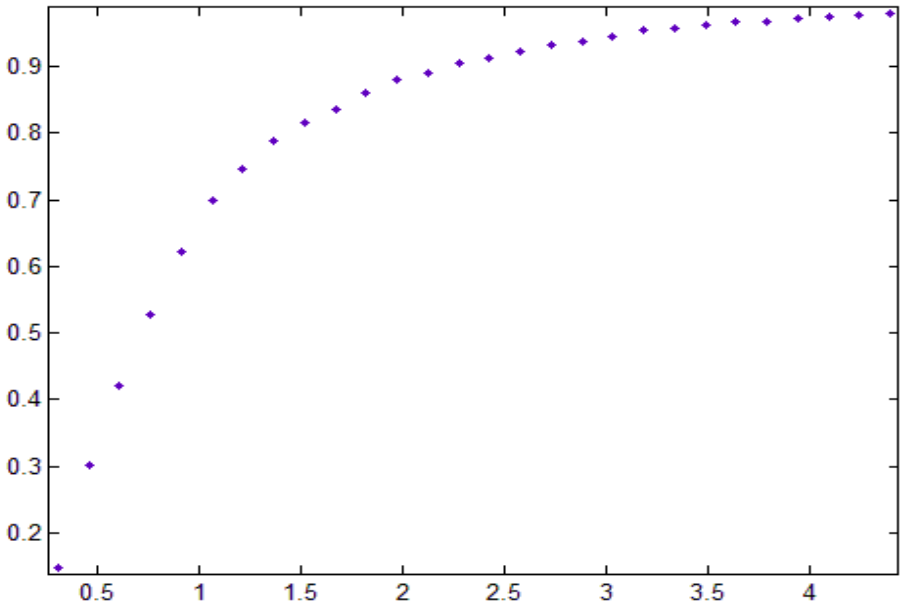
- The probability density function of the skill levels for China:

$$f(n) = \frac{ab^{a}}{n^{a+1}} = \frac{1.501 \times (0.426)^{1.501}}{n^{2.501}} = \frac{0.417}{n^{2.501}}$$

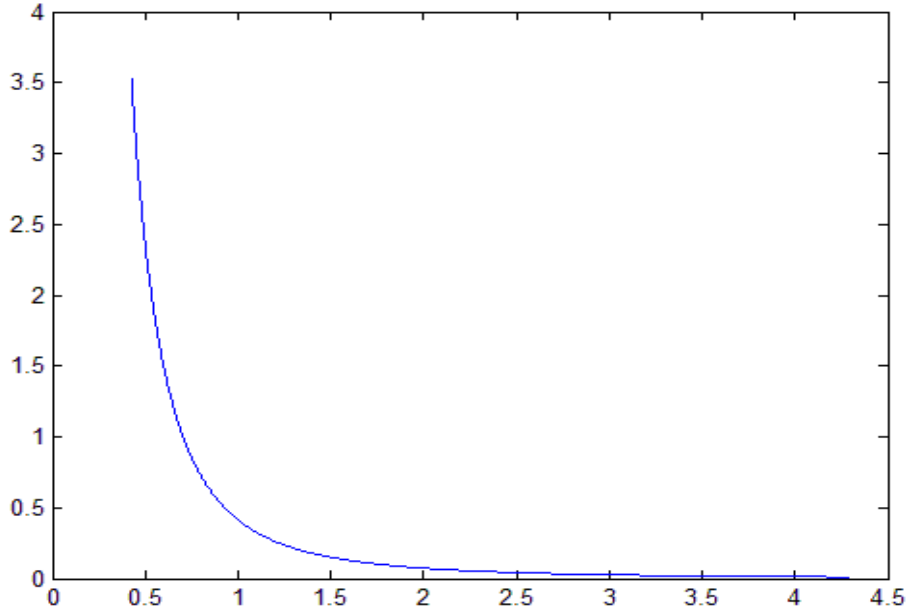
- The density function is illustrated in Figure 2b.

Figure 2. The distribution of skills (labor skill levels) in China

a. $F(n)$ vs. n



b. $f(n)$ vs. n



III. The Model of the Optimal Income Taxation for China

- We now develop a model of optimal income tax introduced by Mirrlees (1971) and extended by Diamond (1998), with China's skill distribution.
- The optimal marginal income tax rate is:

$$T' = 1 + \frac{u_l}{nu_c}$$

- where c and l satisfy equations (8) and (11).
- $$u_n = u_c c_n + u_l l_n = -\frac{lu_l}{n} \qquad (8)$$
- $$n\lambda\left(1+\frac{u_l}{nu_c}\right)f(n) = -\frac{(u_l + lu_{ll})}{n}\int_n^{\bar{n}}\left(\frac{\lambda}{u_c} - G_u\right)f(t)dt \qquad (11)$$

- According to Li, Lin and Zhang (2013), with the utility function, $u(c,l) = c + \alpha(1 - l^k)$,used by Diamond (1998), for some $\alpha > 0$, and $k > 1$, and the density function of skills, $f(n) = ab^a / n^{1+a}$, with, $\underline{n} \le n < \infty, a > 0$, $b > 0, 0 < \underline{n} < \infty$, the optimal marginal tax rates are increasing on $[\underline{n}, \infty)$.
- Using the equations (5) and (11), we can get

$$\frac{T'}{1-T'} = \left(1 + \frac{lu_{ll}}{u_l}\right) \frac{u_c}{n\lambda f(n)} \int_n^{\bar{n}} \left(\frac{\lambda}{u_c} - G_u\right) f(t)dt \qquad (12)$$

- If $u(c,l) = c + \alpha(1 - l^k)$, then $u_c = 1$, $u_l = -\alpha k l^{k-1}$, $u_{ll} = -\alpha k(k-1) l^{k-2}$. Equation (12) can be written as

$$\frac{T'}{1-T'} = \frac{k}{n\lambda f(n)} \int_n^{\infty} (\lambda - G_u) f(t) dt \qquad (13)$$

- Let $D(n) = \frac{1}{nf(n)} \int_n^{\infty} (\lambda - G_u) f(t) dt$ with $f(n) = \frac{ab^a}{n^{1+a}}$, we have

$$D(n) = \frac{1}{nab^a / n^{1+a}} \int_n^{\infty} (\lambda - G_u) \frac{ab^a}{t^{1+a}} dt = \frac{\int_n^{\infty} (\lambda - G_u) t^{-1-a} dt}{n^{-a}}$$

$$D'(n) = \frac{-(\lambda - G_u) n^{-1-a} n^{-a} + \alpha n^{-1-a} \int_n^{\infty} (\lambda - G_u) t^{-1-a} dt}{n^{-2a}} \qquad (14)$$

- Since $\lambda - G_u$ is strictly increasing as n increases, we have the following inequality:

$$an^{-1-a}\int_n^{\infty}(\lambda - G_u)t^{-1-a}dt > an^{-1-a}(\lambda - G_u)\int_n^{\infty}t^{-1-a}dt \qquad (15)$$

- Substituting equation (15) into equation (14)

$$D'(n) > \frac{-(\lambda - G_u)n^{-1-a}n^{-a} + an^{-1-a}(\lambda - G_u)\int_n^{\infty}t^{-1-a}dt}{n^{-2\alpha}}$$

$$= \frac{-(\lambda - G_u)n^{-1-2a} + an^{-1-a}(\lambda - G_u)\frac{1}{-a}(0 - n^{-a})}{n^{-2a}} = 0$$

- Since $k > 0$ and $\lambda > 0$, we have $\frac{d\left(T'/(1-T')\right)}{dn} = \frac{k}{\lambda} D'(n) > 0$ and $\frac{dT'}{dn} > 0$.
- Under this condition, the optimal marginal tax rates are increasing on $[\underline{n}, \infty)$. In order to compute asymptotic marginal tax rates, we write equation (13) as

$$\frac{T'}{1-T'} = k \times \frac{\int_n^{\infty} (\lambda - G_u) f(t) dt}{\lambda (1 - F(n))} \times \frac{1 - F(n)}{n f(n)}$$

- Since the skill density function is $f(n) = \frac{ab^a}{n^{1+a}}$, then $\frac{1-F(n)}{nf(n)} = \frac{1}{a}$.

- As $n \to \infty$, $\int_n^{\infty} (\lambda - G_u) f(t)dt \to 0$, $\lambda(1-F(n)) \to 0$. By the L'Hospital Rule, we have:

$$\lim_{n\to\infty} \frac{\int_n^{\infty} (\lambda - G_u) f(t)dt}{\lambda(1-F(n))} = \lim_{n\to\infty} \frac{(\lambda - G_u) f(n)}{\lambda f(n)} = \lim_{n\to\infty} (1 - \frac{G_u}{\lambda})$$

- So we have

$$\lim_{n\to\infty} \frac{T'}{1-T'} = k \times \lim_{n\to\infty} \frac{\int_n^{\infty} (\lambda - G_u) f(t)dt}{\lambda(1-F(n))} \times \frac{1-F(n)}{nf(n)} = \frac{k}{a} \times \lim_{n\to\infty} (1 - \frac{G_u}{\lambda}) \quad (16)$$

- We now know the optimal marginal tax rates are increasing on $[\underline{n}, \infty)$ and asymptotic marginal tax rates.

- From equations (8) and (11), we can get the system of differential equations for leisure and consumption:

$$l_n = \frac{\alpha k^2 l^k}{k-1}\left\{\frac{2}{\alpha n k^2 l^{k-1}} - \frac{1}{kn^2} + \frac{1}{n^2} + \left(\frac{1}{\alpha k^2 l^{k-1}} - \frac{1}{kn}\right)\frac{f_n}{f} - \frac{\beta}{n^2\lambda} - \frac{1}{n^2\lambda e^{c+\alpha(1-l^k)}}\right\}$$

$$c_n = \frac{\alpha^2 k^3 l^{2k-1}}{k-1}\left\{\frac{2}{\alpha n k^2 l^{k-1}} - \frac{1}{kn^2} + \frac{1}{n^2} + \left(\frac{1}{\alpha k^2 l^{k-1}} - \frac{1}{kn}\right)\frac{f_n}{f} - \frac{\beta}{n^2\lambda} - \frac{1}{n^2\lambda e^{c+\alpha(1-l^k)}}\right\} + \frac{\alpha k l^k}{n}$$

- We now ready to compute and for every *n*.
- First, based on a given , we can obtain a value $l(\underline{n})$.
- Second, based on these results and assumed $c(\underline{n})$, we can know $c(n)$ and $l(n)$ for every *n*, using a Runge-Kutta method to solve the system of differential equations.

- Third, a trial-and-error method is used to compute the value $c(\underline{n})$ given the value of E/Y， and then restart the second step.
- Using the above results, we can obtain the schedules of the optimal marginal income tax rate and optimal average income tax rate .

$$T' = 1 + \frac{u_l}{n u_c} = 1 - \frac{\alpha k l^{k-1}}{n}$$

$$\frac{T}{nl} = 1 - \frac{c}{nl}$$

IV. Optimal Marginal and Average Income Tax Rates for China

IV.1. Values of k and α

- When and E/Y=0.05, the optimal working time will be very different due to the value of k and as showed in Table 4.
- When k is fixed, the value of is smaller, persons tend to work more. When is fixed, the value of k is larger, persons with low level of skill tend to work more.
- According to the actual situation in China, k=3 and k=5 are both not realistic. Here we just set k=4 and α=5.

Table 4. Working time l with different skill n

(β=0.02 and E/Y=0.05)

k	α	$c(n)$	$l(n)$	λ	Asymptotic marginal tax rates	Working time with different skill n					
						$F(n)$=0.1	$F(n)$=0.3	$F(n)$=0.5	$F(n)$=0.7	$F(n)$=0.9	$F(n)$=0.99
3	4	0.1317	0.1884	0.03463	45.78	0.194	0.208	0.227	0.260	0.342	0.646
3	5	0.0986	0.1685	0.02555	30.27	0.174	0.188	0.208	0.241	0.328	0.653
3	6	0.0711	0.1539	0.02209	15.88	0.159	0.173	0.192	0.225	0.316	0.654
4	4	0.1958	0.2986	0.03339	51.66	0.304	0.316	0.334	0.362	0.429	0.654
4	5	0.1563	0.2772	0.02509	35.07	0.283	0.297	0.316	0.347	0.423	0.668
4	6	0.1213	0.2609	0.02192	18.95	0.267	0.281	0.301	0.334	0.416	0.676
5	4	0.2476	0.3820	0.03249	56.15	0.386	0.397	0.411	0.435	0.491	0.672
5	5	0.2035	0.3613	0.02475	38.99	0.367	0.379	0.396	0.424	0.489	0.688
5	6	0.1628	0.3452	0.02180	21.61	0.351	0.364	0.383	0.414	0.486	0.699

IV.2. Value of β

- β represents a degree of inequality aversion, and it has a tremendous impact on tax rates.
- Since $G_u = \beta + e^{-u} = \beta + \frac{1}{e^{c+\alpha(1-l^k)}}$, if β=0, then $\lim_{u\to\infty} G_u = \beta + e^{-u} = 0$, which means marginal social utility will equal zero. If β =1, then $\lim_{u\to\infty} G_u = \beta + e^{-u} = 1$, which means marginal social utility will equal one.
- From Table 5 and Table 6, we can conclude that the value of influences not only the marginal and average tax rates, but also the taxable threshold.

Table 5. The average tax rate

(k=4, α =5and E/Y=0.05)

β	Average tax rate (%) at the point n where						$F(n^*)$ Average tax rate = 0 at n^*
	$F(n)$=0.1	$F(n)$=0.3	$F(n)$=0.5	$F(n)$=0.7	$F(n)$=0.9	$F(n)$=0.99	
0.01	-44.82	-35.69	-25.05	-11.52	10.93	38.76	0.8216
0.02	-29.52	-23.40	-16.27	-7.15	8.58	29.25	0.8113
0.03	-20.69	-16.35	-11.25	-4.63	7.22	23.53	0.7993

Table 6. The marginal tax rate

(k=4, α =5and E/Y=0.05)

β	Marginal tax rate (%) at the point n where					
	$F(n)$=0.1	$F(n)$=0.3	$F(n)$=0.5	$F(n)$=0.7	$F(n)$=0.9	$F(n)$=0.99
0.01	1.49	5.08	10.02	17.47	31.84	46.53
0.02	0.97	3.36	6.77	12.19	23.45	34.80
0.03	0.72	2.52	5.14	9.41	18.64	27.81

IV.3. Government Expenditure, *E/Y*

- *E/Y* is the ratio of government expenditures from personal income tax *E* to gross wage income *Y*.
- From Table 7, we can see that *E/Y* will influence minimum consumption, $c(\underline{n})$, i.e., $c(\underline{n})$ will be smaller when *E/Y* is higher.
- Government expenditure *E/Y* influences the threshold of taxation, i.e., the percentage of people to pay income tax.

Table 7. The income tax rate and budget target

(k=4, α =5and E/Y=0.05)

E/Y (%)	$c(\underline{n})$	$l(\underline{n})$	$F(n^*)$ Average tax rate = 0 at n*	Average tax rate (%) when $F(n^*)=0.99$	Marginal tax rate (%) when $F(n^*)=0.99$
10	0.1349	0.2772	0.7158	29.93	35.19
5	0.1563	0.2772	0.8113	29.25	34.80
0	0.1777	0.2772	0.8543	28.58	34.41
-5	0.1991	0.2772	0.8800	27.90	34.02

- Now given the values of k, α , β and E/Y, we can obtain the optimal tax schedules for the marginal tax rate and the average tax rate.
- From Table 8 and Figure 3, we know that the optimal marginal tax rate always is positive; the optimal average tax rate is negative at low income levels and becomes positive at income levels 4012 yuan; both the optimal marginal and average tax rates for China are increasing as income increases, but increases at decreasing rates, and becomes rather flat at high skills.

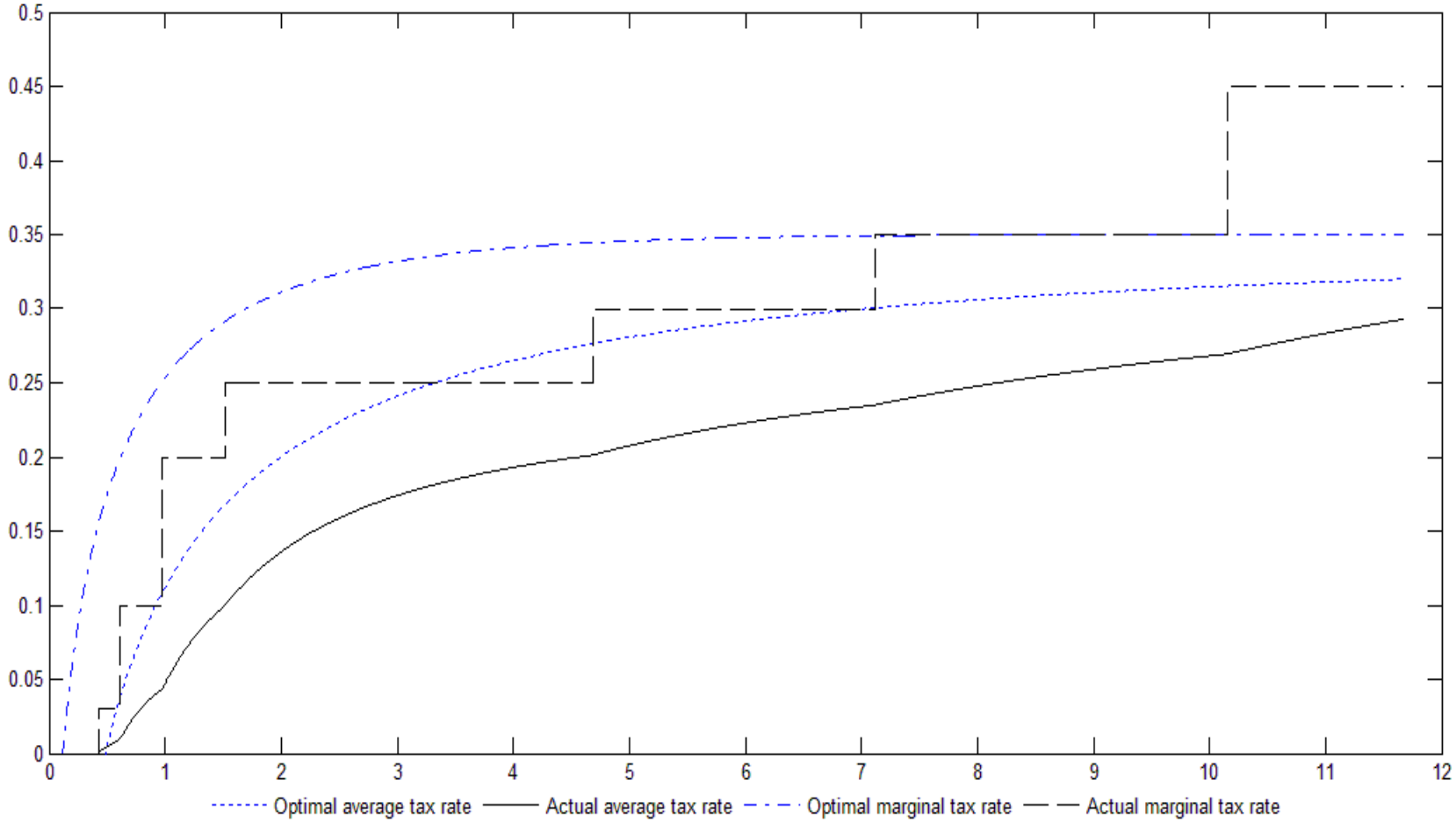
Table 8. The optimal income tax rates

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2757	0.3249	0.5709	8.37	-13.36	0.00	0.00
3500	0.4256	0.4358	0.3663	0.7782	15.43	-2.40	0.00	0.00
4012	0.4879	0.4880	0.3771	0.8113	17.14	0.00	3.00	0.38
5000	0.6080	0.5858	0.3951	0.8545	19.82	3.66	3.00	0.90
8000	0.9728	0.8677	0.4369	0.9164	25.06	10.80	10.00	4.31
12500	1.5200	1.2653	0.4817	0.9505	29.14	16.75	20.00	9.96
38500	4.6817	3.3884	0.6259	0.9864	34.45	27.62	25.00	20.12
58500	7.1137	4.9754	0.6936	0.9916	34.92	30.06	30.00	23.50
83500	10.1538	6.9518	0.7578	0.9944	35.03	31.54	35.00	26.94
90000	10.9442	7.4653	0.7722	0.9948	35.04	31.79	45.00	28.24

Figure 3. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.05)$



- We can draw the following conclusions on the optimal income tax rates for China.
- First, the optimal average tax rate is negative for low-income groups, i.e., the government should subsidize the poor, although the marginal tax rates are nonnegative.
- Second, the optimal tax exemption level is around 4012 yuan, however, the actual income tax exemption level 3,500 yuan, which is lower than the optimal.

- Third, the actual marginal tax rate and average rise too slowly as income increases, i.e., the actual marginal tax rate and average tax rate are too low for income groups between 4,012 to 58,500 yuan.
- Fourth, the actual marginal tax rate seems to coincide with optimal marginal tax rates for income groups with monthly income between 58,500 to 83,500yuan, but the actual average tax rates seems smaller than the optimal average tax rates.
- Fifth, the actual marginal and average tax ratesrise too rapidly at high levels of income, i.e., the actual marginal tax rate is too high for income groups above 83,500.

V. Sensitivity Analyses

- We conduct some sensitivity analyses by varying parameters k, α, β and E/Y.
- **Case 1.** $k = 3$ and $k = 5$
- From Table 9 and Figure 4, it can be seen that the optimal threshold is much lower than the baseline case where $k = 4$. For example, the average tax rate is zero when income is 3294, lower than 4012 yuan with $k = 4$.
- Also, the optimal marginal tax rate is lower than the baseline case with $k = 4$.

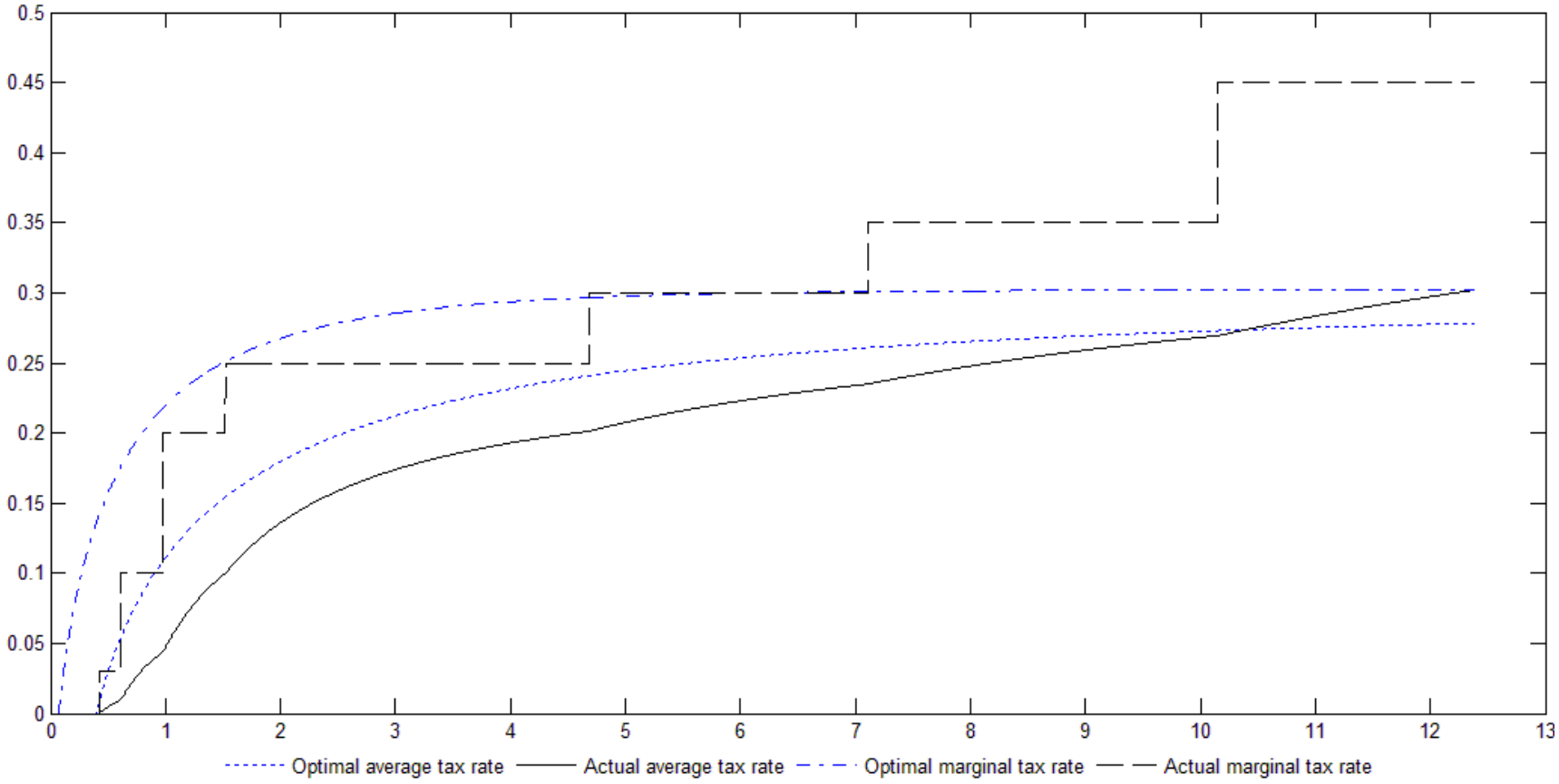
Table 9. The optimal income tax rates

$(k = 3, \alpha = 5, \beta = 0.02, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2613	0.2452	0.7187	9.03	-7.43	0.00	0.00
3294	0.4005	0.4005	0.2846	0.8336	13.62	0.00	0.00	0.00
3500	0.4256	0.4221	0.2898	0.8440	14.20	0.82	0.00	0.00
5000	0.6080	0.5754	0.3221	0.8929	17.55	5.37	3.00	0.90
8000	0.9728	0.8678	0.3703	0.9348	21.72	10.79	10.00	4.31
12500	1.5200	1.2860	0.4234	0.9592	25.08	15.40	20.00	9.96
38500	4.6817	3.5541	0.6033	0.9872	29.66	24.08	25.00	20.12
58500	7.1137	5.2582	0.6921	0.9916	30.09	26.08	30.00	23.50
83500	10.1538	7.3817	0.7789	0.9941	30.19	27.30	35.00	26.94
90000	10.9442	7.9334	0.7986	0.9945	30.19	27.51	45.00	28.24

Figure 4. China's optimal income tax rates vs.*nl*

$(k = 3, \alpha = 5, \beta = 0.02, E/Y = 0.05)$



- **Case 1.** $k = 3$ and $k = 5$
- From Table 10 and Figure 5, it can be seen that the optimal threshold is much higher than the baseline case where $k = 4$. For example, the average tax rate is zero when income is 4571, higher than 4012 yuan with $k = 4$.
- Also, the optimal marginal tax rate is higher than the baseline case with $k = 4$. For example, the marginal tax rate is 27.71% when income is 8,000 yuan, higher than 25.06% with $k = 4$.

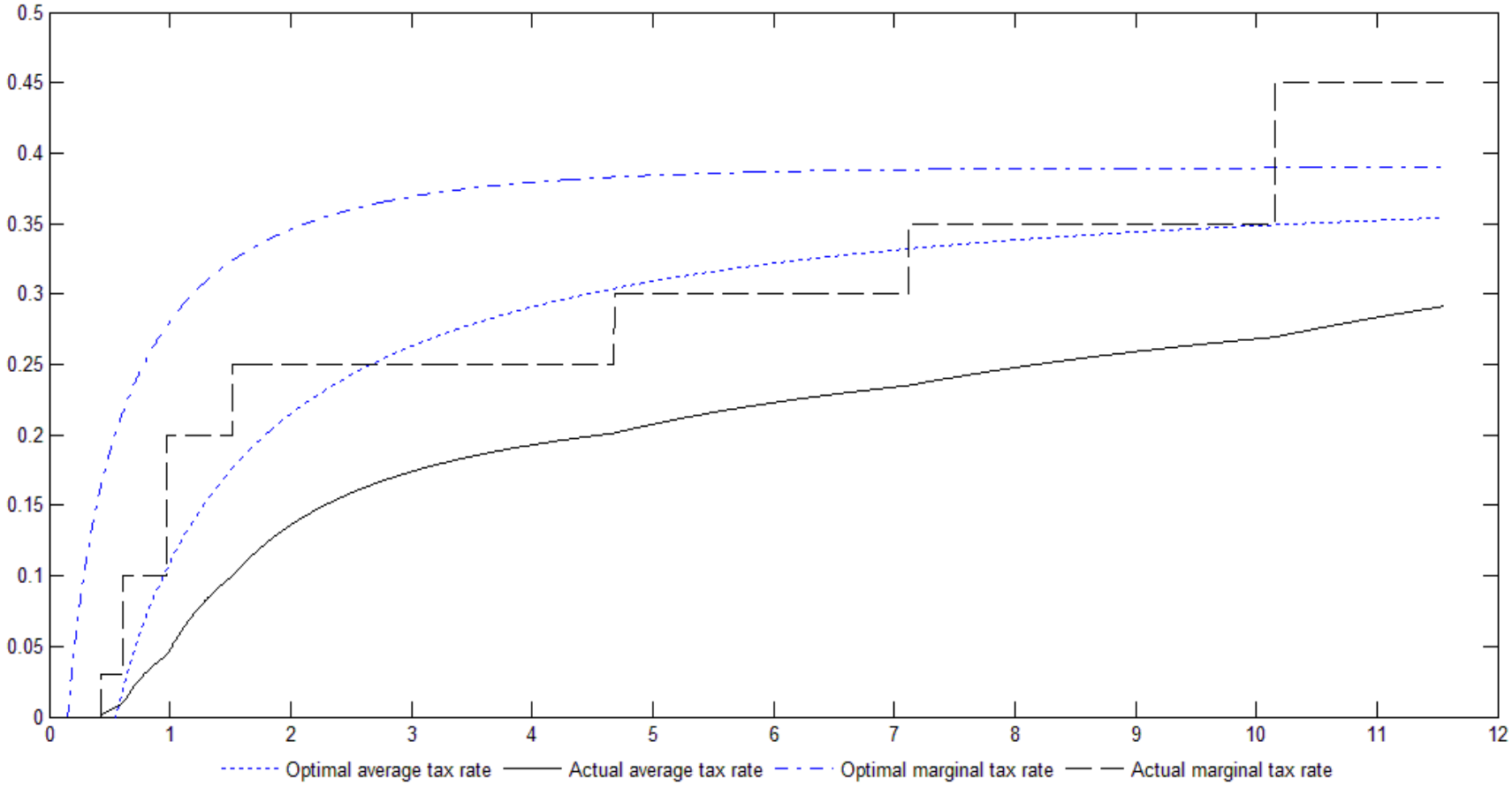
Table 10. The optimal income tax rates

$(k = 5, \alpha = 5, \beta = 0.02, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2893	0.3900	0.4356	7.27	-18.96	0.00	0.00
3500	0.4256	0.4495	0.4274	0.7204	16.21	-5.61	0.00	0.00
4571	0.5558	0.5558	0.4464	0.8001	20.24	0.00	3.00	0.70
5000	0.6080	0.5971	0.4530	0.8214	21.54	1.79	3.00	0.90
8000	0.9728	0.8709	0.4896	0.9009	27.71	10.48	10.00	4.31
12500	1.5200	1.2522	0.5282	0.9432	32.37	17.62	20.00	9.96
38500	4.6817	3.2590	0.6495	0.9857	38.30	30.39	25.00	20.12
58500	7.1137	4.7516	0.7049	0.9914	38.83	33.21	30.00	23.50
83500	10.1538	6.6089	0.7566	0.9944	38.95	34.91	35.00	26.94
90000	10.9442	7.0914	0.7680	0.9949	38.96	35.20	45.00	28.24

Figure 5. China's optimal income tax rates vs.*nl*

$(k = 5, \alpha = 5, \beta = 0.02, E/Y = 0.05)$



- **Case 2.** $\alpha = 4$ and $\alpha = 6$
- From Table 11 and Figure 6, it can be seen that the optimal threshold is higher than the baseline case where $\alpha = 5$. For example, the average tax rate is zero when income is 4257, higher than 4012 yuan with $\alpha = 5$.
- Also, the optimal marginal tax rate is higher than the baseline case with $\alpha = 5$. For example, the marginal tax rate is 37.78% when income is 8,000 yuan, much higher than 25.06% with $\alpha = 5$.

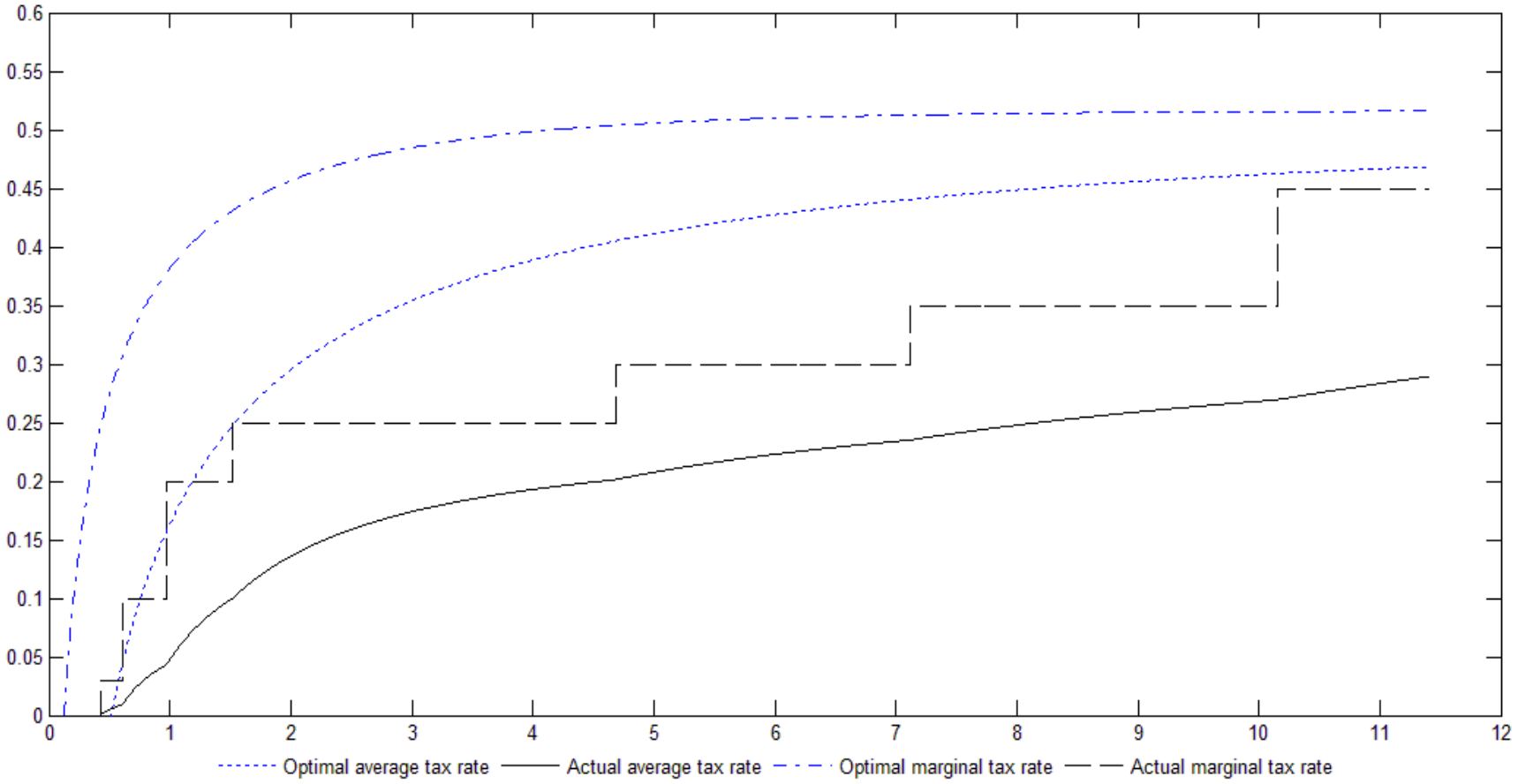
Table 11. The optimal income tax rates

$(k = 4, \alpha = 4, \beta = 0.02, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.3031	0.3386	0.5434	0.3031	13.50	-24.63	0.00	0.00
3500	0.4498	0.3765	0.7689	0.4498	24.47	-5.68	0.00	0.00
4257	0.5176	0.3907	0.8179	0.5176	27.99	0.00	3.00	0.53
5000	0.5814	0.4028	0.8503	0.5814	30.72	4.37	3.00	0.90
8000	0.8198	0.4410	0.9153	0.8198	37.78	15.73	10.00	4.31
12500	1.1441	0.4822	0.9504	1.1441	43.08	24.73	20.00	9.96
38500	2.7861	0.6173	0.9867	2.7861	50.38	40.49	25.00	20.12
58500	3.9800	0.6823	0.9918	3.9800	51.26	44.05	30.00	23.50
83500	5.4563	0.7446	0.9945	5.4563	51.55	46.26	35.00	26.94
90000	5.8391	0.7586	0.9949	5.8391	51.58	46.65	45.00	28.24

Figure 6. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 4, \beta = 0.02, E/Y = 0.05)$



- **Case 2.** $\alpha = 4$ and $\alpha = 6$
- From Table 12 and Figure 7, it can be seen that the optimal threshold is lower than the baseline case where $\alpha = 5$. For example, the average tax rate is zero when income is 3050, lower than 4012 yuan with $\alpha = 5$.
- Also, the optimal marginal tax rate is much lower than the baseline case with $\alpha = 5$. For example, the marginal tax rate is 13.32% when income is 8,000 yuan, much lower than 25.06% with $\alpha = 5$.

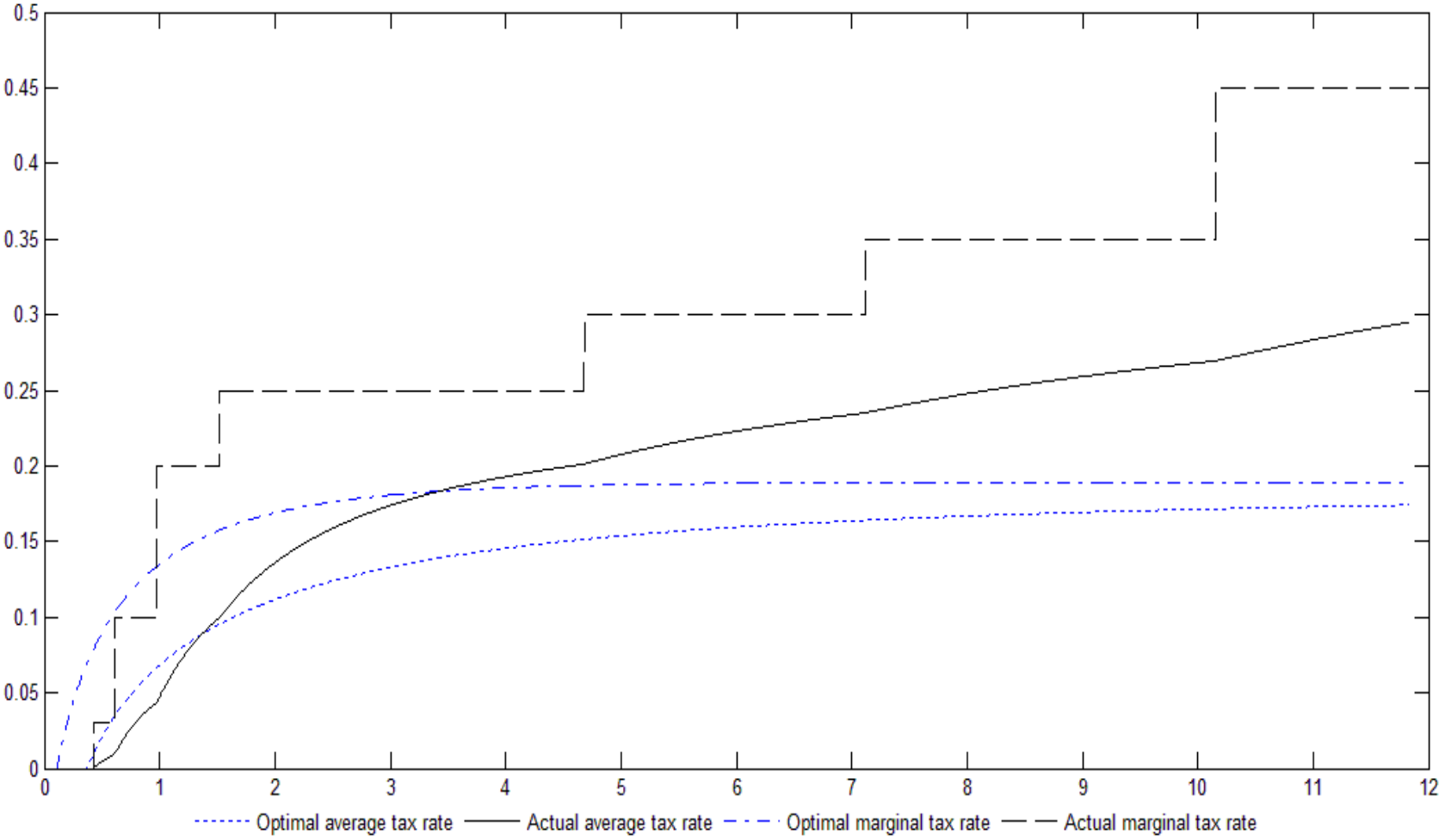
Table 12. The optimal income tax rates

$(k = 4, \alpha = 6, \beta = 0.02, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2504	0.3139	0.5925	4.19	-2.96	0.00	0.00
3050	0.3709	0.3709	0.3463	0.7494	6.92	0.00	0.00	0.00
3500	0.4256	0.4216	0.3575	0.7861	7.85	0.95	0.00	0.00
5000	0.6080	0.5872	0.3883	0.8583	10.28	3.41	3.00	0.90
8000	0.9728	0.9085	0.4329	0.9176	13.32	6.61	10.00	4.31
12500	1.5200	1.3754	0.4806	0.9507	15.74	9.51	20.00	9.96
38500	4.6817	3.9724	0.6310	0.9863	18.71	15.15	25.00	20.12
58500	7.1137	5.9464	0.7002	0.9914	18.90	16.41	30.00	23.50
83500	10.1538	8.4113	0.7653	0.9943	18.93	17.16	35.00	26.94
90000	10.9442	9.0521	0.7798	0.9947	18.93	17.29	45.00	28.24

Figure 7. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 6, \beta = 0.02, E/Y = 0.05)$



- **Case 3.** $\beta = 0.01$ and $\beta = 0.03$
- From Table 13 and Figure 8, it can be seen that the optimal threshold is higher than the baseline case where $\beta = 0.02$. For example, the average tax rate is zero when income is 4084, higher than 4012 yuan with $\beta = 0.02$.
- Also, the optimal marginal tax rate is much higher than the baseline case with $\beta = 0.02$. For example, the marginal tax rate is 34.35% when income is 8,000 yuan, much higher than 25.06% with $\beta = 0.02$.

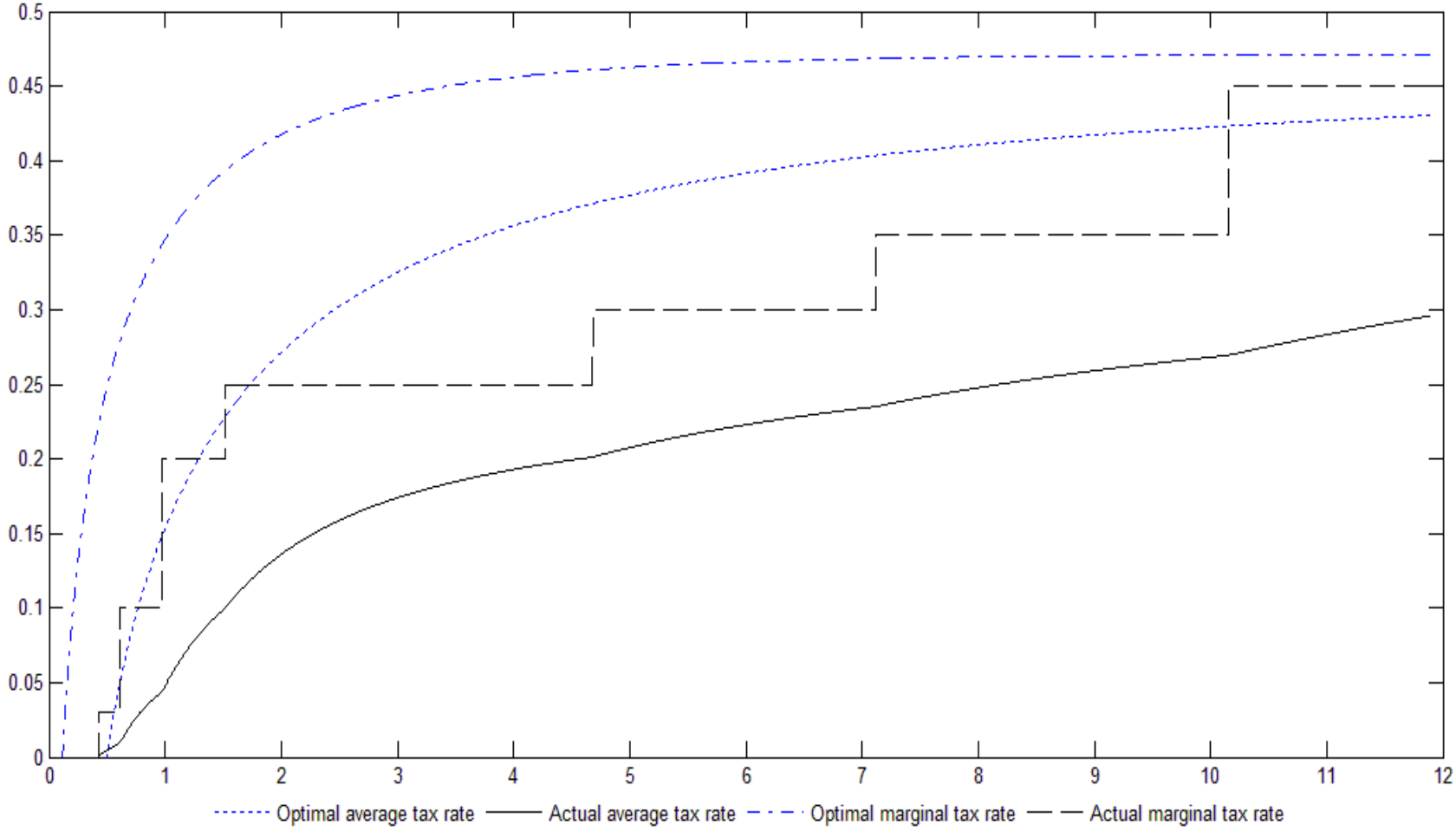
Table 13. The optimal income tax rates

$(k = 4, \alpha = 5, \beta = 0.01, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2924	0.3211	0.5783	12.52	-20.24	0.00	0.00
3500	0.4256	0.4423	0.3588	0.7851	22.16	-3.92	0.00	0.00
4084	0.4967	0.4967	0.3698	0.8216	24.68	0.00	3.00	0.43
5000	0.6080	0.5787	0.3849	0.8602	27.82	4.82	3.00	0.90
8000	0.9728	0.8288	0.4227	0.9205	34.35	14.80	10.00	4.31
12500	1.5200	1.1729	0.4634	0.9533	39.32	22.84	20.00	9.96
38500	4.6817	2.9444	0.5960	0.9874	46.08	37.11	25.00	20.12
58500	7.1137	4.2444	0.6594	0.9922	46.84	40.33	30.00	23.50
83500	10.1538	5.8562	0.7200	0.9948	47.07	42.33	35.00	26.94
90000	10.9442	6.2744	0.7335	0.9952	47.09	42.67	45.00	28.24

Figure 8. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 5, \beta = 0.01, E/Y = 0.05)$



- **Case 3.** $\beta = 0.01$ and $\beta = 0.03$
- From Table 14 and Figure 9, it can be seen that the optimal threshold is lower than the baseline case where $\beta = 0.02$. For example, the average tax rate is zero when income is 3863, lower than 4012 yuan with $\beta = 0.02$.
- Also, the optimal marginal tax rate is much lower than the baseline case with $\beta = 0.02$. For example, the marginal tax rate is 19.80% when income is 8,000 yuan, much lower than 25.06% with $\beta = 0.02$.

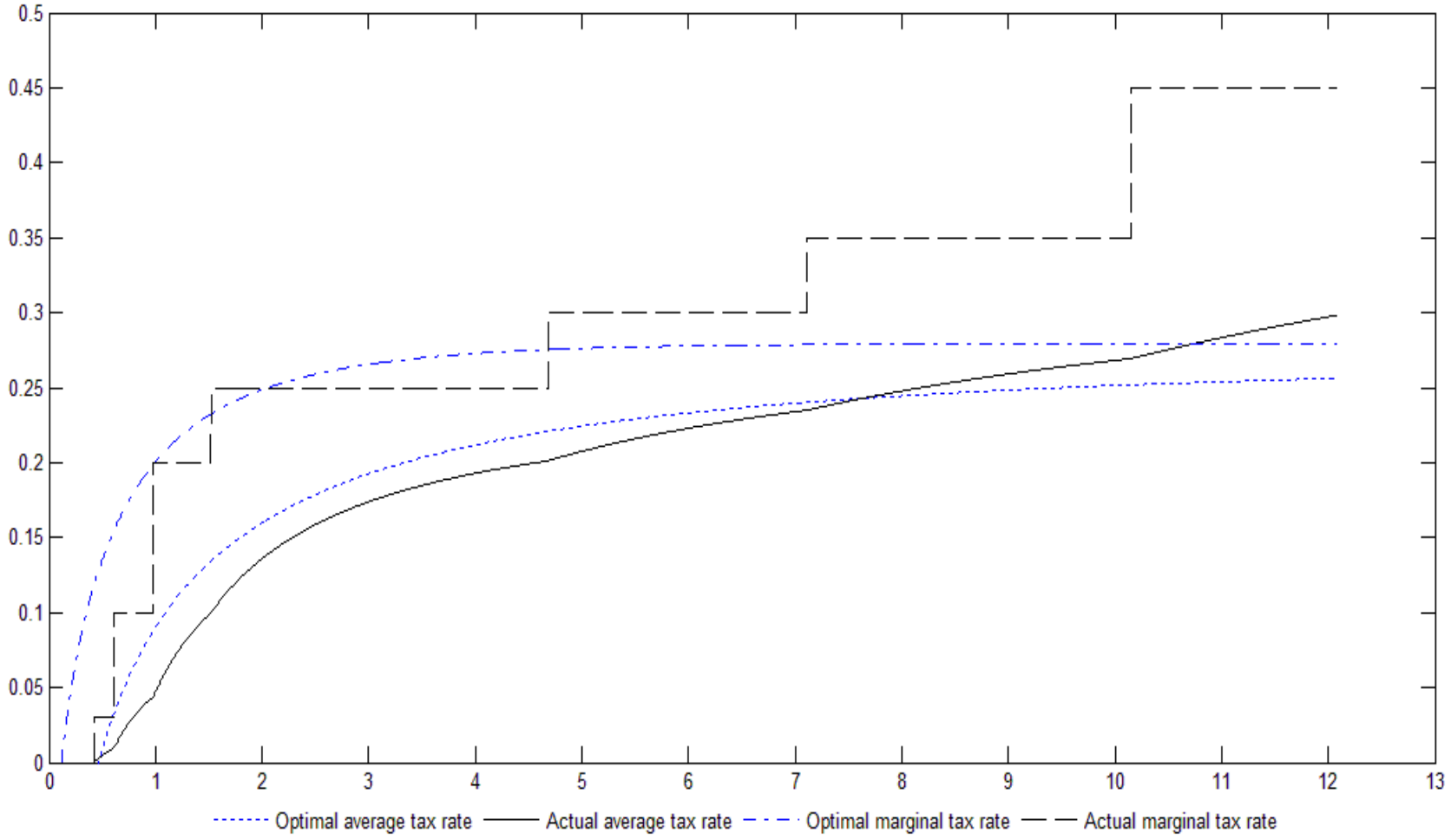
Table 14. The optimal income tax rates

$(k = 4, \alpha = 5, \beta = 0.03, E/Y = 0.05)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2658	0.3267	0.5674	6.32	-9.26	0.00	0.00
3500	0.4256	0.4311	0.3700	0.7748	11.89	-1.29	0.00	0.00
3863	0.4697	0.4698	0.3782	0.7993	12.88	0.00	3.00	0.28
5000	0.6080	0.5883	0.4004	0.8516	15.45	3.23	3.00	0.90
8000	0.9728	0.8881	0.4444	0.9143	19.80	8.71	10.00	4.31
12500	1.5200	1.3165	0.4915	0.9490	23.21	13.39	20.00	9.96
38500	4.6817	3.6477	0.6418	0.9859	27.54	22.09	25.00	20.12
58500	7.1137	5.4048	0.7117	0.9912	27.87	24.02	30.00	23.50
83500	10.1538	7.5964	0.7777	0.9941	27.93	25.19	35.00	26.94
90000	10.9442	8.1660	0.7924	0.9946	27.93	25.39	45.00	28.24

Figure 9. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 5, \beta = 0.03, E/Y = 0.05)$



- **Case 4.** $E/Y = 0.00$ and $E/Y = 0.10$
- From Table 15 and Figure 10, it can be seen that the optimal threshold is higher than the baseline case where $E/Y = 0.00$. For example, the average tax rate is zero when income is 4998, higher than 4012 yuan with $E/Y = 0.00$.
- Also, the optimal marginal tax rate is lower than the baseline case with $E/Y = 0.00$. For example, the marginal tax rate is 24.77% when income is 8,000 yuan, slightly lower than 25.06% with $E/Y = 0.00$.

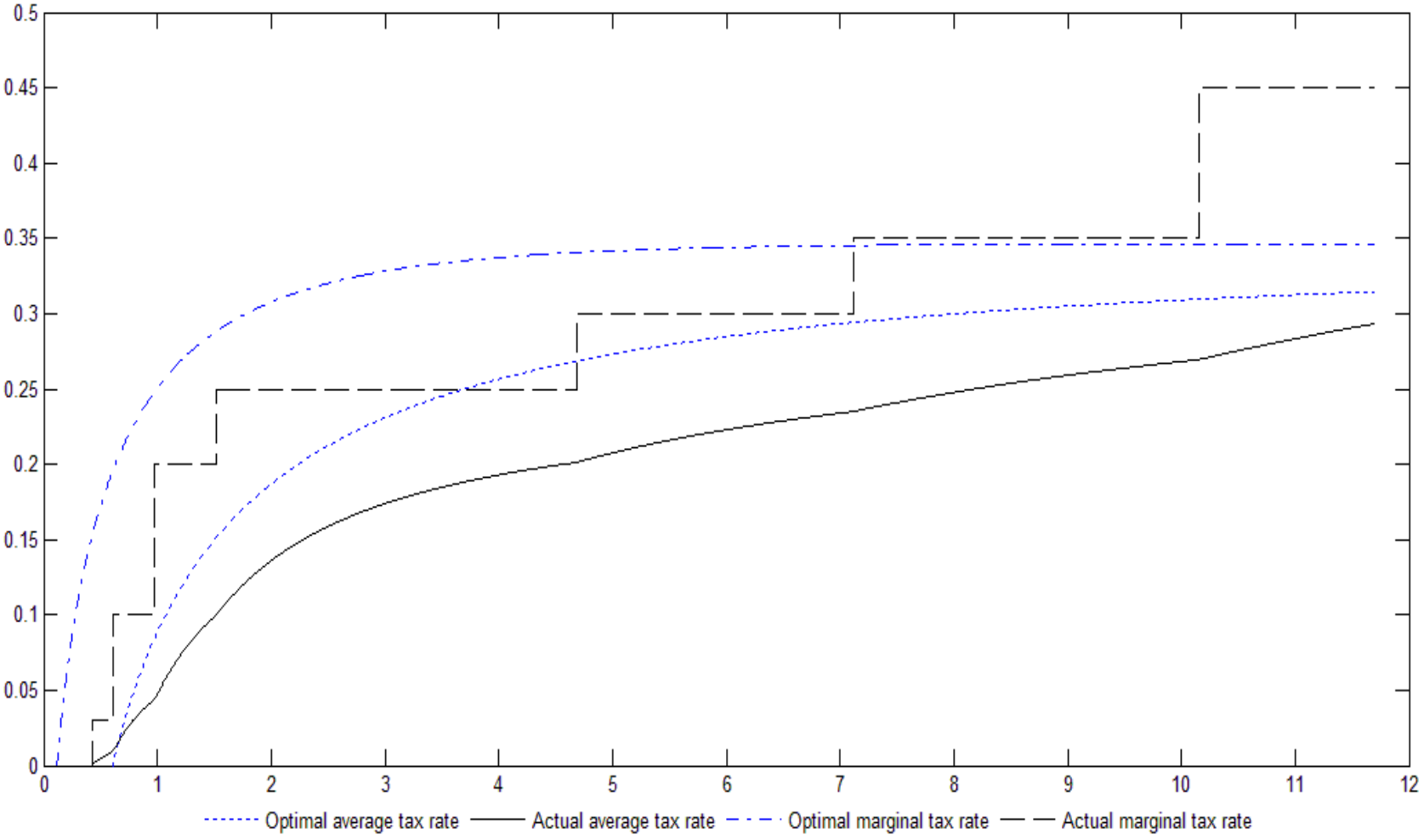
Table 15. The optimal income tax rates

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.00)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2971	0.3250	0.5707	8.25	-22.20	0.00	0.00
3500	0.4256	0.4575	0.3665	0.7780	15.22	-7.52	0.00	0.00
4998	0.6077	0.6078	0.3954	0.8543	19.56	0.00	3.00	0.90
5000	0.6080	0.6080	0.3954	0.8544	19.57	0.00	3.00	0.90
8000	0.9728	0.8909	0.4374	0.9163	24.77	8.42	10.00	4.31
12500	1.5200	1.2903	0.4823	0.9504	28.81	15.11	20.00	9.96
38500	4.6817	3.4249	0.6268	0.9864	34.06	26.85	25.00	20.12
58500	7.1137	5.0215	0.6947	0.9915	34.52	29.41	30.00	23.50
83500	10.1538	7.0099	0.7590	0.9943	34.63	30.96	35.00	26.94
90000	10.9442	7.5266	0.7733	0.9948	34.64	31.23	45.00	28.24

Figure 10. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.00)$



- **Case 4.** $E/Y = 0.00$ and $E/Y = 0.10$
- From Table 16 and Figure 11, it can be seen that the optimal threshold is lower than the baseline case where $E/Y = 0.00$. For example, the average tax rate is zero when income is 2835, much lower than 4012 yuan with $E/Y = 0.00$.
- Also, the optimal marginal tax rate is higher than the baseline case with $E/Y = 0.00$. For example, the marginal tax rate is 25.36% when income is 8,000 yuan, slightly higher than 25.06% with $E/Y = 0.00$.

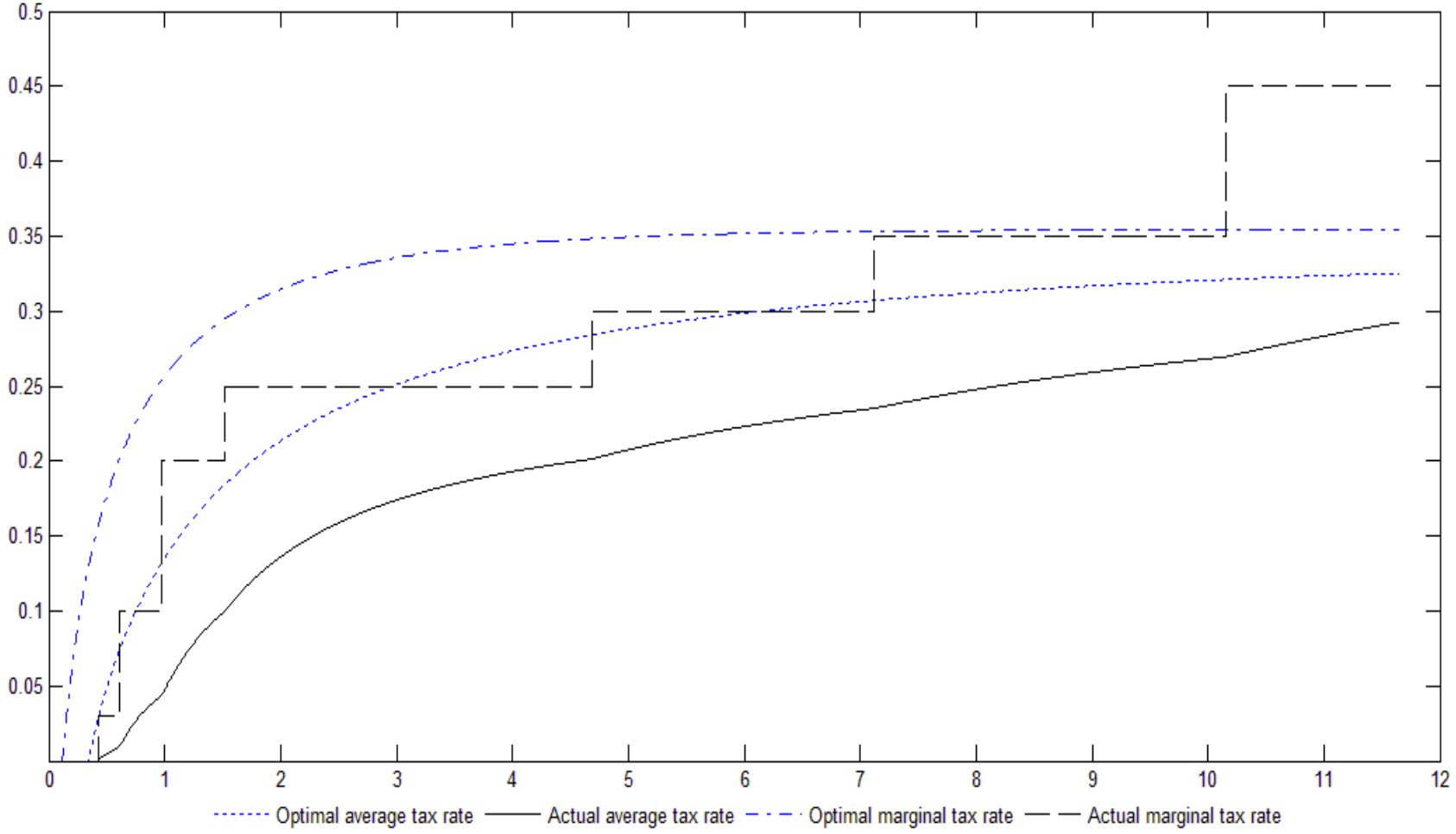
Table 16. The optimal income tax rates

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.10)$

Current monthly income (yuan)	*nl*	*c*	*l*	*F(n)*	Optimal marginal tax rate (%)	Optimal average tax rate (%)	Actual marginal tax rate (%)	Actual average tax rate (%)
2000	0.2432	0.2542	0.3248	0.5711	8.49	-4.53	0.00	0.00
2835	0.3448	0.3447	0.3500	0.7158	12.94	0.00	0.00	0.00
3500	0.4256	0.4140	0.3660	0.7785	15.64	2.73	0.00	0.00
5000	0.6080	0.5636	0.3948	0.8547	20.07	7.31	3.00	0.90
8000	0.9728	0.8445	0.4365	0.9166	25.36	13.19	10.00	4.31
12500	1.5200	1.2404	0.4812	0.9506	29.47	18.39	20.00	9.96
38500	4.6817	3.3519	0.6250	0.9865	34.83	28.40	25.00	20.12
58500	7.1137	4.9295	0.6926	0.9916	35.31	30.70	30.00	23.50
83500	10.1538	6.8939	0.7567	0.9944	35.43	32.11	35.00	26.94
90000	10.9442	7.4042	0.7710	0.9948	35.43	32.35	45.00	28.24

Figure 11. China's optimal income tax rates vs.*nl*

$(k = 4, \alpha = 5, \beta = 0.02, E/Y = 0.10)$



- From Table 17, we can see that *k* and have great influence on both the optimal tax threshold and the asymptotic marginal tax rate. β mainly influences the optimal asymptotic marginal tax rate, and has very limited influence on the optimal tax threshold. On the other hand, *E/Y* has little influence on the optimal asymptotic marginal tax rate, but has a great influence on the optimal tax threshold.
- In general, the optimal tax threshold is less affected by other parameters. In all the circumstances, the marginal tax rate should increase quickly at low skill levels, and keep at a rather flat level at high skill levels.

Table 17. The optimal taxable threshold and asymptotic marginal tax rate

k	α	β	E/Y	$c(n)$	$l(n)$	Taxable threshold (yuan)	Asymptotic marginal tax rate (%)
4	5	0.02	0.05	0.1563	0.2772	4012	35.07
3	5	0.02	0.05	0.0986	0.1685	3294	30.27
5	5	0.02	0.05	0.2035	0.3613	4571	38.99
4	4	0.02	0.05	0.1958	0.2986	4257	51.66
4	6	0.02	0.05	0.1213	0.2609	3050	18.95
4	5	0.01	0.05	0.1760	0.2772	4084	47.14
4	5	0.03	0.05	0.1449	0.2772	3863	27.98
4	5	0.02	0.00	0.1777	0.2772	4998	34.68
4	5	0.02	0.10	0.1349	0.2772	2835	35.47

VI. Conclusions

- Our findings are as follows.
- First, based on the optimal tax schedule, the low income groups should be subsidized.
- Second, the optimal income tax threshold is around 4,000 yuan, compared to current tax threshold of 3500 yuan.
- Third, the optimal highest marginal tax rate is around 35%, compared to the current rate of 45%.

- Fourth, the current marginal tax rate and average rise too slowly for income groups between 4,012 to 58,500 yuan as income rises.
- Fifth, the current marginal tax rate seems to coincide with optimal marginal tax rates for income groups for the income groups with income 58,500 to 83,500 yuan, but the actual average tax rates is smaller than the optimal.
- Sixth, the current marginal tax rate rises too rapidly at high levels of income, i.e., the current marginal tax rate is too high for income groups above 83,500.

- Individuals' preference and government behavior affect the optimal and average tax rates.
- If people value leisure higher, then the optimal tax rate is lower at all levels of income.
- If people do not suffer much from work, i.e., k is larger, then people tend to spend more time working, and the optimal tax rate will be higher.
- If the government values the poor's wellbeing higher than the rich, i.e., is small, then the optimal tax rates will be higher.
- If the government spends a large proportion of the income tax revenue, i.e., E/Y is high, then the optimal threshold will be low and more people will pay taxes.

Thank you !